Relativistic Heavy Ion Collisions

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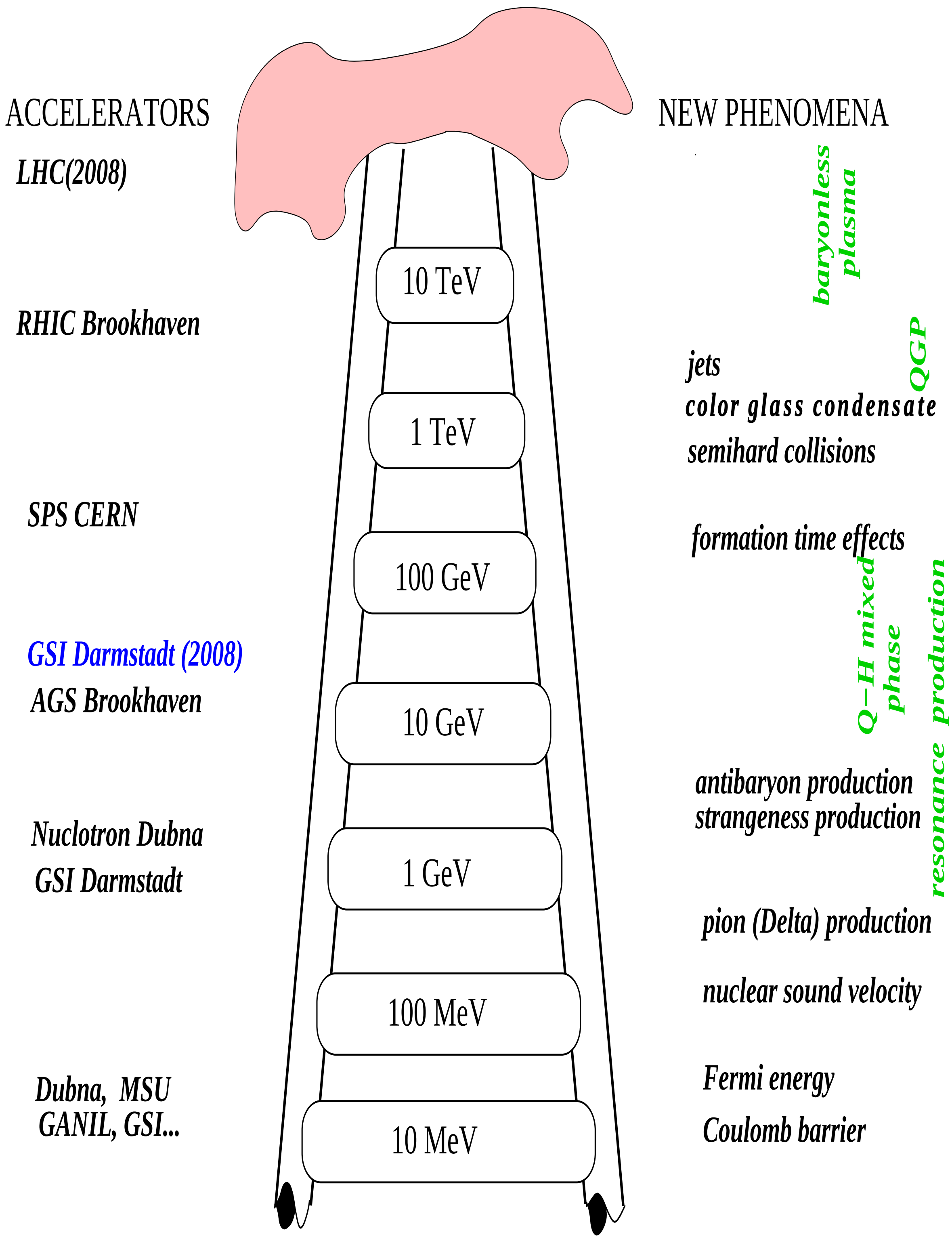
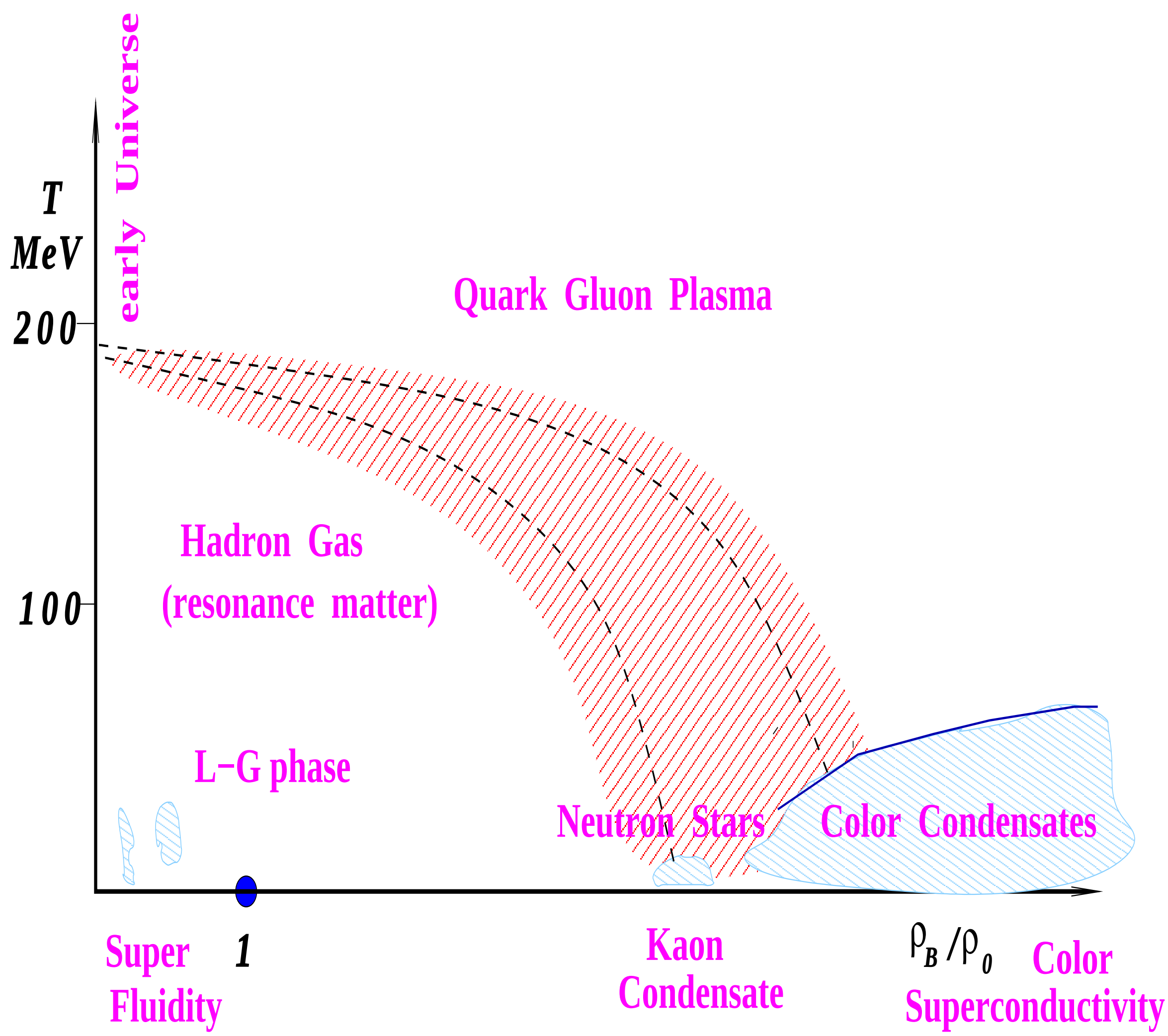
CONTENT

ENERGY "STAIR"



Phase Diagram



General Remarks

Cross section

$$\sigma \sim \int dV^n \; | < f|\mathcal{A}_n|i > |^2 \; \delta(E_f - E_i)$$

$f, i \to A, R, \rho_i, \ldots \qquad \mathcal{A}_n \to g_i, \ldots$

limiting cases:

- elastic, inelastic scattering $\qquad p + A \to p' + A'$

$$\lambda = \frac{\hbar}{p} \gg 1 \qquad \psi(x) \sim \exp(\imath kz) + \mathcal{A}(\vec{q}) \; \exp(\imath \vec{k}\vec{r})/r$$

with the Glauber-Sitenko amplitude

$$\mathcal{A}(\vec{q}) = \frac{\imath}{2\pi\lambda} \int d^2b \; \exp(\imath \vec{q}\vec{b}) \; \Gamma(\vec{b})$$

$$\Gamma(\vec{b}) = \int \mathcal{K}(r) d\vec{r} = \sum_i \eta_i(\vec{b} - \vec{r}_i)$$

- participant-spectator model $\qquad$ (Fermi) phase space

$$| < f|\mathcal{A}_n|i > |^2 \simeq \text{const}$$

$$\sigma \sim V^n \; | < f|i > |^2$$

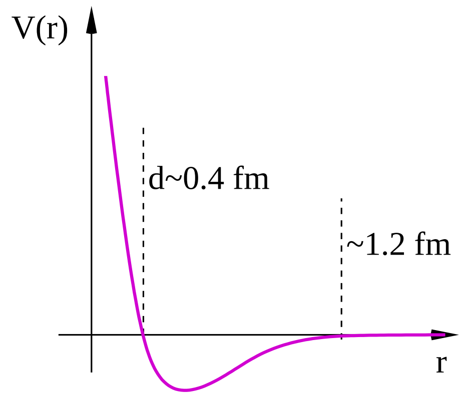
- Pure state → particle ensemble → statistical consideration
- Adiabatic switching on the interaction ? → time evolution

N-body Liouville equation (time reversible !)

$$\frac{d\rho_N}{dt} = \frac{\partial}{\partial t}\rho_N + \frac{1}{\imath\hbar}[H, \; \rho_N] = 0$$

to solve it, justified approximations are needed

Length Scales



d - attractive NN force range

$\Lambda = \frac{1}{\sigma\rho}$ - (nucleon) mean free path

$\rho_0 \simeq 0.16\ fm^{-3}\quad \sigma \simeq 40\ mb \rightarrow \Lambda \sim 1.5$ fm

Pauli principle

compression ...

L - "macroscopic" length, 2-8 fm

units	d	Λ	L	d/Λ	$Kn = \Lambda/L$
air (10^{-8} cm)	1	10^5	10^8	10^{-3}	10^{-3}
liquid (10^{-8} cm)	1	2-10	10^8	0.1-0.5	10^{-7}
nuclei (0.4/1.2 fm)	1	1.5-2	2-8	0.2-0.6	1-0.2

$$\boxed{d \ll \Lambda \ll L}$$

$\Leftarrow$ kinetics hydrodynamics $\Rightarrow$

For nuclear case (intermediate energies) : $d < \Lambda < L$

Intermediate Energies

- A-body problem in a classical picture

[Quantum] Molecular Dynamics

$$\dot{\vec{x}}_i = \frac{\partial}{\partial \vec{p}_i} H(i=1,\dots A)$$
$$\dot{\vec{p}}_i = -\frac{\partial}{\partial \vec{x}_i} H(i=1,\dots A)$$

with $H = -\sum \bigtriangledown^2_{p_i} + \sum_{i>k} V_{ik}$

nuclear stability

$V \to V^{Pauli}(p)$

NN-scattering ?

- Fermionic Molecular Dynamics

$q = \{\vec{p}, \vec{x}, s \dots\}$

$$\sum \mathcal{A}_{\mu\nu} \, \dot{q^\mu} = -\frac{\partial}{\partial q^\mu} \, H$$

with $\quad \mathcal{A}_{\mu\nu} := \dfrac{\partial^2 \mathcal{L}_0}{\partial \dot{q}^\mu \, \partial q_\nu} - \dfrac{\partial^2 \mathcal{L}_0}{\partial \dot{q}^\nu \, \partial q_\mu}$

QMD limit

$$\mathcal{A}_{\mu\nu} \Rightarrow \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$$

BBGKY-Hierarchy

- Non-relativistic kinetic models

$$H = T + V = \sum \epsilon_i a_i^\dagger a_i + \sum V(ij, i'j')\ a_i^\dagger a_j^\dagger a_{i'} a_{j'}$$

n-particle density :

$\rho_n(x_1, x_2, \dots x_n) = V^n \int dx_{n+1} \dots dx_N \rho(x_1 \dots x_N)$

$$\begin{aligned} i\hbar \frac{\partial \rho_1(1)}{\partial t} &= [T_1, \rho_1(1)] + Tr_{(2)}[V_{12}, \rho_2(1,2)] \\ i\hbar \frac{\partial \rho_2(1,2)}{\partial t} &= [(T_1 + T_2 + V_{12}), \rho_2(1,2)] \\ &+ Tr_{(3)}[(V_{13} + V_{23}), \rho_3(1,2,3)] \\ \dots\dots\dots\dots &\dots \ \dots\dots\dots\dots \end{aligned}$$

$\rho_1 \Rightarrow f^W(\vec{p}, \vec{x}, t) =< n(\vec{p}, \vec{x}) >_t$

with $n(\vec{p}, \vec{x}) = \int \frac{d^3k}{(2\pi\hbar)^3}\ e^{\imath \vec{k}\vec{x}}\ a^\dagger_{\vec{p}-\vec{k}/2} a_{\vec{p}+\vec{k}/2}$

and $\frac{1}{\Delta\mu} \int f^W(\vec{p}, \vec{x}, t)\ d\mu = f(\vec{p}, \vec{x}, t) + O(\frac{\hbar}{\Delta\mu})$

Generalized kinetic equation :

$$\boxed{\frac{\partial f(\vec{p},\vec{x},t)}{\partial t} = -D(f) + C(ff)}$$

- Driving Vlasov term (classical limit) (Hartree approximation, no exchange terms)

$$D(\vec{p}, \vec{x}, t) = \frac{\vec{p}}{m} \frac{\partial}{\partial \vec{x}} f(\vec{p}, \vec{x}, t) - \frac{\partial}{\partial \vec{x}} U(x)\ \frac{\partial}{\partial \vec{p}} f(\vec{p}, \vec{x}, t)$$

with an effective potential

$$U(x) = \int \frac{d^3x_1\ d^3p_1}{(2\pi\hbar)^3}\ V(\vec{x} - \vec{x}_1)\ f(\vec{p}_1, \vec{x}_1, t)$$

phenomenologically (Skyrme) $U(x) = -a\rho + b\rho^2$

BBGKY-Hierarchy (continuation)

