LINEAR DYNAMICS OF MANY-BODY SYSTEMS: NUCLEI, ATOMIC CLUSTERS, BEC

- Linear collective dynamics: $\delta\rho(\vec{r},t) \propto V_{ext}(\vec{r},t)$

  atomic nuclei $\Rightarrow$ giant resonances of nucleons

  atomic clusters $\Rightarrow$ plasmons of electrons

  BEC $\Rightarrow$ coll. modes of Bose atoms

- Atomic clusters $\Rightarrow$ - bridge between one atom and bulk
  - similarity with nuclei
  - nanotechnologies

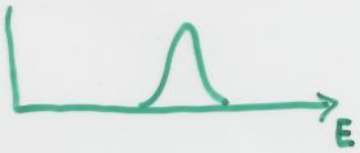
- BEC $\Rightarrow$ brilliant collection of various physics (superfluidity, vortices, optical lattices, ...)

- Origin of giant resonances and their variety in different systems (E1, ..., scissor M1)

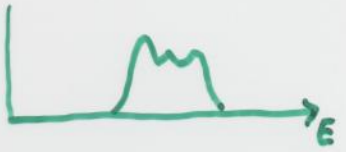
- Main models: TDHF, RPA, hydrodynamics, ...

Linear regime $\Rightarrow$ Random Phase Approximation (RPA)

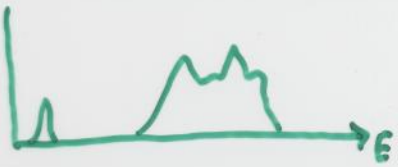
Sum Rules (= one oscillator)
collective strength is concentrated in one peak



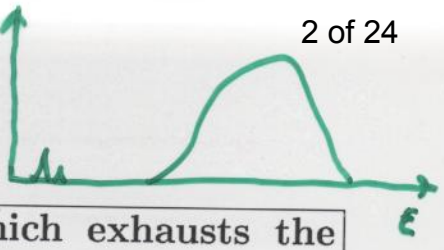
Local RPA (= several coupled oscillators)
several peaks, gross-structure, ...



Full RPA (= many coupled oscillators)
fine structure, Landau damping, ...



Definition of giant resonances (GR)



Giant resonance is a collective excitation which exhausts the main part of the corresponding sum rule:

$$m_1^{E\lambda} = \sum_j \omega_j < j|\hat{F}^{E\lambda}|0>^2 \Rightarrow \sim \omega_{GR} < GR|\hat{F}^{E\lambda}|0>^2$$

Isoscalar (IS) GR: protons and neutrons oscillate in phase

Isovector (IV) GR: protons and neutrons oscillate in opposite phases

Sum rules:

$H = T+V \Rightarrow [F,H] = [F,T] + \underbrace{[F,V]}_{=0}$

$$\begin{aligned} m_1^{E\lambda} &= \sum_j \omega_j < j|\hat{F}^{E\lambda}|0>^2 \\ &= < 0|[\hat{F}^{E\lambda},[\hat{F}^{E\lambda},\hat{H}]]|0> = < 0|[\hat{F}^{E\lambda},[\hat{F}^{E\lambda},\hat{T}]]|0> \\ &= \frac{\hbar^2}{8\pi m}\lambda(\lambda+1)Z < 0|r^{2\lambda-2}|0> \quad \Leftarrow \boxed{\text{model independent!}} \end{aligned}$$

where

$$\begin{aligned} \hat{F}^{E\lambda} &= e\sum_{i=1}^{Z} r_i^\lambda Y_{\lambda\mu}(\Omega_i) \\ &= e\sum_{i=1}^{A} \frac{1-\tau_i}{2} r_i^\lambda Y_{\lambda\mu}(\Omega_i) \\ &= \frac{e}{2}\sum_{i=1}^{A} r_i^\lambda Y_{\lambda\mu}(\Omega_i) - \frac{e}{2}\sum_{i=1}^{A} \tau_i r_i^\lambda Y_{\lambda\mu}(\Omega_j) \\ &= \hat{F}^{\lambda,IS} - \hat{F}^{\lambda,IV} \end{aligned}$$

$\tau = \begin{cases} -1, & p \\ 1, & n \end{cases}$

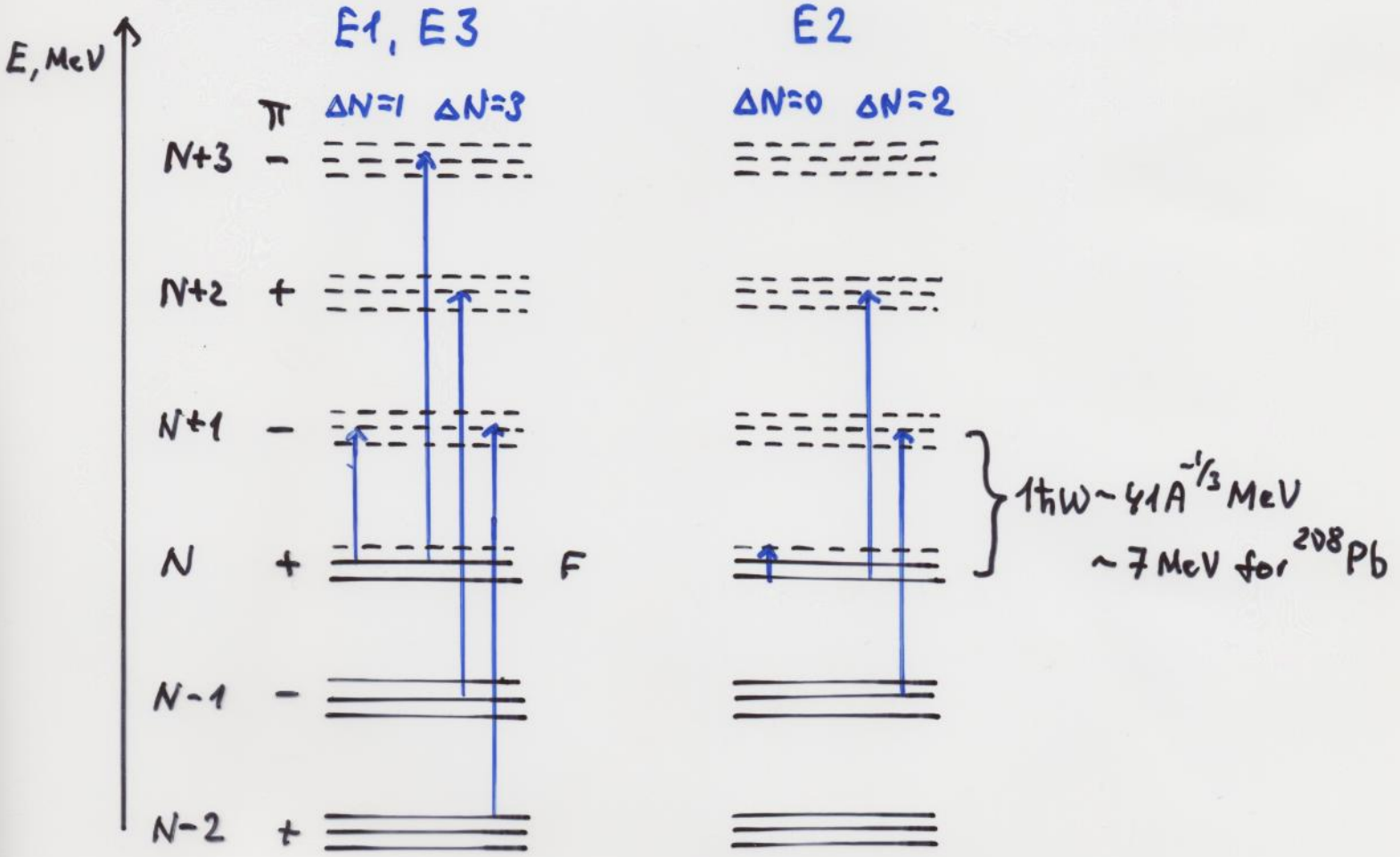
$$m_1^{\lambda,IS} = \sum_j \omega_j < j|\hat{F}^{\lambda,IS}|0>^2 = \frac{\hbar^2}{8\pi m}\lambda(\lambda+1)\frac{Z^2}{A} < 0|r^{2\lambda-2}|0>$$

