Nuclear Structure Models

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- 72 years of nuclear structure; (1932 y. W. Heisenberg, D. Ivanenko) proton - neutron nuclear model ($p-\alpha-e^-$ model before)

- Practically immediately after invention of the proton - neutron nuclear model Bartlett and especially Elsasser suggested a shell model approach to nuclei by analogy with atoms. In the paper "On the Pauli principle in nuclei" Elsasser wrote: "This principle suggests that the field due to (N-1) nucleons acting on the Nth nucleon leads to the shell structure of nucleons similar to the shell structure of e^- in atom. Probably, this mean field has spherical symmetry. Since in nucleus there is no massive center of force like in atom, the order of s.p. levels in nuclei may be different"... !!!

He concluded also that the potential well have a flat bottom

It was assumed also that LS coupling scheme is working in nuclei. This statement was unchanged up to 1949.

^{4}He - closed s-shell
^{16}O - closed p-shell

In 1934 Elsasser considered heavier nuclei and indicate on closed shells with Z or N = 50 and 82. To explain this magic numbers he assumed the following s.p. level scheme.

1i	108
1h	82
2d	60
1g	50
1f	32
1d	18
1p	8
1s	2

Problems:

1) N = 126 as the magic number.
   1i → 108
2) ^{40}Ca, N = 20 and Z = 20 are magic numbers.
   2s orbit must be below 1f?

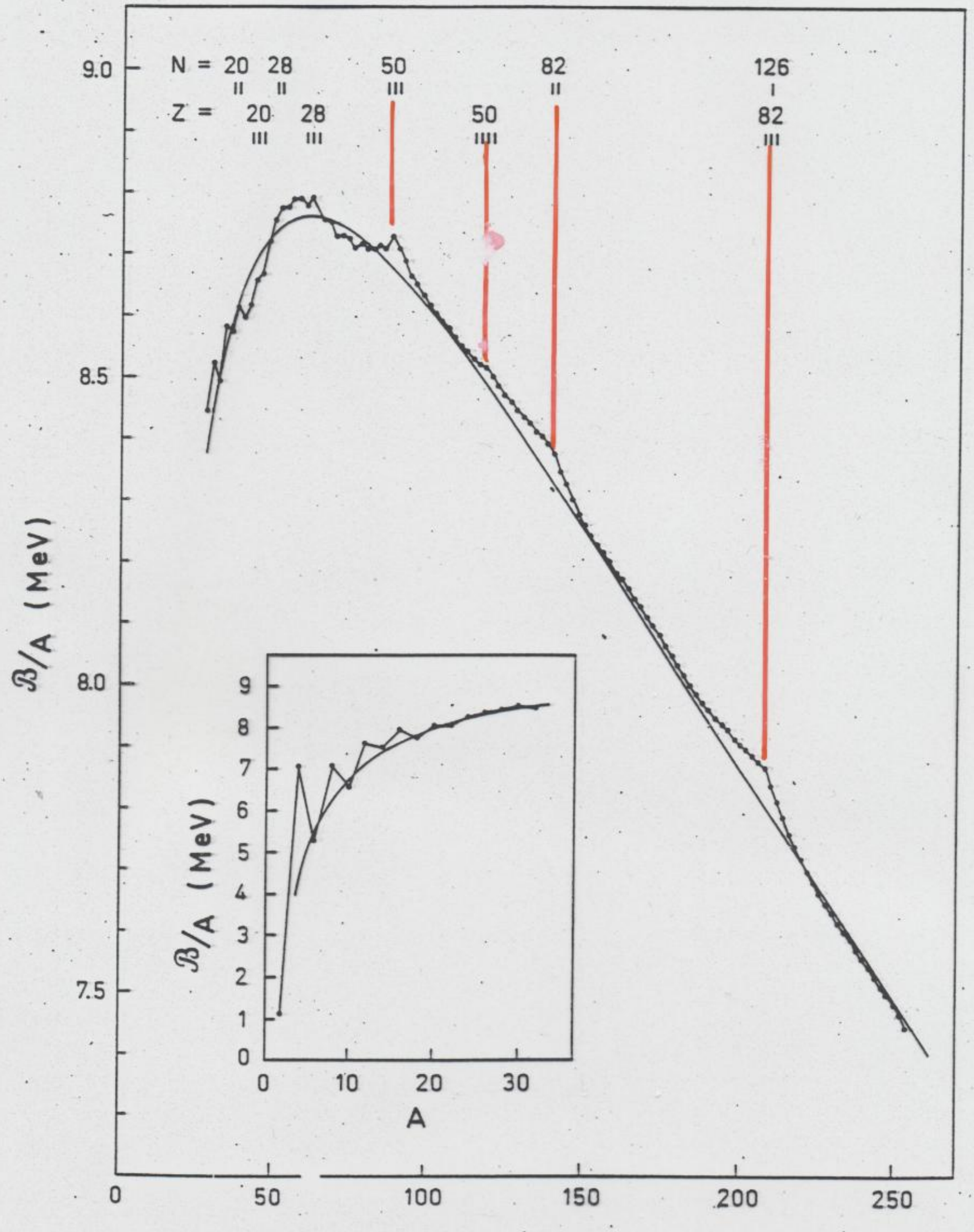


Figure 2-4 The experimental binding energies are taken from the compilation by J. H. E. Mattauch, W. Thiele, and A. H. Wapstra, *Nuclear Phys.* **67**, 1 (1965). The smooth curve represents the semi-empirical mass formula, Eq. (2-12), with the constants given by A. E. S. Green and N. A. Engler, *Phys. Rev.* **91**, 40 (1953).

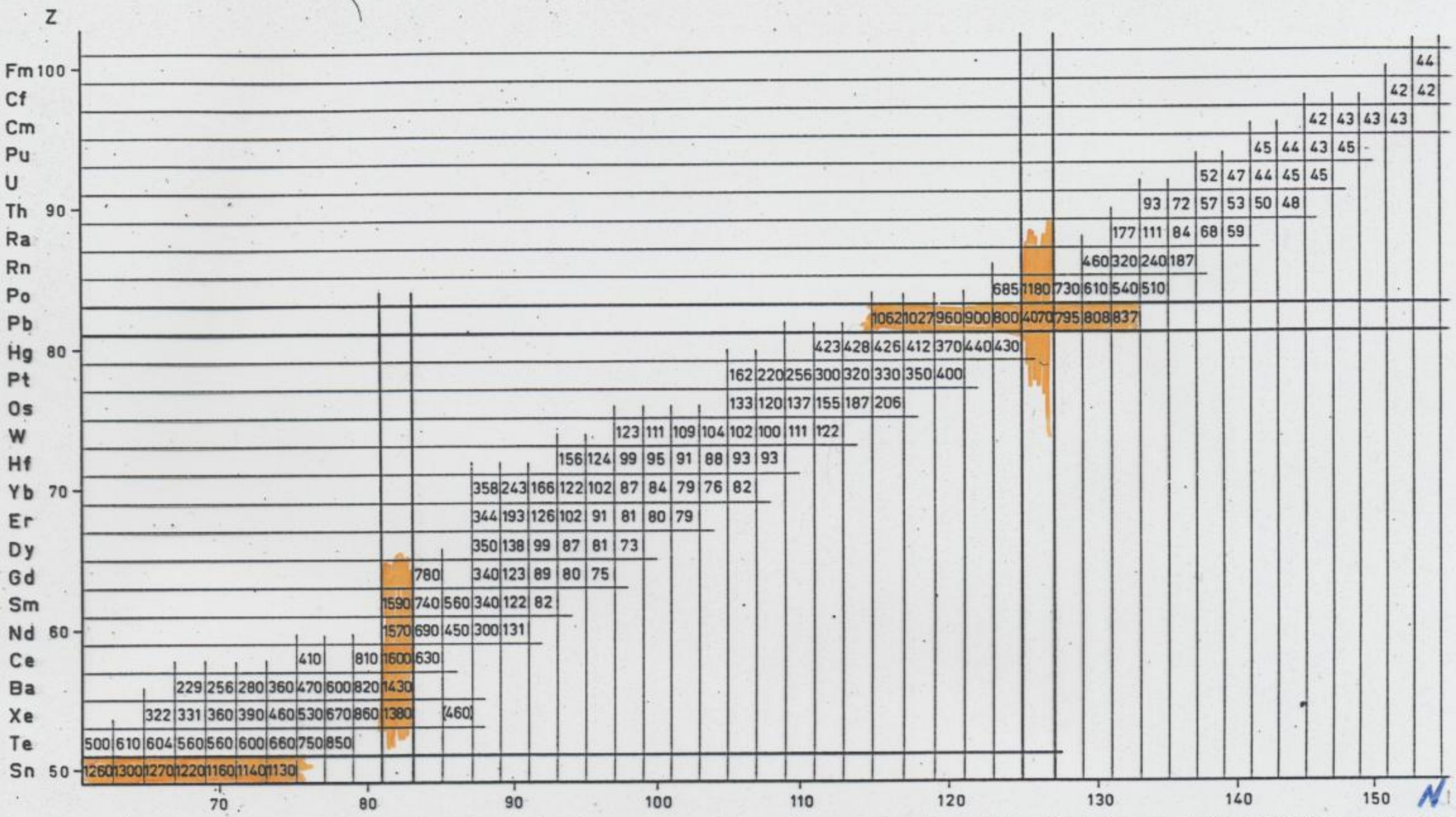
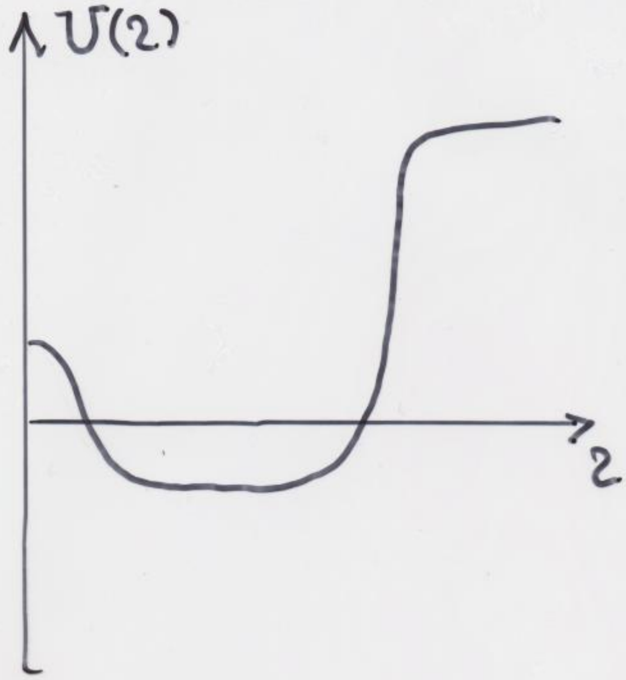


Figure 2-17b The energies of the first excited $2+$ states of even-even nuclei are given as a function of N and Z. In Fig. 2-17a ($Z \leq 50$), the energies are in MeV, while in Fig. 2-17b ($Z \geq 50$), the energies are in keV. The data are taken from *Nuclear Data Sheets* and the *Table of Isotopes* by Lederer *et al.* (1967). The striking systematics in the energies of the first excited states of even-even nuclei was emphasized at an early stage in the development of nuclear spectroscopy (Stähelin and Preiswerk, 1951; Rosenblum and Valadares, 1952; Asaro and Perlman, 1952; Scharff-Goldhaber, 1952, 1953).



- The situation in nuclear physics at that time was presented in the famous article by Bethe and Bacher (1936) with the following main points

  - The opposite extreme to the assumption of α-particles as nuclear subunits is that of independent motion of the individual protons and neutrons.