- Collision term

$\vec{p} + \vec{p}_2 \Rightarrow \vec{p}_1' + \vec{p}_2'$, no correlation and retardation effects

$$\begin{aligned} C(\vec{p},\vec{x},t) &= \int \frac{d^3p_2 d^3p_1' d^3p_2'}{(2\pi\hbar)^6} \, |T_2(\vec{p}\vec{p}_2;\vec{p}_1'\vec{p}_2') - T_2(\vec{p}\vec{p}_2;\vec{p}_2'\vec{p}_1')|^2 \\ &\times \; \delta(E_p + E_{p_2} - E'_{p_1} - E'_{p_2}) \, \delta(\vec{p} + \vec{p}_2 - \vec{p}_1' - \vec{p}_2') \\ &\times \; \left[f_p f_{p_2}(1 - f_{p_1'})(1 - f_{p_2'}) - f_{p_1'} f_{p_2'}(1 - f_p)(1 - f_{p_2})\right] \end{aligned}$$

⇑ gain ⇑ lost

no exchange, no im-medium effects, ladder approximation for T_2

$$\begin{aligned} C(\vec{p},\vec{x},t) &= \int \frac{d^3p_2 d^3p_2'}{(2\pi\hbar)^3} \, \delta(\vec{p} + \vec{p}_2 - \vec{p}_1' - \vec{p}_2') \, v_{12} \, \frac{d\sigma^{el}}{d\Omega} \\ &\times \; \left[f_p f_{p_2}(1 - f_{p_1'})(1 - f_{p_2'}) - f_{p_1'} f_{p_2'}(1 - f_p)(1 - f_{p_2})\right] \end{aligned}$$

$$\boxed{\{\frac{\partial}{\partial t} + \frac{\vec{p}}{m}\frac{\partial}{\partial \vec{x}} + \frac{\dot{\vec{p}}}{m}\frac{\partial}{\partial \vec{p}}\} f(\vec{p},\vec{x},t) = C(\vec{p},\vec{x},t)}$$

BUU ⇒ events generators

$f \ll 1$ ⇒ Boltzmann equation

account for fluctuations ⇒ BL equation

$$\{\frac{\partial}{\partial t} + \frac{\vec{p}}{m}\frac{\partial}{\partial \vec{x}} + \frac{\dot{\vec{p}}}{m}\frac{\partial}{\partial \vec{p}}\} f(\vec{p},\vec{x},t) = C(\vec{p},\vec{x},t) + \delta C$$

random force ⇑

Relativistic Kinetic Equations

Lagrangian density for the Walecka $\sigma - \omega$ model

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_0 + \mathcal{L}_{int} \\
\mathcal{L}_0 &= \bar{\psi}(i\gamma_\mu\partial^\mu - m_N)\psi + \frac{1}{2}(\partial_\mu\sigma\ \partial^\mu\sigma - m_S\ \sigma^2) \\
&- \frac{1}{4}F_{\mu\nu}\ F^{\mu\nu} + \frac{1}{2}m_V^2\ \omega_\mu\omega^\mu \\
\mathcal{L}_{int} &= g_S\bar{\psi}\psi\sigma - g_V\bar{\psi}\gamma^\mu\psi\omega_\mu
\end{aligned}$$

equations of motion

$$\begin{aligned}
(\partial_\mu\partial^\mu + m_S^2)\ \sigma &= g_S\bar{\psi}\psi && \text{Klein-Gordon} \\
\partial F^{\mu\nu} + m_V^2 &= g_V\bar{\psi}\gamma^\nu\psi && \text{Proka} \\
\gamma^\mu(i\partial_\mu + g_V\omega_\mu) &- (m_N - g_S\sigma)\psi = 0 && \text{Dirac}
\end{aligned}$$

with $F^{\mu\nu} = \partial^\mu\omega^\nu - \partial^\nu\omega^\mu$

in the mean-field approximation

$$\begin{aligned}
\sigma_0 &= \frac{g_S}{m_S^2} < \bar{\psi}\psi > \equiv \frac{g_S}{m_S^2}\rho_s \\
\omega_0 &= \frac{g_V}{m_V^2} < \bar{\psi}\gamma_0\psi > \equiv \frac{g_V}{m_V^2}\rho_B
\end{aligned}$$

$$\boxed{\left[p_\mu\partial^\mu - m_N^*\dot{p}^\nu\frac{\partial}{\partial p^\nu}\right] f(p,x) = C^{rel}(p,x)}$$

with $m_N^*\dot{p}^\nu = g_V p_\mu F^{\mu\nu} + m_N^*(\partial^\nu m_N^*)$

and quasiparticle parameters

$m_N^* = m_N - g_S\ \sigma_0$ effective mass

$p_\mu \to p_\mu - g_V\ \omega_\mu$ kinetic momentum

RBUU $\Rightarrow$ events generators

STEP TO HIGHER ENERGIES

Relativistic Boltzmann equation ($\psi, \sigma, \omega \Rightarrow 0; f \ll 1$)

$$(p_\mu \partial^\mu)\ f(p,x) = C^{rel}(ff)$$

- multiple particle production

$$\frac{d\sigma^{el}}{d\Omega} \Rightarrow \frac{d\sigma^{h_1+h_2\to h+X}}{dp^3}$$

  $\Rightarrow$ coupled set of equations for $\{h\}$

- finite formation time $\theta(t-\tau\gamma),\ \tau \sim 1$ fm;
  memory (retarded) effect
- new degrees of freedom (QCD) : quark/gluons (Nambu-Jona-Lasinio), strings, formation of color rope

solution $\Rightarrow$ Monte Carlo Methods:
event generators $\Rightarrow$ UrQMD, QGSM, HSM ...

quark-gluon transport theory

Basic Kinetic Idea

HIC ⇒ subsequent collisions
between quasiparticles
(Boltzmann-like equations)

Physics : What is a quasiparticle ?

non-relativistic

$$(\frac{\partial}{\partial t} + \vec{v}\vec{\bigtriangledown}_x \quad + \quad \frac{d\vec{p}}{dt}\vec{\bigtriangledown}_p) f(\vec{p},\vec{x},t) = C(f,f)$$

$$\Uparrow \quad \frac{d\vec{p}}{dt} = -\vec{\bigtriangledown}_x \frac{d\vec{p}}{dt} V(\vec{r},t)$$

$(p-h)$

$N + V(r)$

$free\ N$

relativistic – QHD

$$(p_\mu \partial^\mu + m^* \dot{p}_\mu \partial^\mu) f(p,x) = C^{rel}(ff)$$

$$m^* \dot{p}_\mu = g_V p_\nu F^{\mu\nu} + m^* (\partial^\mu_x m^*) + \text{field eqs.}$$

$hadrons + \psi$

$(Walecka-like)$

Boltzmann : $(p_\mu \partial^\mu) f(p,x) = C^{rel}(ff)$

$resonances$

$strings$

$color\ rope$

non-abelian fields (color) – QCD

$p, x \Rightarrow p, x, \mathcal{Q}$

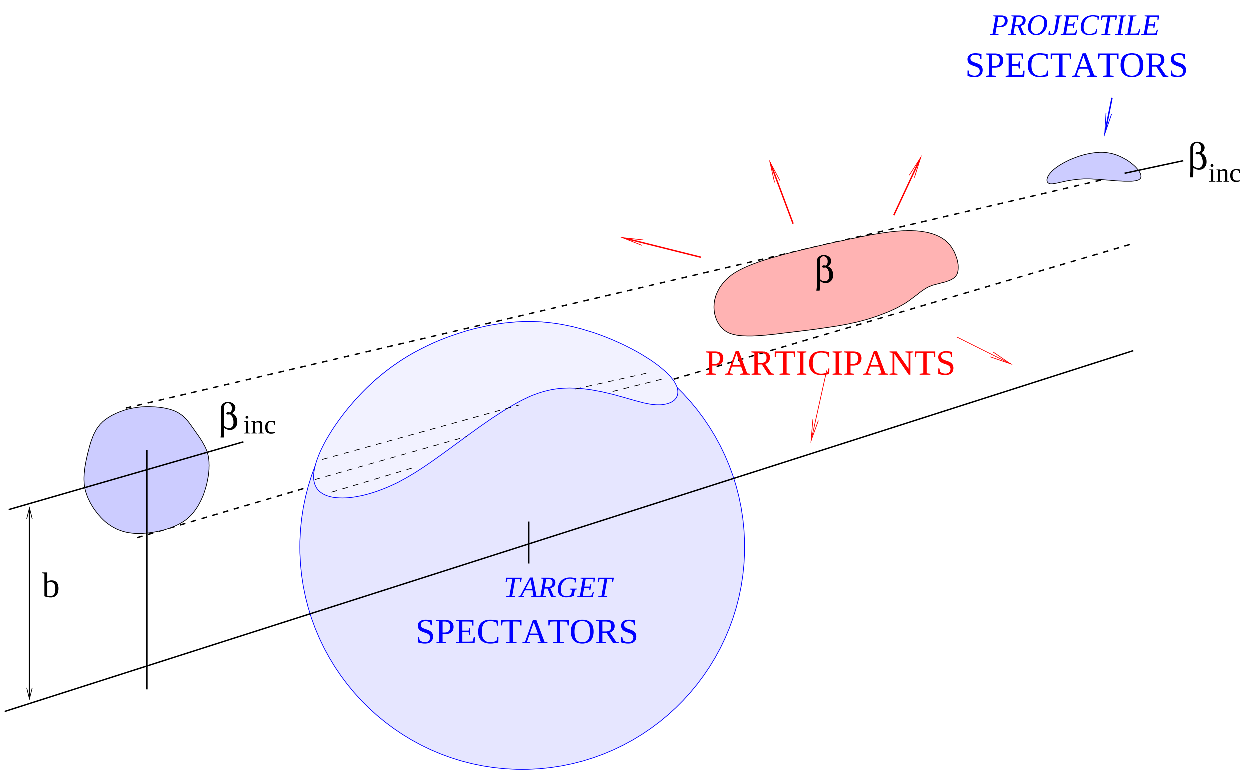
flow term +source term

$quarks/gluons$

extreme case: free rescattering of quarks and gluons partons

"FROZEN" HYDRODYNAMICS

Participant-spectator picture



Nuclear Hydrodynamics

- Non-relativistic case

$\Lambda/L \ll 1$

$$< \begin{pmatrix} \rho \\ \vec{v} \\ \epsilon \end{pmatrix} >= \int d^3p \begin{pmatrix} 1 \\ \vec{p}/m_N \\ p^2/2m_N \end{pmatrix} f(\vec{p},\vec{x},t)$$

Boltzmann equation + local equilibrium hypothesis

$$\begin{aligned} \vec{v} &= \vec{u} + \vec{c} \\ \vec{u} &= < \vec{v} > \ , \quad < \vec{c} >= 0 \\ \rho < c_i c_k > &= P\ \delta_{ik} + \Pi_{ik}\ , \quad \rho < c^2 c_k >= Q_k \end{aligned}$$

HYDRO $\Rightarrow$ codes

$$\begin{aligned} \frac{\partial \rho}{\partial t} - \frac{\partial}{\partial x_k}\rho u_k &= 0 \\ \frac{\partial \rho u_i}{\partial t} - \frac{\partial}{\partial x_k}\rho u_i u_k &= \{\frac{\partial}{\partial x_k}\Pi_{ik} - \frac{\partial}{\partial x_i}P\} \\ \frac{\partial \epsilon}{\partial t} - \frac{\partial}{\partial x_k}\epsilon u_k &= \{\frac{\partial}{\partial x_k}\Pi_{ik}u_i - \frac{\partial}{\partial x_i}P u_k - \frac{\partial}{\partial x_k}Q_k\} \end{aligned}$$

for perfect gas $\{\ldots\} \Rightarrow 0$

$\rho, \vec{u}, \epsilon, T \Rightarrow$ hydro eq.+ EOS

initial conditions

freeze-out

turbulent regime $Re \simeq 10^2 - 10^3$

$$Re = \frac{inertial}{viscous} \simeq \frac{M}{\Lambda/L} \simeq 4 - 10$$

with $M = v/c_s$ and $c_s = \sqrt{\partial P/\partial \rho}|_s \approx 0.2$

Motivations (Why 3-Fluids ?)

♠ Conservation laws (Gauss theorem) ⇒ Fluid dynamics

$$\partial_\mu \, J^\mu = 0 \quad \text{net charge} \quad \boxed{4}$$
$$\partial_\mu \, T^{\mu\nu} = 0 \quad \text{energy momentum} \quad \boxed{10}$$

♠ Tensor decomposition of J^μ and $T^{\mu\nu}$ with respect to u^μ

$$J_i^\mu = n_i u^\mu + \nu_i^\mu$$
$$T^{\mu\nu} = \varepsilon \, u^\mu u^\nu - P\Delta^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \pi^{\mu\nu} + ...$$

with $\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$ and $\nu_i^\nu \equiv \Delta_\nu^\mu J_i^\nu$

- Perfect hydro in local thermodynamical equilibrium

$$J_i^\mu = n_i u^\mu \qquad \boxed{+\text{ EoS}}$$
$$T^{\mu\nu} = \varepsilon \, u^\mu u^\nu - P\Delta^{\mu\nu}$$

with
$$J_i^\mu(x) = \int \frac{d^3p}{p^0} \, p^\mu \, [f_i(x,p) - \bar{f}_i(x,p)]$$
$$T^{\mu\nu}(x) = \int \frac{d^3p}{p^0} \, p^\mu p^\nu \, [f(x,p) + \bar{f}(x,p)]$$

where
$$f_i(x,p) = \frac{g_i}{(2\pi)^3}[\exp\left((u_\mu p^\mu(x) - \mu(x))/T(x)\right) \pm 1]^{-1}$$

- First order dissipative corrections (viscosity, heat capacity)

⇒ acasuality

- Second order corrections ⇒ + 14 Grad equations

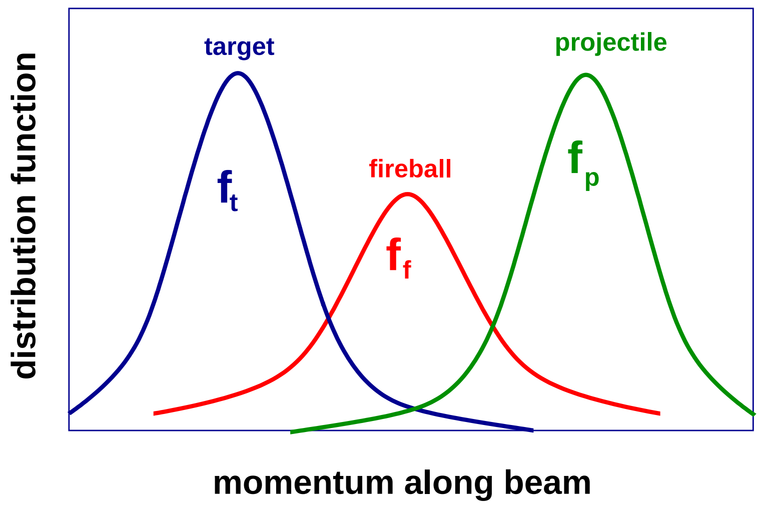
Spatial-temporal variation of the macro fields have to be small

♠ Many fluid dynamics

$$f(x,p) = \sum_j^M \, f_j(x,p)$$

A single fluid may consist of several particle species. Different fluids may be of the same particle species.