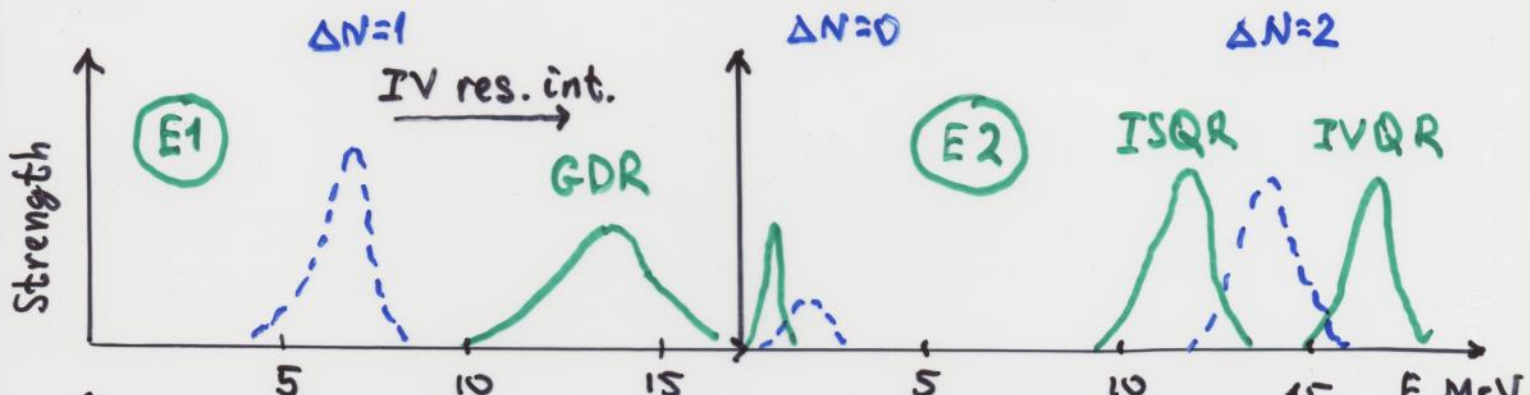
$$m_1^{\lambda,IV} = \sum_j \omega_j < j|\hat{F}^{\lambda,IV}|0>^2 = \frac{\hbar^2}{8\pi m}\lambda(\lambda+1)\frac{NZ}{A} < 0|r^{2\lambda-2}|0>$$

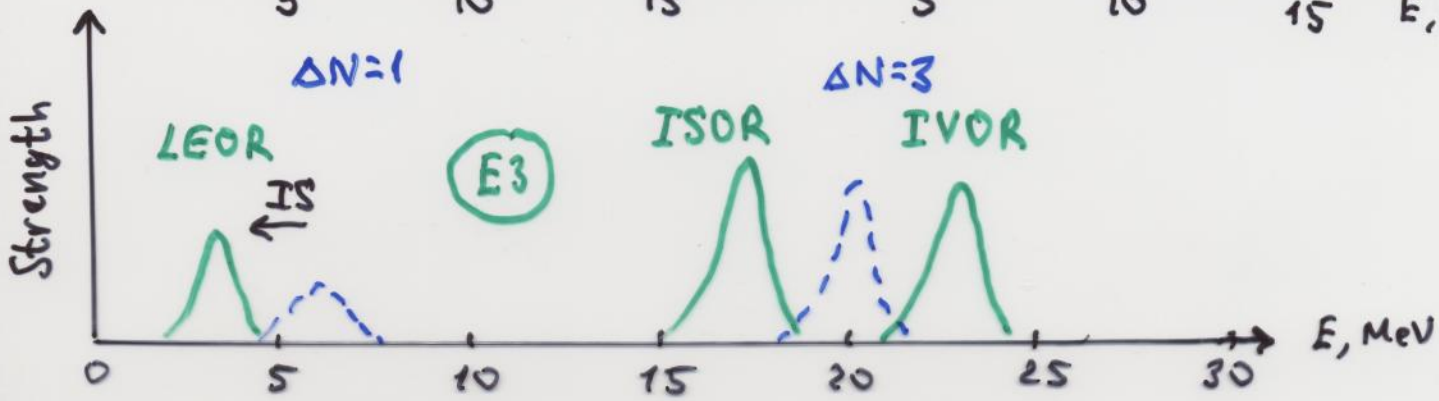
$$m_1^{\lambda,IS} + m_1^{\lambda,IV} = m_1^{E\lambda}$$

GIANT RESONANCES

Mean field



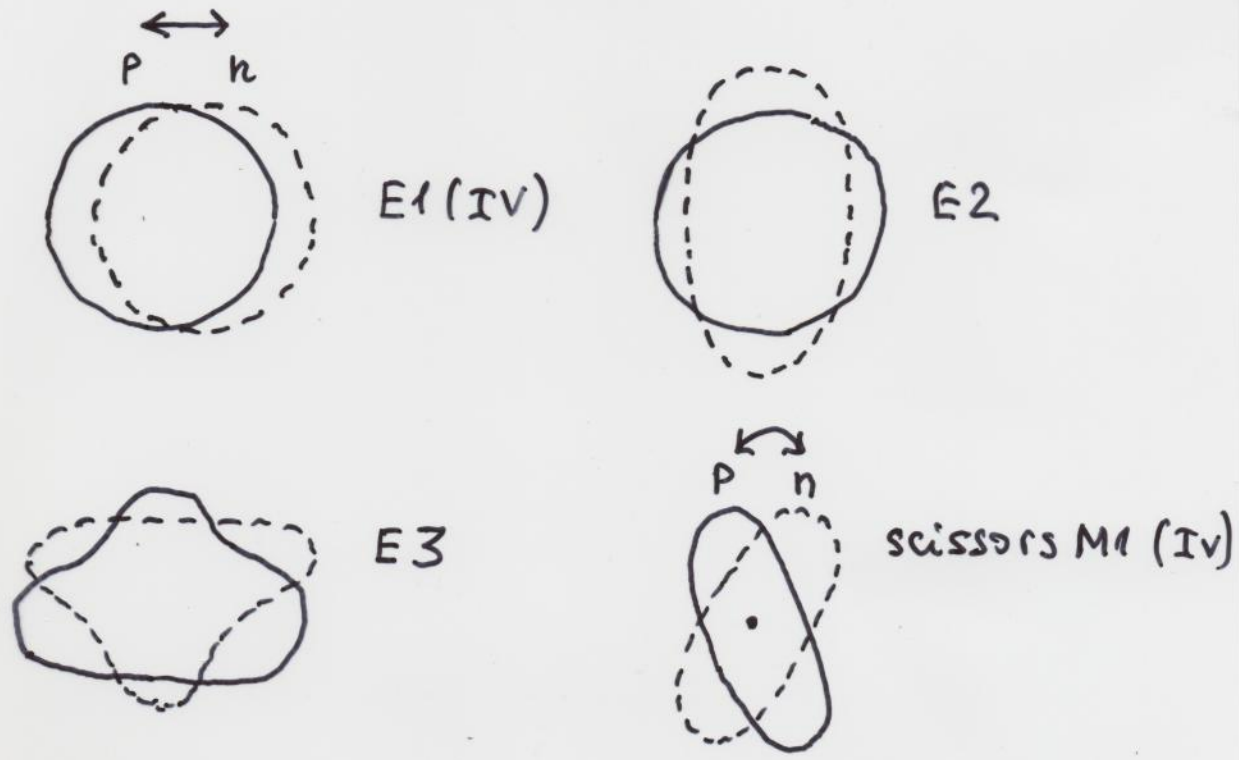




GR origin: quantum shells + resid. inter.