  - Whenever shell is completed we should expect a nucleus of particular stability. When a new shell is begun the binding energy of a newly added particles should be less than that of a preceding particles which serve to complete the preceding shell.

— Before considering the heavy nuclei they stress: necessary to give a strong warning against taking proton and neutron shells too literally... Zero approximation must be completed by considering the mutual interaction of nucleons and also configuration mixing.

Thus,

1. There exist magic numbers of nucleons and for their explanations we are needed in shells.

2. There is a problem to derive s.p. level scheme producing this magic numbers

3. Residual interaction of nucleons !

What is the mean field and the residual interaction?

$$\hat{H} = \sum_{i=1}^{A} T(\bar{r}_i) + \frac{1}{2} \sum_{i \neq j} V(\bar{r}_i - \bar{r}_j)$$

$$\hat{\psi}(\bar{r}_i) = \sum_s \psi_s(\bar{r}_i)\, a_s^+$$

$a_s^+(a_s)$ - nucleon creation (annihilation) operators;

$$\hat{H} = \sum_{s,s'} t_{ss'} a_s^+ a_{s'} + \frac{1}{2} \sum_{s,s',t,t'} V(ss'|tt') \times a_s^+ a_{s'} a_t^+ a_{t'}$$

$$= \sum_{s,s'} \Big(t_{ss'} + \sum_{t,t'} V(ss'|tt') \langle a_t^+ a_{t'} \rangle \Big) a_s^+ a_{s'}$$

$$+ \sim (a^+a - \langle a^+a \rangle)^2$$

$$= \sum_s E_s \tilde{a}_s^+ \tilde{a}_s + \text{residual interaction}$$

s.p. energy levels in the mean nuclear field

Fluctuations of the mean nuclear field.

Let us introduce into consideration imaginary time evolution operator

$$e^{-\beta \hat{H}} \quad , \beta > 0 \text{ and real}$$

$$|\Psi\rangle = \sum_i C_i |\Phi_i\rangle$$

$$H|\Phi_i\rangle = E_i |\Phi_i\rangle$$

$$e^{-\beta \hat{H}} |\Psi\rangle = \sum_i e^{-\beta E_i} C_i |\Phi_i\rangle$$

For large β the amplitudes of the g.s. becomes the largest one. This means that $\exp(-\beta \hat{H})$ behaves as a projection operator. Consider

$$\hat{H} = \sum_s E_s a_s^+ a_s + \frac{1}{2} \sum_{ss'tt'} V(ss'|tt') \times (a_s^+ a_{s'})(a_t^+ a_{t'})$$

$$st = \alpha \quad , \quad a_s^+ a_t \equiv \hat{O}_\alpha$$

$$\hat{H} = \sum_\alpha E_\alpha \hat{O}_\alpha + \frac{1}{2} \sum_{\alpha,\beta} V_{\alpha\beta} \hat{O}_\alpha \hat{O}_\beta$$

Diagonalizing $V_{\alpha\beta}$ we obtain

$$\hat{H} = \sum_{\alpha} E_{\alpha} \hat{O}_{\alpha} + \frac{1}{2} \sum_{\alpha} V_{\alpha} \hat{O}_{\alpha} \hat{O}_{\alpha}$$

For simplicity, consider the case of one α

$$\hat{H} = E\hat{O} + \frac{1}{2} V \hat{O}\hat{O}$$

All the difficulty arises from the two-body interaction. To linearize the operator in the exponent of the evolution operator we employ the Gaussian identity

$$\int_{-\infty}^{+\infty} d\sigma\, e^{-a(\sigma + c)^2} = \sqrt{\frac{\pi}{a}} \quad , \quad a > 0$$

$$e^{ac^2} = \sqrt{\frac{a}{\pi}} \int_{-\infty}^{+\infty} d\sigma\, e^{-a\sigma^2 - 2a\sigma c}$$

Substitute

$$c \to \hat{O} \quad , \quad a \to -\frac{1}{2}\beta V$$

$$e^{-\frac{1}{2}\beta V \hat{O}^2} = \sqrt{-\frac{1}{2}\frac{\beta V}{\pi}} \int_{-\infty}^{+\infty} d\sigma\, e^{\frac{1}{2}\beta V \sigma^2 + \beta V \sigma \hat{O}}$$

$V < 0$

$$e^{-\beta \hat{H}} = e^{-\beta E \hat{O} - \frac{1}{2}\beta V \hat{O}^2}$$

$$= \sqrt{\frac{\beta |V|}{2\pi}} \int_{-\infty}^{+\infty} d\sigma\, e^{-\frac{1}{2}\beta |V| \sigma^2} e^{-\beta (E\hat{O} + |V|\sigma \hat{O})}$$

If $\sigma = -\langle\hat{O}\rangle$ then one-body operator $E\hat{O} + |V|\sigma\hat{O}$ is the operator of the selfconsistent mean field. Indeed

$$\hat{H} = E\hat{O} + \frac{1}{2} V\hat{O}\hat{O} \rightarrow E\hat{O} + V\langle\hat{O}\rangle\hat{O}$$

Selfconsistent field realize a minimum of the energy. Thus, the main contribution to the integral in the expression of the evolution operator gives the region of the values of σ near the selfconsistent value. Selfconsistent value of σ corresponds to the shell model hamiltonian of the independent nucleons moving in the selfconsistent field. Deviations of σ from the selfconsistent value correspond to the fluctuations of the nuclear mean field.

We have in general many $\hat{O}_\alpha$, i.e. many σ_α. These fluctuations correspond to both coherent motion of many nucleons (collective excitations) and noncoherent motion.

For realistic case of many $\hat{O}_\alpha$ in general

$$[\hat{O}_\alpha, \hat{O}_\beta] \neq 0$$

and we cannot apply directly Habbard-Stratanovich transformation. We should split the interval β into N slices of length $\Delta\beta = \beta / N$

$$e^{-\beta \hat{H}} = \left[e^{-\Delta\beta \cdot \hat{H}} \right]^N$$

In this case H-S transformation is applied to an infinitisimally small interval and we obtain

$$e^{-\beta\hat{H}} =$$

$$= \prod_{n=1}^{N} \int_{-\infty}^{+\infty} \prod_\alpha d\sigma_{\alpha,n} \left(\frac{\Delta\beta |V_\alpha|}{2\pi} \right)^{1/2} e^{-\frac{\Delta\beta}{2} |V_\alpha| \sigma_{\alpha,n}^2}$$

$$\times e^{-\Delta\beta \left(E_\alpha + V_\alpha \sigma_{\alpha,n} \right) \hat{O}_\alpha}$$

For $N \to \infty$ we obtain functional integral.

Compound nucleus model.

N. Bohr in a lecture given at the Royal Danish Academy presented a compound nucleus model. Based on the experimental data he explained that when a neutron hits a nucleus energy of incident neutron will be rapidly divided among all nucleons with the result that for some time afterwards no single particle will possess sufficient kinetic energy to leave the nucleus. Nuclear states become more and more complicated and after some time (relaxation time) statistical equilibrium will be achieved. The wave function becomes a mixture of a large number of the configurations.

From this N. Bohr reaches the following conclusion:

as we have seen energy exchange between nucleons is a decisive factor... <u>In the atom and in the nucleus we have to do with two extreme cases for which approximation resting on one-body problem so effective for atom loses any validity in nucleus.</u>

Bohr's criticism had a profound effect on the development of the nuclear shell model. His strong objections discouraged theoretical physicists from using it. In the book of L. Rosenfeld (1948) the magic numbers 50, 82 and 126 were not even mentioned.

The renaissance of the nuclear shell model began by the paper of Maria Göppert - Mayer (1948). She presented strong experimental evidence for the reality of magic numbers 20, 50, 82 and 126. Experimental data at that time were much more abundant than those available to Elsasser. She based her conclusion not only on binding energies but also on isotopic abundance. Her paper drew attention of nuclear physicists to existance of magic numbers in heavy nuclei. Some of them tried to obtain shell closures at 50, 82 and 126.

The paper of M. G - Mayer was published in August 1948. On December 27, 1948 two manuscript were received by the Physical Review. Both presented s.p. level schemes which reproduce the magic numbers 50, 82.

— Feenberg and Hammack adopted "wine bottle potential of Elsasser to push 2S level in heavy nuclei.

1h	82
2d	60
1g	50
1f	32
1d	18
1p	8
1S	2

They suggest that repulsion at the center of the potential well effective only for heavy nuclei. 2S orbit is fully occupied in the shell closure at 20. But lies higher than 1f and 1g at 50 and 1h, 2d at 82.

— The other paper of Nordheim (1949) presents a very different level scheme

2f	82
1g	
2d	50
2p	
1f	
2S	20
1d	
1P	8
1S	2

Thus, the problem of the construction of the mean field potential describing correctly magic numbers was not solved!

Up to this moment people believe in LS-coupling. The spell of LS-coupling which made it difficult to find level scheme with shell closures at 50, 82 and 126 was broken by Fermi's question who, after a seminar given by M.G-Mayer, asked her: "Is there any indication of spin-orbit coupling in nuclei?" She thought and answered "Yes, and it explains everythings". In her paper (1949) she introduced spin-orbit interaction

$$-C_{ls}\,\bar{l}\cdot\bar{s}$$

which gives rise to the modern level scheme reproducing all magic numbers. This version of the shell model explains angular and magnetic momenta of odd nuclei, electromagnetic transitions and, in particular "islands of isomerism" near shell closures where s.p. states may have widely different j-values.

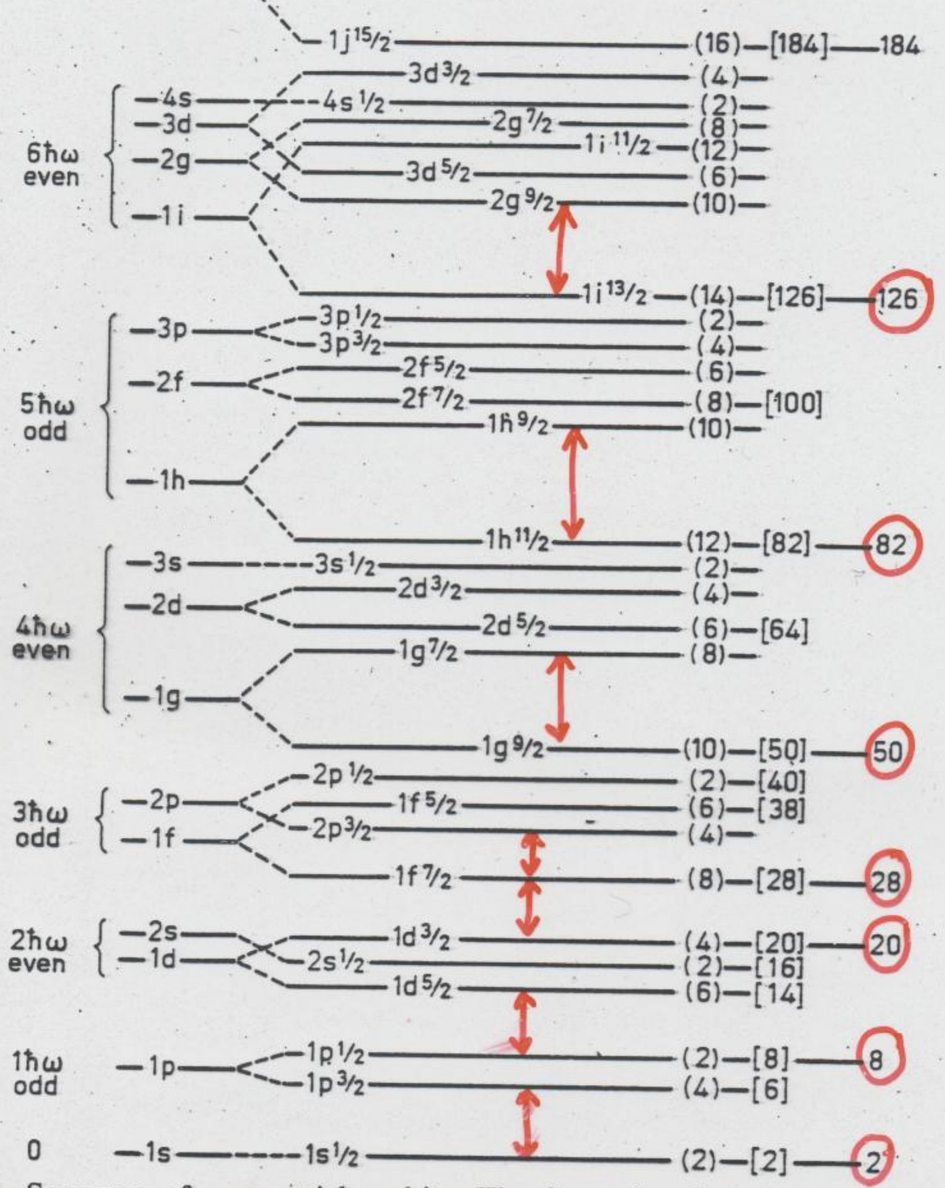


Figure 2-23 Sequence of one-particle orbits. The figure is taken from M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure*, p. 58, Wiley, New York, 1955.