From Kinetics to Multi-Fluids



- Distribution functions are separated in momentum space
  ⇒ can be associated with different fluids
- Leading particles carry baryon charge
  ⇒ 2 baryon-rich fluids: projectile-like and target-like
- Produced particles populate mid-rapidity region
  ⇒ fireball fluid
- Intra-fluid equilibration is faster than inter-fluid stopping
  ⇒ local equilibrium in each fluid

$$
\begin{aligned}
p_\mu \partial_x^\mu f_p &= C_p(f_p, f_p) + C_p(f_p, f_t) + C_p(f_p, f_f) \\
p_\mu \partial_x^\mu f_t &= C_t(f_t, f_t) + C_t(f_p, f_t) + C_t(f_t, f_f) \\
p_\mu \partial_x^\mu f_f &= C_f(f_f, f_f) + C_f(f_p, f_t) + C_f(f_p, f_f) + C_f(f_t, f_f)
\end{aligned}
$$

C_α = collision integral (having lost and gain terms)

$C_p(f_p, f_p)$, etc. = intra-fluid collision terms $= 0 \Rightarrow f^{(equilib.)}$

$C_{p/t}(f_p, f_t) \Rightarrow$ projectile-target friction/emission into fireball

$C_{p/t}(f_{p/t}, f_f)$ and $C_f(f_{p/t}, f_f) \Rightarrow$ friction

$C_f(f_p, f_t) \Rightarrow$ particle production in mid-rapidity (fireball) region

Derivation of Multi-Fluid Equations

Baryon number conservation:

$$\sum_{\text{“}baryons''} \int \frac{d^3p}{p^0}\, p_\mu \partial_x^\mu f_p \quad = \quad \partial_\mu J_p^\mu = 0$$

$$\sum_{\text{“}baryons''} \int \frac{d^3p}{p^0}\, p_\mu \partial_x^\mu f_t \quad = \quad \partial_\mu J_t^\mu = 0$$

$$\sum_{\text{“}baryons''} \int \frac{d^3p}{p^0}\, p^\mu f_f \quad = \quad J_f^\mu = 0$$

Energy–momentum exchange

$$\sum_{species} \int \frac{d^3p}{p^0}\, p_\nu p_\mu \partial_x^\mu f_p \quad = \quad \partial_\mu T_p^{\mu\nu} = \text{Friction} + \text{Emission}$$

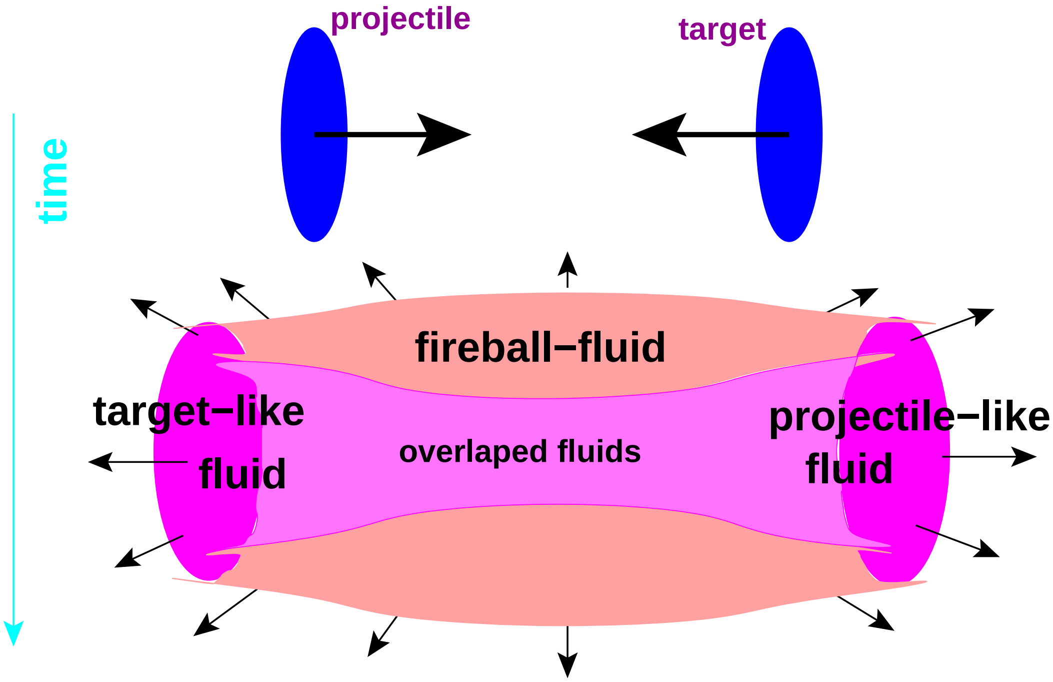
$$\sum_{species} \int \frac{d^3p}{p^0}\, p_\nu p_\mu \partial_x^\mu f_t \quad = \quad \partial_\mu T_t^{\mu\nu} = \text{Friction} + \text{Emission}$$

$$\sum_{species} \int \frac{d^3p}{p^0}\, p_\nu p_\mu \partial_x^\mu f_f \quad = \quad \partial_\mu T_f^{\mu\nu} = \text{Friction} + \text{Production}$$

with using "sum rules" for hadron-hadron $ab \to cX$ collisions

$$\sum_{j\in c} b_j \int d\sigma_{ab\to cX} \quad = \quad (b_a + b_b)\,\sigma_{ab}$$

$$\sum_{c} \int d\sigma_{ab\to cX} p_c^i \quad = \quad (p_a + p_b)^i\,\sigma_{ab}$$



Finally:

TARGET-LIKE FLUID:

$$\partial_\mu J^\mu_t = 0 \qquad \partial_\mu T^{\mu\nu}_t = -F^\nu_{tp} + F^\nu_{ft}$$

⇑ Baryon conservation — Lead. particles carry b-charge

⇑ Energy–momentum exchange/emission

PROJECTILE-LIKE FLUID:

$$\partial_\mu J^\mu_p = 0, \qquad \partial_\mu T^{\mu\nu}_p = -F^\nu_{pt} + F^\nu_{fp}$$

FIREBALL FLUID:

$$J^\mu_f = 0, \qquad \partial_\mu T^{\mu\nu}_f = F^\nu_{pt} + F^\nu_{tp} - F^\nu_{fp} - F^\nu_{ft}$$

⇑ Baryon-free fluid

⇑ Source term ⇑ Exchange

The source term is delayed due to a formation time $\tau \sim 1$ fm/c

TOTAL ENERGY-MOMENTUM CONSERVATION:

$$\partial_\mu (T^{\mu\nu}_p + T^{\mu\nu}_t + T^{\mu\nu}_f) = 0$$

3-Fluid Hydrodynamics with Delayed Formation of Fireball

For baryon-rich fluids ($\alpha =$p and t):

$$J^{\mu}_{\alpha} = u^{\mu}_{\alpha} n_{\alpha}$$

$$T^{\mu\nu}_{\alpha} = (\varepsilon_{\alpha} + P_{\alpha})\; u^{\mu}_{\alpha}\; u^{\nu}_{\alpha} - g^{\mu\nu} P_{\alpha}$$

$n_{\alpha} =$ proper baryon density
$\varepsilon_{\alpha} =$ proper energy density
$P_{\alpha} =$ pressure
$u_{\alpha} =$ hydro 4-velocity

For fireball fluid, only thermalized part is of hydrodynamic form: $n_{\alpha} = 0$ baryon-free fluid

$$T^{(eq)\mu\nu}_{f} = (\varepsilon_{f} + P_{f})\; u^{\mu}_{f}\; u^{\nu}_{f} - g^{\mu\nu} P_{f}$$

Its evolution is defined by a retarded source term

$$\begin{aligned} \partial_{\mu} T^{(eq)\mu\nu}_{f}(x) \;&=\; \int d^4x' \delta^4 \left(x - x' - U_F(x')\tau\right) \left[F^{\nu}_{pt}(x') + F^{\nu}_{tp}(x')\right] \\ &- \quad F^{\nu}_{fp}(x) - F^{\nu}_{ft}(x) \end{aligned}$$

where $\tau =$ formation time, and

$$U^{\nu}_{F}(x') = \frac{F^{\nu}_{pt}(x') + F^{\nu}_{tp}(x')}{|F_{pt}(x') + F_{tp}(x')|}$$

is a free-streaming 4-velocity of the produced fireball matter.
The residual, free-streaming part of fireball matter

$$T^{(fs)\mu\nu}_{f} = T^{\mu\nu}_{f} - T^{(eq)\mu\nu}_{f}$$

is not formed and hence not thermalized.

Friction

Projective-Target Friction:

$$F_{pt}^{\nu} = \rho_p \rho_t \left[\left(u_p^{\nu} - u_t^{\nu} \right) D_P + \left(u_p^{\nu} + u_t^{\nu} \right) D_E \right]$$

EoS depend.

enh. $\xi_h^2(s_{pt})$

⇑ heating ⇑ fireball production

ρ_α = scalar density of α fluid

$$D_{P/E} = m_N \ V_{rel}^{pt} \ \sigma_{P/E}(s_{pt}),$$

m_N = nucleon mass

$s_{pt} = m_N^2 \left(u_p^{\nu} + u_t^{\nu} \right)^2$ = mean invariant energy squared

$V_{rel}^{pt} = [s_{pt}(s_{pt} - 4m_N^2)]^{1/2}/2m_N^2$ = mean relative velocity

$\sigma_{P/E}(s_{pt})$ = proton-proton cross sections integrated with certain weights (L.M. Satarov, Sov. J. Nucl. Phys. **52**, 264 (1990))

V_{rel}^{pt} < thermal or Fermi velocity ⇒ Unification of p and t fluids

Projective(Target)-Fireball Friction:

Absorption of a fireball matter by baryon-rich fluids (estimated by pion-nucleon resonance cross sections)

$$F_{fp}^{\nu} = D_{fp} \frac{T_f^{(eq)0\nu}}{u_f^0} \rho_p$$

where

$$D_{fp} = V_{rel}^{fp} \ \sigma_{tot}^{N\pi \to R}(s_{fp}).$$

$V_{rel}^{fp} = [(s_{fp} - m_N^2 - m_\pi^2)^2 - 4m_N^2 m_\pi^2]^{1/2}/(2m_N m_\pi)$

$s_{fp} = (m_\pi u_f + m_N u_p)^2$

Freeze-out

- Criterion:

$$\underbrace{Local}_{(at\ x\ position)}\ \underbrace{proper}_{(in\ local\ rest\ frame)}\ \underbrace{energy\ density\ of\ matter}_{(summed\ over\ all\ fluids)}$$

is less than ε_{frz}

Other criteria are available but not used.

- Shock-like freeze-out:

T_{hydro} and μ_{hydro} are mapped to T_{gas} and μ_{gas} proceeding from baryon, energy and momentum conservations.

Energy accumulated in "mean fields" is released.

- Freeze-out *a là* Milekhin

$$E\frac{dN}{d^3p} = \int f_{gas}(x,p)p^\mu d\sigma_\mu, \qquad d\sigma_\mu = u_\mu(d^3x)_{proper}$$

u_μ =hydro 4-velocity   proper = in the frame, where u_μ = (1,0,0,0)

◦ In "space-like regions" it is very similar to Cooper-Frye

◦ In "time-like regions" there is no problem with energy conservation, because $P = 0$ on the system boundary

◦ In fact, there is no "time-like freeze-out" in the code. Only tiny fireballs are frozen out.

◦ Therefore, there is no problem with Cooper-Frye's negative contributions into particle numbers

◦ Baryon number, energy and momentum are exactly conserved!

◦ Problem of shadowing still persists

◦ Further study of Freeze-out is needed!

Hadronic EoS (gasEoS)

Energy density:

$$\varepsilon(n_B,T) = \underbrace{\varepsilon_{gas}(n_B,T)}_{\text{gas of free hadrons}} + \underbrace{W(n_B)}_{\text{mean field}}$$

Pressure:

$$P(n_B,T) = \underbrace{P_{gas}(n_B,T)}_{\text{gas of free hadrons}} \underbrace{+ n_B \frac{dW(n_B)}{dn_B} - W}_{\text{mean field}}$$

$$W(n_B) = n_B m_N \left[-b \left(\frac{n_B}{n_0}\right) + c \left(\frac{n_B}{n_0}\right)^{\gamma+1} \right]$$

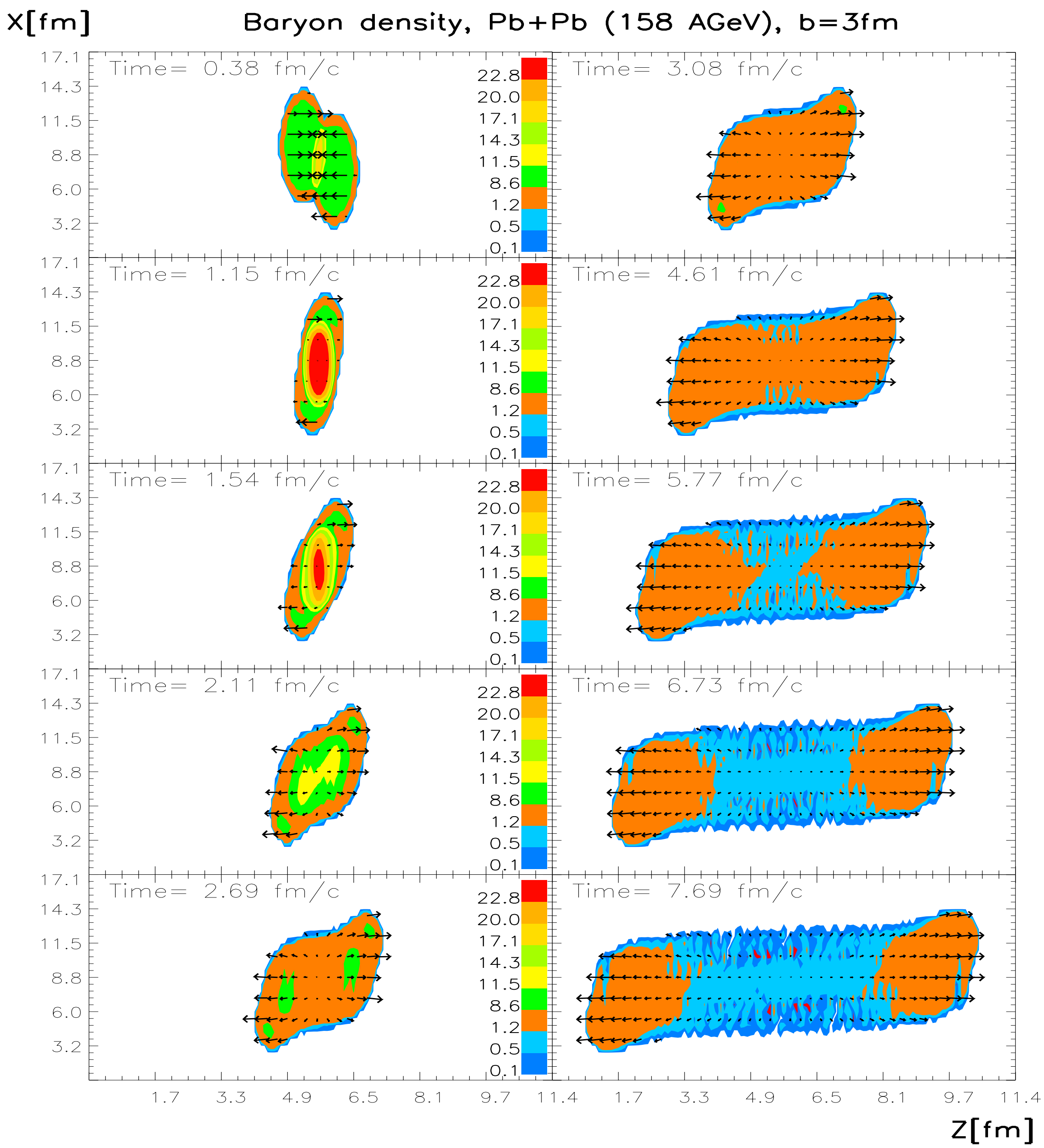
$W(n_B)$ saturates the cold nuclear matter at $n_0 = 0.15$ fm^{-3} and $\varepsilon(n_0, T=0)/n_0 - m_N = 16$ MeV, and provides incompressibility of nuclear matter $K = 235$ MeV.

