SPIN-ISOSPIN TRANSITIONS AND NUCLEAR MUON CAPTURE

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Convention for isospin quantum numbers

$$t^0|\text{neutron}\rangle = \frac{1}{2}|\text{neutron}\rangle \quad \text{and} \quad t^0|\text{proton}\rangle = -\frac{1}{2}|\text{proton}\rangle$$

β-decay

$$g_V^2 B^-(F) + g_A^2 B^-(GT) = \frac{K}{ft},$$

where

$$B^\pm(F) \equiv \frac{1}{2J_i+1} \sum_{M_i,M_f} \left| \langle J_f M_f | \sum_{k=1}^{A} t^\pm(k) | J_i M_i \rangle \right|^2$$

$$B^\pm(GT) \equiv \frac{1}{2J_i+1} \sum_{M_i,M_f} \left| \langle J_f M_f | \sum_{m=-1}^{1} \sum_{k=1}^{A} \sigma_m(k)\, t^\pm(k) | J_i M_i \rangle \right|^2$$

Sum rule for Gamow-Teller transitions

$$\sum_f B_f^- - \sum_{f'} B_{f'}^+$$

$$= \sum_m \left\{ \sum_f \left| \langle f | \sum_{k=1}^{A} \sigma_m(k) t_k^- | g.s.\rangle \right|^2 - \sum_{f'} \left| \langle f' | \sum_{k=1}^{A} \sigma_m(k) t_k^+ | g.s.\rangle \right|^2 \right\}$$

$$= \sum_m \left\{ \sum_f \langle f | \sum_{k=1}^{A} \sigma_m(k) t_k^- | g.s.\rangle^* \langle f | \sum_{k=1}^{A} \sigma_m(k) t_k^- | g.s.\rangle \right.$$
$$\left. - \sum_{f'} \langle f' | \sum_{k=1}^{A} \sigma_m(k) t_k^+ | g.s.\rangle^* \langle f' | \sum_{k=1}^{A} \sigma_m(k) t_k^+ | g.s.\rangle \right\}$$

$$= \sum_m \left\{ \sum_f \langle g.s. | \sum_{k=1}^{A} \sigma_m(k) t_k^+ | f\rangle \langle f | \sum_{k=1}^{A} \sigma_m(k) t_k^- | g.s.\rangle \right.$$
$$\left. - \sum_{f'} \langle g.s. | \sum_{k=1}^{A} \sigma_m(k) t_k^- | f'\rangle \langle f' | \sum_{k=1}^{A} \sigma_m(k) t_k^+ | g.s.\rangle \right\}$$

$$= \sum_m \left\{ \langle g.s. | \sum_{k,k'=1}^{A} \sigma_m(k) t_k^+ \sigma_m(k') t_{k'}^- | g.s.\rangle - \langle g.s. | \sum_{k',k=1}^{A} \sigma_m(k') t_{k'}^- \sigma_m(k) t_k^+ | g.s.\rangle \right\}$$

$$= \sum_m \langle g.s. \left| \sum_{k,k'=1}^{A} \sigma_m(k) \sigma_m(k') \left(t_k^+ t_{k'}^- - t_{k'}^- t_k^+ \right) \right| g.s.\rangle$$

$$= \sum_m \langle g.s. \left| \sum_{k,k'=1}^{A} \sigma_m(k) \sigma_m(k') \delta_{k,k'} 2 t_k^0 \right| g.s.\rangle = 3(N-Z)$$

(p, n) reaction and GT strength

$$\frac{d\sigma}{d\omega}(0^\circ) = \frac{\mu}{\pi\hbar^2}\frac{k_f}{k_i}\left[N_\tau J_\tau^2 B^-(F) + N_{\sigma\tau} J_{\sigma\tau}^2 B^-(GT)\right]$$

$$\sigma_\alpha = \hat{\sigma}_\alpha(E_p, A) F_\alpha(q, w) B(\alpha) \qquad (\alpha = F, GT).$$

$$w = E_{\mathrm{x}} + (M_B - M_A + M_n - M_p)$$

For nuclei with large neutron excess

$$N > Z \longrightarrow S^- >> S^+ \quad \text{and} \quad S^- \approx 3(N - Z)$$

The quenching of GT transition strength:

$$S^-_{\mathrm{exp}} \approx (0.6 - 0.7) S^-_{\mathrm{th}}$$

The calculations in Random Phase Approximation (interacting particle-hole excitations) show that almost 80% of total theoretical strength are collected in one resonance state.

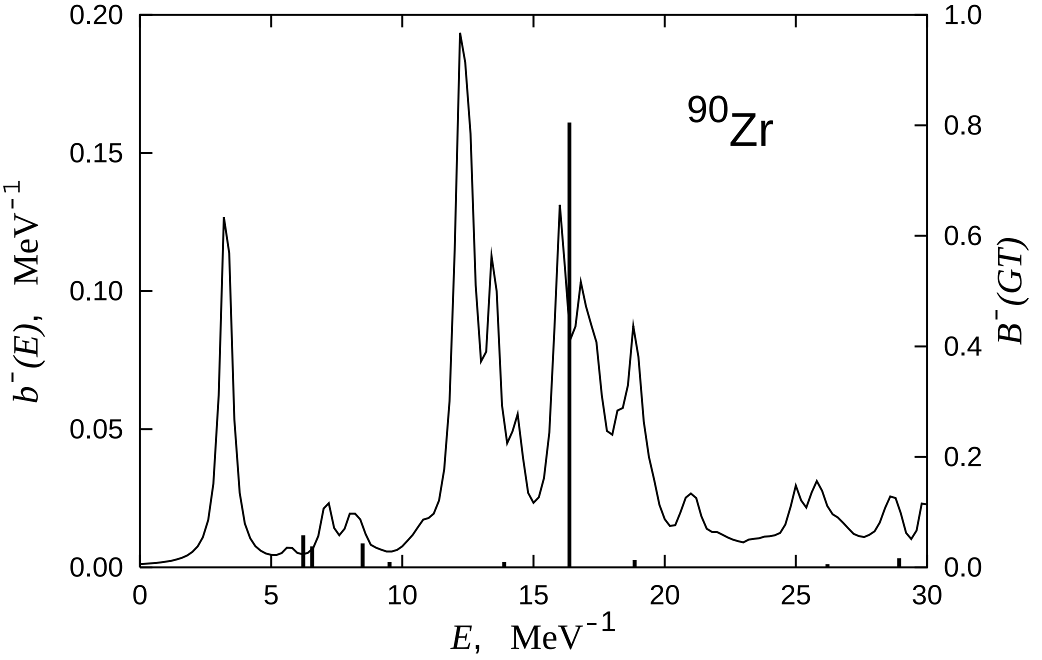


Figure 1: Distributions $\boldsymbol{\sigma t}^{-}$-strength $^{90}\mathbf{Zr}$ caluclated in RPA (bars) and with inclusion of two-phonon admixtures into wave function of excited states.

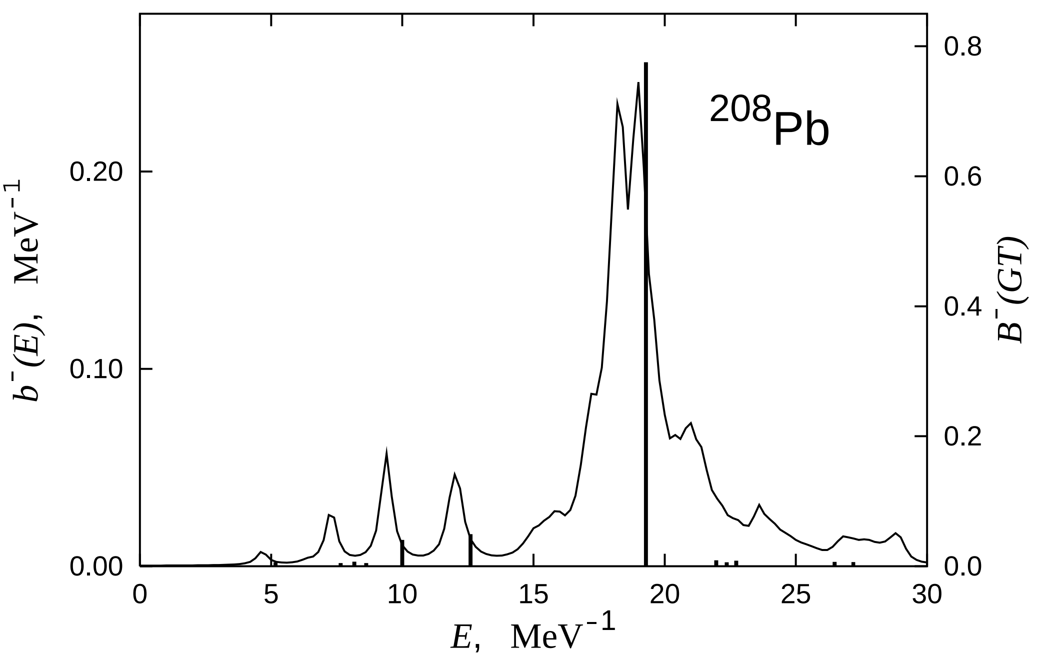


Figure 2: Distributions of $\boldsymbol{\sigma t}^{-}$-strenth in $^{208}\mathbf{Pb}$.