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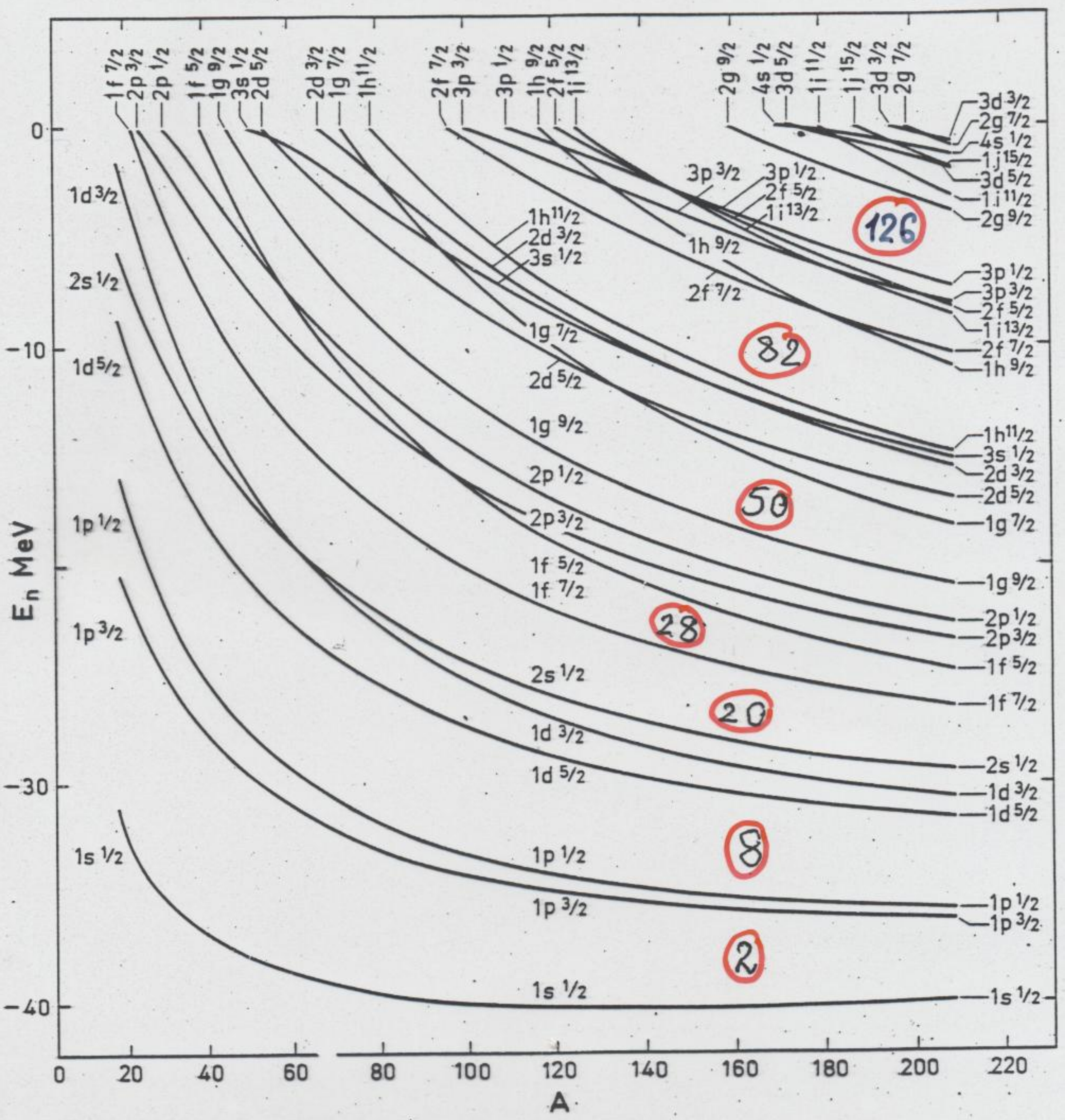


Figure 2-30 Energies of neutron orbits calculated by C. J. Veje (private communication).

- The single nucleon level scheme based on strong spin-orbit coupling was discovered independently by Maxel, Jensen and Suess (4 February - 12 February)

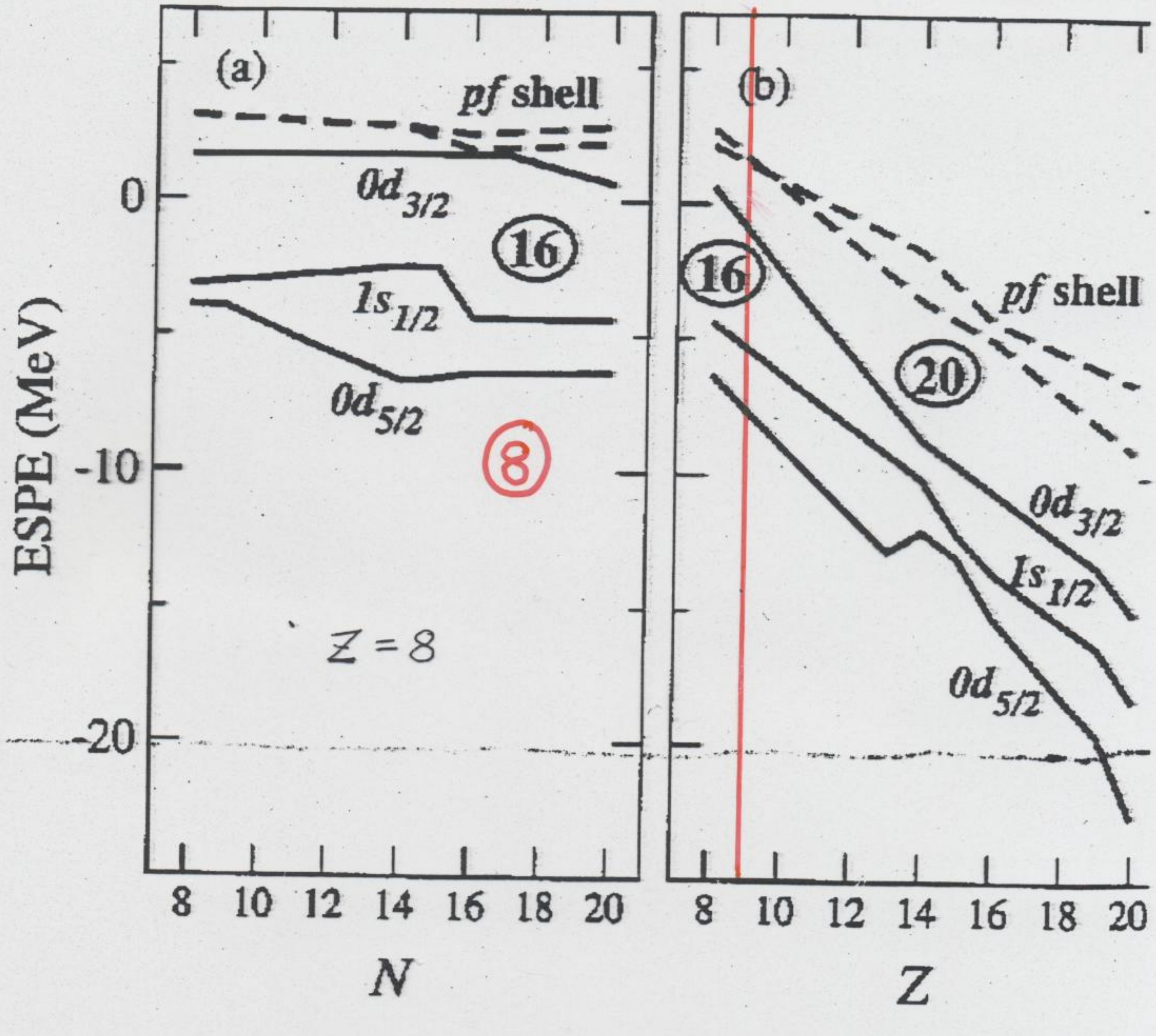
- Since the first papers of M.G-Mayer and Jensen at al. the shell model made enormous impact on almost all aspects of Nuclear Physics.

- The shell model is marvelous creation, especially when one realizes it exists in spite of relatively strong nucleon-nucleon forces.

We now realize that this mean field feature is characteristic of complex systems and relatively independent of the character of the interparticle forces

- The theoretical foundation of the shell model is a problem. We can say that the shell model is an energy average of the many-body Hamiltonian (H. Feshbach).

Radioactive beams.



Potential with spherical symmetry.

The motion separates into radial and angular components.

The one-particle energies depend on the two quantum numbers: ℓ and n (radial nodes)

Shell structure occurs if one-particle energy $\varepsilon(n,\ell)$ is approximately stationary with respect to certain variations of the quantum numbers

$$\varepsilon(n,\ell) = \varepsilon(n_0,\ell_0) + (n-n_0)\left(\frac{\partial\varepsilon}{\partial n}\right)_0 + (\ell-\ell_0)\left(\frac{\partial\varepsilon}{\partial\ell}\right)_0$$

$$+\frac{1}{2}(n-n_0)^2\left(\frac{\partial^2\varepsilon}{\partial n^2}\right)_0 + (n-n_0)(\ell-\ell_0)\left(\frac{\partial^2\varepsilon}{\partial n\,\partial\ell}\right)_0 + \frac{1}{2}(\ell-\ell_0)^2\left(\frac{\partial^2\varepsilon}{\partial\ell^2}\right)_0 + \ldots$$

Series of approximately degenerate levels occur in the spectrum if the first derivatives of ε are in the ratio of two integers "a" and "b"

$$b\left(\frac{\partial\varepsilon}{\partial n}\right)_0 = a\left(\frac{\partial\varepsilon}{\partial\ell}\right)_0 .$$

When this relation is fulfilled, the levels with constant value of $(an+b\ell)$ differ in energy by terms involving second and higher order derivatives of ε. The successive shells can be labeled by the quantum number

$$N_{sh} = a(n-1) + b\cdot\ell$$

For harmonic oscillator $a=2$, $b=1$

This characterization of the shell structure can be extended to any potential that permits a separation of the motion in the three dimensions. The separability implies that the eigenstates of the single particle motion can be characterized by three quantum (n_1, n_2, n_3). Shells occur if

$$\left(\frac{\partial \varepsilon}{\partial n_1}\right)_0 : \left(\frac{\partial \varepsilon}{\partial n_2}\right)_0 : \left(\frac{\partial \varepsilon}{\partial n_3}\right)_0 = a : b : c$$

where a, b, c are integers.

