A constructive formula for function of matrix. Analytical and numerical aspects.

V. N. Robuk

Institute of Electrophysics and Radiation Technologies of NAS of Ukraine, Kharkov, Ukraine Laboratory of Information Technologies, Joint Institute for Nuclear Research, Dubna, Russia

Instead of the elegant functions (formula of Lagrange and Sylvester) depending on badly-defined arguments (eigenvalues) we suggest "cumbersome" functions (hypergeometrical series) depending on well-defined arguments (matrix invariants) to construct the function on matrix. Explicit expressions for the hypergeometric series of an arbitrary finitedimension matrix are presented. The function of matrix M is a sum of any formal finite or infinite power series of M with arbitrary coefficients. It is also true for any operator with a finite discrete spectrum. This work is a generalization of the main result of the paper [1] for the complex matrix M, and possibility of using the elements of matrix M as arguments of hypergeometrical functions is demonstrated. The obtained results can be used also in the analytical solution of the linear differential equations and in the group representation theory. At the same time, if there is a need in numerical calculations, the suggested explicit expressions for the hypergeometric series present the simplest algorithm. The use of only operations of additions and multiplications for the initial parameters (coefficients of linear differential equation, invariants of matrix or elements of matrix) is of fundamental importance for suggested approach.

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