Figures and table from *V. A. Kuz'min and V. G. Soloviev, J. Phys. G, 10 (1984) 1507.*

Table 1: **Gamow-Teller Strength Distributions**

Nucleus	Maximum resonance region (in MeV)	Theory Approximation	Strength distribution in % of $3(N-Z)$: Below max. GT res.	Region of max. GT res.	Above res. max., but below 25 MeV	25–30 MeV	GT str. below 30 MeV, in % of $3(N-Z)$
^{90}Zr	12 − 18	RPA	15	81	1	2	99
		$\Omega^\dagger + Q^\dagger\Omega^\dagger$	22	49	18	9	98
^{120}Sn	12 − 19	RPA	26	50	2	1	89
		$\Omega^\dagger + Q^\dagger\Omega^\dagger$	33	34	12	9	88
^{124}Sn	14 − 20	RPA	17	67	2	3	89
		$\Omega^\dagger + Q^\dagger\Omega^\dagger$	29	36	15	8	88
^{124}Te	13 − 19	RPA	25	62	3	3	93
		$\Omega^\dagger + Q^\dagger\Omega^\dagger$	31	40	13	7	91
^{140}Ce	15 − 20	RPA	12	81	0	1	94
		$\Omega^\dagger + Q^\dagger\Omega^\dagger$	30	46	11	6	93
^{208}Pb	17 − 22	RPA	11	78	3	1	93
		$\Omega^\dagger + Q^\dagger\Omega^\dagger$	21	59	7	5	92

Moments of Strength Function

Strength function

$$b(E) \equiv \sum_n B_n\, \delta(E - E_n)$$

and its energy-weighted moments

$$S_k \equiv \int_0^\infty E^k\, b(E)\, dE = \sum_n E_n^k\, B_n\,, \quad \text{where } k = 0, 1, 2, \ldots$$

In the <u>fragmentation problem</u> excited state wave functions are solutions of

$$H\psi_\nu = E_\nu \psi_\nu,$$

and ground state, $|0\rangle$, is fixed in some way.

$$H = H_v + H_{vq},$$

H_v describes simple modes (single-particle, 1p–1h, one-phonon, ...) modes, and H_{vq} contains the interaction between simple and and more complicated ("particle plus phonon", 2p–2h, two-phonon, ...), "complex" modes.

$$\psi_\nu = \sum_m c_m^\nu \phi_m + \sum_n d_n^\nu \rho_n,$$

where

$$H_v \phi_m = \omega_m \phi_m$$

with norm condition $\langle \phi_m | \phi_{m'} \rangle = \delta_{m,m'}$, and ρ_n are "complex" states, so

$$\langle \phi_m | \rho_n \rangle = 0 \quad \text{and} \quad \langle \rho_n | \rho_{n'} \rangle = \delta_{n,n'}.$$

Projection operators P and Q:

$$P\psi_\nu = \sum_m c_m^\nu \phi_m\,, \qquad Q\psi_\nu = \sum_n d_n^\nu \rho_n.$$

$$PQ\psi = QP\psi = 0, \qquad (P + Q)\psi = \psi.$$

Using them

$$H\psi_\nu = E_\nu \psi_\nu \Longrightarrow \begin{array}{ccccc} PH_vP\psi_\nu & + & PH_{vq}Q\psi_\nu & = & E_\nu P\psi_\nu, \\ QH_{vq}P\psi_\nu & + & QHQ\psi_\nu & = & E_\nu Q\psi_\nu \end{array}.$$

In maxrix notation

$$\hat{H} \begin{Vmatrix} P\psi_\nu \\ Q\psi_\nu \end{Vmatrix} \equiv \begin{Vmatrix} PH_vP & PH_{vq}Q \\ QH_{vq}P & QHQ \end{Vmatrix} \cdot \begin{Vmatrix} P\psi_\nu \\ Q\psi_\nu \end{Vmatrix} = E_\nu \begin{Vmatrix} P\psi_\nu \\ Q\psi_\nu \end{Vmatrix},$$

as a system of linear equations

$$\begin{array}{rcrcl} \omega_m c_m^\nu & + & \sum_p \langle \phi_m | H_{vq} | \rho_p \rangle d_p^\nu & = & E_\nu c_m^\nu, \\ \sum_r \langle \rho_n | H_{vq} | \phi_r \rangle c_r^\nu & + & \sum_p \langle \rho_n | H_{vq} | \rho_p \rangle d_p^\nu & = & E_\nu d_n^\nu. \end{array}$$

For transition operator B

$$\langle \psi_\nu | B | 0 \rangle = \sum_m \langle \phi_m | B | 0 \rangle \bar{c}_m^\nu + \sum_n \langle \rho_n | B | 0 \rangle \bar{d}_n^\nu,$$

$\bar{d}_n^\nu$ is the complex-conjugated d_n^ν. In fragmentation problem there is no transitions from ground state to the "complex" ones:

$$\langle \rho_n | B | 0 \rangle = 0$$

$$\langle \psi_\nu | B | 0 \rangle = \sum_m \langle \phi_m | B | 0 \rangle \bar{c}_m^\nu = \sum_m b_m \bar{c}_m^\nu, \quad \text{where} \quad b_m \equiv \langle \phi_m | B | 0 \rangle.$$

Energy-weighted moments

$$S_k = \sum_\nu E_\nu^k \sum_{m,m'} \bar{b}_m c_m^\nu \, \bar{c}_{m'}^\nu b_{m'} = \sum_{m,m'} \bar{b}_m b_{m'} \sum_\nu E_\nu^k c_m^\nu \bar{c}_{m'}^\nu.$$

The second sum does not depend on transition operator.

For each ν numbers c_m^ν and d_n^ν form a vector $\eta^\nu \equiv \left(\ldots, c_m^\nu, \ldots, d_n^\nu, \ldots \right)$, which is the eigen vector of hermitian matrix $\hat{H}$, corresponding to the eigenvalue E_ν.

$$\bar{\eta}^\mu \eta^\nu = \sum \bar{\eta}_m^\mu \eta_m^\nu = \delta_{\mu,\nu}.$$

The spectral projection matrix

$$(R_\nu)_{k,k'} \equiv \eta_k \bar{\eta}_{k'}, \qquad R_\nu^\dagger = R_\nu \quad \text{and} \quad R_\mu R_\nu = \delta_{\mu,\nu} R_\nu$$

$$\sum_\nu R_\nu = \hat{1}, \qquad \sum_\nu E_\nu \, R_\nu = \hat{H} \quad \rightarrow \quad \hat{H}^k = \sum_\nu E_\nu^k \, R_\nu$$

$$\sum_\nu E_\nu^k c_m^\nu \bar{c}_{m'}^\nu = \left(\hat{H}^k \right)_{m,m'}$$

$$S_k = \sum_{m,m'} \left(\hat{H}^k \right)_{m,m'} \bar{b}_m b_{m'} = \langle 0 | B^\dagger P H^k P B | 0 \rangle.$$