The levels in the shell have the same value of the quantum number

$$N_{sh} = a \cdot n_1 + b \cdot n_2 + c \cdot n_3$$

Anisotropic harmonic oscillator

$$V = \frac{1}{2} M \left(\omega_1^2 x_1^2 + \omega_2^2 x_2^2 + \omega_3^2 x_3^2 \right)$$

If the frequences $\omega_1, \omega_2, \omega_3$ are in the ratios of integers, the energy depends only on the quantum number N_{sh}

$$E_{rot} = \frac{1}{2} \mathcal{J} \omega^2 \qquad E_{rad} = \frac{1}{2} m v^2$$

$$L = \mathcal{J} \omega \qquad p = m v$$

$$L = \hbar l \qquad P = \hbar k$$

$$k R = n$$

$$E_{rot} = \frac{1}{2\mathcal{J}} L^2 \qquad E_{rad} = \frac{P^2}{2m}$$

$$\frac{\partial E_{rot}}{\partial l} = \hbar \frac{L}{\mathcal{J}} = \hbar \omega \qquad \frac{\partial E_{rad}}{\partial n} = \hbar \frac{P}{mR} = \hbar \frac{v}{R}$$

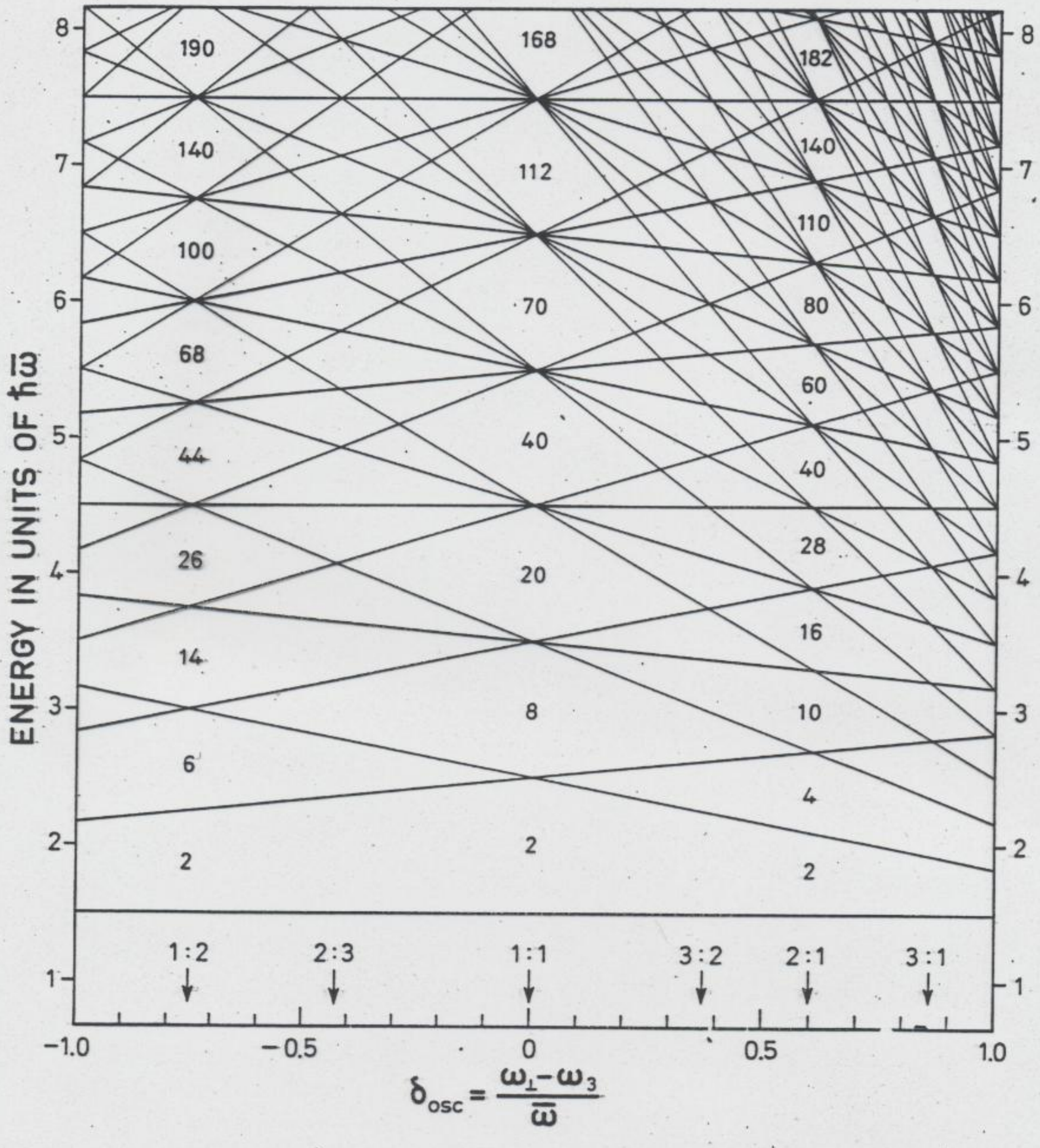
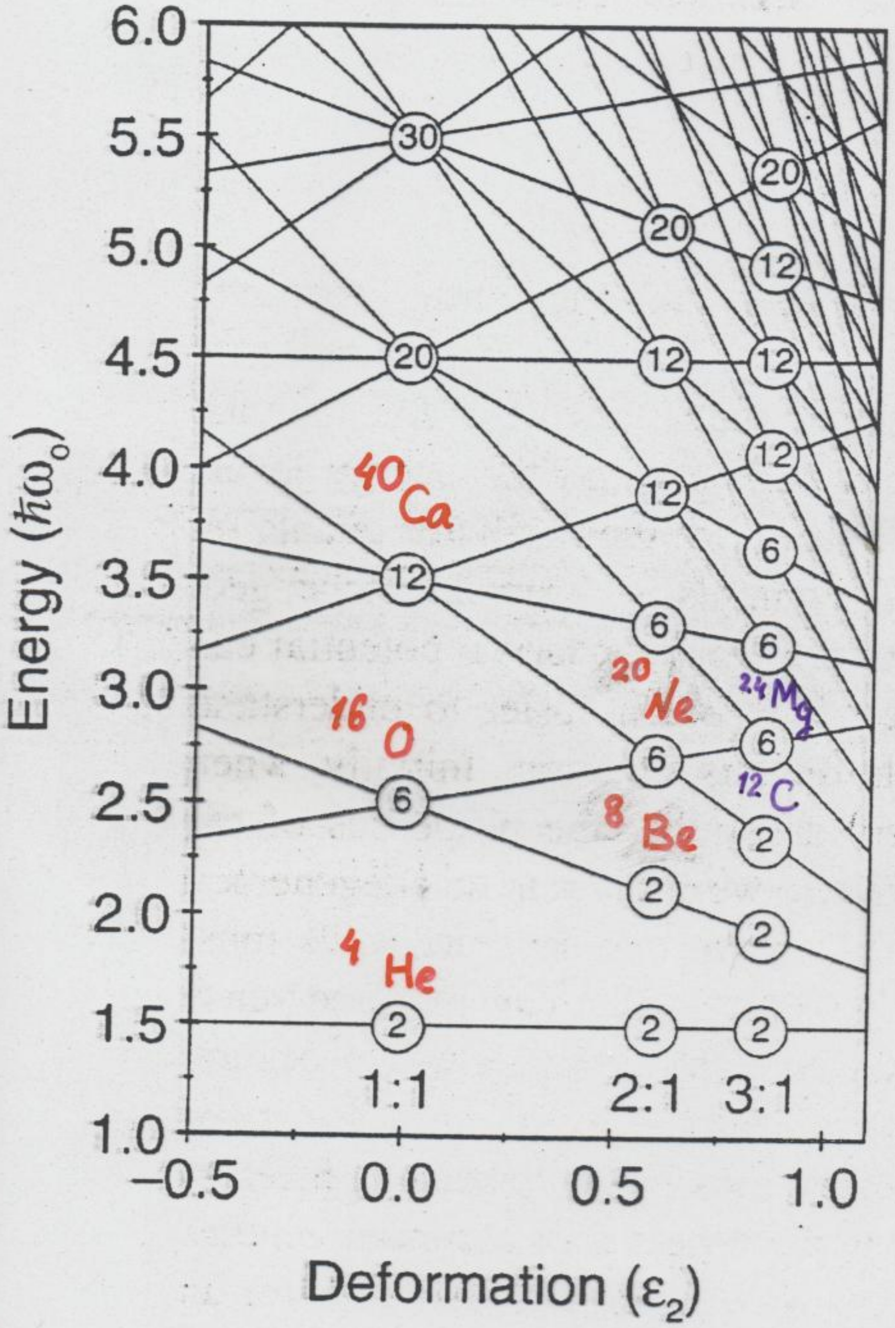


Figure 6-48 Single-particle spectrum for axially symmetric harmonic oscillator potentials. The eigenvalues are measured in units of $\overline{\omega} = (2\omega_\perp + \omega_3)/3$, and the deformation parameter δ_{osc} is that defined by Eq. (5-11). The arrows mark the deformations corresponding to the indicated rational ratios of frequencies $\omega_\perp : \omega_3$.



$^{8}Be \to {}^{4}He - {}^{4}He$

$^{20}Ne \to {}^{16}O - {}^{4}He$

$^{12}C \to {}^{4}He - {}^{4}He - {}^{4}He$

$^{24}Mg \to {}^{4}He - {}^{16}O - {}^{4}He$

The deformation process can be viewed as the division of the original spherical potential into a series of small potentials aligned along the deformation axis.

Figure 1. The deformed-harmonic oscillator energy levels plotted as a function of the quadrupole deformation (ϵ_2). The numbers shown inside the circles indicate the degeneracy of the levels for the various crossing points.

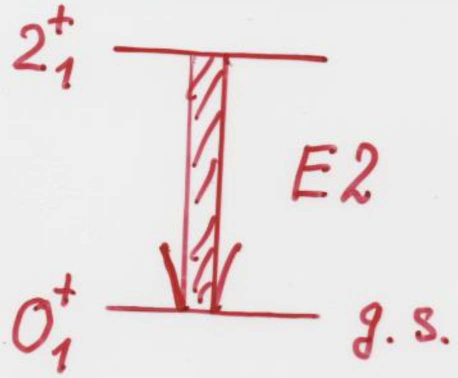
Collective model of nuclear excitations.

(A. Bohr and B. Mottelson, 1952)

We mentioned before coherent and noncoherent fluctuations of the nuclear mean field. They are connected with excitations of nuclei (increase of the energy).