GR: macroscopic view

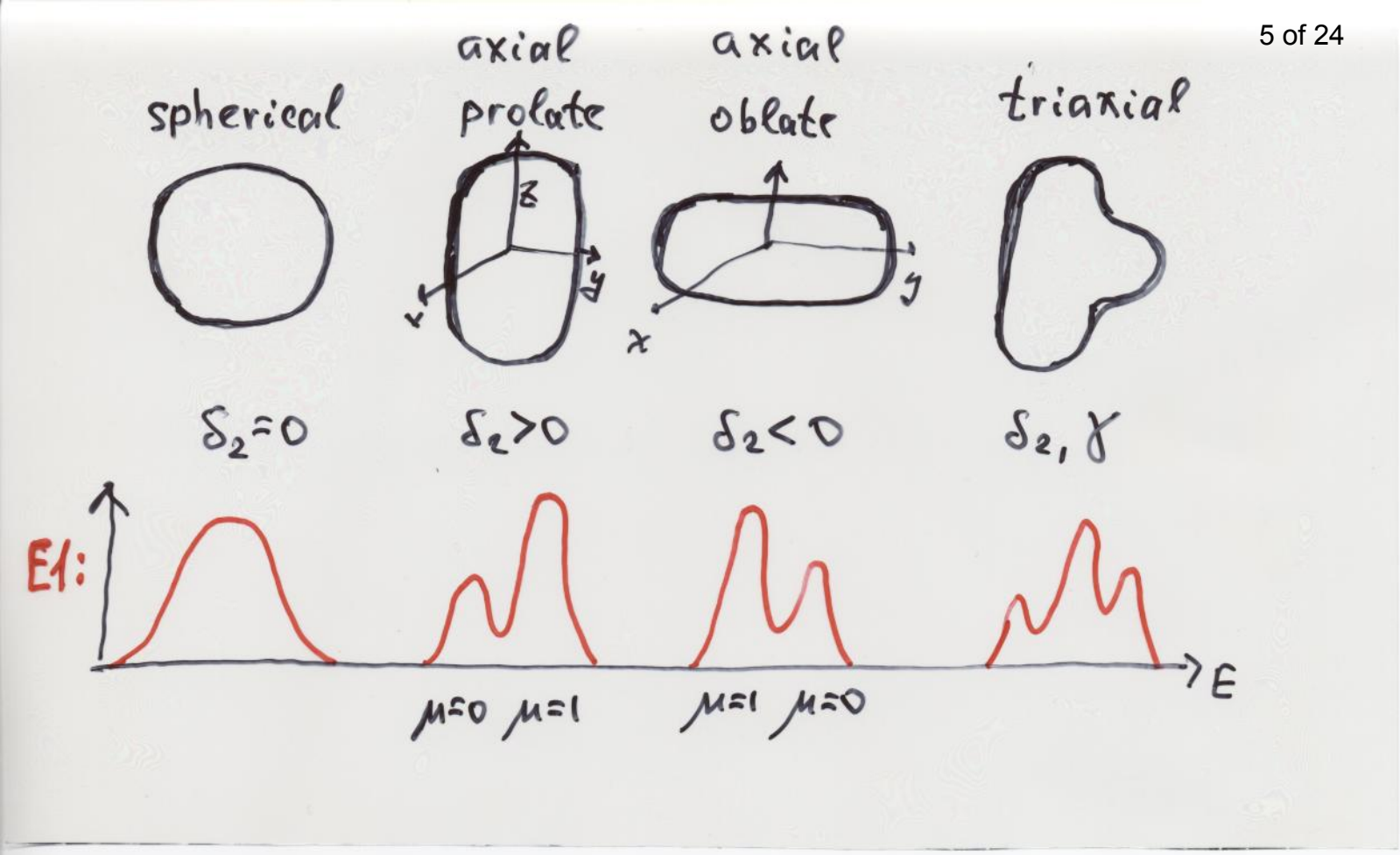
Treatment in hydro/fluid dynamical models:



- The relevant microscopic and macroscopic models give similar velocity fields which justifies that, in the first approximation, both treatments are similar.

- Some of GR, e.g. E2(IS), are not described by familiar hydrodynamics but are well treated in fluid dynamics where deformation of the Fermi sphere in the momentum space (or nondiagonal components of the stress tensor) are taken into account.
This corresponds to treatment of GR as oscillations of the elastic globe (G.F. Bertsch, 1974).

- BEC: mainly hydro/fluid dynamical models.


spherical
axial
prolate
axial
oblate
triaxial
z
y
x
$\delta_2 = 0$
$\delta_2 > 0$
$\delta_2 < 0$
δ_2, γ
E1:
E
μ=0 μ=1
μ=1 μ=0

VARIETY OF GIANT RESONANCES

Atomic nuclei

isovector E1

E2, E3 ⇒ nuclei as elastic globe

E0 ⇒ nuclear incompressibility

spin M1, M2

orbital M1 (scissors) and M2 (twist) ⇒ orbital magnetism

isoscalar E1 (compressional, toroidal) ⇒ vorticity

pygmy E1 ⇒ neutron skin osc.

Actual problems:

– GR in rotating nuclei,

– exotic vortex GR (toroidal, compressional E1, pygmy, ...)

– GR in halo nuclei

Atomic clusters

E1 ⇒ many different cases, applications

other GR ⇒ only theor. predictions

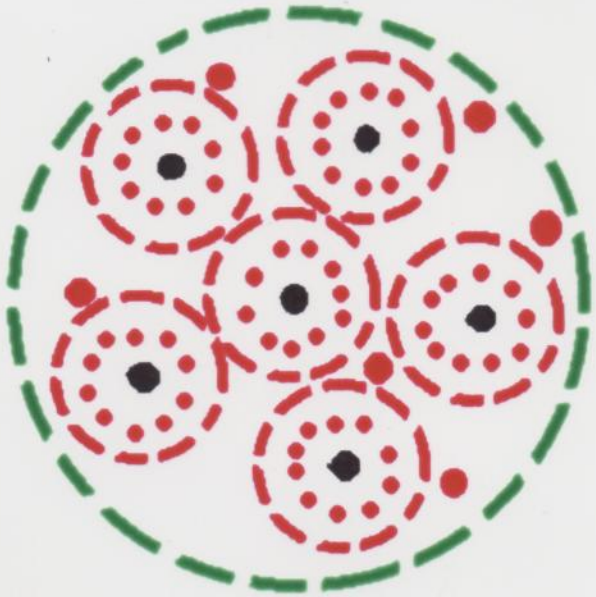
BEC

E1, E2, ...