For $k = 0, 1, 2$

$$\begin{aligned} S_0 &= \langle 0|B^\dagger P B|0\rangle = \sum_m |b_m|^2 \,, \\ S_1 &= \langle 0|B^\dagger P H_v P B|0\rangle = \sum_m \omega_m |b_m|^2 \,, \\ S_2 &= \langle 0|B^\dagger P (H_v^2 + H_{vq} Q Q H_{vq}) P B|0\rangle \\ &= \sum_{m,m'} \bar{b}_m \Big(\delta_{m,m'} \omega_m^2 + \sum_n \langle \phi_m | H_{vq} | \rho_n \rangle \langle \rho_n | H_{vq} | \phi_{m'} \rangle \Big) b_{m'} \end{aligned}$$

If $Q\psi = (Q_1 + Q_2)\psi$ and $Q_2 H_{vq} P\psi = P H_{vq} Q_2 \psi = 0$

$$\hat{H} = \begin{Vmatrix} P H_v P & P H_{vq} Q_1 & 0 \\ Q_1 H_{vq} P & Q_1 H Q_1 & Q_1 H Q_2 \\ 0 & Q_2 H Q_1 & Q_2 H Q_2 \end{Vmatrix} .$$

By matrix block multiplication

$$\begin{aligned} S_2 = &\quad \langle 0|B^\dagger P \Big(H_{vq}^2 + H_{vq} Q_1 H_{vq} \Big) P B|0\rangle \,, \\ S_3 = &\quad \langle 0|B^\dagger P \Big(H_{vq}^3 + H_v P H_{vq} Q_1 H_{vq} + H_{vq} Q_1 H_{vq} P H_v \\ &\quad + H_{vq} Q_1 H Q_1 H_{vq} \Big) P B|0\rangle \,. \end{aligned}$$

Random Phase Approximation

- **Starting point: the leve;s in the single-particle space are marked by i, j if they are occupied and by p, q if they are unoccupied. Φ_0 shall be a Slater determinant constructed out of occupied states only.**
- **Phonon operators**

$$O_\alpha^\dagger = \sum_{j,p} \left(\psi_{jp}^\alpha a_j^\dagger a_p - \phi_{jp}^\alpha a_p^\dagger a_j \right)$$
$$O_\alpha = \sum_{j,p} \left(\psi_{jp}^{\alpha\,*} a_p^\dagger a_j - \phi_{jp}^{\alpha\,*} a_j^\dagger a_p \right)$$

- **Phonon amplitudes $\psi_{j,p}^\alpha$, $\phi_{j,p}^\alpha$ are determined by**

$$\langle \Phi_0 | [[O_\alpha, H] a_j^\dagger a_p] | \Phi_0 \rangle = E_\alpha \langle \Phi_0 | [O_\alpha, a_j^\dagger a_p] | \Phi_0 \rangle$$
$$\langle \Phi_0 | [[O_\alpha, H] a_p^\dagger a_j] | \Phi_0 \rangle = E_\alpha \langle \Phi_0 | [O_\alpha, a_p^\dagger a_j] | \Phi_0 \rangle$$

After complex conjugate of these equations

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} \psi^\alpha \\ \phi^\alpha \end{pmatrix} = E_\alpha \begin{pmatrix} \psi^\alpha \\ -\phi^\alpha \end{pmatrix},$$

where A and B are square matrices with elements

$$\begin{aligned} A_{jp,j'p'} &= \langle \Phi_0 | a_p^\dagger a_j H a_{j'}^\dagger a_{p'} | \Phi_0 \rangle - \langle \Phi_0 | H | \Phi_0 \rangle \delta_{j,j'} \delta_{p,p'} \\ B_{jp,j'p'} &= \langle \Phi_0 | a_p^\dagger a_j a_{p'}^\dagger a_{j'} H | \Phi_0 \rangle. \end{aligned}$$

If the Hamiltonian H of the system is hermitian then the matrix A is also hermitian and B is a symmetric matrix. The properties of the eigenvalue problem allow the following normalization of the non-trivial solutions

$$\langle \Phi_0 | [O_\alpha, O_\beta^\dagger] | \Phi_0 \rangle = \mathrm{sign}(E_\alpha)\, \delta_{\alpha,\beta}. \tag{1}$$

Those positive E_α's, corresponding to non-trivial solutions are the approximate excitation energies of the system.

- **The matrix elements of any one-body transition operator R**

$$R_\alpha \equiv \langle \alpha | R | 0 \rangle_{\mathrm{RPA}} \equiv \langle \Phi_0 | [O_\alpha, R] | \Phi_0 \rangle.$$

This equation defines also the normalization of the (positive energy, E_α) one-phonon states

$$\langle \alpha | O_\beta^\dagger | 0 \rangle_{\mathrm{RPA}} = \langle \Phi_0 | [O_\alpha, O_\beta^\dagger] | \Phi_0 \rangle = \delta_{\alpha,\beta}.$$

Sum rules for RPA

From the spectral decomposition

$$\sum_{\alpha} \mathrm{sign}(E_\alpha)\, E_\alpha^k \begin{pmatrix} \psi^\alpha \\ \phi^\alpha \end{pmatrix} \begin{pmatrix} \psi^{\alpha\dagger} & \phi^{\alpha\dagger} \end{pmatrix} = \begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix}^k \begin{pmatrix} \hat{1} & \hat{0} \\ \hat{0} & -\hat{1} \end{pmatrix}$$

for any positive k follows

$$\sum_{\alpha:\, E_\alpha > 0} E_\alpha^k \left(|R_\alpha|^2 - (-1)^k |R_\alpha^\dagger|^2 \right) = \begin{pmatrix} R^\dagger & \widetilde{R^+} \end{pmatrix} \begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix}^k \begin{pmatrix} R \\ -R^{+*} \end{pmatrix},$$

This expression is valid for any positive integer k. $\hat{1}$ and $\hat{0}$ are unit and zero square matrices. The vectors (R) and (R^+) are constructed from the matrix elements $\langle \Phi_0 | a_p^\dagger a_j R | \Phi_0 \rangle$ and $\langle \Phi_0 | a_p^\dagger a_j R^\dagger | \Phi_0 \rangle$.

Second RPA

Phonon operators

$$\begin{aligned} \mathcal{O}_\alpha^\dagger = & \sum_{j,p} \left(\psi_{jp}^\alpha a_j^\dagger a_p - \phi_{jp}^\alpha a_p^\dagger a_j \right) \\ & + \sum_{j<j',p<p'} \left(\psi_{jj',pp'}^\alpha a_j^\dagger a_{j'}^\dagger a_{p'} a_p - \phi_{jj',pp'}^\alpha a_p^\dagger a_{p'}^\dagger a_{j'} a_j \right) \end{aligned}$$

"Equations of motion"

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{B}^* & \mathcal{A}^* \end{pmatrix} \begin{pmatrix} \chi^\alpha \\ \rho^\alpha \end{pmatrix} = E_\alpha \begin{pmatrix} \chi^\alpha \\ -\rho^\alpha \end{pmatrix}$$

where

$$\mathcal{A} = \begin{pmatrix} A_{jp,j'p'} & A_{jp,j_1'j_2'p_1'p_2'} \\ A_{j_1j_2p_1p_2,j'p'} & A_{j_1j_2p_1p_2,j_1'j_2'p_1'p_2'} \end{pmatrix},$$

$$\mathcal{B} = \begin{pmatrix} B_{jp,j'p'} & B_{jp,j_1'j_2'p_1'p_2'} \\ B_{j_1j_2p_1p_2,j'p'} & B_{j_1j_2p_1p_2,j_1'j_2'p_1'p_2'} \end{pmatrix}$$

and

$$\chi^\alpha = \begin{pmatrix} \psi_{jp}^\alpha \\ \psi_{j_1j_2p_1p_2}^\alpha \end{pmatrix}, \qquad \rho^\alpha = \begin{pmatrix} \phi_{jp}^\alpha \\ \phi_{j_1j_2p_1p_2}^\alpha \end{pmatrix}.$$

The general structure, however, remains the same as in RPA. This is also true regarding the formal definition of the matrix elements of the transition operator R. The only non-vanishing matrix elements of any one-body transition operator are those between Φ_0 and the 1p–1h excited states.

$$\sum_{\alpha:\, \mathcal{E}_\alpha > 0} \mathcal{E}_\alpha^k \left(|\mathcal{R}_\alpha|^2 - (-1)^k |\mathcal{R}_\alpha^\dagger|^2 \right) = \begin{pmatrix} \mathcal{R}^\dagger & \widetilde{\mathcal{R}^+} \end{pmatrix} \begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B}^* & -\mathcal{A}^* \end{pmatrix}^k \begin{pmatrix} \mathcal{R} \\ -\mathcal{R}^{+*} \end{pmatrix}.$$

As a consequence the zeroth energy-weighted moment (which is simply equal to the difference of the lengths of the vectors (R) and (R^+))

$$
\begin{aligned}
S_0 - S_0^\dagger &= \sum_{\alpha:\,\mathcal{E}_\alpha>0} \left(|\mathcal{R}_\alpha|^2 - |\mathcal{R}_\alpha^\dagger|^2\right) = \sum_{j,p} \left(|\langle\Phi_0|a_p^\dagger a_j R|\Phi_0\rangle|^2 - |\langle\Phi_0|a_p^\dagger a_j R^\dagger|\Phi_0\rangle|^2\right) \\
&= \sum_{\beta:\,E_\beta>0} \left(|R_\beta|^2 - |R_\beta^\dagger|^2\right),
\end{aligned}
$$

GT transitions $(R_\mu = \sigma_\mu t^-)$: $\quad S_0^- - S_0^+ \Rightarrow 3(N-Z)$.

$$
\begin{aligned}
S_1 + S_1^\dagger &= \sum_{\alpha:\,\mathcal{E}_\alpha>0} \mathcal{E}_\alpha \left(|\mathcal{R}_\alpha|^2 + |\mathcal{R}_\alpha^\dagger|^2\right) = (\,\mathcal{R}^\dagger \quad \widetilde{\mathcal{R}^+}\,) \begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B}^* & -\mathcal{A}^* \end{pmatrix} \begin{pmatrix} \mathcal{R} \\ -\mathcal{R}^{+*} \end{pmatrix} \\
&= (\,R^\dagger \quad \widetilde{R^+}\,) \begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} R \\ -R^{+*} \end{pmatrix} = \sum_{\beta:\,E_\beta>0} E_\beta \left(|R_\beta|^2 + |R_\beta^\dagger|^2\right) .
\end{aligned}
$$

$$
S_1^- + S_1^+\Big|_{\mathrm{RPA}} = S_1^- + S_1^+\Big|_{\mathrm{SRPA}}
$$

If $N >> Z$ then $S_0^-(GT) >> S_0^+(GT)$

$$
S_0^- - S_0^+\Big|_{\mathrm{RPA}} = S_0^- - S_0^+\Big|_{\mathrm{SRPA}} \quad\Longrightarrow\quad S_0^-\Big|_{\mathrm{RPA}} \approx S_0^-\Big|_{\mathrm{SRPA}}
$$

$$
S_1^- + S_0^+\Big|_{\mathrm{RPA}} = S_1^- + S_0^+\Big|_{\mathrm{SRPA}} \quad\Longrightarrow\quad S_1^-\Big|_{\mathrm{RPA}} \approx S_1^-\Big|_{\mathrm{SRPA}}
$$

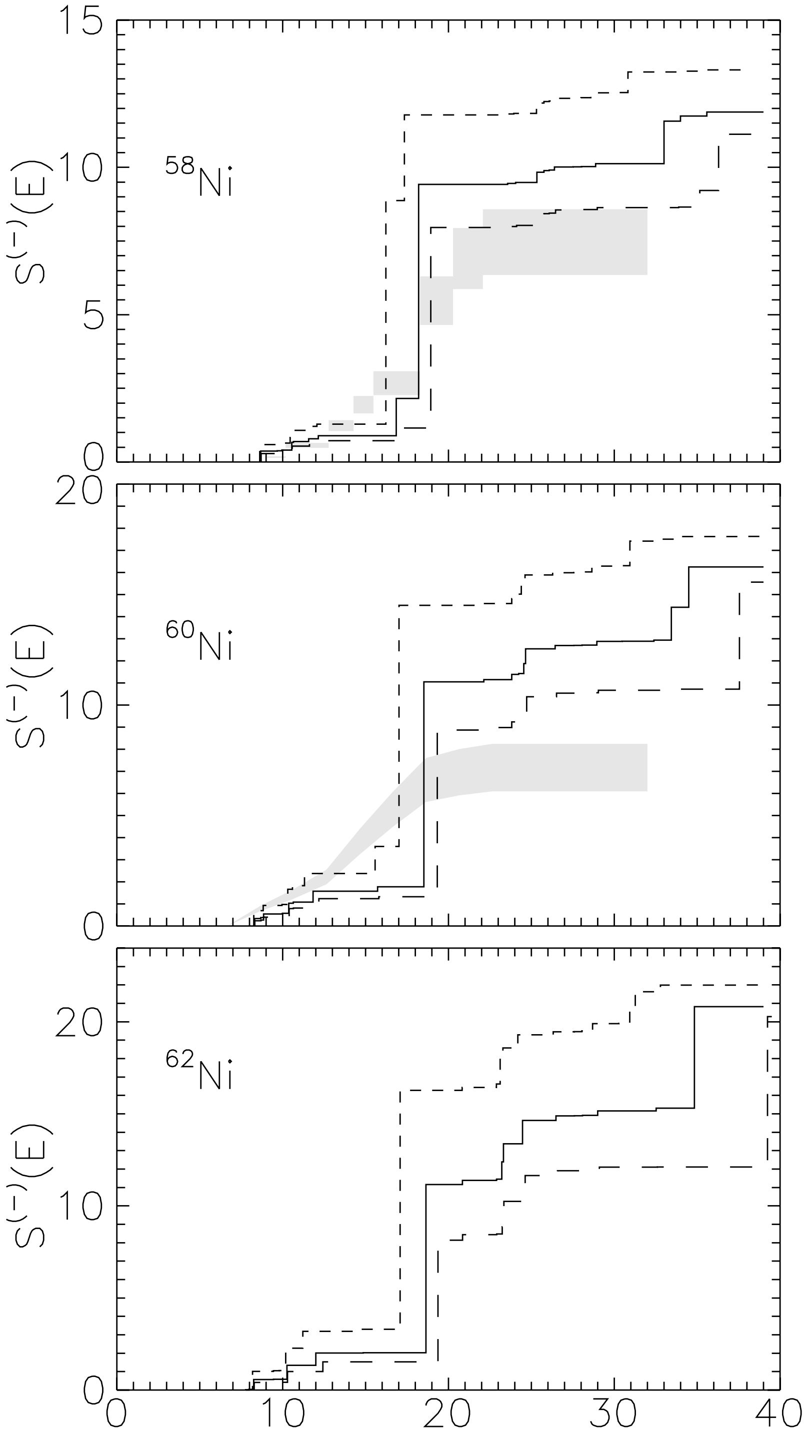
The conservation of zeroth and first moments has important consequence. If on behalf of an interaction between 1p–1h states and more complex states a large fraction of the strength of the giant resonance could be shifted to higher energies, then some strength has necessarily to be shifted into the low energy region in order to conserve the first moment. As a result the strength distribution in the giant resonance region and below it would change completely.

Therefore, the quenching of GT strength (or shifting of the part of GT strength to the highest excitaion energy) should be explained by the interaction between 1p–1h states only.

The high excited 1^+ states can be formed around particle-hole states having different number of nodes in the radial part of their single-particle wave functions, i.e. belonging to different major shells.

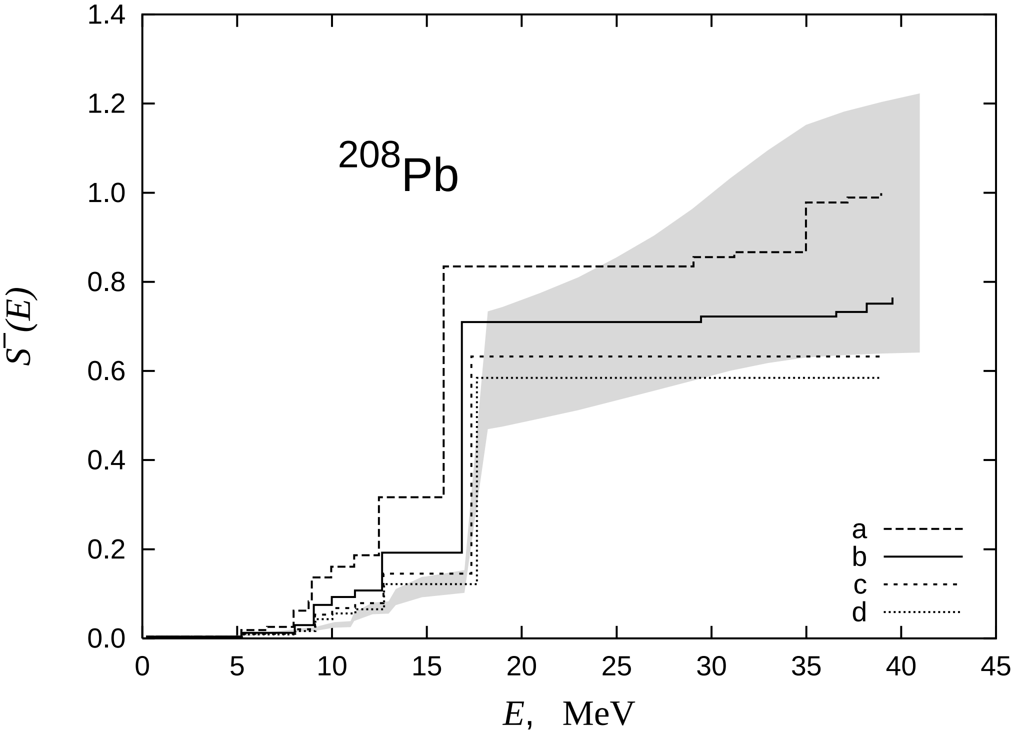
Therefore, the residual interaction should mix $\Delta N = 0$ and $\Delta N \geq 2$ configurations.

Detail explanations in *K. Junker, V. A. Kuz'min, and T. V. Tetereva, Eur. J. Phys. A* 5 *(1999) 37*



The σt^- transitions in three isotopes of Ni.

R. A. Eramzhyan, V. A. Kuz'min, and T. V. Tetereva, Nucl. Phys. A* 642 *(1998) 428.

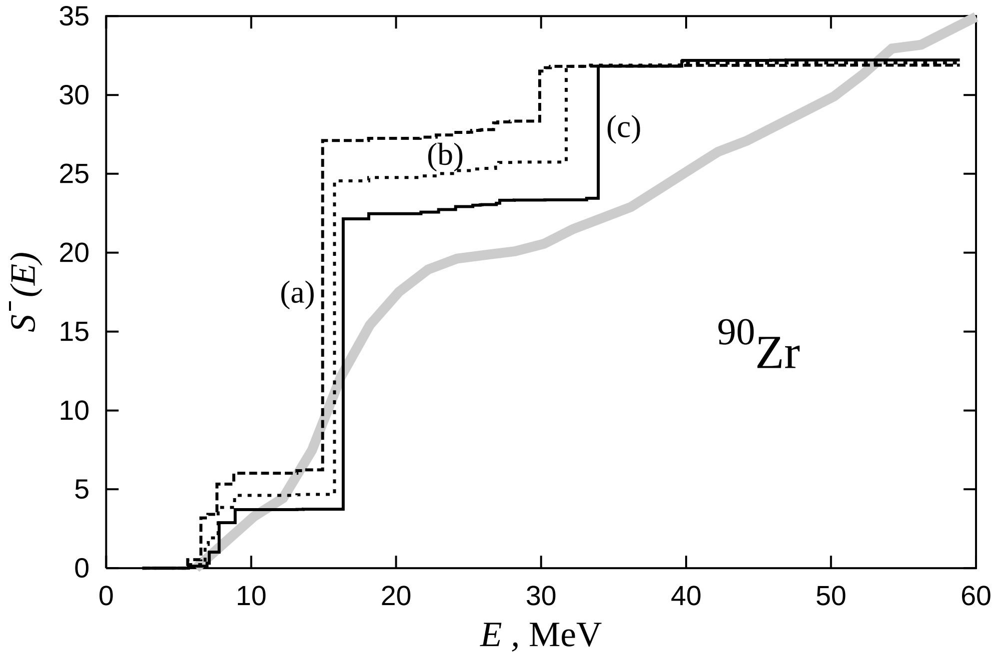


Distribution of σt^- strength in ^{208}Pb calculated in RPA with residual interaction which mixs $\Delta N = 0$ and $\Delta N \geq 0$ particle-hole excitations.

$$S^-(E) = \frac{1}{3(N-Z)} \sum_{k:E_k \leq E} B_k^-(GT)$$

	a	b	c	d
$\kappa_1^{01} =$	$-0.23/A$	$-0.43/A$	$-0.63/A$	$-0.83/A$

V. A. Kuz'min, Phys. At. Nucl. 58 *(1995) 368.*



Distribution of σt^- strength in ^{90}Zr. Thick gray line — experimental distribution GT strength from T. Wakasa *et al.*, **Phys. Rev. C 56 (1997) 2909, where $(93 \pm 5)\%$ of $3(N - Z) = 30$ were observed.**

	a	b	c
$\kappa_1^{01} =$	$-0.23/A$	$-0.43/A$	$-0.63/A$

V. A. Kuz'min, and T. V. Tetereva, Bulletin of the Russian Academy of Sciences, Physics* 67 *(2004) 708.

The quenching of GT strength requires specific feature of nuclear residual interaction:
mixing of $\Delta N = 0$ and $\Delta N \geq 2$ particle-hole configurations.

The Rates of Ordinary Muon Capture by Nuclei

$$\mu^- + \mathrm{A}(N,Z) \to \nu_\mu + \mathrm{B}(N+1, Z-1)$$

Main sources:

M. Morita and A. Fujii, Phys. Rev., 118 (1960) 606

V.V. Balashov, R.A. Eramzhyan, Rev. At. En. (Vienne), 5 (1967) 3

- V–A theory of weak interaction. Hadron weak axial current:

$$\langle \mathrm{N}(p')|A_a^\alpha(0)|\mathrm{N}(p)\rangle =$$
$$= \bar{u}(p')\,[g_A(q^2)\gamma^\alpha + \frac{g_P(q^2)}{m_\mu}q^\alpha + i\frac{g_T(q^2)}{2M}\sigma^{\alpha\beta}q_\beta]\,\gamma_5\frac{\tau_a}{2}\,u(p)\,.$$

- Non-relativistic reduction for muon and nucleon spinors.
- Impulse approximation.

For $0^+ \to 1^+$ transition when the σt^+ matrix element dominates

$$\Lambda \approx \quad \frac{2}{3}\,V\,g_A^2\,[101]^2\Big\{1 + \frac{2}{3}\eta + \frac{8}{3}\frac{g_V+g_M}{g_A}\eta - \frac{2}{3}\frac{g_P}{g_A}\eta + \frac{1}{3}(\frac{g_P}{g_A}\eta)^2$$
$$+\frac{[121]}{[101]}\sqrt{\frac{8}{9}}\Big[2(1+\frac{g_V+g_M}{g_A} - \frac{g_P}{g_A})\eta + (\frac{g_P}{g_A}\eta)^2\Big]$$
$$+\frac{[011p]}{[101]M}2(1-\frac{g_P}{g_A})\eta - \frac{[111p]}{[101]M}\sqrt{\frac{8}{9}}\frac{g_V}{g_A}\Big\},\quad \eta = \frac{E_\nu}{2\,M_p}\,.$$

Here nuclear matrix elements are

$$[101] \quad = \sqrt{\frac{3}{4\pi}}\langle J_f \parallel \sum_{k=1}^{A}\varphi_\mu(r_k)\,j_0(E_\nu r_k)\,\sigma_k\,t_k^+ \parallel J_i\rangle\frac{1}{\sqrt{2J_f+1}}\,,$$
$$[121] \quad = \sqrt{\frac{3}{4\pi}}\langle J_f \parallel \sum_{k=1}^{A}\varphi_\mu(r_k)\,j_2(E_\nu r_k)\,[Y_2(\hat{r}_k)\,\sigma_k]_1\,t_k^+ \parallel J_i\rangle\frac{1}{\sqrt{2J_f+1}}\,,$$
$$[111p] \quad = \sqrt{\frac{3}{4\pi}}\langle J_f \parallel \sum_{k=1}^{A}\varphi_\mu(r_k)\,j_1(E_\nu r_k)\,[Y_1(\hat{r}_k)\,\nabla_k]_1\,t_k^+ \parallel J_i\rangle\frac{1}{\sqrt{2J_f+1}}\,,$$
$$[011p] \quad = \sqrt{\frac{1}{4\pi}}\langle J_f \parallel \sum_{k=1}^{A}\varphi_\mu(r_k)\,j_1(E_\nu r_k)\,Y_1(\hat{r}_k)\,(\nabla_k,\sigma_k)\,t_k^+ \parallel J_i\rangle\frac{1}{\sqrt{2J_f+1}}\,.$$

Muon wave function $\varphi_\mu(r) \approx$ const inside the light nuclei.

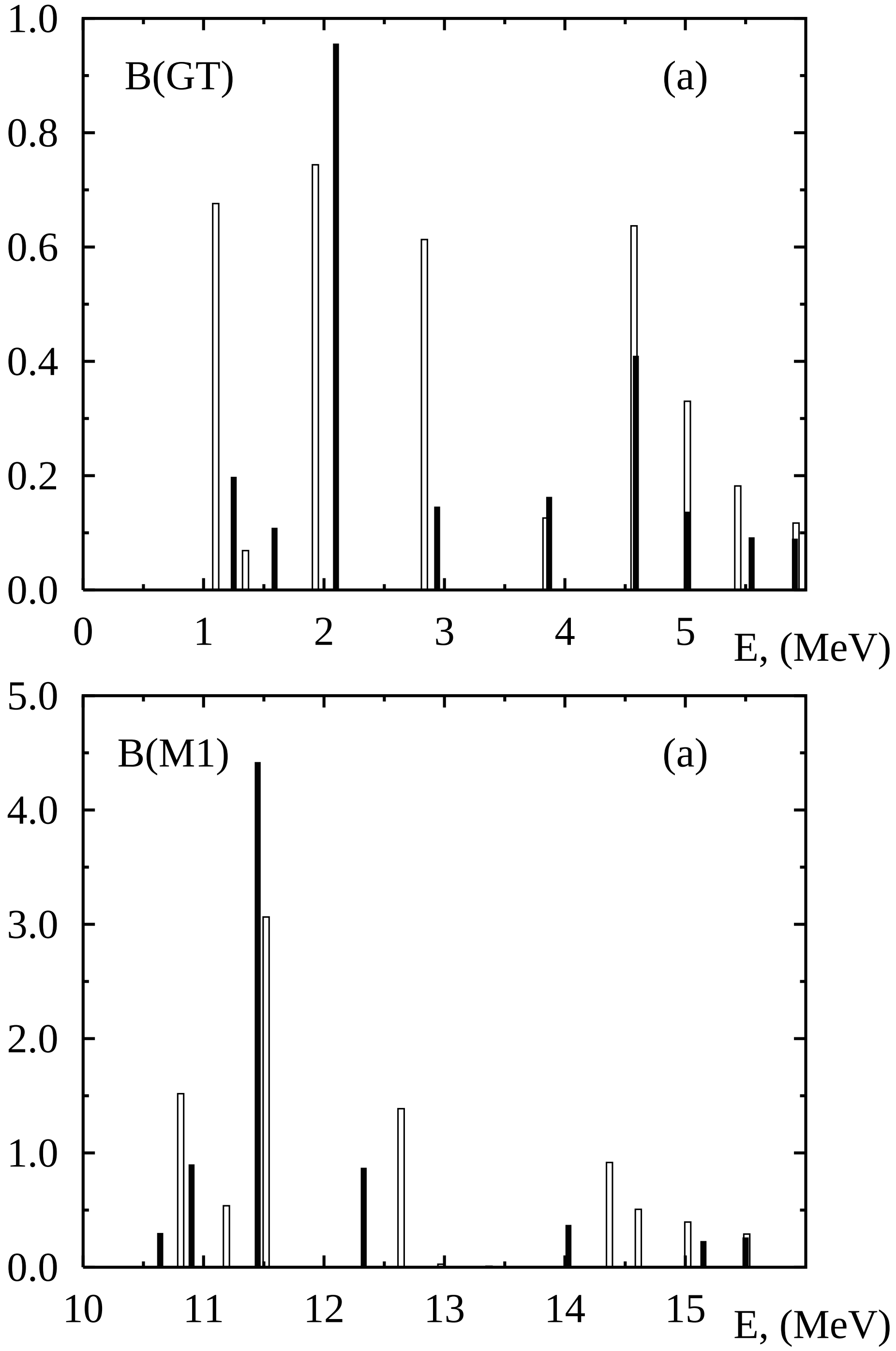


Figure 3: Excitation energies and strength of isovector $\mathbf{0^+ \rightarrow 1^+}$ transitions in $\boldsymbol{A = 28}$ nuclei, calculated with wave functions of Wildenthal Hamiltonian (open bars). The corresponding experimental data are presented by the dark bars. The references to experiment are given on the next slide.

Calculation results										
k	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$E_k - E_1$	0.0	0.38	0.71	1.83	2.16	2.56	3.56	3.80	4.21	4.70
E_k	10.81	11.19	11.52	12.64	12.97	13.37	14.37	14.61	15.02	15.51
$b^-(GT)$	0.822	0.262	0.862	−0.783	0.014	−0.355	0.798	−0.574	−0.426	0.342
$b(M1)$	1.232	0.733	1.750	−1.178	0.162	−0.091	0.957	−0.712	0.629	0.538
$B^-(GT)$	0.676	0.069	0.744	0.613	0.000	0.126	0.637	0.330	0.182	0.117
$B(M1)$	1.518	0.538	3.064	1.387	0.026	0.008	0.917	0.507	0.395	0.290

The experimental GT strength function obtained from ^{28}Si$(p,n)^{28}$P. Cited from †

$E_k - E_1$	0.0	0.34	0.85	1.69		2.62	3.34	3.77	4.30	4.64
E_k	1.25	1.59	2.10	2.94		3.87	4.59	5.02	5.55	5.91
$B^-(GT)$	0.198	0.109	0.956	0.146		0.163	0.410	0.137	0.092	0.090
err.	0.002	0.002	0.005	0.003		0.002	0.004	0.041	0.004	0.003

The experimental $M1$ strength function obtained from ^{28}Si$(e,e')^{28}$Si. Cited from ‡.

$E_k - E_1$	0.0	0.26	0.80	1.69			3.39		4.50	4.86
E_k	10.64	10.90	11.45	12.33			14.03		15.15	15.50
$B(M1)$	0.30	0.90	4.42	0.87			0.37		0.23	0.26
err.	0.04	0.02	0.20	0.06			0.02		0.02	0.03

† B.D. Anderson, *et al.* Phys. Rev. C 43(1991)50; corrected $B(GT)$ for ^{28}Si are in P. von Neumann-Cosel, *at al.* Phys. Rev. C 55(1997)532

‡ C. Lüttge, *at al.* Phys. Rev. C, 53(1996)127; Y. Fujita, *at al.* Phys. Rev. C, 55(1996)1137

The many-particle shell model with Wildenthal Hamiltonian *(Progr. Part. Nucl. Phys.* 11 *(1984) 11)* describes qualitatively the main features of GT- and $M1$-strength functions in the sense that small theoretical $B(\mathrm{GT})$'s and $B(M1)$'s correspond to small experimental values. However, the theoretical distributions of the transition strength over the states which absorb the largest part of the total strenght differ considerably from the experimental strength function.

In order to use the available experimental information about GT- and $M1$-strength funtions in the calculations of muon capture rates the orthogonal transformation in the space spanned by the wave functions of isovector 1^+- states is carried out. The parameters of transformation should be chosen such that the GT- and $M1$-strength functions calculated with transformed wave functions coincide in shape (within a constant factor) with the experimental strength functions.

The transformation of wave functions of excited states

$$\phi_k \longrightarrow \psi_k = U_{k,k'}\phi_{k'} \qquad (k = 1, 2, \ldots, N)$$

causes the transformation of transition matrix elements

$$\langle\phi_k|\mathcal{O}|\Phi\rangle \longrightarrow \langle\psi_k|\mathcal{O}|\Phi\rangle = U^*_{k,k'}\langle\phi_{k'}|\mathcal{O}|\Phi\rangle = \langle\phi_{k'}|\mathcal{O}|\Phi\rangle U^\dagger_{k',k}.$$

An orthogonal $N \times N$ matrix could be fixed by $N(N-1)/2$ free parameters. The simplest orthogonal transformation of a vector is the reflection in the plane:

$$v = v_\parallel + v_\perp \longrightarrow v' = v_\parallel - v_\perp \ .$$

For the plane given by equation

$$(b, x) = b_k x_k = 0 \quad \text{where} \quad b \ \text{is a constant vector} \ (b, b) \neq 0$$

the transformation is

$$v_k \longrightarrow v'_k = v_k - 2\,\frac{b_k b_l}{(b, b)}\,v_l \ .$$

If $|u| = |w|$ one can convert $u \rightleftharpoons w$ by using $b = u - w$.

V. A. Kuz'min and T. V. Tetereva, Phys. At. Nucl. 63 *(2000) 1874.*

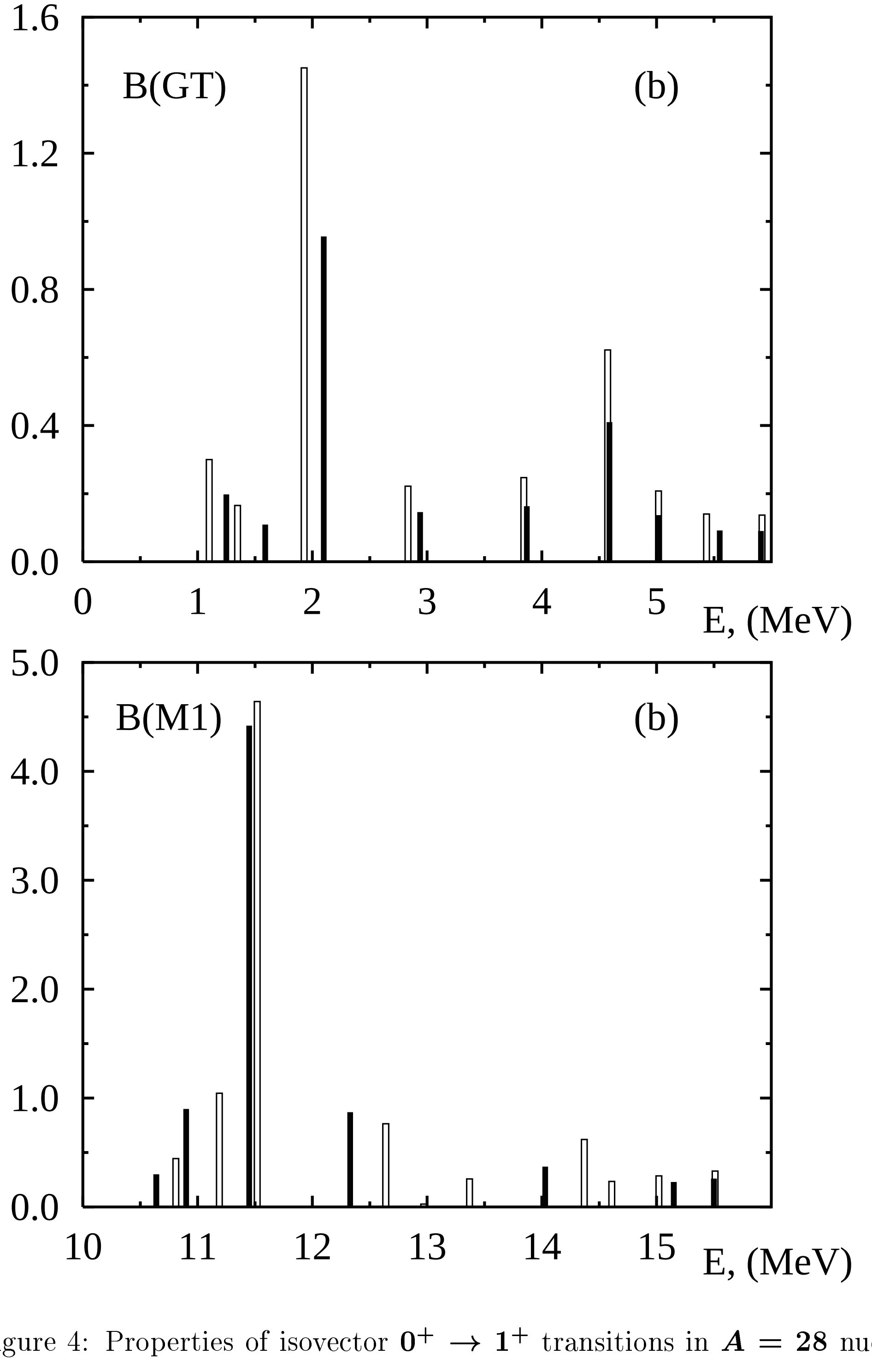


Figure 4: Properties of isovector $\mathbf{0^+} \rightarrow \mathbf{1^+}$ transitions in $\boldsymbol{A} = \mathbf{28}$ nuclei, calculated with transformed wave functions.

Transformation matrix.

	(1)	(2)	(3)	(4)	(6)	(7)	(8)	(9)	(10)
(1)	0.952	0.024	-0.293	-0.013	-0.080	-0.032	-0.003	0.006	0.001
(2)	0.023	0.988	0.146	0.004	0.030	0.014	-0.005	-0.003	-0.001
(3)	-0.293	0.146	-0.885	0.021	-0.153	-0.149	0.248	0.042	0.011
(4)	-0.013	0.004	0.021	0.903	-0.340	-0.050	-0.258	-0.002	-0.005
(6)	-0.080	0.030	-0.153	-0.340	-0.216	-0.194	-0.877	-0.004	-0.018
(7)	-0.032	0.014	-0.150	-0.050	-0.194	0.961	-0.116	0.002	-0.002
(8)	-0.003	-0.005	0.248	-0.258	-0.877	-0.116	0.299	-0.011	-0.015
(9)	0.006	-0.003	0.043	-0.002	-0.004	0.002	-0.011	0.999	-0.000
(10)	0.001	-0.001	0.011	-0.005	-0.018	-0.002	-0.015	-0.000	1.000

GT and $M1$ strength functions in ^{28}Si calculated with transformed wave functions of excited states.

k :	(1)	(2)	(3)	(4)	(6)	(7)	(8)	(9)	(10)
$B^{-}(GT)$	0.300	0.165	1.451	0.222	0.247	0.622	0.208	0.140	0.137
$B(M1)$	0.445	1.044	4.641	0.764	0.258	0.620	0.234	0.286	0.330

Life-times of 1^+ states in ^{28}Al ($10^{-15}\,s$.)

k	E_k	(a)	(b)	E'_k	(a′)	(b′)	(c)	expt.
1	1.373	239	152	10.810	184	117	235	320 ± 50
2	1.620	465	531	11.192	285	279	590	120 ± 60
3	2.201	66	44	11.519	70	48	65	65 ± 35
4	3.105	21	12	12.643	17	10	22	
5	3.542	9.7	9.8	12.970	9.1	9.2	7.9	
6	4.115	0.94	7.0	13.771	1.1	8.6	0.9	
7	4.846	0.69	0.88	14.374	0.64	0.80	0.7	
8	5.017	1.2	0.48	14.605	1.0	0.41		
9	5.435	0.94	0.93	15.024	0.77	0.76		
10	5.919	1.8	1.8	15.507	1.5	1.5		

(a) – Calculated with Wildenthal Hamiltonian;
(b) – calculated with transformed wave functions of 1^+ states;
(c) – results of calculations of P.M. Endt, J.G.L. Booten, Nucl. Phys. A 555(1993)499 utilizing Wilgenthal Hamiltonian
expt. – experimental data. Cited from Endt and Booten paper.
The known experimental values of excitation energy of 1^+ states have been used in (a,b) calculations. The excitation energies calculated with model Hamiltonian have been used in the calculations (a′, b′). In that case all excitation energies are measured from the ^{28}Si ground state.

Branching rations for γ-decay of 1^+ states in ^{28}Al.

J^π_f:		$3^+_{g.s.}$	2^+_1	0^+_1	3^+_1	1^+_1	1^+_2	2^+_2	2^+_3	1^+_3	2^+_4	0^+_2	1^+_4
$E_i \setminus E_f$:		0.000	0.031	0.972	1.014	1.372	1.620	1.623	2.139	2.201	2.486	3.012	3.105
1.372	(a)	4.4	73.2	22.4									
	(b)	2.9	75.8	21.3									
	(c)	4.4	74	22									
	expt.	4.7 *3*	55 *1*	40 *1*									
1.620	(a)	3.8	77.6	18.0		0.5							
	(b)	4.2	58.7	36.7		0.3							
	(c)	4.7	93	2.1									
	expt.	6	92	< 2									
2.201	(a)	0.2	2.1	80.4		14.5		2.6					
	(b)	0.0	63.3	35.2		1.4							
	(c)	0.2	2.2	80.1		14		2.7					
	expt.	< 6	79 *3*	16 *2*		< 3		< 5					
3.105	(a)	0.5	41.4	30.5	0.1	0.4	3.7	17.7	1.3	0.3	4.1		
	(b)	0.2	76.1	2.6	0.0	1.2	4.9	7.4	3.7	0.1	3.7		
	(c)	0.5	43	32	0.2	0.4	3.8	18	1.4	0.3			
	expt.		75 *3*							25 *3*			
3.542	(a)	1.6	30.2	25.7		11.9	1.5	22.2	2.6	0.0	4.1		
	(b)	1.6	30.4	25.8		11.1	1.6	22.3	2.7	0.1	4.2		
4.115	(a)	0.0	52.3	36.0		3.2	2.7	0.1	4.4	0.1	0.6		
	(b)	0.1	9.9	2.1		30.0	0.1	18.5	26.1	5.0	6.3		
4.846	(a)	0.1	77.0	1.2		4.9	4.5	2.1	1.2	5.1	0.2	2.0	1.1
	(b)	0.2	71.8	0.3		4.6	5.3	4.3	0.3	8.2	0.8	3.1	0.4
5.017	(a)	0.3	36.5	29.0		7.9	0.8	4.9	3.4	0.0	12.1	1.3	0.3
	(b)	0.1	56.4	36.1		0.4	0.9	0.7	0.3	1.4	0.2	0.0	2.3
5.435	(a)	1.1	1.1	14.0	0.3	11.0	0.0	61.5	0.2	0.3	4.0	0.4	0.6
	(b)	1.1	1.0	12.0	0.3	8.2	0.3	61.7	0.6	4.0	4.0	0.5	0.7

^{28}Si. The partial OMC rates (in $10^3\,\mathrm{s}^{-1}$), calculated with initial (a) and transformed (b) one-body transition densities ($g_A = -1.26$ and $g_P/g_A = 7.0$).

k	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(a)	29.9	3.1	34.1	26.1	0.0	3.0	20.6	11.5	8.4	3.5
(b)	12.8	7.6	63.6	11.2		8.5	18.7	7.3	6.6	4.2
$\pm$	0.2	0.2	2.4	0.5		0.4	0.2	2.1	0.2	0.1

Properties of Spin-Isospin Transitions in $A = 28$ Nuclei.

E_f MeV	Experiment	Theory (a)	Theory (b)
Λ_f (in $10^3\,\mathrm{s}^{-1}$) from $^{28}\mathrm{Si}(0^+_{\mathrm{g.s.}})\,(\mu,\nu)\,^{28}\mathrm{Al}(1^+_f)$			
1.62	12.9 ± 2.1	3.1	7.6 ± 0.2
2.20	62.8 ± 7.4	34.1	63.6 ± 2.4
3.11	14.7 ± 2.6	26.1	11.2 ± 0.5
$B_f(M1)$ (in μ_N) from $^{28}\mathrm{Si}(0^+_{\mathrm{g.s.}})\,(e,e')\,^{28}\mathrm{Si}(1^+_f)$			
10.90	0.90 ± 0.02	0.538	1.044
11.45	4.42 ± 0.20	3.064	4.461
12.33	0.87 ± 0.06	1.387	0.764
$B^-_f(GT)$ from $^{28}\mathrm{Si}(0^+_{\mathrm{g.s.}})\,(p,n)\,^{28}\mathrm{P}(1^+_f)$			
1.59	0.109 ± 0.002	0.069	0.165
2.10	0.956 ± 0.005	0.774	1.451
2.94	0.146 ± 0.003	0.613	0.222

V.A. Kuz'min, T.V. Tetereva, and K. Junker Phys. At. Nucl. 64 *(2001) 1169.*

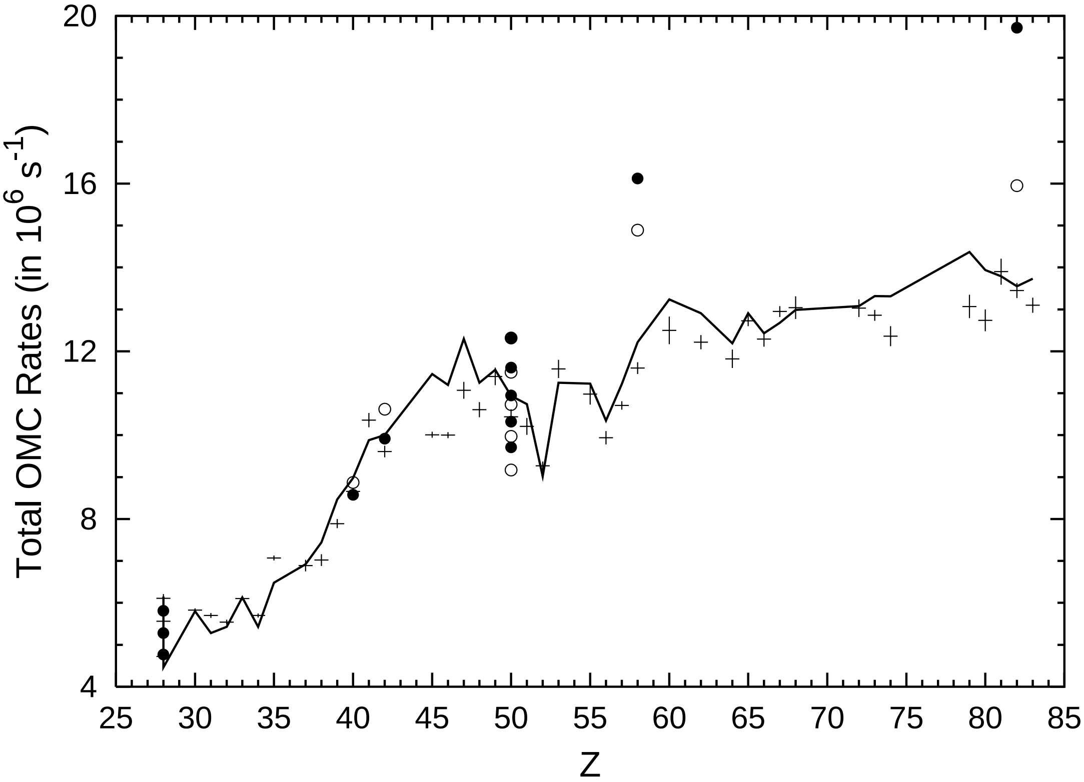


Figure 5: Total rates of OMC on complex nuclei.

V. A. Kuz'min, T. V. Tetereva, K. Junker, and A. A. Ovchinnikova, J. Phys. G* 28 *(2001) 665.