To preserve causality at high n_B

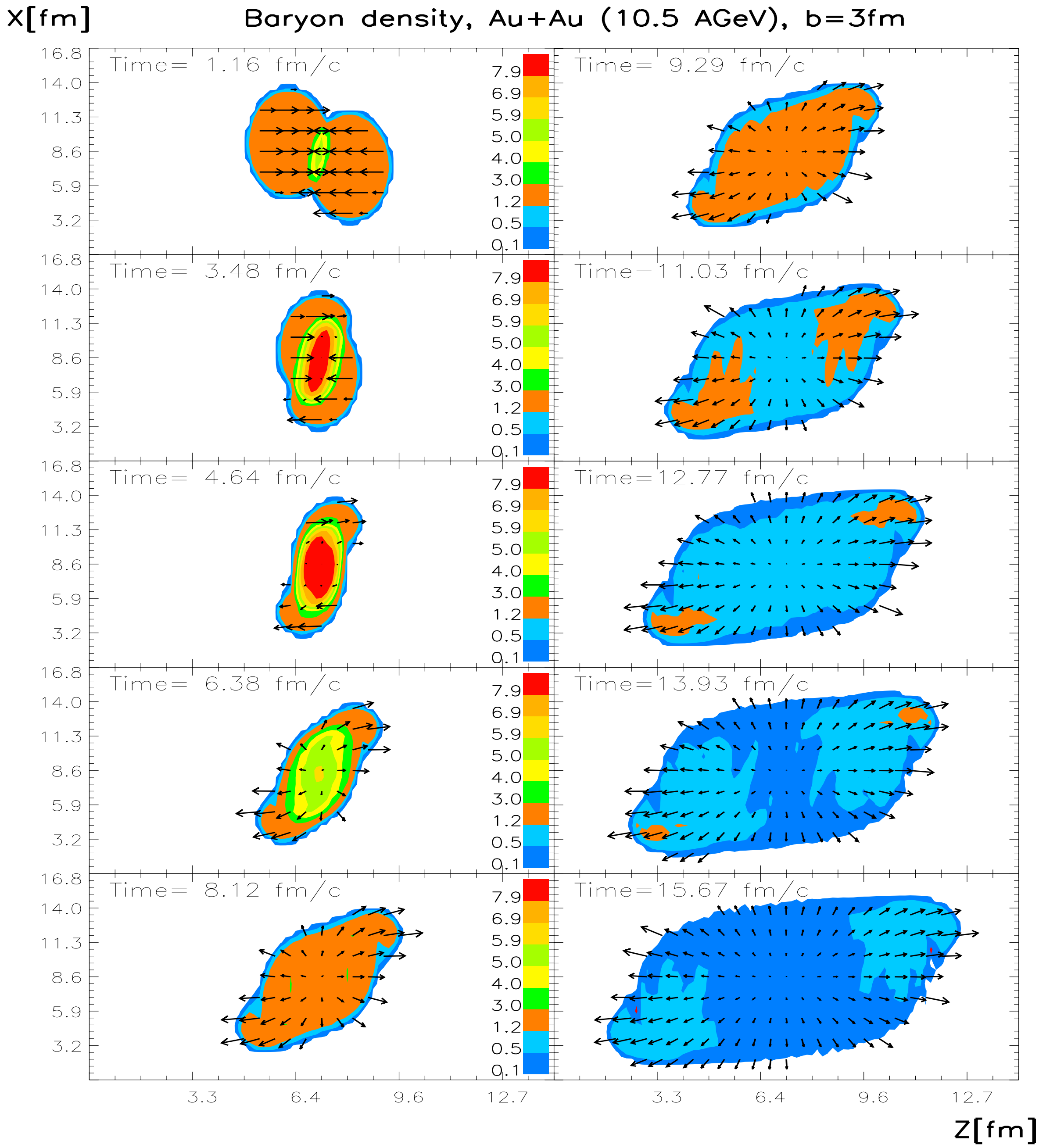
$$\varepsilon(n_B, T=0) = n_0 m_N \left[A \left(\frac{n_B}{n_0}\right)^2 + C + B \left(\frac{n_B}{n_0}\right)^{-1} \right], \quad n_B > n_c \approx 6 n_0$$

Parameters are determined on the condition that $\varepsilon(n_B, T=0)$ and its two first derivatives are continues at n_c.