scissors M1 ⇒ direct signature of superfluidity

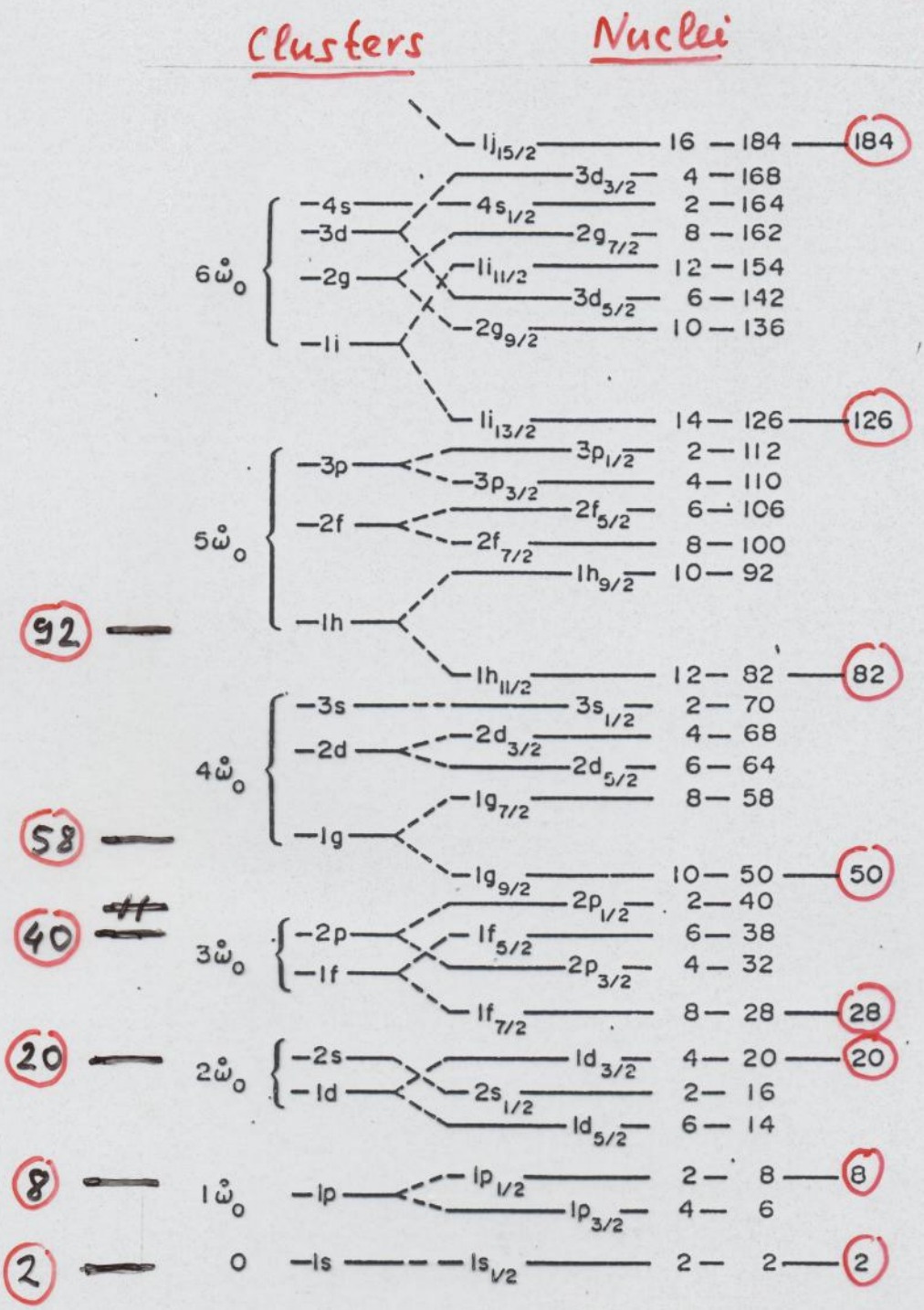
1. W.D. Knight, K. Clemenger et all, PRL 52 (1984) 2141 - exper.
2. W. Ekardt, PR, B29 (1984) 1558 - theor.

Metal Cluster Na_6



- MC is a bound system of atoms of some metals.
- Alkali metal clusters are very similar to atomic nuclei.
- In alkali metals, valence electrons are weakly coupled with ions and thus are not strongly localized in space. As a result, the mean free path of valence electrons is of the same order of magnitude as the size of cluster (like for nucleons in atomic nuclei) → good conditions to form in MC a mean field of the same kind as in nuclei.
- Valence electrons:
  -- are counterparts of nucleons in a nucleus,
  -- determine quantum properties of MC.
- Ions can be replaced by a uniform distribution of the positive charge over cluster volume: jellium approximation.
  Ionic jellium is "frozen", i.e. has no any intrinsic excitations.
- Collective oscillations of valence electrons are counterparts of giant resonances in atomic nuclei.

Mean field and magic numbers

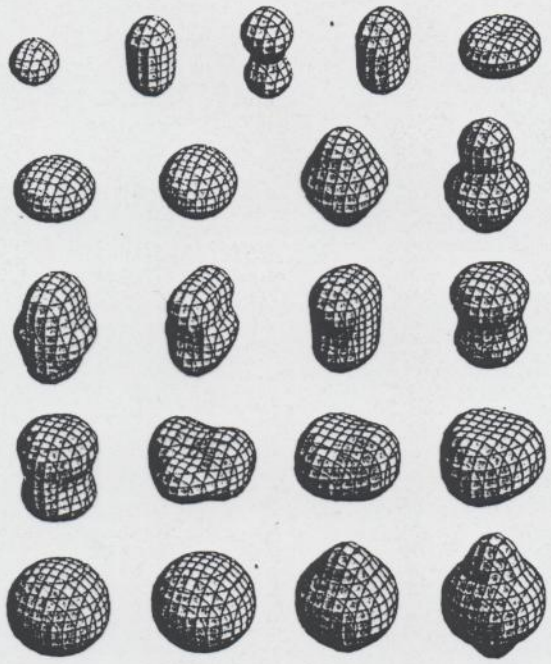


Difference in magic numbers in cluster and nuclei is mainly because of the spin-orbital coupling in nuclei

	s	p	d	f	g
ℓ =	0	1	2	3	4

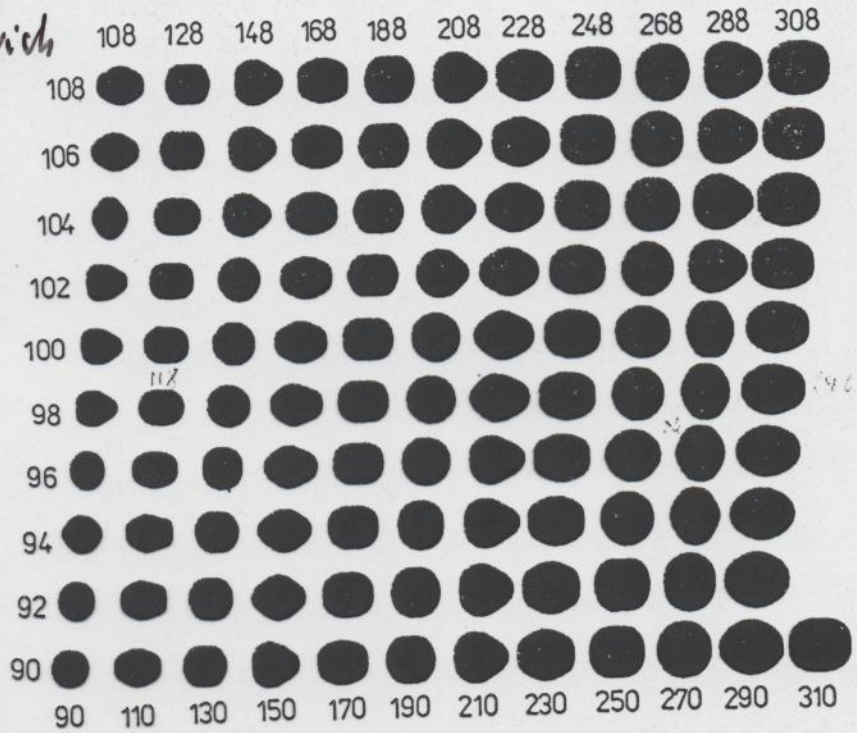
M. Koskinen et al

$Na_2 - Na_{22}$



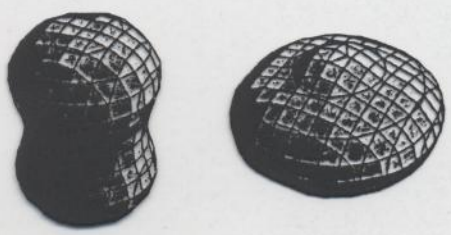
S. Frauendorf and V.V. Pashkevich

$Na_{108} - Na_{310}$



M. Koskinen et al

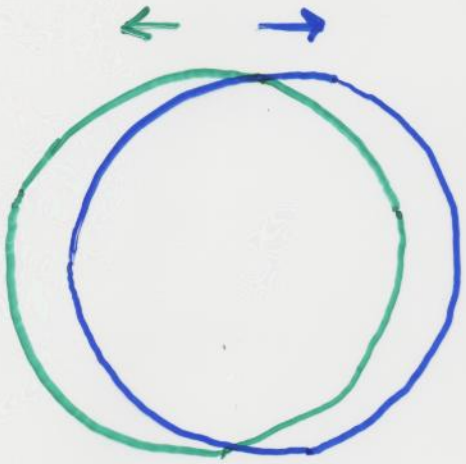
Na_{14}



ground isomer

$\Delta E = 0.1$ eV

E1 Giant Resonance



Similar nature:

- nuclei : GDR $n \leftrightarrow p$
  clusters : E1 -plasmon $e \leftrightarrow$ ions
- Landau fragmentation
- deformation splitting

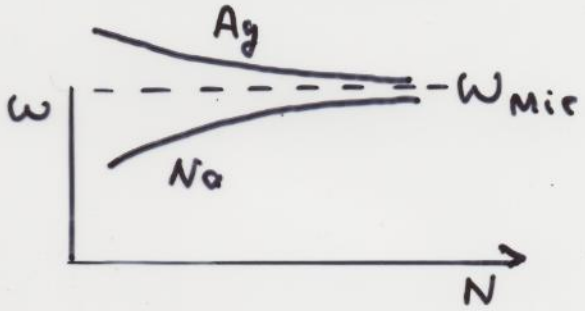
Differencies

Nuclei

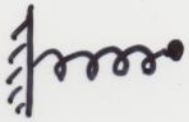
- IV nuclear inter.
- $\omega_{E1} \sim 81 A^{-1/3}$ MeV
- $M_n \approx M_p$
  isoscalar spurious state
-

Clusters

Coulomb inter.



Na: $\omega = \omega_{Mie}\left(1 - \frac{\Delta N}{N}\right)$, ΔN-spill-out

$M_e \ll M_i$ 

no problem of spurious state

- much more GDR versions
- interesting practical applications

Fullerenes

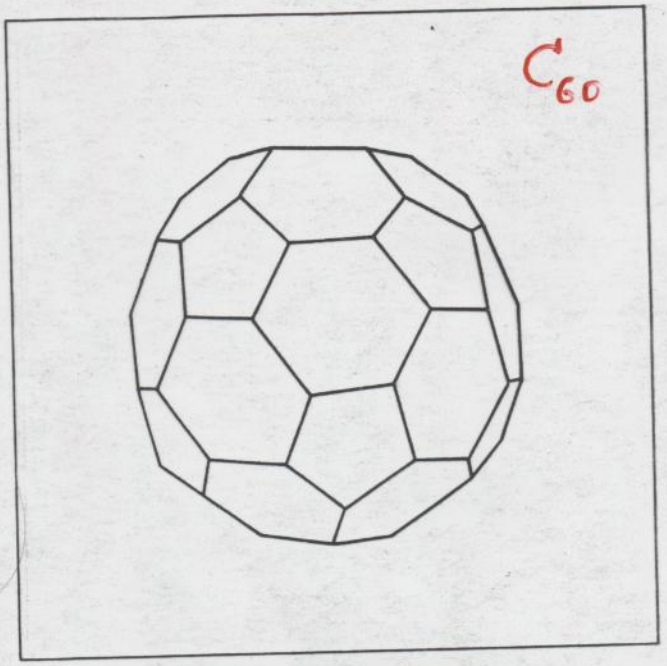


Figure 14. The European football structure ascribed to the exceptionally stable carbon sixty cluster (Hopkins *et al.* 1983). This classical structure was first studied by Leonardo da Vinci in the early sixteenth century.

R.S. Smalley et al
Rice University, Texas

The Making of the Fullerenes

The first recorded observation of the fullerenes was made using an apparatus similar to that shown in Figure 1. Here a pulsed laser beam (the green second harmonic of a Nd:YAG laser, 532 nm) is directed onto the surface of a rotating/translating graphite disk. Although the intensity of the laser is only enough to deposit a few tens of millijoules onto a 1-mm-diameter spot on the surface of the disk, it does this in an extremely short period of time: only 5 ns. This is sufficient to generate a plasma of carbon vapor over the irradiated spot in which temperatures over 10 000 °C are readily obtained. In order to cool this superhot plasma, and generate clusters, a burst of helium gas is used from a pulsed gas nozzle. As the carbon atoms and small radicals are cooled in this carrier gas, clustering reactions occur. By controlling the relative timing between the vaporization laser and helium gas pulses and adjusting the flow geometry, the residence time in the source can be extended to allow these growing clusters to "cook" in the soup of carbon radicals before the carrier gas expands out the end of the source and forms a supersonic free jet in a vacuum chamber. Collimated beams skimmed from this supersonic free jet then provide an excellent environment for probing both the size distribution and detailed nature of the carbon clusters by time-of-flight (TOF) mass spectrometry and

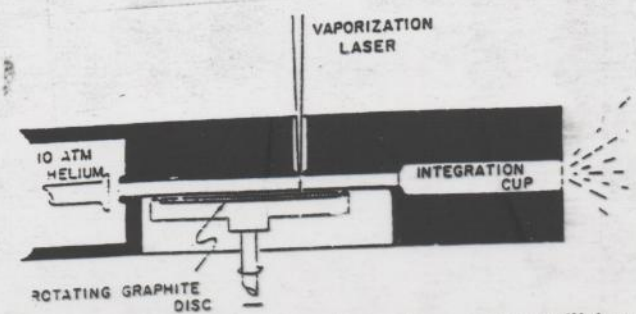


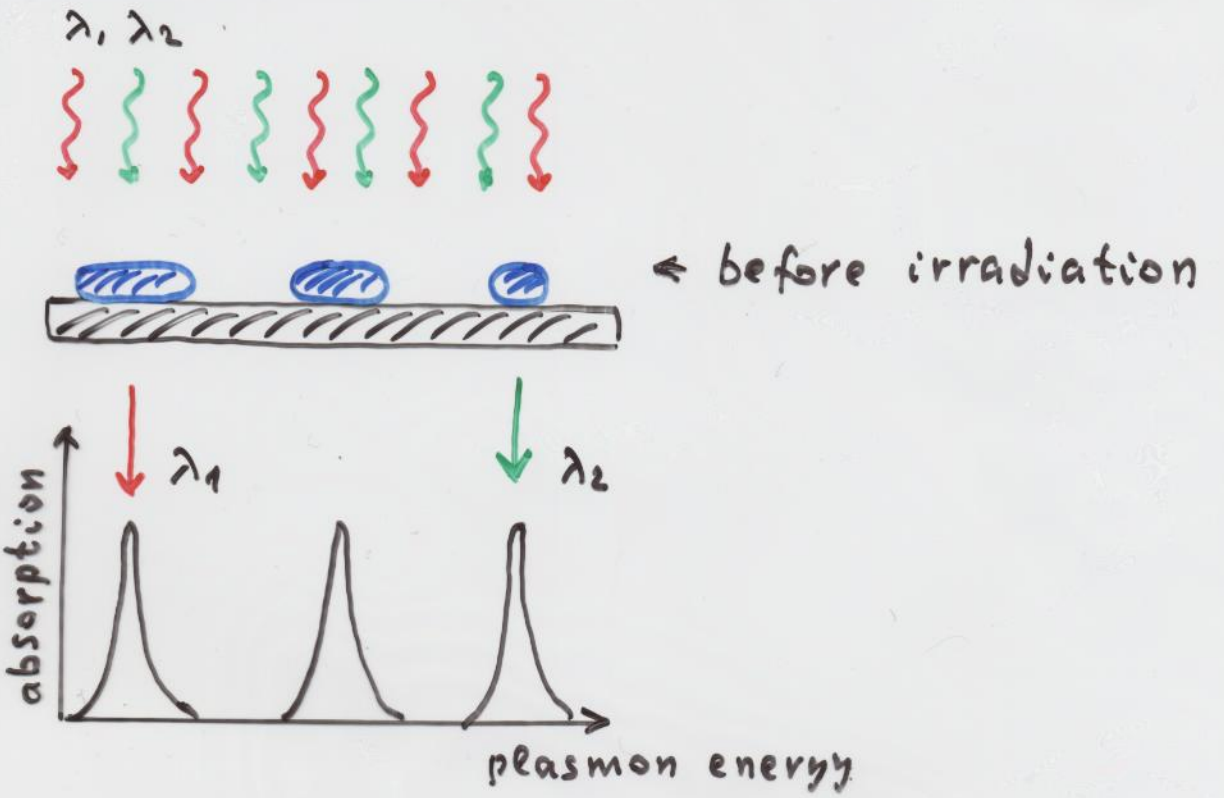
Figure 1. Supersonic carbon cluster beam source utilizing a pulsed laser to vaporize a graphite disk and a pulsed jet of helium to control clustering. Reprinted with permission from ref 8. Copyright 1985 Macmillan Magazines Ltd.

laser spectroscopy. The combination of laser vaporization with a pulsed gas flow permits the intermediate steps involved in carbon condensation to be studied in a controlled, systematic way. This fact beyond all others was what finally led to the discovery of the fullerenes.

Control of size and deformation of clusters on dielectic substrates

F. Stietz, F. Träger (Kassel, Germany)

Monitoring of the size:



Independent monitoring of size and deformation is now possible.

Free clusters: $N \rightarrow \beta$

Supported clusters: N $\beta = 0$, $\beta = -0.5$

New kind of finite Fermi system!

ATOMIC CLUSTERS: MAIN POINTS

- Fabrication of atomic clusters from any element of the periodic table is possible
- Bridge between one atom and bulk
- Multidisciplinary field
- Similarity to atomic nuclei:

– mean field, quantum shells,
– deformation,
– GR or plasmons,
– fission

but

– Coulomb interaction,
– much wider size region,
– neutral and charged,
– free, supported, embedded,
– different thermodynamics (canonical ensemble),
– distorted ionic lattice ⇒ new materials

- Main results, contemporary activities:

– supershells (in systems with N$\sim 10^3$ particles),
– irradiation of clusters by intense femtosecond lasers (enhanced energy accumulation, new ionization regimes, ...)

- Practical applications ⇒ a world of nanotechnologies !!!
- Crossover with other fields:

– fullerenes, nanotubes,
– atomic and molecular physics (next lecture of T. Kunert),
– quantum transport

Tsukuba, Japan 1997

Application of atomic clusters

Creation of new materials (semiconductor fabrication, etc.)

- depositing on a surface at very high density with low energy per atom
- „soft" deposition
- fine regulation of the size and charge of the projectile

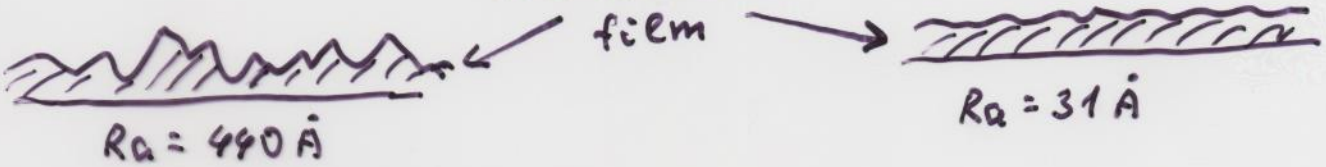
Precise machining of hard materials

(hard metal oxids, hard metals, carbon compounds, silicon and silicon compounds, ...)

Before irradiation

CVD diamond film

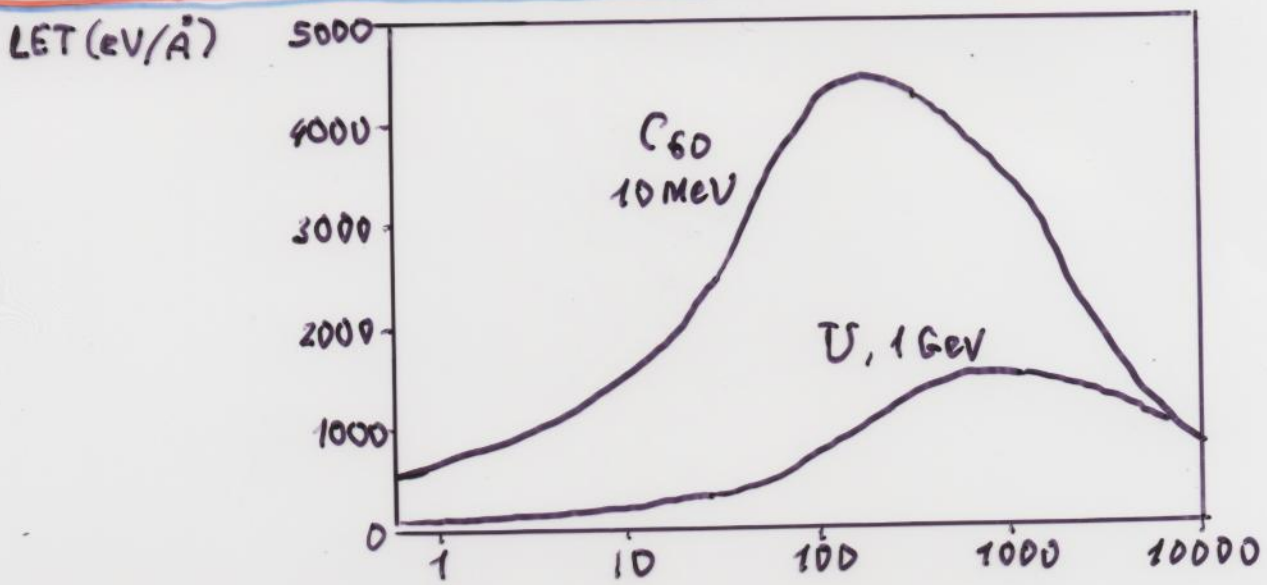
After radiation by Ar_{3000}, 20 keV



R_a = 440 Å

R_a = 31 Å

Exstremely high energy densities



- Ultra-high-density memory elements
- microstructuring
- high-temperature superconductivity
- catalis
- spectroscopy in helium drops
- .....

ATOMIC CLUSTERS: APPLICATIONS

- medicine:
  - cluster beams to treat caner,
  - versatile fluorescent clusters for labeling proteins within cells

- Catalysis: strongly depends on the size and surface preparation of nanoparticles

- Nanostructures at the surface

- Monitoring size and shape of supported clusters

- New alloys: combination of materials not mixed in bulk

- New magnetic materials: transformation of magnetic properties due to distorted ionic lattice

- Molecular and cluster nanoelectronics and nanodevices:
  - circuits, switches, transistors, ...
  - ultrahigh density magnetic recording disks,
  - nanotermometers, microcoolers, ...

- Helium clusters as nanoscale cryostats ($t^o < 0.37K$), creation of cold clusters inside He droplets

BOSE - EINSTEIN CONDENSATE

• Dilute ultracold gas of weakly interacting Bose atoms
(^{87}Rb, ^{23}Na, ^{7}Li, ^{4}He, ^{41}K, $\sim 10^6$ atoms)
in magnetic or electric trap.

• Nonlinear Gross-Pitaevskii equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = \left(-\frac{\hbar^2\nabla^2}{2m} + V_{ext}(\vec{r},t) + g|\Psi(\vec{r},t)|^2\right)\Psi(\vec{r},t)$$

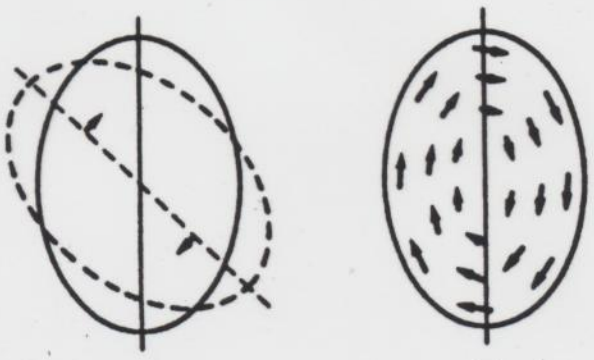
where

$\Psi(\vec{r},t)$	– order parameter,
$V_{ext}(\vec{r},t)$	– trap potential,
$g = 4\pi\hbar^2 a/m$	– interatomic interaction,
a	– s-scattering length.

• Superfluidity in the finite Bose system (before only in bulk He)

• Modeling and monitoring a world of different physical cases:

– variation of interaction through Feshbach resonance,
– initiation of different collective modes by the relevant shake of the trap,
– rotating BEC ⇒ vortices ⇒ vortex lattice ⇒ elastic modes,
– gas of Fermi atoms ⇒ superconductivity,
– mixtures of Bose and Fermi gases,
– molecular gas,
– molecular gas with competing dipolar interaction,
– spinor BEC in electric traps,
– optical lattices.



(a) (b)

macroscopic view

N. Lo Iudice E. Lipparini

SCISSORS MODE

- **General feature of elastic deformed two-component Fermi (Bose) systems:**

Atomic nuclei

N. Lo Iudice and F. Palumbo, Phys. Rev. Lett. 41 (1978) 1532.

E. Lipparini and S. Stringari, Phys. Lett. 130B (1983) 139.

Atomic clusters

E. Lipparini and S. Stringari, Phys. Rev. Lett. 63 (1989) 570.

V.O. Nesterenko, W. Kleinig, F.F. de Souza Cruz, and N. Lo Iudice, Phys. Rev. Lett., 83 (1999) 57.

Bose-Einstein condensate

D. Guéri and S. Stringari, Phys. Rev. Lett., 83 (1999) 4452.

Quantum dots

Ll. Serra, A.Puente, and E. Lipparini, Phys. Rev. B20 (1999) R13966.

- **Low-energy (0.1-1 eV) 1^+ excitations with strong M1 transitions to the ground state:**

$$\omega = \frac{30.7}{r_s^2} N_e^{-1/3} \delta \; eV, \quad B(M1) = N_e^{4/3} \delta \; \mu_b^2$$

- **Exists only in deformed systems (in jellium appr.)**

BEC: SCISSORS M1 MODE AS INDICATOR OF SUPERFLUIDITY

- How to distinguish normal and superfluid regimes?

– specific expansion of atomic cloud after switching off the trap,

– frequency shifts of collective modes,

– moment of inertia in rotating BEC,

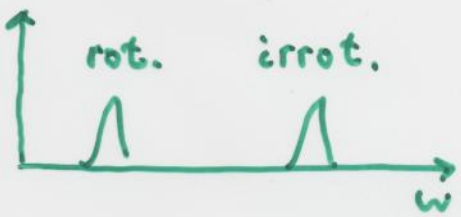
– scissors mode!

- Theory (D. Guéry-Odelin, S. Stringari, 1999)

$$V_{ext}(\vec{r}) = \frac{m}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2)$$

$$\omega_x^2 = \omega_0^2(1+\epsilon), \quad \omega_y^2 = \omega_0^2(1-\epsilon)$$

$$\vec{v}(\vec{r}) = \sim \vec{\Omega} \times \vec{r} + \sim \vec{\nabla}(xy)$$



Normal regime ($T > T_0$): two modes

$\vec{v}(\vec{r}) \sim \vec{\Omega} \times \vec{r} \quad \Rightarrow \quad \omega = |\omega_x - \omega_y| = \epsilon\omega_0$ low-energy rotational

$\vec{v}(\vec{r}) \sim \vec{\nabla}(xy) \quad \Rightarrow \quad \omega = |\omega_x + \omega_y| = 2\omega_0$ high-energy irrotational

Superfluid regime ($T < T_0$): one mode

$\vec{v}(\vec{r}) \sim \vec{\nabla}(xy) \quad \Rightarrow \quad \omega = \sqrt{\omega_x^2 + \omega_y^2} = \sqrt{2}\omega_0$ only irrotational!

- Generation of the scissors mode in experiment:
  sudden rotation of the trap by a small angle

Scissors mode in Bose gas: experiment

O. M. Maragò et al, PRL 84 2056 (2000) Oxford, UK

^{87}Rb, magnetic trap $\omega_x = 90$ Hz

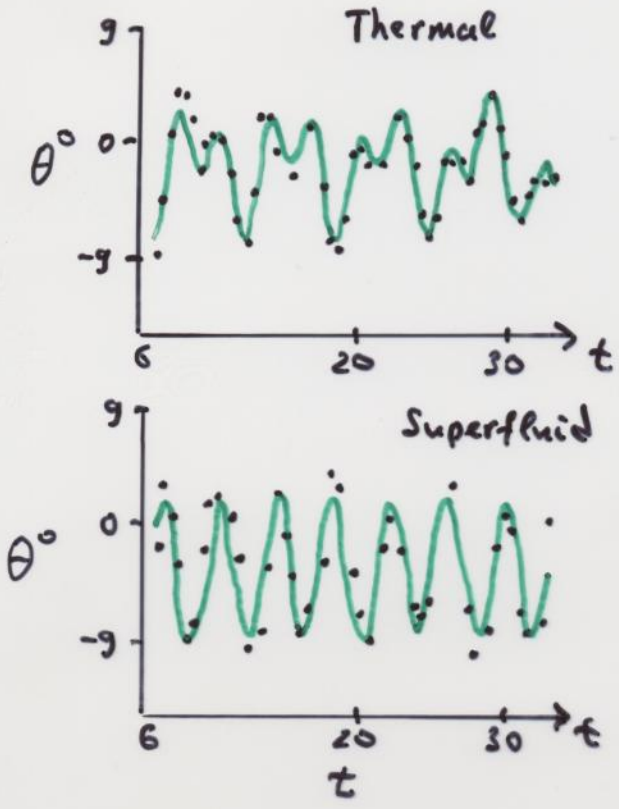
$\omega_z = \sqrt{8}\,\omega_x$

Thermal cloud

Exp.	Stringari	
159.1 ± 0.8 Hz	159 ± 2 Hz	$\omega_z - \omega_x$
338.5 ± 0.8 Hz	339 ± 3 Hz	$\omega_z + \omega_x$

Superfluid cloud

Exp.	Stringari	
265.6 ± 0.8 Hz	265 ± 2 Hz	$\sqrt{\omega_x^2 + \omega_z^2}$



HIERARCHY OF THEORETICAL APPROACHES

- Main models: TDHF, RPA, hydrodynamics, ...

Linear TDHF limit ⇒ Random Phase Approximation (RPA)

$C_g^\dagger = \frac{1}{2}\sum_{ph}(\psi_{ph}^g a_p^\dagger a_h - \phi_{ph}^g a_h^\dagger a_p)$ ⇒ creation operator of one-phonon state

$[C_{g'}, C_g^\dagger] = \delta_{g,g'} + <|a_h^+ a_p|>$ ⇒ quasiboson approxination

small!