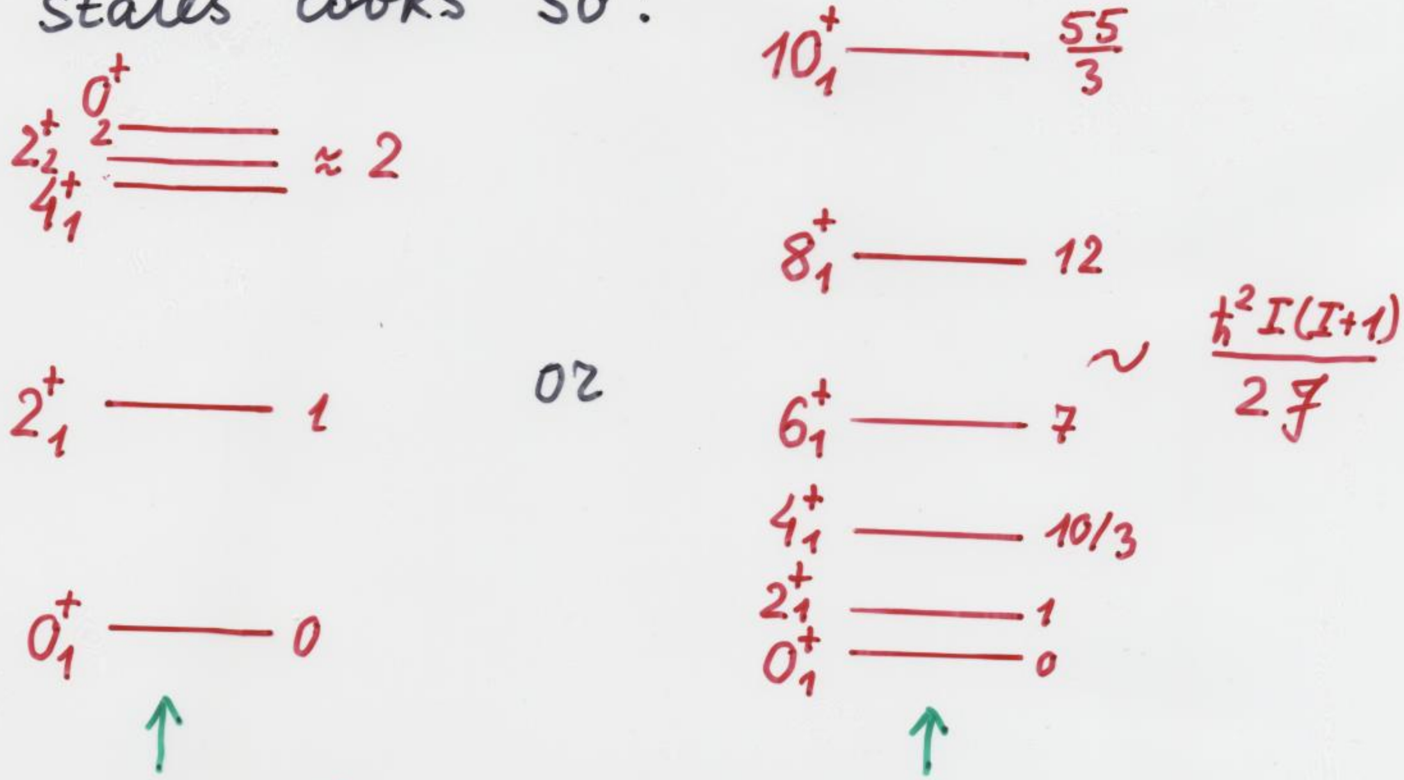
Among low lying nuclear excitations there are states which can not be described in the framework of the independent particle model of nuclei.

These are 2_1^+ states in even-even nuclei



$(10 \div 100)$ s.p.u.

The spectra of the higher lying states looks so:



Quadrupole vibrator

Rotor

Similar spectra has been known for molecules.

Because of enormously strong E2-transitions to these states from the ground states their excitation is connected with changing of motion of many nucleons. Therefore, their description by the shell model with inclusion correlations of nucleons (necessary to explain large E2 transition probabilities) is very complicated.

Qualitative description has been obtained basing on analogy with liquid drop.

For low energy excitations nuclear liquid can be considered as incompressible. However, shape of a nucleus can be changed

$$R = R_0 \left[1 + \sum_{\lambda,\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta, \varphi) \right]$$

$\alpha_{2\mu}$ are most important

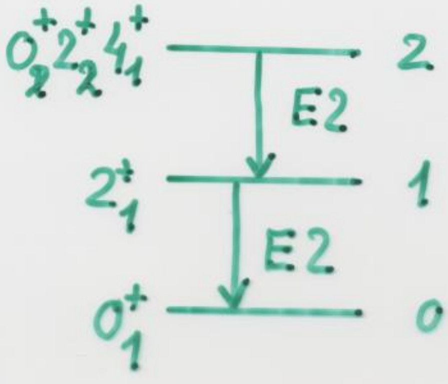
$$H = T + V$$

$$T = \frac{1}{2} \sum_{\lambda,\mu} B_\lambda |\dot{\alpha}_{\lambda\mu}|^2$$

$$V = V(\alpha)$$

- Harmonic oscillations of quadrupole degree of freedom:

$$V = \frac{1}{2} C_2 \sum_\mu |\alpha_{2\mu}|^2$$



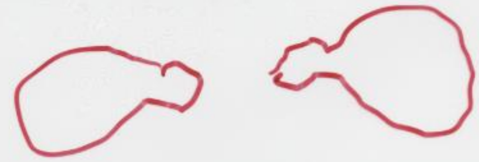
$$\frac{E(0_2^+, 2_2^+, 4_1^+)}{E(2_1^+)} = 2$$

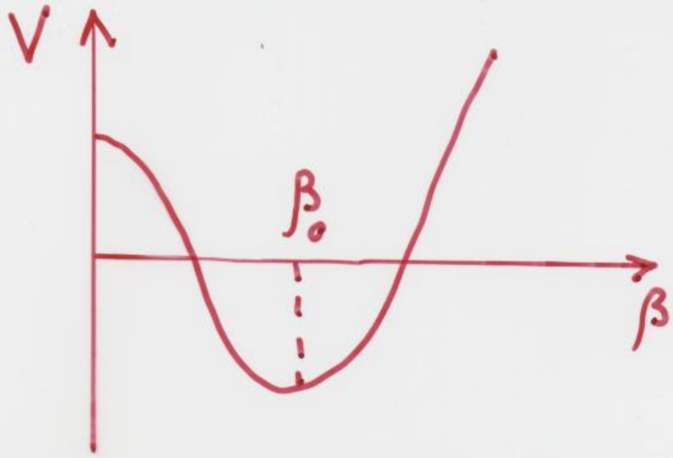
$$\frac{B(E2;\, 0_2^+, 2_2^+, 4_1^+ \to 2_1^+)}{B(E2;\, 2_1^+ \to 0_1^+)} = 2$$

- Rotor

$$\alpha_{2\mu} = D^2_{\mu 0}(\bar{\theta})\,\beta\cos\gamma + \frac{1}{\sqrt{2}}\left(D^2_{\mu 2} + D^2_{\mu -2}\right)\beta\sin\gamma$$

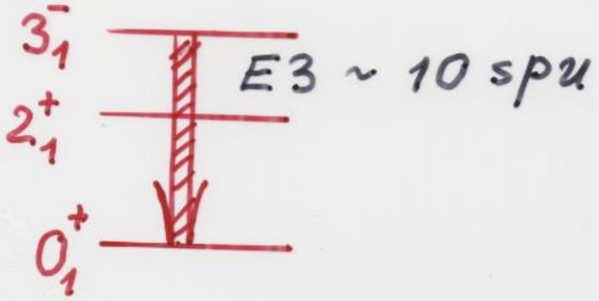
$$V = V(\beta^2, \beta^3\cos 3\gamma)$$

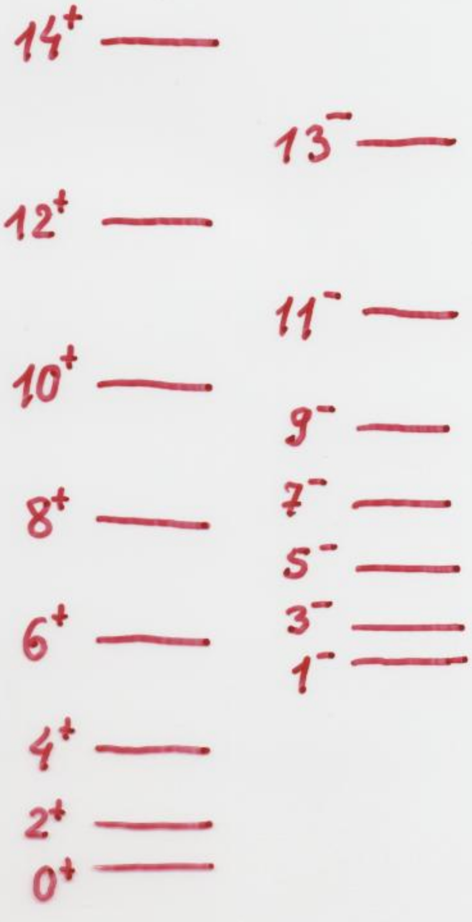




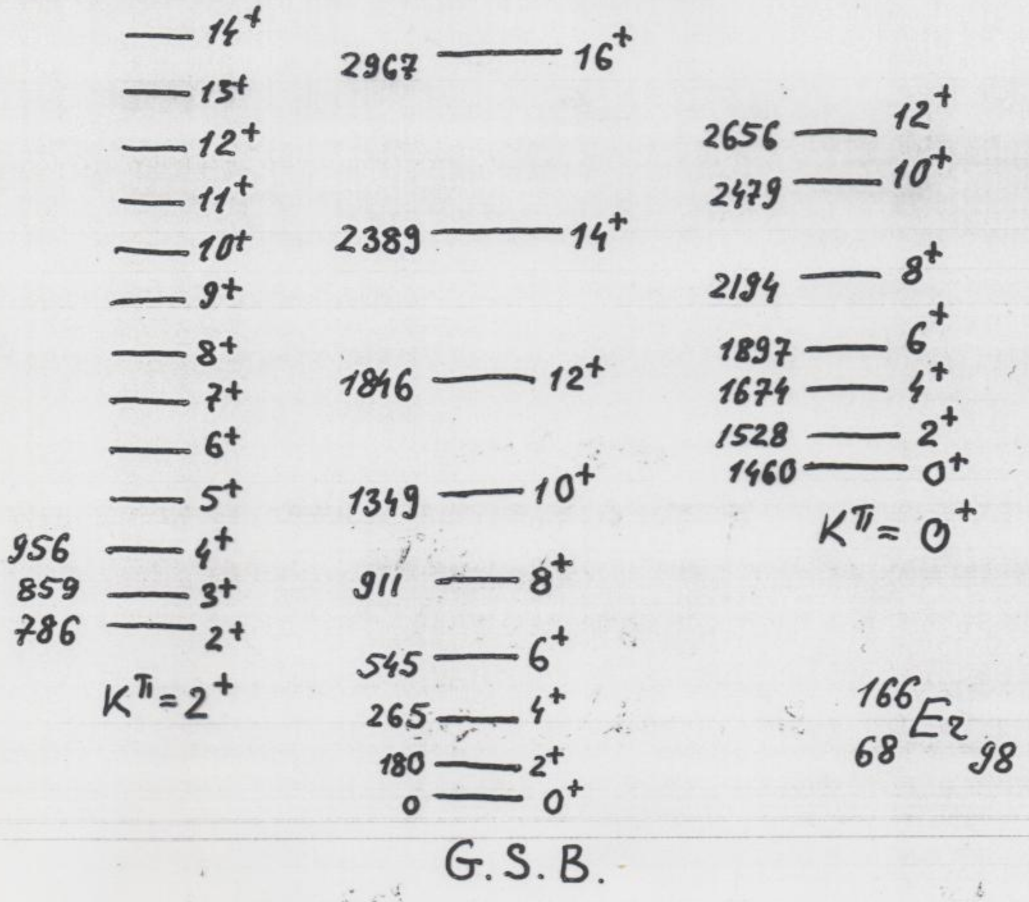
$$E_{rot} = \frac{\hbar^2 I(I+1)}{2\cdot 3B\beta_0^2}$$