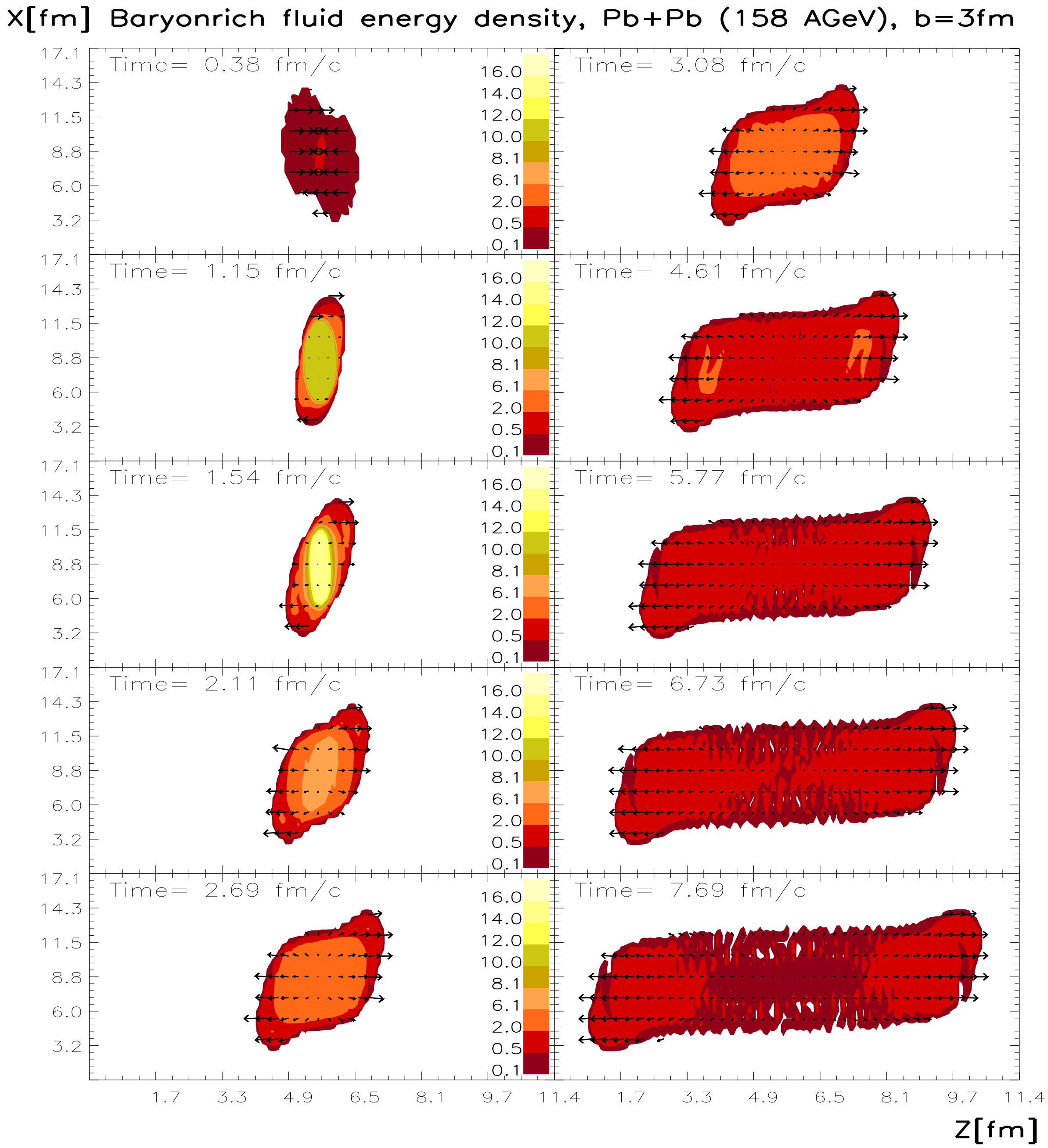
GLOBAL DYNAMICS



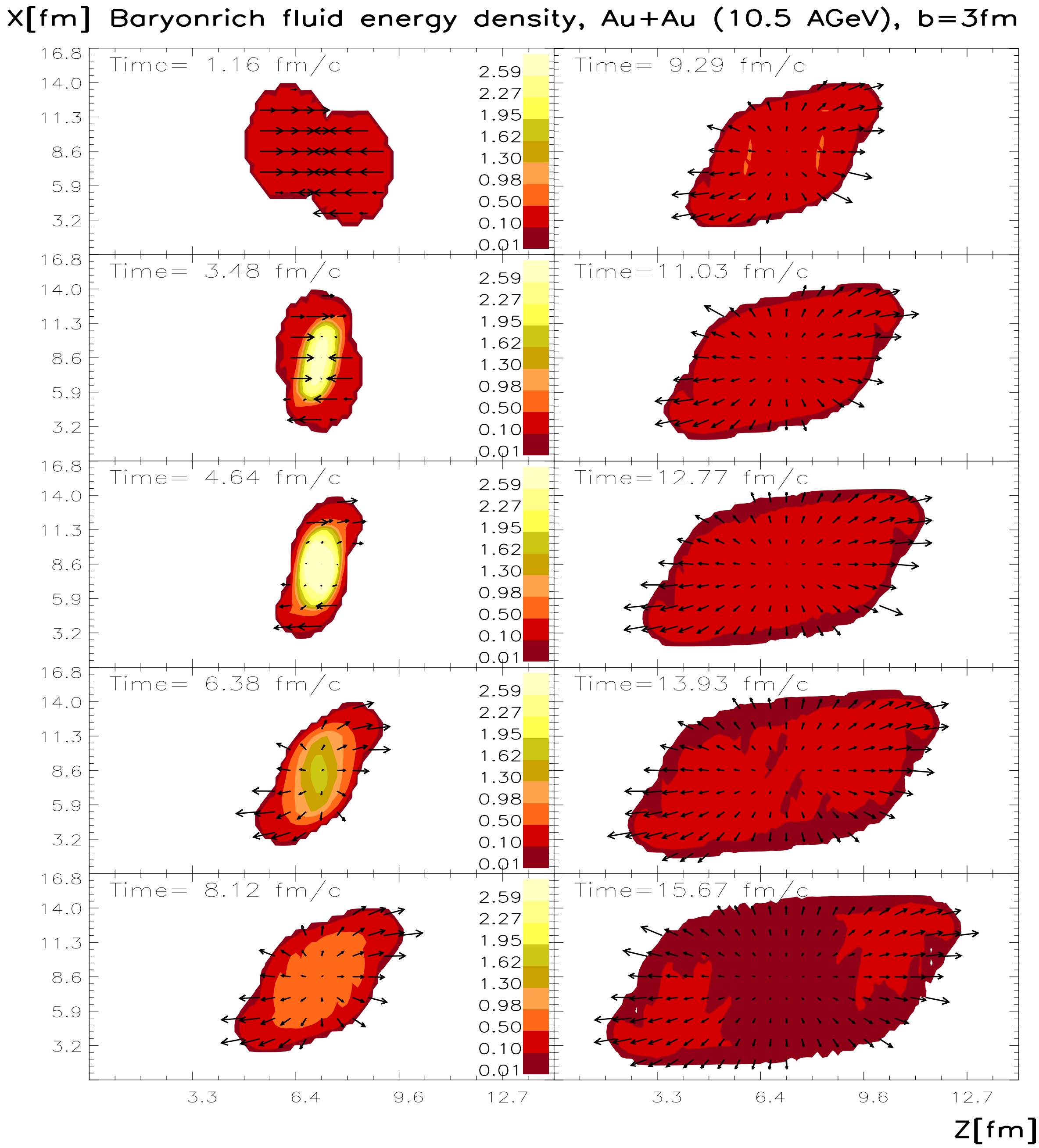
Global Dynamics



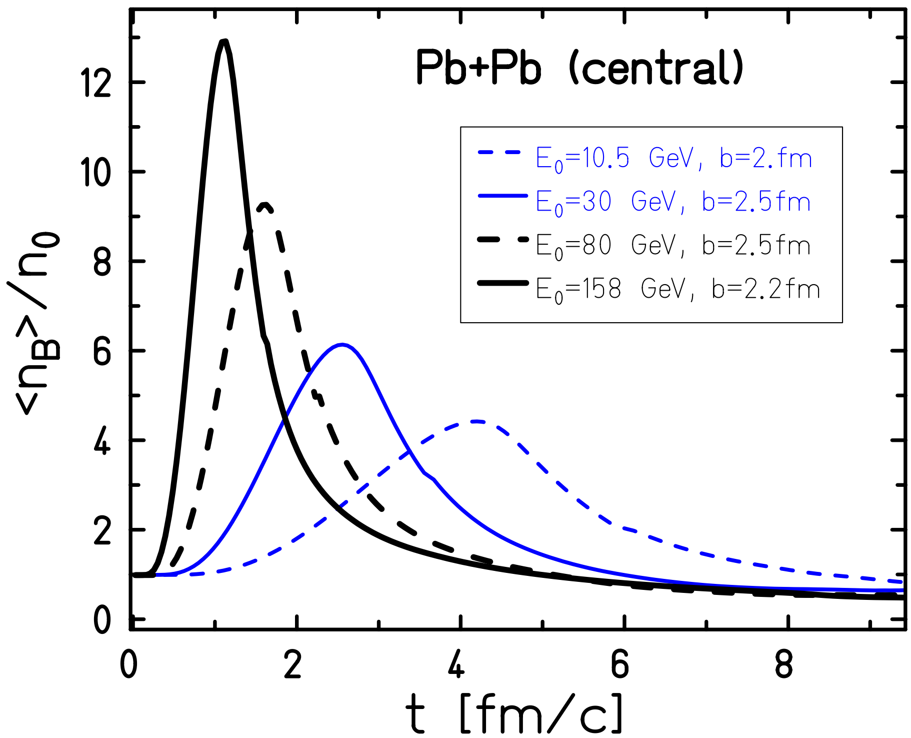
Global Dynamics

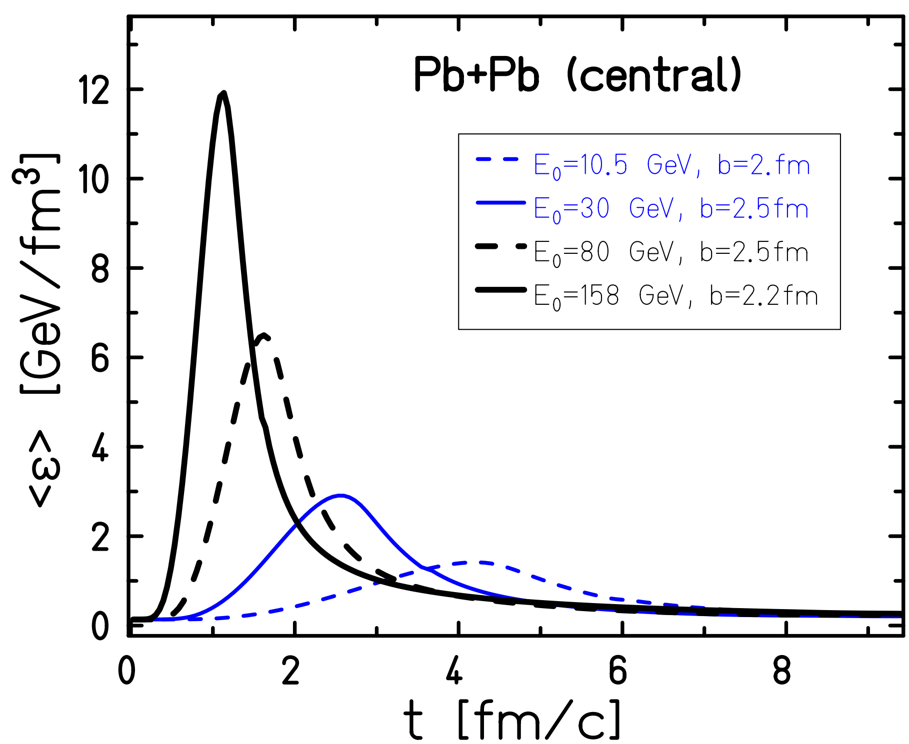


Global Dynamics

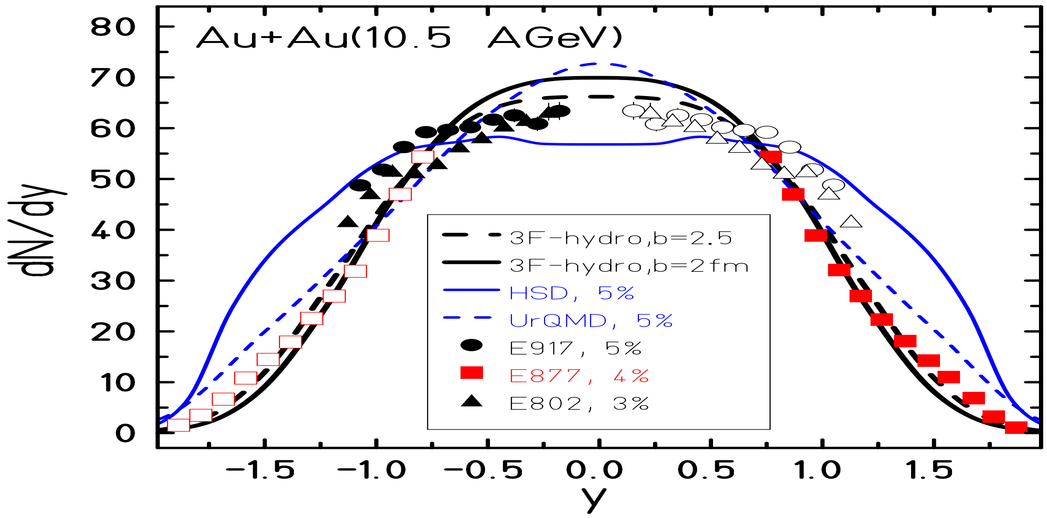


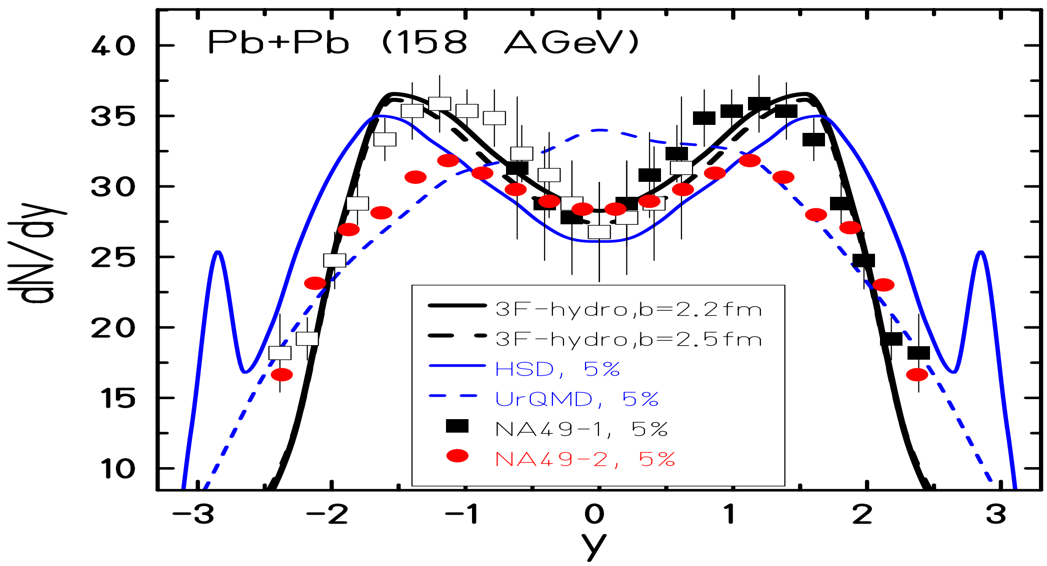
Evolution of Thermodynamic Quantities





Experimental Data & Other Models
Proton Rapidity Distributions



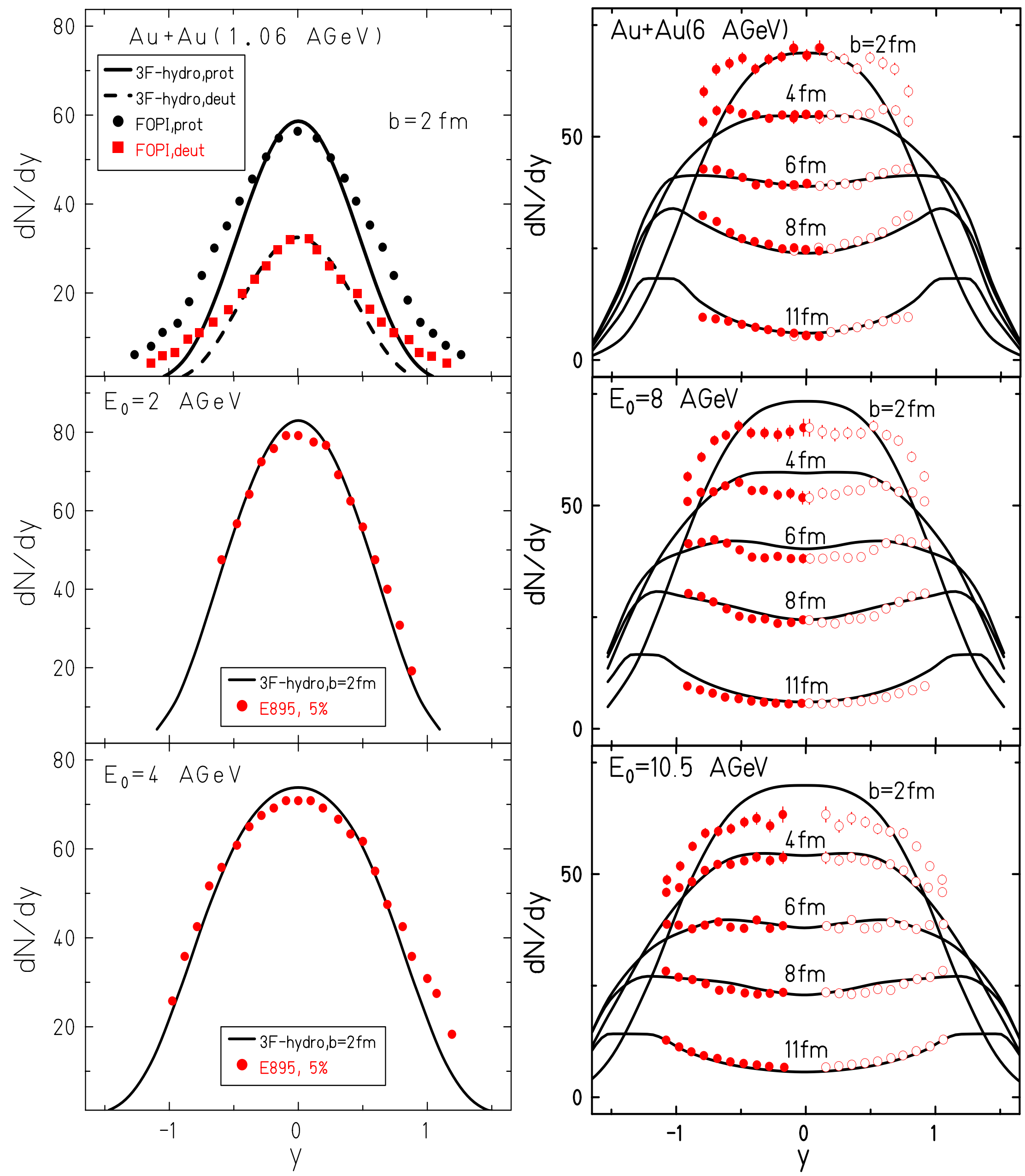


3-Fluids: gasEoS

b = 2 fm for Au+Au(10 AGeV) and b = 2.2 fm for Pb+Pb(158 AGeV) are experimental estimates.

NA49 (prot.): Phys. Rev. **C69** (2004) 024902
NA49–1: Phys. Rev. Lett. **82** (1999) 2471
NA49–2(preiminary): Nucl. Phys. **A661** (1999) 362c
Models: H. Weber, E.L. Bratkovskaya, W. Cassing and H. Stöcker, Phys. Rev. **C67** (2003) 014904
E802: Phys. Rev. **C60** (1999) 064901
E877: Phys. Rev. **C62** (2000) 024901
E917: Phys. Rev. Lett. **86** (2001) 1970

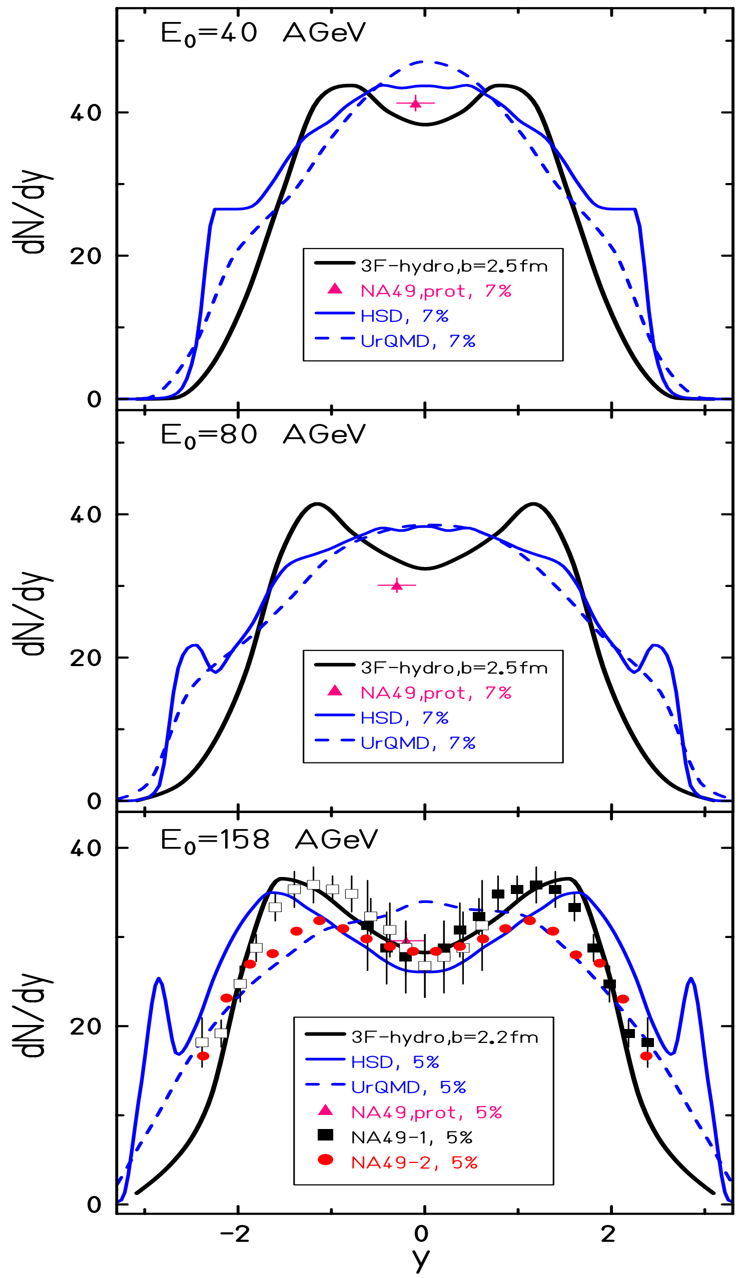
SIS&AGS Data
Proton Rapidity Distributions