$C_g|0>_{RPA} = 0$ ⇒ correlated RPA ground state

– number of RPA states is equal to the number of 1ph pairs

– describes both collective and 1ph states

– describes Landau damping

Landau damping is dissipation of the collective mode into neighbouring 1ph excitations

$H = H_0 + V_{res} = \sum_g \omega_g C_g^\dagger C_g \quad \Rightarrow \quad [H, C_g^\dagger] = \omega_g C_g^\dagger$

$[H, C_g] = -\omega_g C_g$

$$Q_g = \sqrt{\frac{1}{2\omega_g}}(C_g^\dagger + C_g), \quad P_g = i\sqrt{\frac{\omega_g}{2}}(C_g^\dagger - C_g), \quad [Q_g, P_{g'}] = i\delta_{g,g'}$$

$$Q_g^\dagger = Q_g, \qquad P_g^\dagger = P_g, \quad \hat{T}Q_g\hat{T}^{-1} = Q_g, \quad \hat{T}P_g\hat{T}^{-1} = -P_g$$

$$H = \frac{1}{2}\sum_g(\omega_g^2 Q_g^2 + P_g^2) \quad \Rightarrow [H, P_g] = i\omega_g^2 Q_g, \quad [H, Q_g] = -iP_g$$

– Self-consistent RPA restores conservation laws (translational [H,P]=0 and rotational [H,J]=0 invariances) violated by the mean field Hamiltonian H_0. Then the spurios RPA state connected with the motion of the whole nucleus is fully concentrated in $\omega_{spur} = 0$ root and thus does not contribute to the intrinsic RPA excitations.

SUM RULES

- SR: the collective strength is concentrated in one peak
- SR are defined as moments

$$m_p = \int \omega^p S(\omega) d\omega = \sum_g \omega_g^p | < g|F|0 > |^2$$

of the strength function

$$S(\omega) = \sum_g \delta(\omega - \omega_g) | < g|F|0 > |^2$$

where $\hat{F}$ is the excitation operator.

- SR are expressed through commutators and anticommutators:

$$m_0 = \frac{1}{2} < 0|\{F, F\}|0 > - < 0|F|0 >^2, \quad m_1 = \frac{1}{2} < 0|[F, [H, F]]|0 >$$

$$m_2 = \frac{1}{2} < 0|\{[F, H], [H, F]\}|0 >, \quad m_3 = \frac{1}{2} < 0|[[F, H], [H, [H, F]]]|0 >$$

- If the collective strength is concentrated in one peak, then one SR gets estimations for GR energy and width:

$$m_p^{E\lambda} = \sum_g \omega_g^p < g|F|0 >^2 \quad \Rightarrow \quad \omega^p < GR|F|0 >^2$$

$$\omega_k = \sqrt{\frac{m_k}{m_{k-2}}}, \quad \omega_1 \leq \bar{\omega} \leq \omega_3, \quad \sigma^2 \leq \frac{1}{4}(\omega_3^2 - \omega_1^2)$$

Practicable case $\omega_3 = \sqrt{\frac{m_3}{m_1}}$

- SR resembles oscillator with:

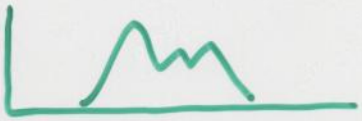
  m_3 – spring parameter (restoring force),

  $m_1 \sim N$ – inertia or mass parameter.

- Thouless: $m_1^{RPA} = \sum_g \omega_g^p | < g|F||\Psi_0 > |^2 = \frac{1}{2} < \Phi_0|[F, [H, F]]|\Phi_0 >$

  the same for m_3^{RPA} $\Leftarrow$ insensitivity to g.s. correlations!

LOCAL RPA



- LRPA is convenient if:

– gross structure of GR is important,

– collective strength is shared between several peaks.

- Time-dependent wave function of the collective state: through scaling of the ground state (Slater determinant $|\Psi_0>$):

$$|\Psi(t)>=\prod_{k=1}^{K} exp\{-iq_k(t)\hat{P}_k\}\, exp\{ip_k(t)\hat{Q}_k\}|\Psi_0>$$

$$\hat{P}_k = i[H,\hat{Q}_k]/B, \qquad B =<\Psi_0|[\hat{Q}_k,[H,\hat{Q}_k]]|\Psi_0>$$

Then the classical Hamiltonian

$$\mathfrak{H} =<\Psi(t)|H|\Psi(t)> - <\Psi_0|H|\Psi_0>$$

in small-amplitude limit (linear regime) describes a system of coupled oscillators

$$\mathfrak{H} = \frac{1}{2}\sum_{k,k'}(B_{kk'}p_k(t)p_{k'}(t) + C_{kk'}q_k(t)q_{k'}(t))$$

$B = m_1(kk') =<\Psi_0|[\hat{Q}_k,[H,\hat{Q}_{k'}]]|\Psi_0>$ $\Leftarrow$ mass parameter

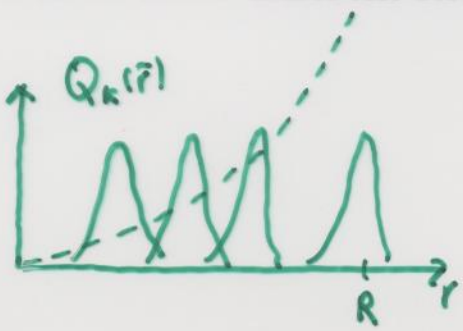
$C = m_3(kk') =<\Psi_0|[\hat{P}_k,[H,\hat{P}_{k'}]]|\Psi_0>$ $\Leftarrow$ spring constant

Mixed sum rules!

- Choice of coordinate operators for $\lambda\mu$ modes

$$Q_1(\vec{r}) = r^\lambda(Y_{\lambda\mu}+Y^\dagger_{\lambda\mu}), \quad Q_2(\vec{r}) = j_\lambda(q_2 r)(Y_{\lambda\mu}+Y^\dagger_{\lambda\mu}), \quad ...$$

so as to cover different slices (surface and interior) of the system



DENSITY FUNCTIONALS

• For density-dependent forces it is convenient to use density functionals

$$E[\rho(\vec{r},t)] = < \Psi_0 | H[\rho(\vec{r},t]| \Psi_0 >$$

where the density reads

$$\rho(\vec{r},t) = \sum_i |\phi_i(\vec{r})|^2.$$

For small time-dependent perturbations $\rho(\vec{r},t) = \rho(\vec{r}) + \delta\rho(\vec{r},t)$ we have

$$E[\rho(\vec{r},t)] = E|_{t=0} + \frac{\partial E}{\partial \rho}\,|_{t=0}\,\delta\rho + \frac{\partial^2 E}{\partial\rho_1\partial\rho_2}|_{t=0}\delta\rho_1\delta\rho_2 + \ldots$$

and thus obtain self-consistently the static mean field

$$h_0 = T + \frac{\partial E}{\partial \rho}|_{t=0}$$

and time-dependent residual interaction

$$h_{res}(t) = \frac{\partial^2 E}{\partial\rho_1\partial\rho_2}|_{t=0}\delta\rho_2(t) = \frac{\partial h_0}{\partial\rho}|_{t=0}\delta\rho(t)$$

Then $h_{TDHF} = h_0 + h_{res}(t)$ constitutes the TDHF Hamiltonian to be used in TDHF equation.

• Some typical functionals:

– Skyrme functional for nuclei,

– Kohn-Sham functional for electron systems (atomic clusters, atoms, molecules, ...)

• Hohenberg and Kohn (1964) has proved that

For interacting inhomogeneous electron system in an external potential $v(\vec{r})$ there exists a functional $F[\rho(\vec{r})]$, independent on $v(\vec{r})$, such that

$$E = \int v(\vec{r})\rho(\vec{r})d\vec{r} + F[\rho(\vec{r})]$$

is the minimal g.s. energy when $\rho(\vec{r})$ is the g.s. density.

CONCLUSIONS

• Collective oscillations in linear regime:

giant resonances (nuclei), plasmons (clusters), coll. modes (BEC)

– definition,

– origin,

– GR family (E1, ..., scissors M1)

• Theoretical tools:

– TDHF, sum rules, local RPA, full RPA, hydrodynamics;

– density functionals: Kohn-Sham (electron systems),
Skyrme (nuclei)

• GR:

common features & models

but

greate variety in different systems:

– nuclei,

– atomic clusters,

– BEC

• Activity in several areas!

– mutual exchange of ideas and methods,

– deeper understanding of physics,

– loosed, flexible mentality