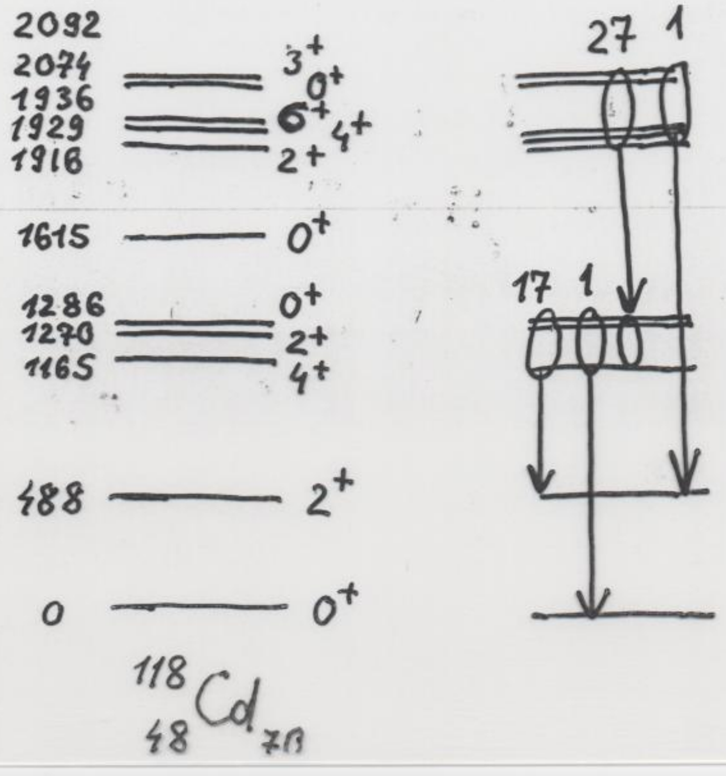
- Collective octupole mode was also ob-served



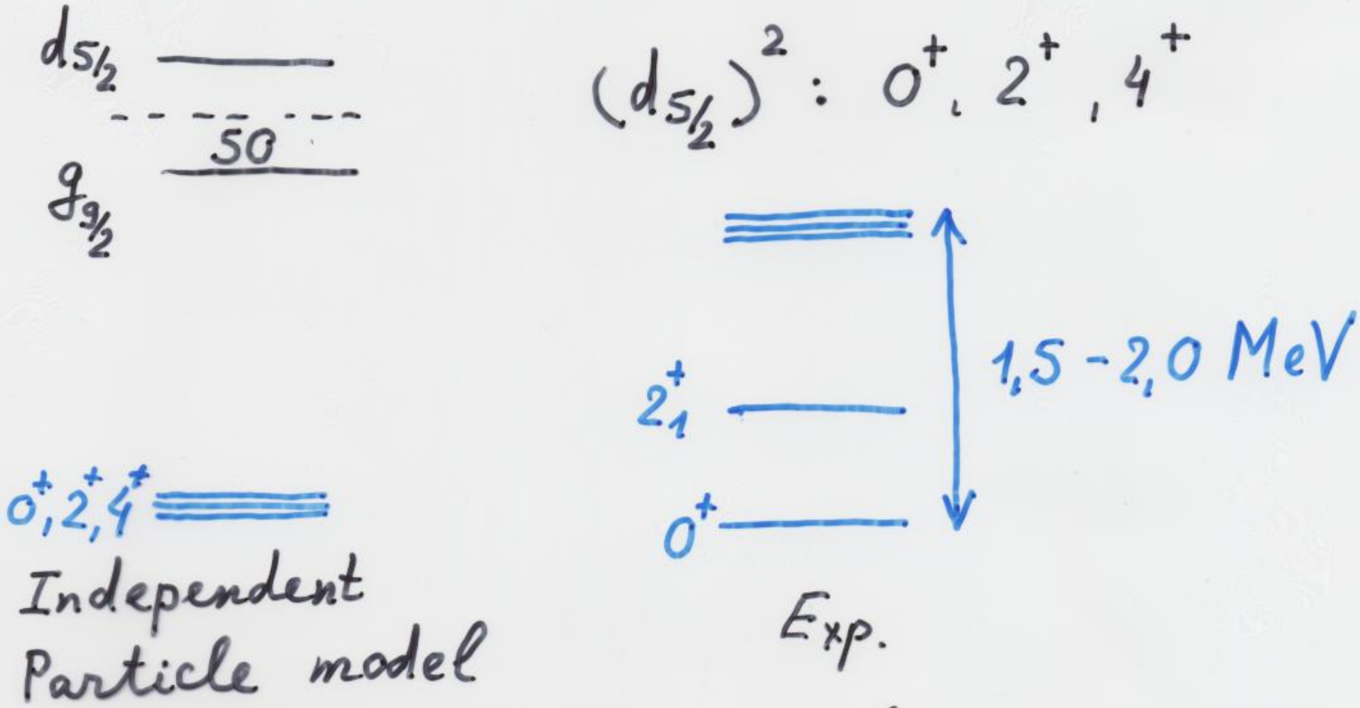


Alternative parity rotational bands like in asymmetric molecules has been observed


14+
13+
12+
11+
10+
9+
8+
7+
6+
5+
956 4+
859 3+
786 2+
K^π = 2+
2967 16+
2389 14+
1846 12+
1349 10+
911 8+
545 6+
265 4+
180 2+
0 0+
G.S.B.
2656 12+
2479 10+
2194 8+
1897 6+
1674 4+
1528 2+
1460 0+
K^π = 0+
166 68 Er 98


2092
2074
1936
1929
1916
3+
0+
6+
4+
2+
1615 0+
1286 0+
1270 2+
1165 4+
488 2+
0 0+
27
1
17
1
118 48 Cd 70

Pairing (Pair correlations of nucleons)



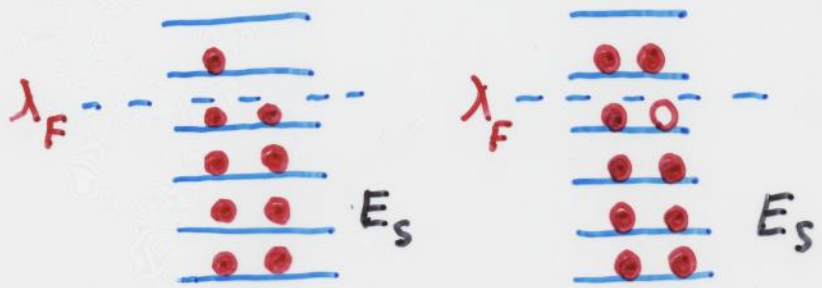
Energy gap like in a superconductor!

To describe this picture attractive monopole pair forces has been introduced

$$-G \sum_s a_s^+ a_{\bar{s}}^+ \cdot \sum_t a_{\bar{t}} a_t$$

$$s, t \equiv n, l, j, m$$
$$\bar{s}, \bar{t} \equiv n, l, j, -m$$

Particles and Holes



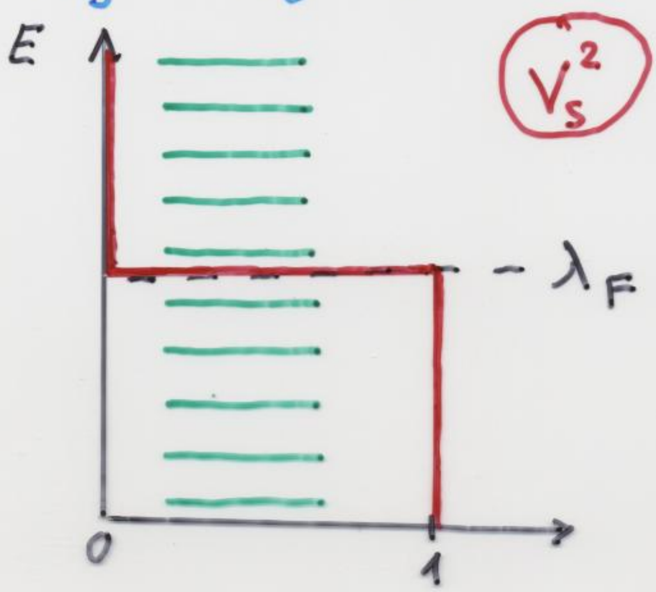
Quasiparticles are introduced to describe excitations

$$\alpha^+_s = u_s\, a^+_s - v_s\, a_{\bar{s}}$$

$$u_s = \begin{cases} 0\,, & E_s < \lambda_F \\ 1\,, & E_s > \lambda_F \end{cases}$$

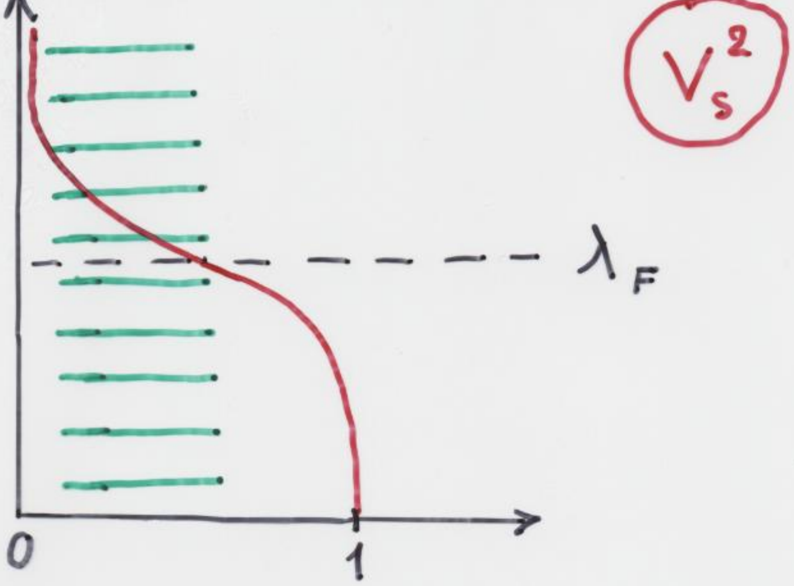
$$v_s = \begin{cases} 1\,, & E_s < \lambda_F \\ 0\,, & E_s > \lambda_F \end{cases}$$

$$u_s^2 + V_s^2 = 1$$



Generalization:

u_s^2, v_s^2 can take any values between zero and one



Let us find the wave function of the ground state which must satisfy to the following condition

$$\alpha_s |g.s.\rangle = 0$$

$$|g.s.\rangle = \prod_s (u_s + V_s a_s^+ a_{\bar{s}}^+) |0\rangle$$

$$\alpha_t |g.s.\rangle = (u_t a_t - V_t a_{\bar{t}}^+) \prod_s (u_s + V_s a_s^+ a_{\bar{s}}^+) |0\rangle$$

$$= (u_t V_t a_{\bar{t}}^+ - u_t V_t a_{\bar{t}}^+) \prod_{s \neq t} (u_s + V_s a_s^+ a_{\bar{s}}^+) |0\rangle$$

$$= 0$$

$$|g.s.\rangle \equiv |BCS\rangle$$

Interacting Boson Model

- From the shell model point of view collective vibrational states are superpositions of a large number of the particle - hole excitations

To treat them in a harmonic approximation **RPA** has been invented. Inclusion of the anharmonic effects or, especially, treatment of transitional nuclei requires shell model consideration in a very large configurational space which is cumbersome. This creates a need in a simplified model which, however, should take into account Pauli principle in some way.

Application of the Bohr - Mottelson model to consideration of transitional nuclei requires too many parameters!

We know, however, that usually introduction of symmetry into consideration reduces the number of parameters in the models!

What was known experimentally at the beginning of 70th?