3-Fluids: gasEoS

FOPI: N.Herrmann,Nucl.Phys. **A610** (1996) 49c [Au(1.06 GeV/nucl.)+Au]
E895: Phys. Rev. **C68** (2003) 054905 [Au(2 and 4 GeV/nucl.)+Au]
E917: Phys. Rev. Lett. **86** (2001) 1970 [Au(6, 8 and 10.5 GeV/nucl.)+Au]

SPS Data
$(p - \bar{p})$ Rapidity Distributions



Pb + Pb

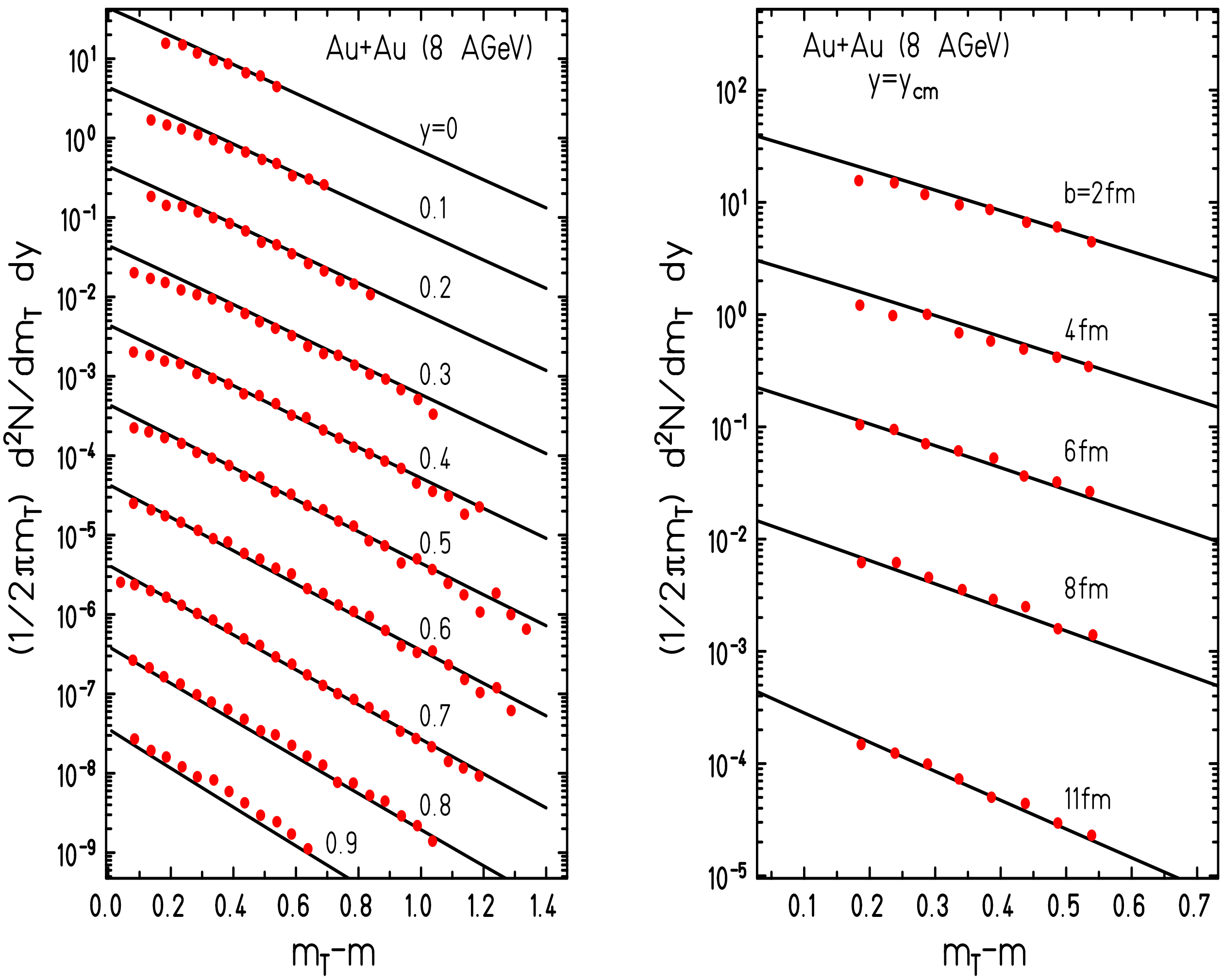
3-Fluids: gasEoS

b = 2.2 fm for 158 AGeV, and b = 2.5 fm for 40 and 80 AGeV are experimental estimates.

NA49 (prot.): Phys. Rev. **C69** (2004) 024902
NA49–1: Phys. Rev. Lett. **82** (1999) 2471
NA49–2(preiminary): Nucl. Phys. **A661** (1999) 362c
Models: H. Weber, E.L. Bratkovskaya, W. Cassing and H. Stöcker, Phys. Rev. **C67** (2003) 014904

AGS Data: Proton p_T Spectra

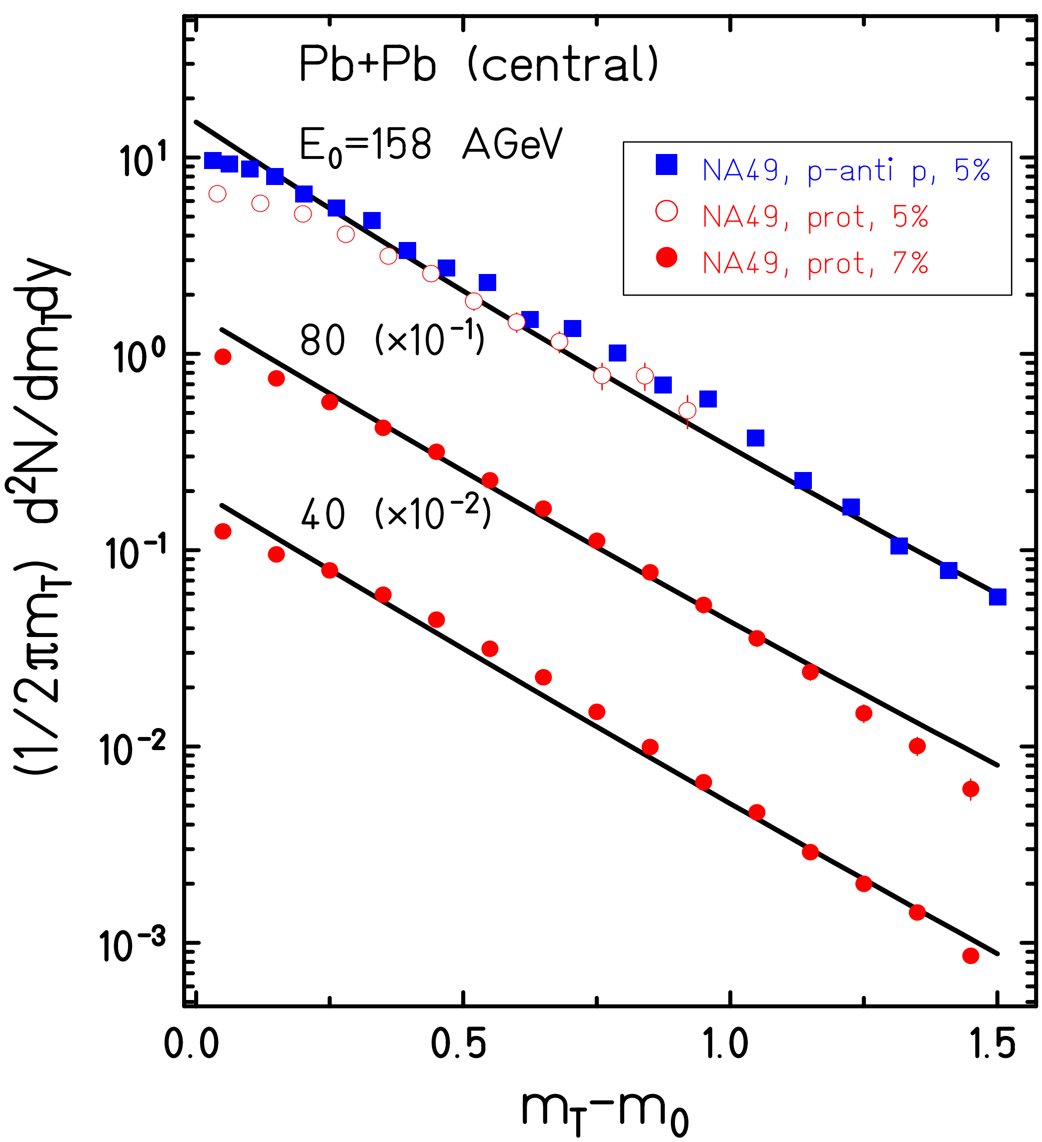
b = 2 fm



3-Fluids: gasEoS

E917: Phys. Rev. Lett. **86** (2001) 1970 [Au(6, 8 and 10.5 GeV/nucl.)+Au]

SPS DATA: PROTON p_T SPECTRA



Pb + Pb

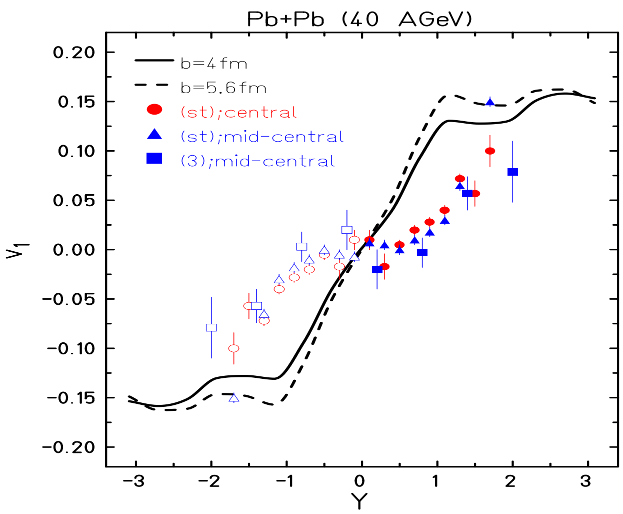
3-Fluids: gasEoS

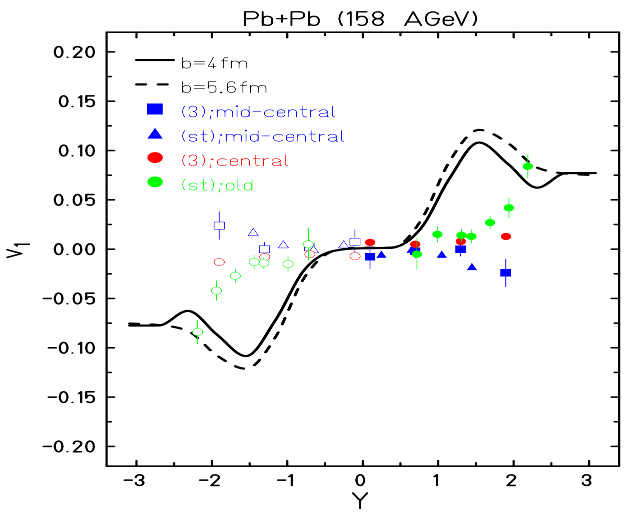
b = 2.2 fm for 158 AGeV, and b = 2.5 fm for 40 and 80 AGeV are experimental estimates.

NA49: Phys. Rev. Lett. **82** (1999) 2471
NA49: Nucl. Phys. **A715** (2003) 166c

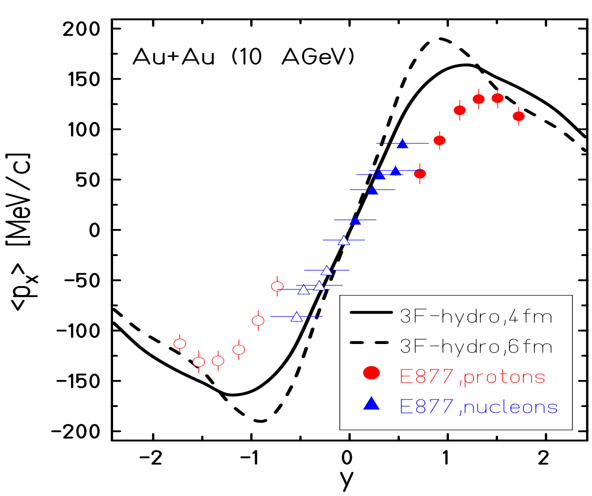
SPS/AGS Data: Nucleon v_1/p_x Flow

$$v_1(y) = \int d^2p_T \frac{p_x}{p_T}\frac{dN}{d^3p} \Big/ \int d^3p \frac{dN}{d^3p}$$





b = 3.7 fm for central coll., and b ≈ 7 fm for midcentral coll. are experimental estimates.
(st): standard method; (3): 3-particle correlation method
NA49: Phys. Rev. Lett. **80** (1998) 4136
NA49: Phys. Rev. **C68** (2003) 034903

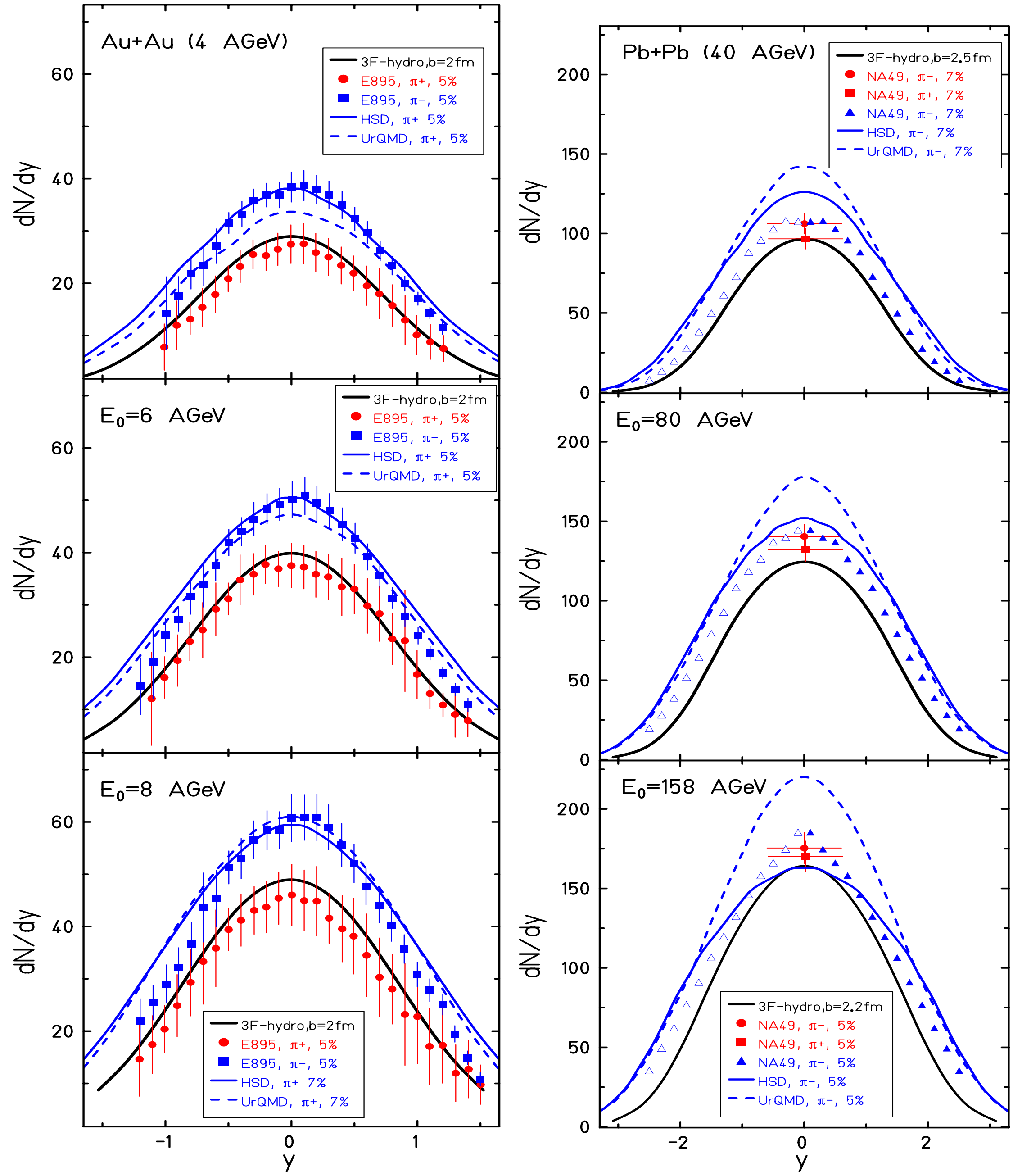


$$\langle p_x\rangle(y) = \frac{\int d^2p_T p_x (dN/d^3p)}{\int d^3p(dN/d^3p)}$$

E877: Phys. Rev. **C56** (1997) 3254

3-Fluids: gasEoS is too hard

AGS&SPS Data
Pion Rapidity Distributions

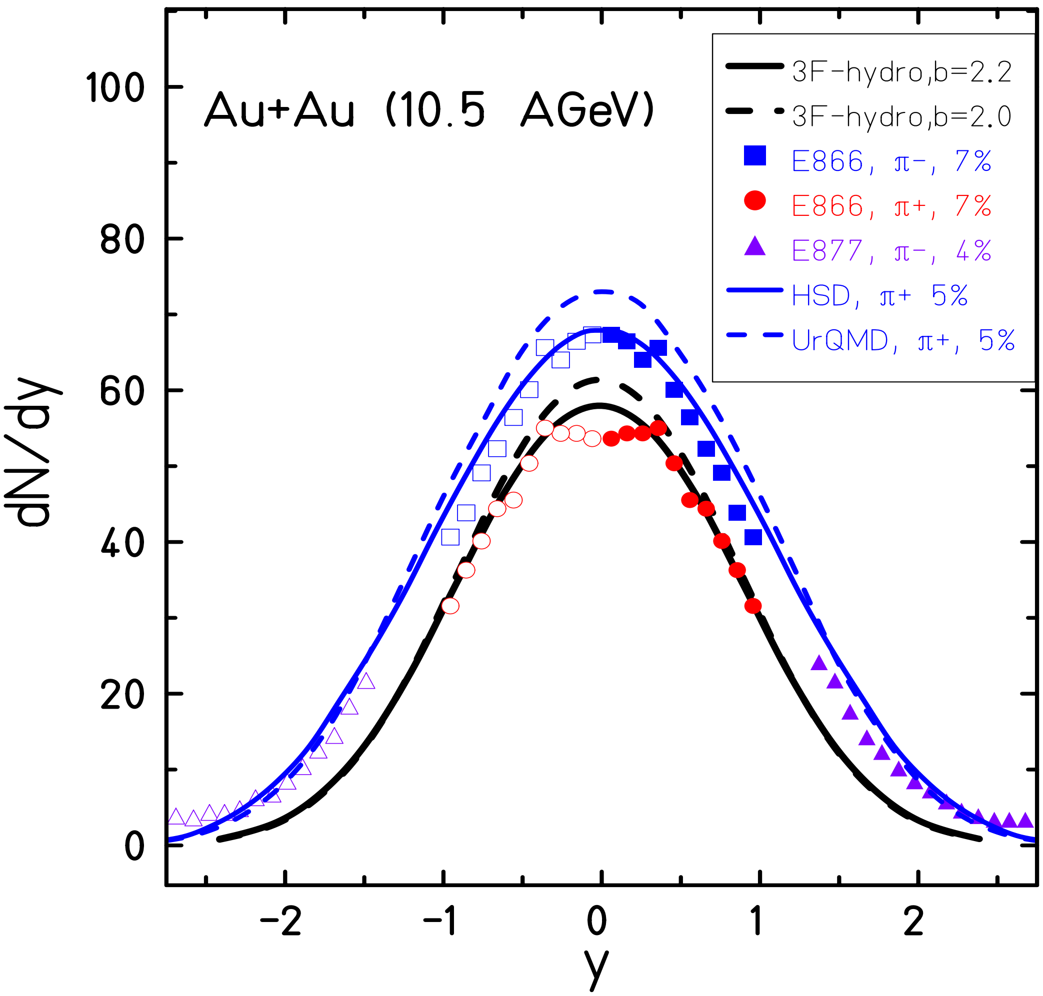


3-Fluids: gasEoS is too hard

b = 2.0 fm for 4, 6 and 8 AGeV, b = 2.2 fm for 158 AGeV, and b = 2.5 fm for 40 and 80 AGeV, are experimental estimates.

NA49: Phys. Rev. **C66** (2002) 054902
E895: Phys. Rev. **C68** (2003) 054905
Models: H. Weber, E.L. Bratkovskaya, W. Cassing and H. Stöcker, Phys. Rev. **C67** (2003) 014904

AGS Data
Pion Rapidity Distributions

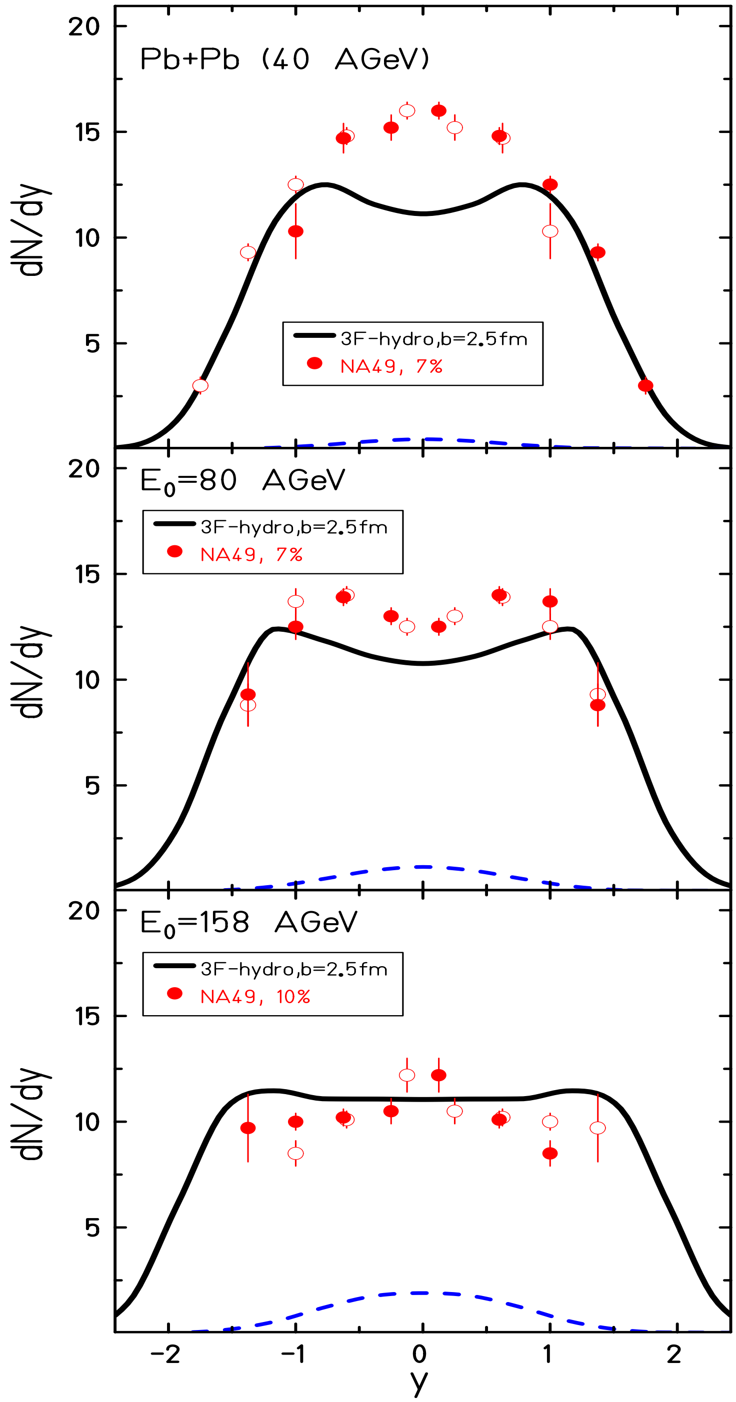


3-Fluids: gasEoS is too hard

b = 2.0 fm for 7%σ and b = 1.5 fm for 4%σ are experimental estimates.

E895: Phys. Rev. **C68** (2003) 054905
E877: Phys. Rev. **C62** (2000) 024901
Models: H. Weber, E.L. Bratkovskaya, W. Cassing and H. Stöcker, Phys. Rev. **C67** (2003) 014904

SPS Data: $\Lambda + \Sigma^0$ Rapidity Distributions Preliminary

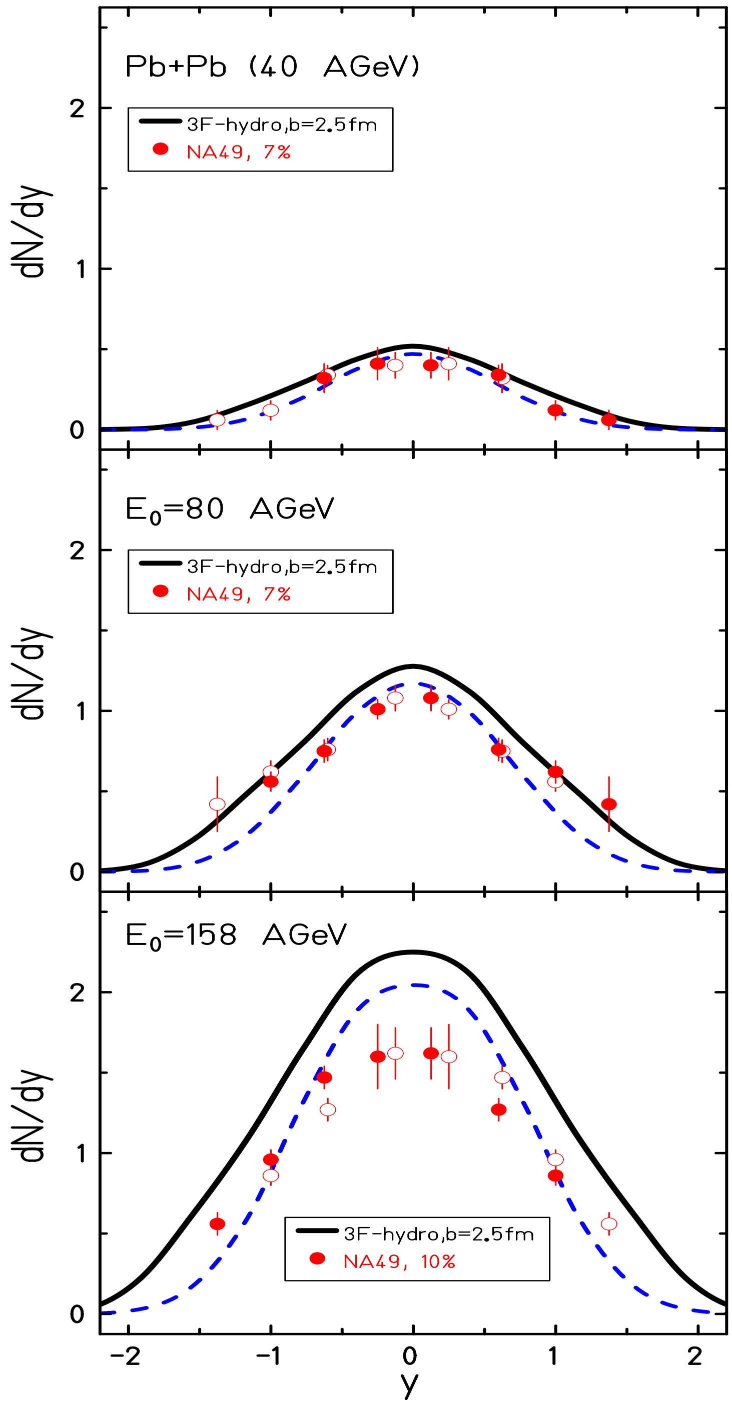


3-Fluids: gasEoS

dashed line = contribution from the fireball fluid

NA49: nucl-ex/0311024

SPS Data: $\bar{\Lambda}+\bar{\Sigma}^0$ Rapidity Distributions Preliminary



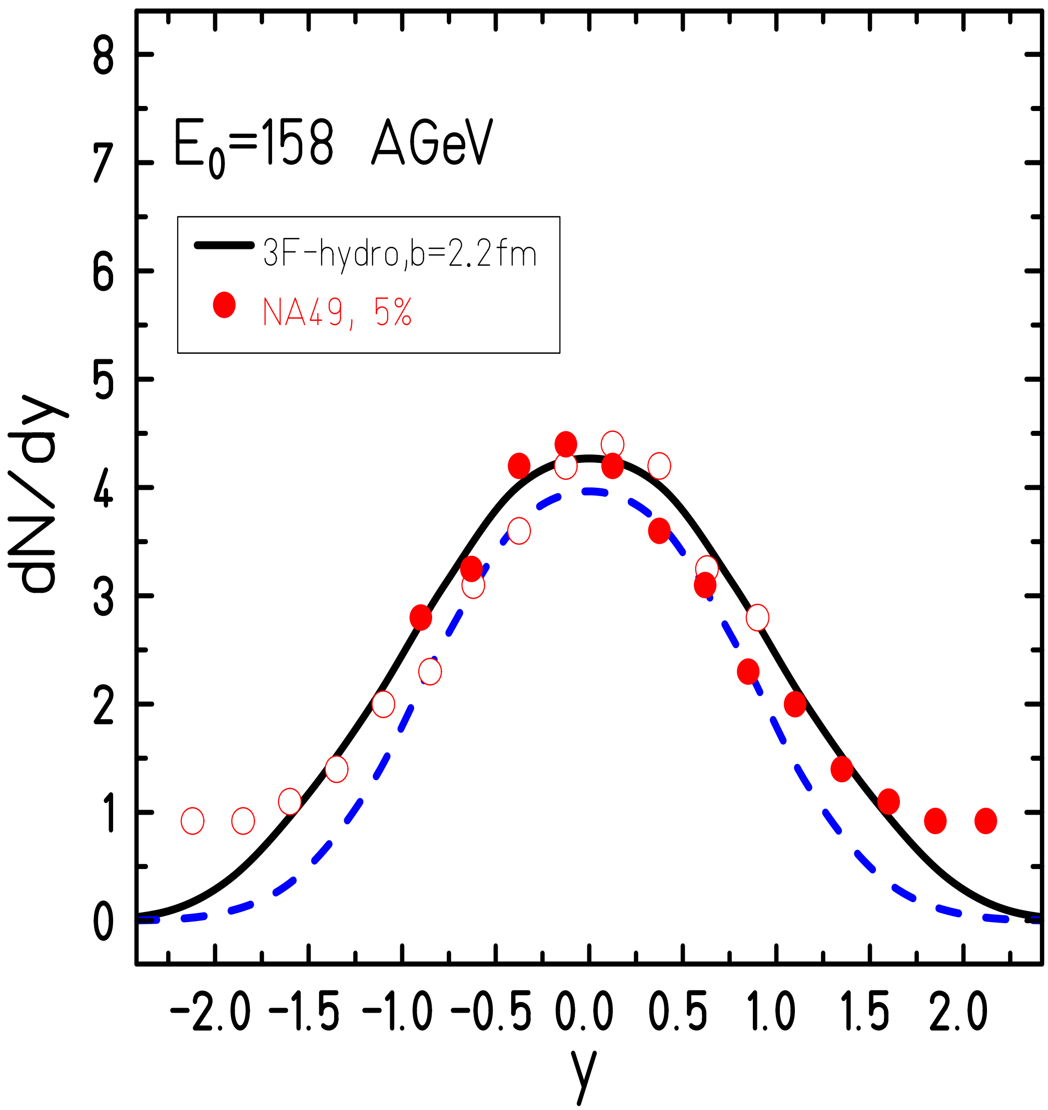
3-Fluids: gasEoS

dashed line = contribution from the fireball fluid

NA49: nucl-ex/0311024

SPS Data: $\bar{p}$ Rapidity Distributions Preliminary

$\bar{p} = \frac{1}{2}\bar{N}$
($\bar{\Lambda} \rightarrow \bar{N} + \pi$ is excluded)



3-Fluids: gasEoS

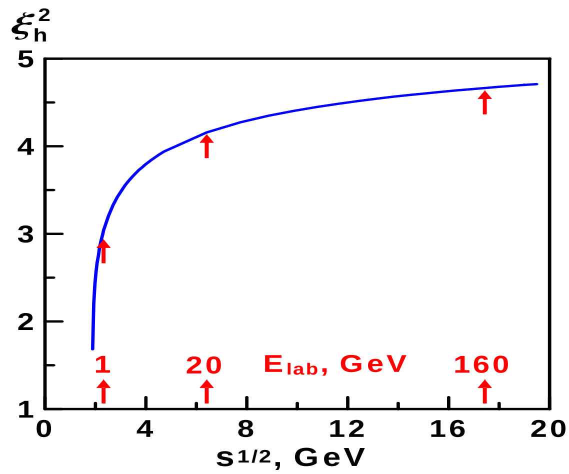
dashed line = contribution from the fireball fluid

NA49: Nucl. Phys. **A661** (1999) 45c.

Summary

• All global observables, considered up to now (!!!), are reasonably reproduced with a simple hadronic EoS, provided the friction is enhanced as follows

This enhancement reproduces the observable Stopping Power.



• Is it reasonable enhancement in view of model uncertainties?

(medium effects, multiparticle collisions, poor knowledge of various σ)

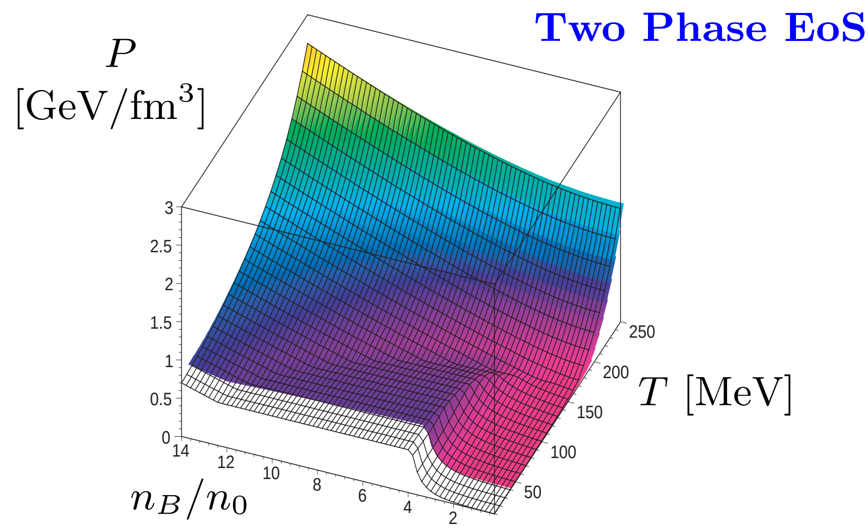
∘ Mixed quark/hadron phase formation $\Rightarrow$ at $T \sim T_c$ the scattering length for $q - \bar{q}$ (quasi-)mesons and gluons goes through ∞ ? (at RHIC the enhancement factor $10 - 10^2$ is needed for partonic σ !; E.Shuryak and I.Zahed, hep-ph/0307276; "sticky moalesses" : G.E.Brown, C.-H.Lee, M.Rho, hep-ph/0402207)

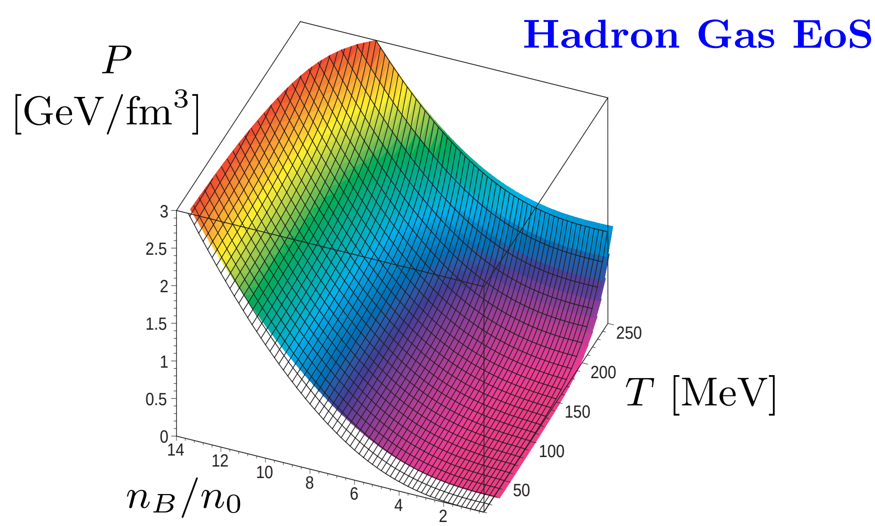
• Different EoS (with different order of phase transition) should be probed

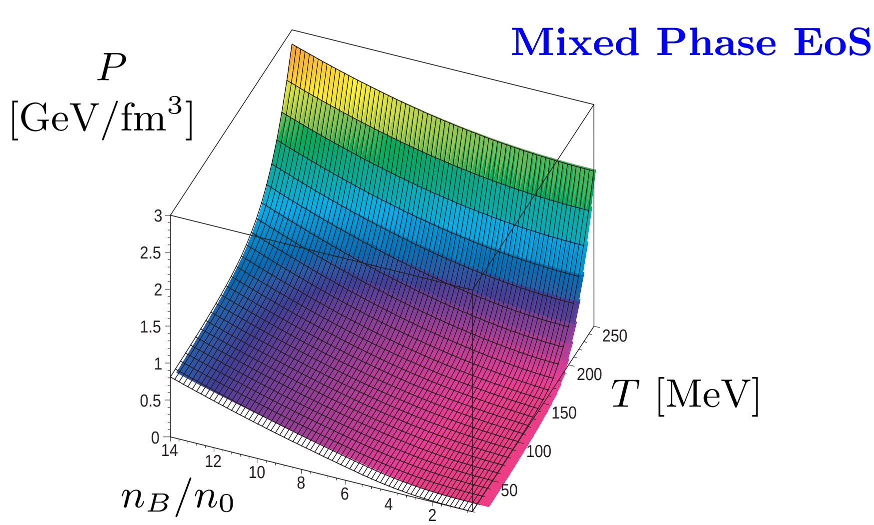
• Observable Stopping Power $\Rightarrow$ there are certain windows of incident energies, where a matter with desired properties is most efficiently produced, e.g.

∘ 15 GeV/nucl. $< E_{lab} <$ 80 GeV/nucl. is preferable for production of thermalized baryonic matter with $n_B > 8n_0$

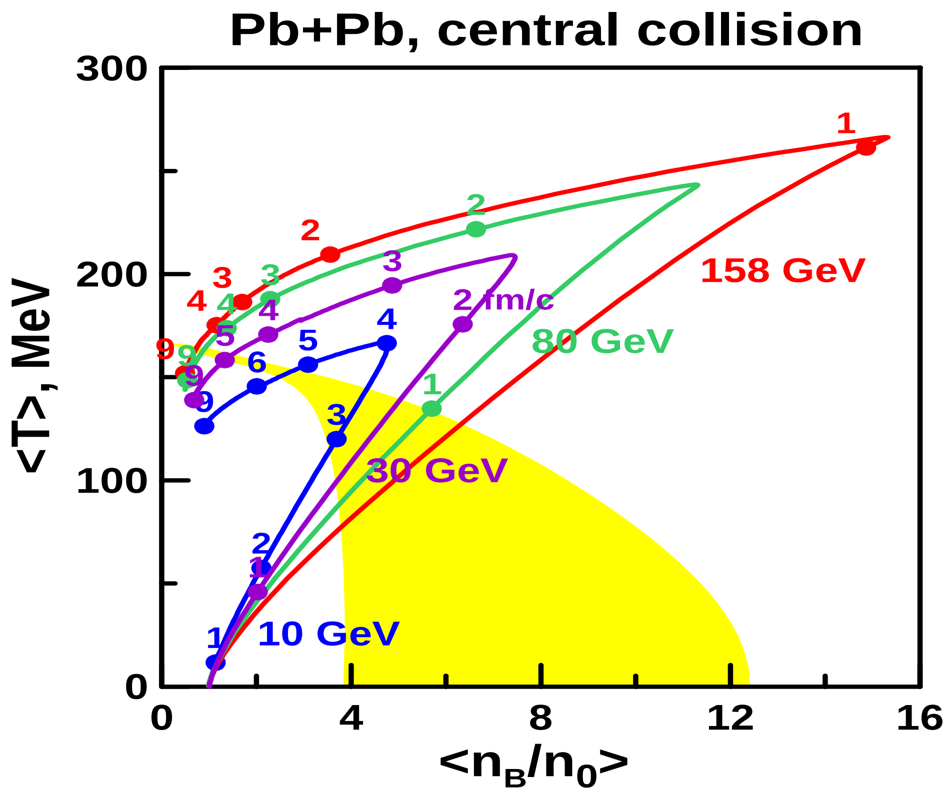
Pressure (Thermodynamic Potential)

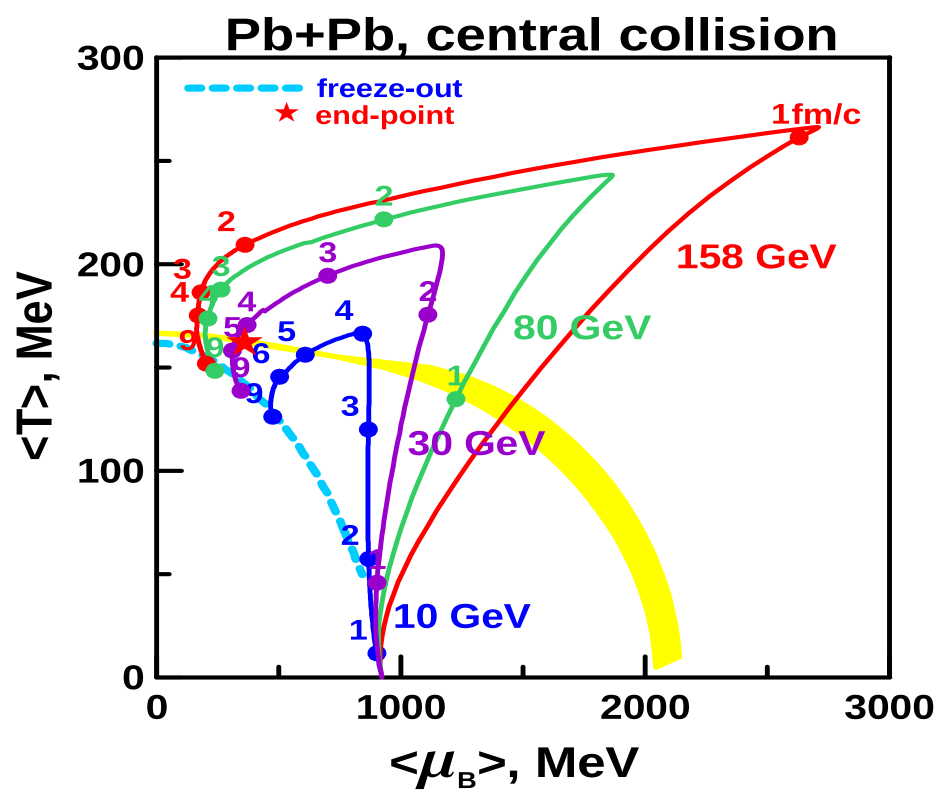






Phase Diagrams





Critical end-point: Z.Fodor, S.D.Katz, hep-lat/0402006.

Further Studying

♠ Within 3F hydrodynamics, to repeat comprehensive analysis of experimental data with Two Phase MIT bag model (first order phase transition) and Mixed Phase model EoS (crossover) for finding "friction enhancement factor"

♠ To disentangle different EoS through 3F-hydro analysis of excitation functions in the SIS-SPS energy range :

- Directed v_1 and elliptic v_2 flow
- Strangeness production n_s/n_π
- Transverse temperature $T^\star$
- Dilepton production
- ...