- Quasirotational bands in nuclei from vibrational to rotational limits:

  $^{166}_{68}Er$ — rotor ;

  $^{118}_{48}Cd$ — vibrator ;

  large variety of transitional nuclei ;

$$R_{4/2} \equiv \frac{E(4_1^+)}{E(2_1^+)}$$

	$^{146}_{64}Gd_{82}$	^{148}Gd	^{150}Gd	^{152}Gd	$^{154}_{64}Gd_{90}$	^{156}Gd	^{158}Gd	^{160}Gd
$R_{4/2}$	1,33	1,81	2,02	2,19	3,02	3,24	3,29	3,30

As a reply on these requirements IBM was created

$$S^{\dagger} = \sum_{j} \alpha_j A^{\dagger}(jj;0,0) \rightarrow S^{+} \qquad (2{\cdot}1)$$

and

$$D^{\dagger}_M = \sum_{jj'} \beta_{jj'} A^{\dagger}(jj';2,M)\ , \rightarrow d^{+}_{M} \qquad (2{\cdot}2)$$

where α_j and $\beta_{jj'}$ are normalized amplitudes, and the pair creation operator is

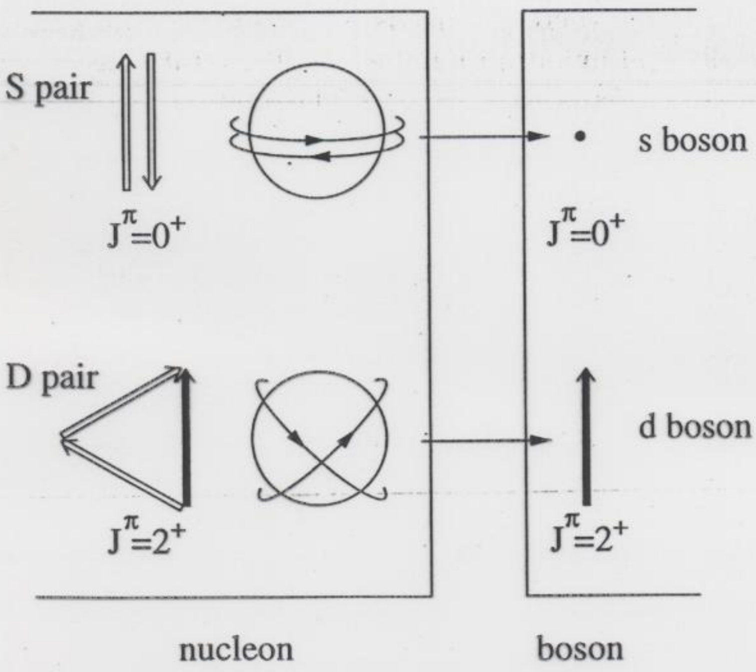


Fig. 1. Correspondence between nucleon pairs S and D, and bosons s and d.

$N = s^{+}s + \sum_{\mu} d^{+}_{\mu} d_{\mu}$ = half of the number of valent nucleons

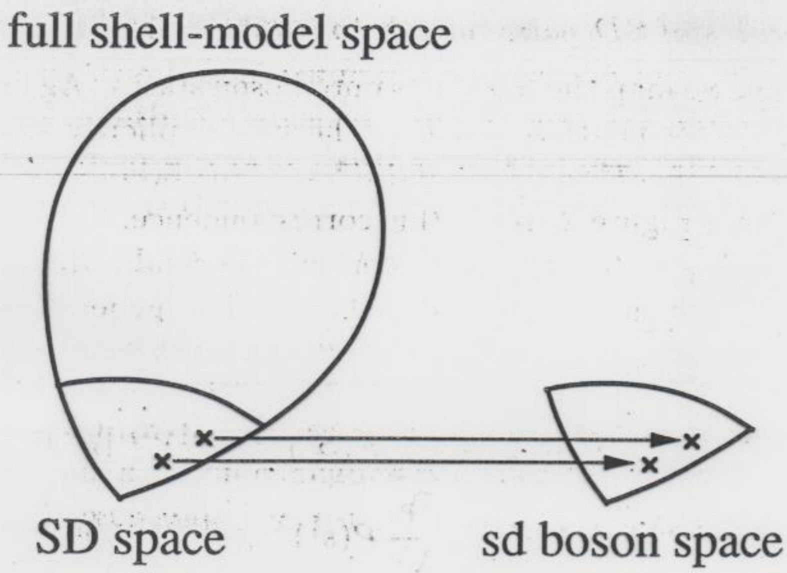


Fig. 2. Mapping from the SD subspace of the full shell-model space onto the sd boson space.

- 37 -

The process for determining $\mathcal{O}^B$ is illustrated in detail below. We shall consider spherical nuclei, where the states in Eqs. (2·12)∼(2·14) should be the dominant components in the lowest states, and are used as the *SD* states Ψ and Ψ' in Eq. (2·21). The IBM Hamiltonian is written in general for one kind of bosons as

$$H^B = E_0^{(N)} + \varepsilon \hat{N}_d + V \tag{2·22}$$

and

$$\begin{aligned} V = & \frac{1}{2} \sum_{L=0,2,4} c_L([d^\dagger d^\dagger]^{(L)} \cdot [\tilde{d}\tilde{d}]^{(L)}) \\ & + \frac{1}{\sqrt{2}} y\{([d^\dagger d^\dagger]^{(2)} \cdot [s\tilde{d}]^{(2)}) + \text{h.c.}\} \\ & + \frac{1}{2} w\{([d^\dagger d^\dagger]^{(0)} [ss]^{(0)}) + \text{h.c.}\}, \end{aligned} \tag{2·23}$$

where $\hat{N}_d$ denotes the d-boson number operator, N stands for the total boson number, and $u_0, u_1, u_2, \varepsilon, c_L, y$ and w are constants. Note that the total boson number is conserved. The quantity $E_0^{(N)}$ in Eq. (2·22) is the energy of the state $|s^N; J=0)$, and is fixed by the energy of *SD* state $|S^N; J=0\rangle$;

$$\begin{aligned} E_0^{(N)} &= (s^N; J=0|H^B|s^N; J=0) \\ &= \langle S^N; J=0|H|S^N; J=0\rangle, \end{aligned} \tag{2·24}$$

where H is the nucleon Hamiltonian. The quantity $E_0^{(N)}$ is a constant for a given nucleus. As long as one is interested in excitation energies and/or wave functions, one does not have to pay attention to $E_0^{(N)}$. We shall, in fact, ignore $E_0^{(N)}$, hereafter, while it should be regarded as the origin point from which to measure the excitation energy. One should bear in mind, however, that $E_0^{(N)}$ has to be treated explicitly in describing binding energies.

The constant ε is determined by

$$\varepsilon = \langle S^{N-1}D; J=2|H|S^{N-1}D; J=2\rangle - E_0^{(N)}, \tag{2·25}$$

because of

$$\langle S^{N-1}D; J=2|H|S^{N-1}D; J=2\rangle = (s^{N-1}d; J=2|H^B|s^{N-1}d; J=2) = E_0^{(N)} + \varepsilon. \tag{2·26}$$

The constant c_L is determined similarly by

$$c_L = \langle S^{N-2}D^2; J=L|H|S^{N-2}D^2; J=L\rangle - 2\varepsilon - E_0^{(N)}, \tag{2·27}$$

because of

$$\begin{aligned} \langle S^{N-2}D^2; J=L|H|S^{N-2}D^2; J=L\rangle &= (s^{N-2}d^2; J=L|H^B|s^{N-2}d^2; J=L) \\ &= E_0^{(N)} + 2\varepsilon + c_L. \end{aligned} \tag{2·28}$$

- Operators

$$d^+_\mu d_{\mu'} + d^+_\mu s + s^+ d_\mu = 25+5+5$$
$$= 35$$

used to construct Hamiltonian

form SU(6) algebra.

This algebra has subgroups

1. SU(5)

$d^+_\mu d_{\mu'}$ 25

harmonic vibrator

$$n_d \equiv \sum_\mu d^+_\mu d_\mu = \text{const}$$

2. SU(3), $\overline{\text{SU(3)}}$

rotor

$$I_\mu \sim (d^+ d)_{1\mu}$$
$$Q_{2\mu} = d^+_\mu s + s^+ d_{-\mu} (-)^\mu \pm \frac{\sqrt{7}}{2} (d^+ d)_{2\mu}$$

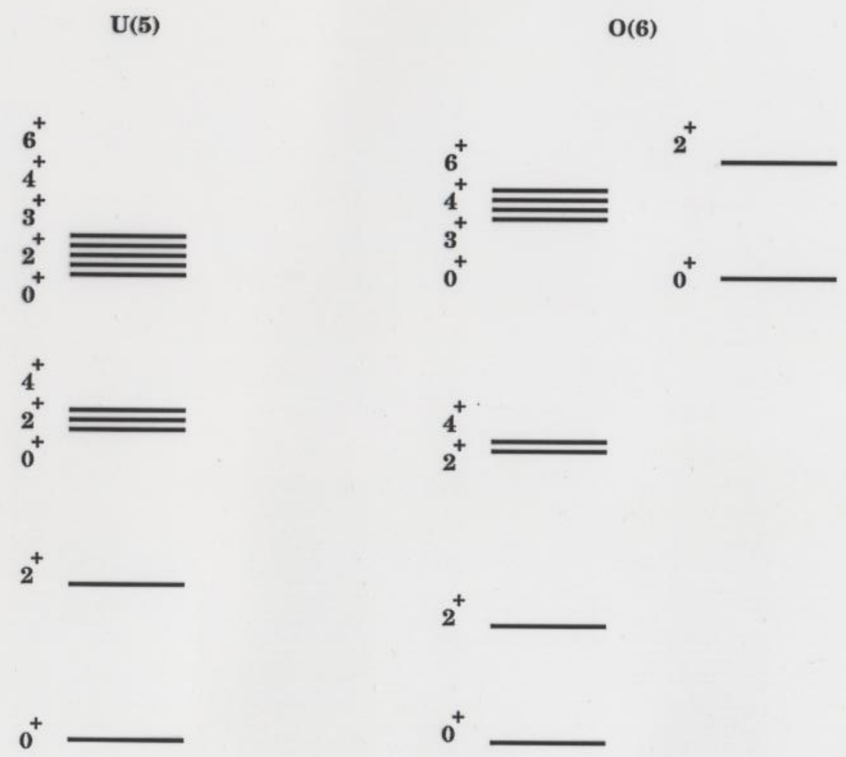
8

3. O(6)

$$(d^+ d)_{1\mu}, \ (d^+ d)_{3\mu}, \ d^+_\mu s + s^+ (-)^\mu d_{-\mu}$$

15

γ-unstable nucleus


U(5)
O(6)
6+
4+
3+
2+
0+
4+
2+
0+
2+
0+
6+
4+
3+
0+
4+
2+
2+
0+
2+
0+

Classical limit for the quantum system

-Theorem of Gilmore: U(r) → r-1
-IBM: (d^+_μ, s^+) r=6

$$H = \epsilon \hat{n}_d - \varkappa \hat{Q}\cdot\hat{Q} \quad , \quad \hat{Q}_\mu = d^+_\mu s + s^+ \tilde{d}_\mu + \chi (d^+ \tilde{d})_{2\mu}$$

$$\varkappa / \epsilon = (1-\eta)/\eta N$$

$$H = \frac{\epsilon}{\eta}\left(\eta \cdot \hat{n}_d - \frac{(1-\eta)}{N}\hat{Q}\cdot\hat{Q}\right)$$

$$0 \leq \eta \leq 1 \quad , \quad -\frac{\sqrt{7}}{2} \leq \chi \leq \frac{\sqrt{7}}{2}$$

$\eta \to 1$ spherical limit

$\eta \to 0$ deformed limit

$$|N,\alpha\rangle \sim \left(s^+ + \sum_\mu \alpha_\mu d^+_\mu\right)^N |0\rangle$$

$$E(N,\eta,\chi,\alpha) \equiv \langle N,\alpha|\hat{H}|N,\alpha\rangle / \langle N\alpha|N\alpha\rangle$$

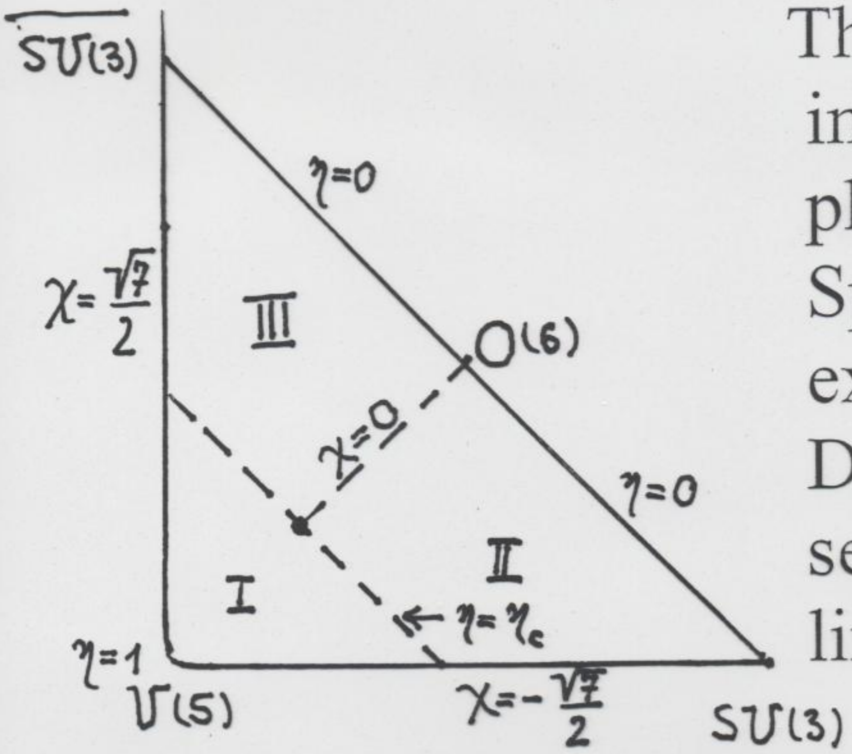
$$\frac{\delta E(N,\eta,\chi,\alpha)}{\delta\alpha} = 0$$ - condition for equilibrium shape

In the case of IBM the coefficient $B(\eta,\chi)$ in the expansion

$$E(N,\eta,\chi;\beta) = E_0(N,\eta,\chi) + A(N,\eta,\chi)\,\beta^2 + B(N,\eta,\chi)\,\beta^3 + C(N,\eta,\chi)\,\beta^4 + \dots$$

is not equal to zero in general case. For this reason phase transition of the 2nd kind between spherical and deformed nuclear shapes can take place only at the isolated point of the plane $\eta \otimes \chi$. This point is located at

$$\chi = 0 \text{ and } \eta = \eta_{trip}(N) = (4N-8)/(5N-8)$$



This is a triple point on the nuclear phase diagram. Spherical phase exists at $\eta > \eta_{trip}$. Deformed phases separated by the line $\chi = 0$ - at $\eta < \eta_{trip}$

$$\eta_c(N,\chi) = \frac{4 + 2\chi^2/7}{5 + 2\chi^2/7} + O\left(\frac{1}{N}\right)$$

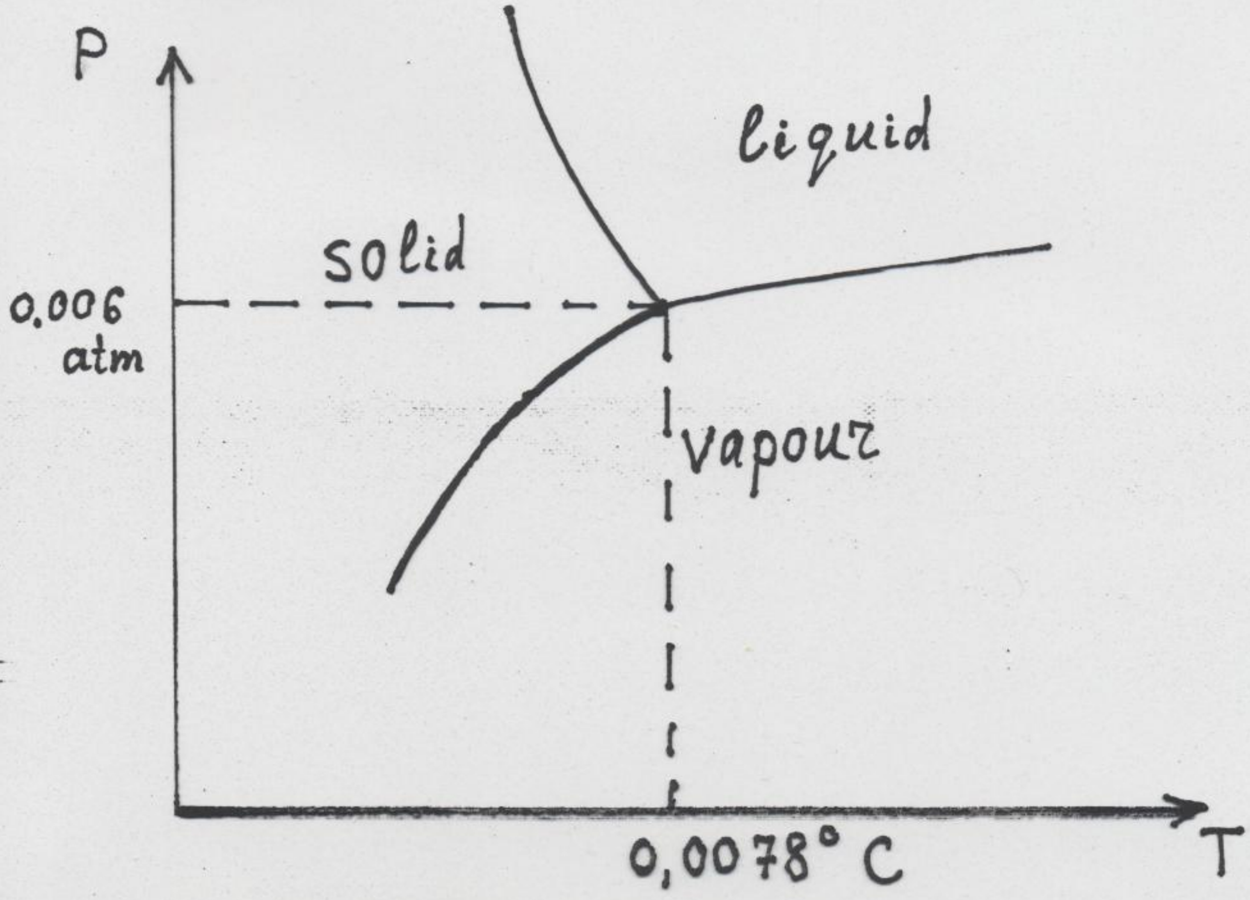
$$\eta_c(N, \chi=0) = \eta_{trip}.$$

NATURE

A TRIPLE POINT IN NUCLEI

D.Warner

The most familiar example of a triple point Is in water: on a graph of pressure and temperature, the three lines separating the Vapour-liquid, liquid-solid and solid-vapo-Ur phases all cross at temperature of 273.15 K



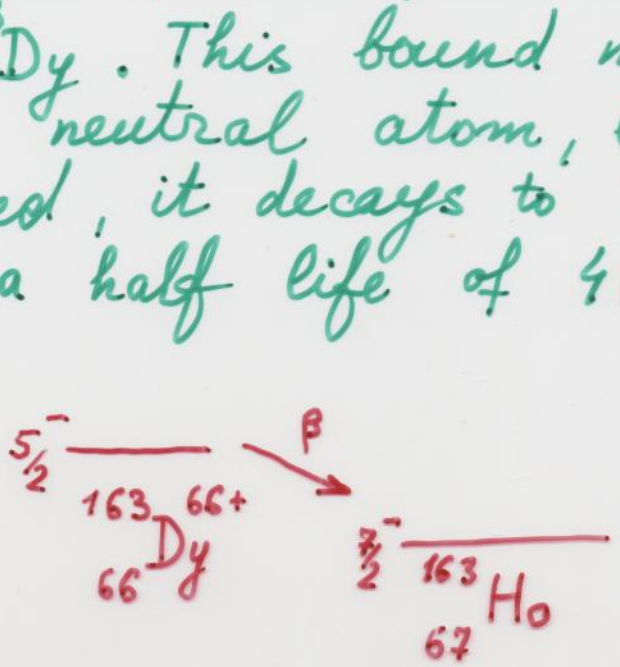
Neutral and ionised atoms and nuclear processes

Ionisation of atom can delay or accelerate nuclear radioactive decay. such situation can be observed in nuclear β - decay

$$n \rightarrow p + e^- + \bar{\nu}_e$$

An emitted e^- can leave atom or occupy an emty place in electron shell. In β^--decay to the bound e^- state the energy is saved because e^- is bound. This increase decay probability which is proportional to E^5. Moreover, other can be opened other decay channels.

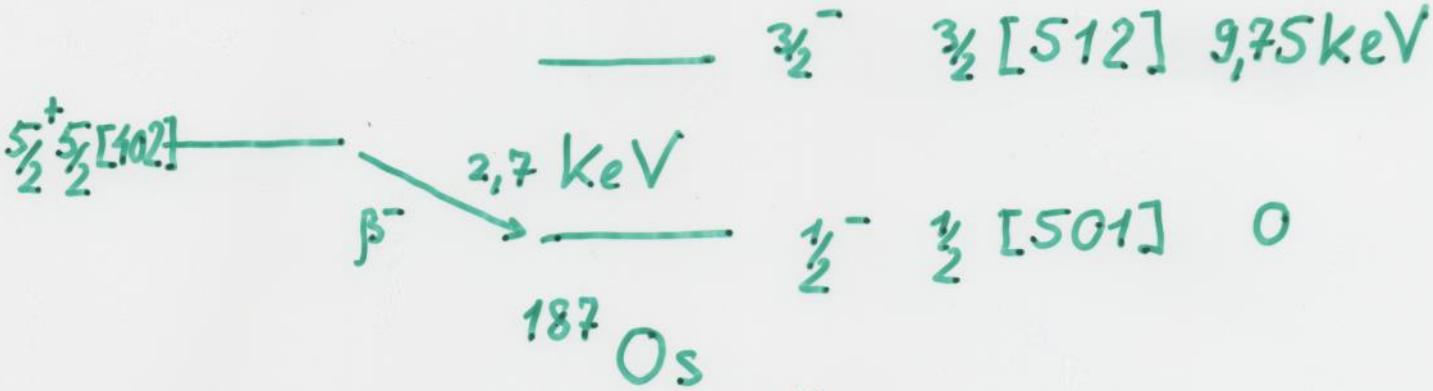
- Experimentally, β-decay was observed for the first time in the case of ^{163}Dy. This bound nucleus is stable as a neutral atom, but when fully ionized, it decays to ^{163}Ho ($Q_\beta = +50,3$ KeV) with a half life of 47 days:



The experiment has been done with a storage ring of GSI.

Other example is β^--decay of ^{187}Re into ^{187}Os.

Decay scheme of neutral atoms



Lifetime : $5 \cdot 10^{10}$ years.

Decay scheme of fully ionized ^{187}Re: