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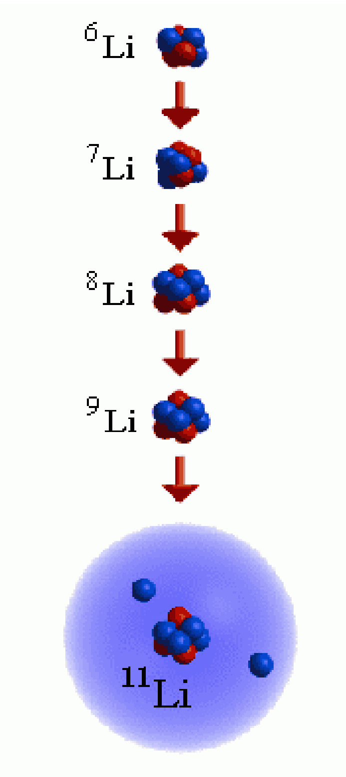
Joint Institute for Nuclear Research

Halo Nuclei: Structure and Reactions



Frontiers of Nuclear Physics

⇩

nucleonic matter under *extreme* conditions

(temperature, angular momentum, *very proton / neutron reach nuclei*, …)

⇩

Physics of Radioactive Ion Beams

Nuclei ⇨ { line of β-stability to the limits of stability
~ zero energy to more than 1 GeV/u }

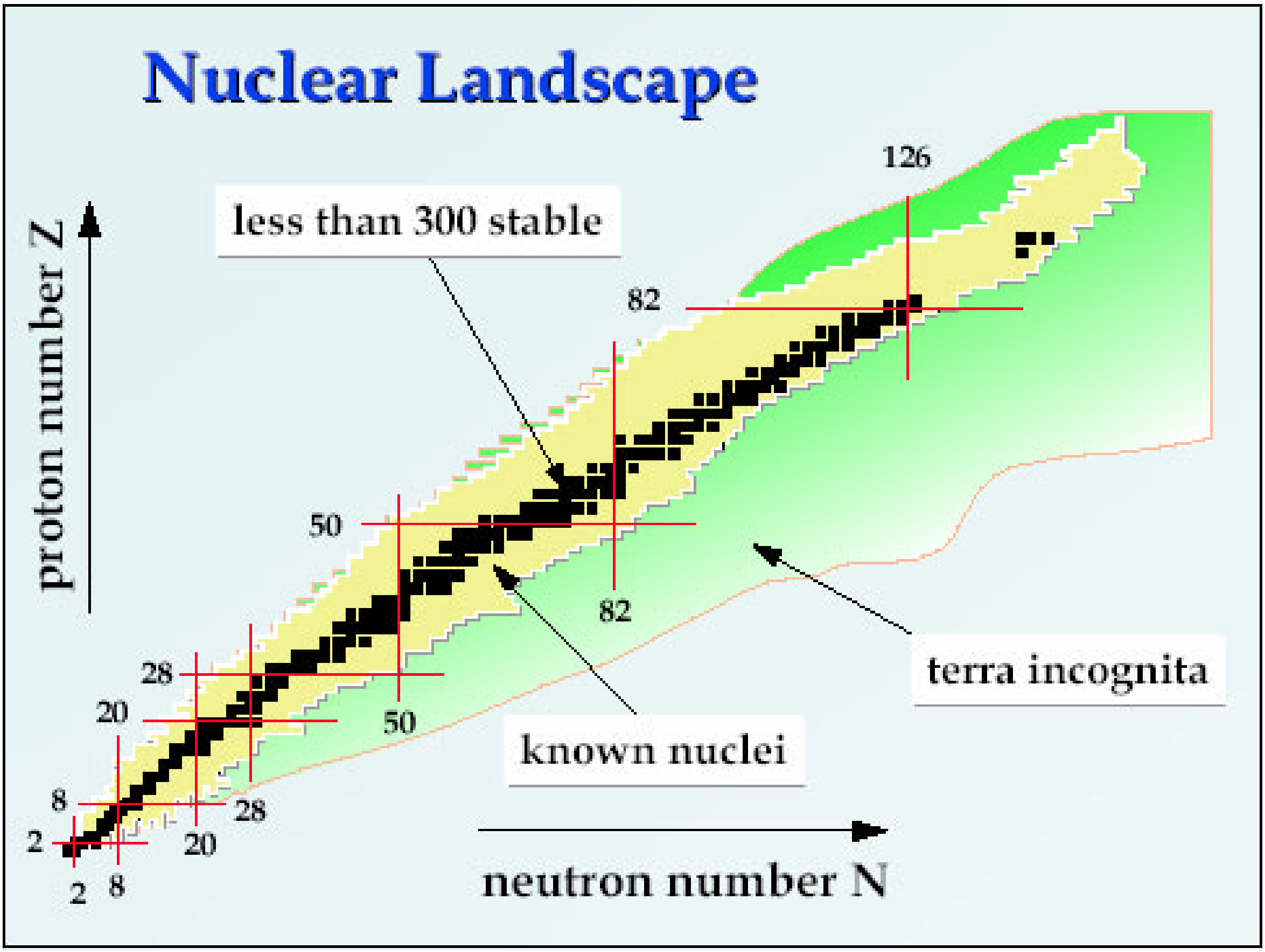
- exact *locations* of the neutron and proton driplines
- producing the *heaviest bound* nuclei
- learning about the *astrophysical* r- and rp- processes
- exploring the *evolution* of shell structure
  (vanishing of magic numbers, new magic numbers, …)
- resonances (nuclei) *beyond* the driplines

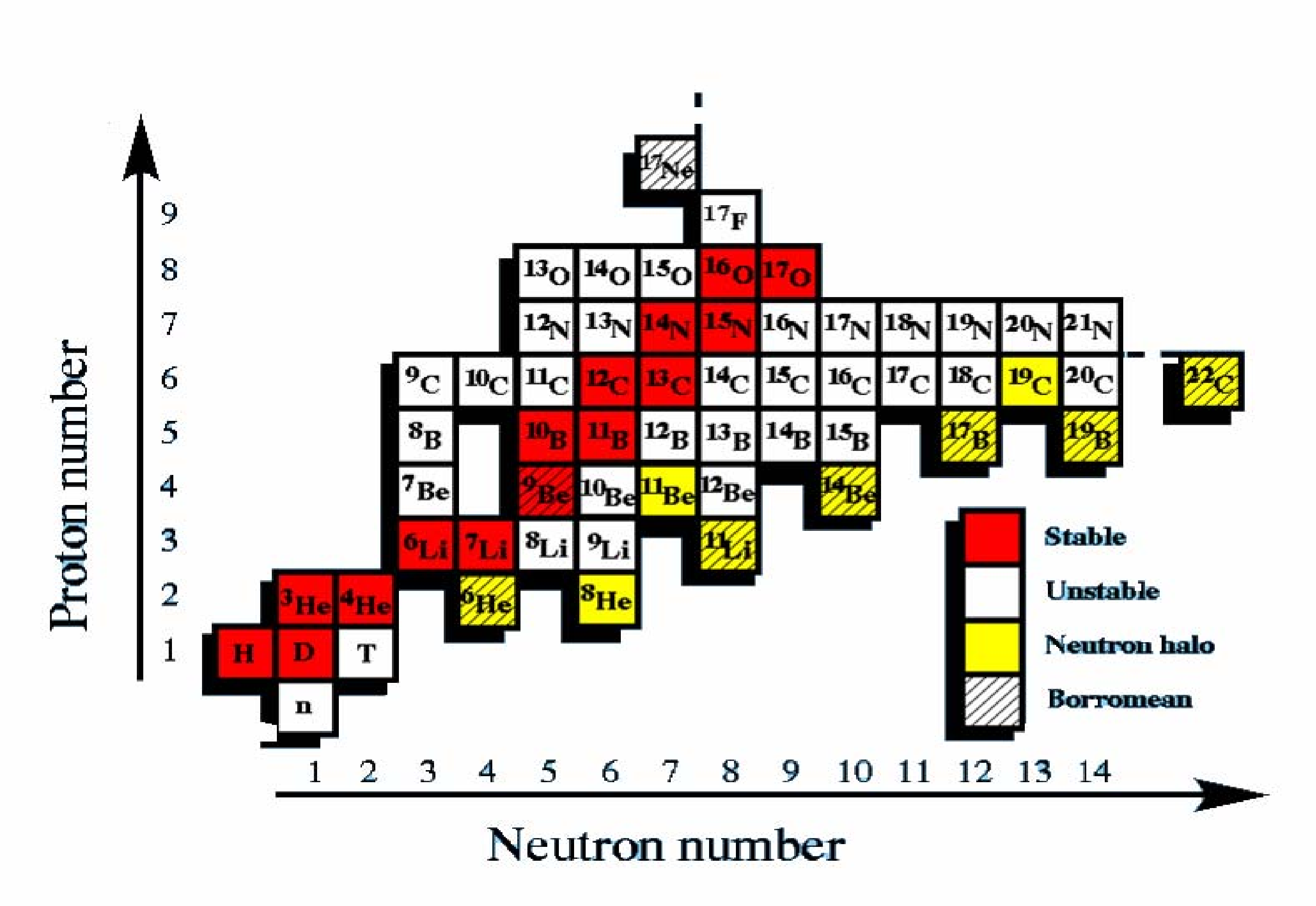
……………..

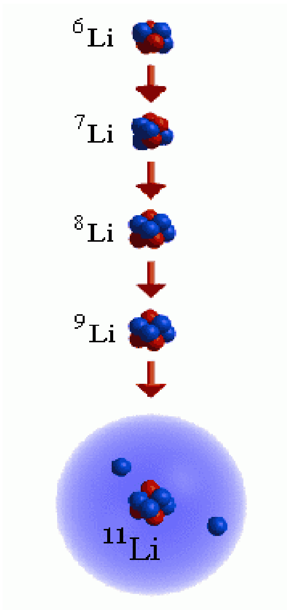
Remarkable *discoveries* have already been made with RIBs

HALO :

new structural dripline phenomenon with clusterization into an ordinary core nucleus and a veil of halo nucleons – forming very dilute neutron matter


Nuclear Landscape
less than 300 stable
proton number Z
neutron number N
known nuclei
terra incognita
126
82
50
28
20
8
2
82
50
28
20
8
2





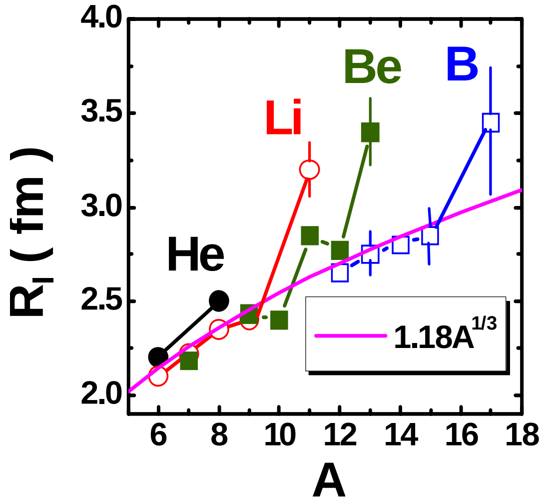
Chains of the lightest isotopes (He, Li, Be, B, …) end up with two neutron halo nuclei

Two neutron halo nuclei (^{6}He , ^{11}Li , ^{14}Be , …) break into three fragments and are all Borromean nuclei

One neutron halo nuclei (^{11}Be , ^{19}C) break into two fragments

Experimental evidence of halo structure

Reaction cross sections



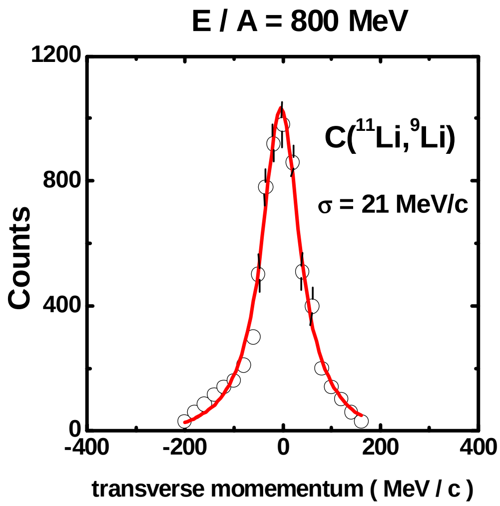
Interaction radii

$$\sigma_I = \pi\left(R_I(proj) + R_I(targ)\right)^2$$

E / A = 790 MeV, *light targets*

I. Tanihata et al., Phys. Rev. Lett., 55 (1985) 2676

Fragment momentum distributions



(naive picture)

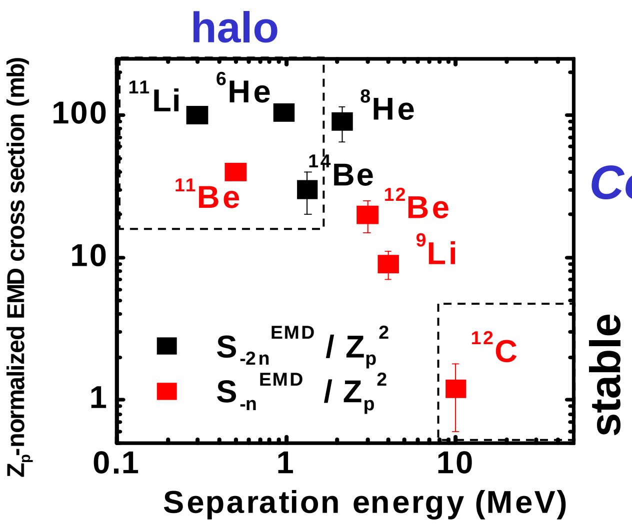
narrow momentum distributions

⇩

large spatial extensions

I. Tanihata et al., Phys. Lett., B297 (1992) 307

Electromagnetic dissociation cross sections



Large

***Coulomb dissociation* cross sections for core + neutron(s) channel**

T. Kobayashi, Proc. 1st Int. Conf. On Radiactive Nuclear Beams, 1990.

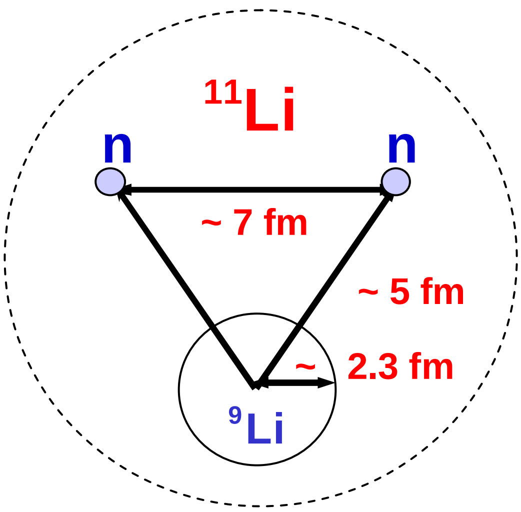
Neutron halo nuclei

(^{11}Li, ^{6}He, ^{11}Be, ^{14}Be, ^{17}B, . . .)

Halo → **weakly bound systems with large extension and space granularity**

"Residence in *forbidden* regions"

Appreciable probability for dilute nuclear matter extending far out into *classically forbidden* region



Separation energies of last neutron (s)

halo : < 1 MeV
stable : ~ 6 - 8 MeV

$\varepsilon(^{11}\mathrm{Li}) = 0.3$ MeV

$\varepsilon(^{11}\mathrm{Be}) = 0.5$ MeV

$\varepsilon(^{6}\mathrm{He}) = 0.97$ MeV

***Large size* of halo nuclei** $\left\{ \begin{array}{l} \langle r^2(^{11}\mathrm{Li})\rangle^{1/2} \sim 3.5 \text{ fm} \\ (\sim \text{r.m.s. for } A \sim 48) \end{array} \right.$

***Two-neutron* halo nuclei (^{11}*Li*, ^{6}*He*, ^{11}*Be*, ^{14}*Be*, ^{17}*B*, . . .)** → ***Borromean* systems**

Borromean* system is *bound → **none of the constituent *two-body* subsystems *are* bound**

Stable nuclei

N / Z ~ 1 - 1.5

ε_S ~ 6 - 8 MeV

ρ_0 ~ 0.15 fm $^{-3}$

proton and neutrons homogeneously mixed, no decoupling of proton and neutron distributions

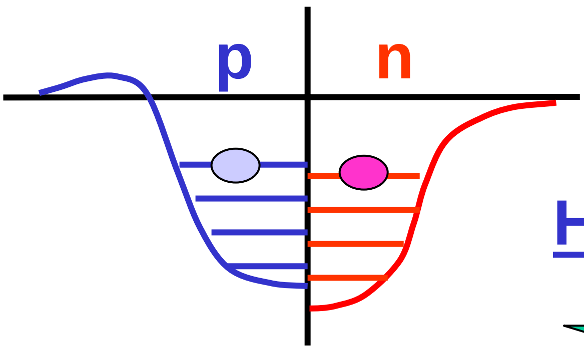
Unstable nuclei

N / Z ~ 0.6 - 4

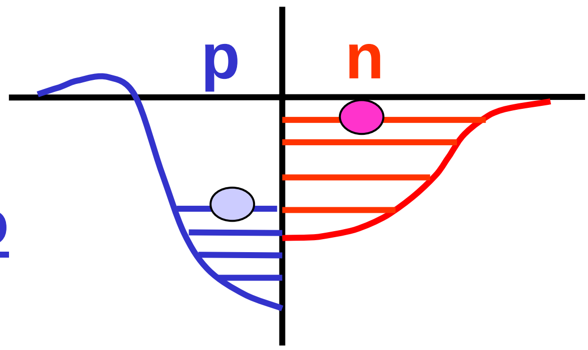
ε_S ~ 0 - 40 MeV

decoupling of proton and neutron distributions

neutron halos and neutron skins

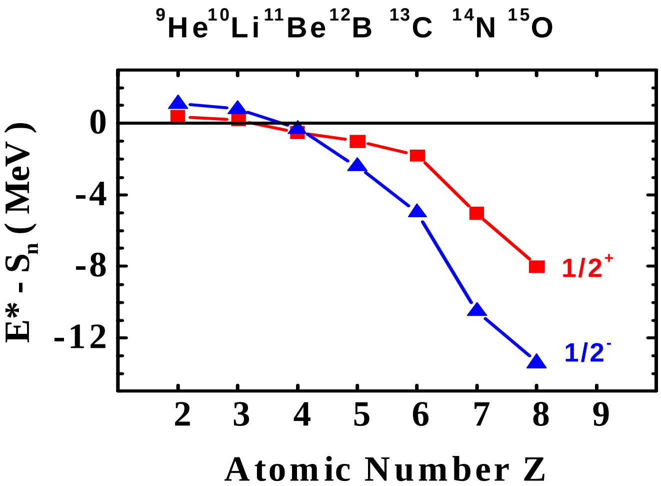


Halo



new structure with clusterization into an ordinary core nucleus and veil of halo nucleons
the only example of dilute nuclear matter

Prerequisite ⇨ low angular momentum motion for halo particles and few-body dynamics



1s - intruder level

^{11}Be parity inversion of g.s.

^{10}Li g.s. : $\left[\pi 0p_{\frac{3}{2}} \otimes \nu 1s_{\frac{1}{2}}\right]_{2^-}$

Peculiarities of halo

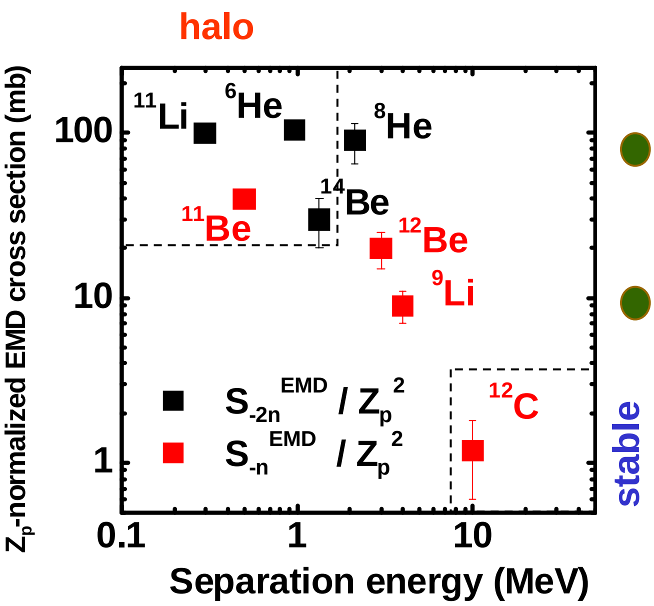
- **in *ground* state**

  weakly bound,
  with large extension
  and space granularity

- **in *low-energy* continuum**

concentration of the transition strength near break up threshold - soft modes

***Large* EMD cross sections** ⇒ **specific nuclear property of extremely neutron-rich nuclei**



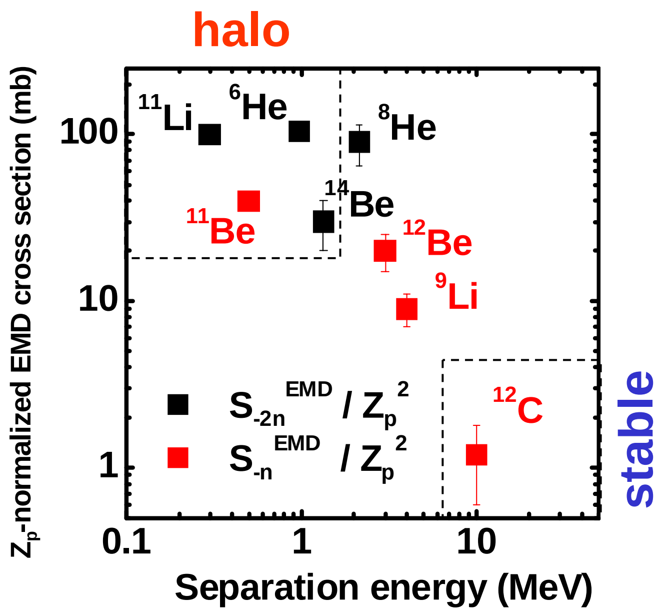
- **excitations of soft modes with different multipolarity**
- **collective excitations *versus* direct transition from weakly bound to continuum states**

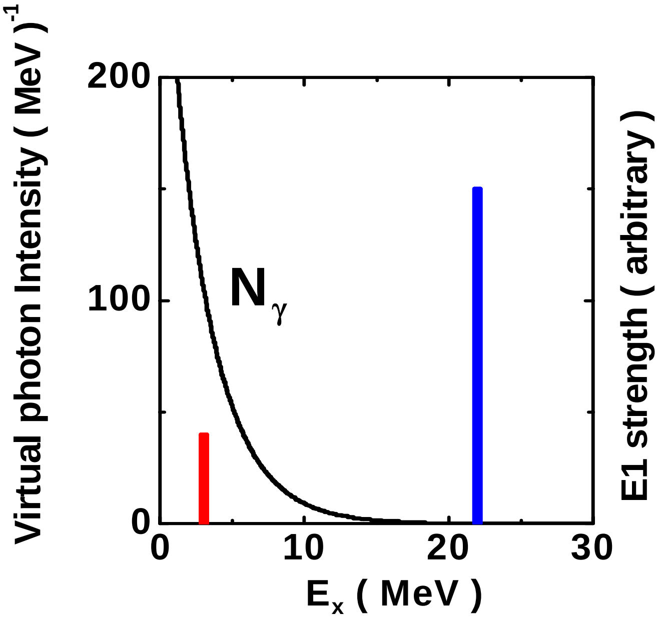
T. Kobayashi, Proc. 1st Int. Conf. On Radiactive Nuclear Beams, 1990.

Soft Excitation Modes

(peculiarities of low energy halo continuum)

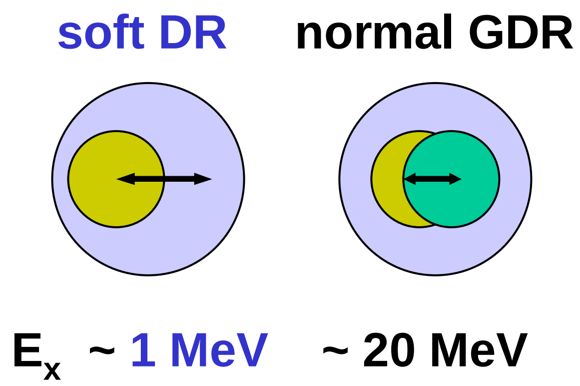
Large **EMD cross sections** ⇨ specific nuclear property of extremely neutron-rich nuclei

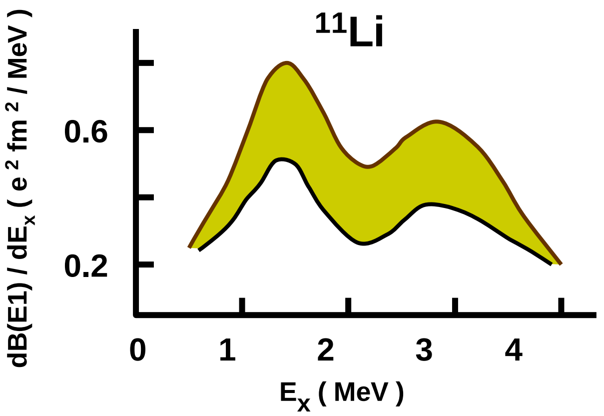




$$\sigma_{\mathrm{EMD}} = \int \mathrm{N}(E_{\mathrm{X}})\, \sigma_\gamma(E_{\mathrm{X}})\, dE_{\mathrm{X}}$$

$$\sigma_\gamma(E_{\mathrm{X}}) = \frac{16\pi^3}{9\hbar c} E_{\mathrm{X}} \frac{d\mathrm{B}(\mathrm{E1})}{dE_{\mathrm{X}}}$$





M. Zinser et al., Nucl. Phys. A619 (1997) 151

- excitations of soft modes with different multipolarity
- collective excitations *versus* direct transition from weakly bound to continuum states

BASIC **dynamics of halo nuclei** → ***Decoupling*** **of halo and nuclear core degrees of freedom**

$$|\Psi\rangle = |\psi\rangle_{\text{halo}} |\Phi\rangle_{\text{core}}$$

EVIDENCES

- **weakly bound** → **break up into three fragments**

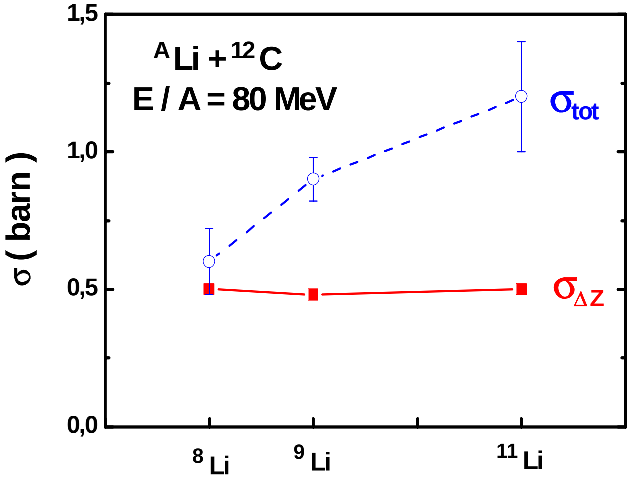
- **Relations between interaction and neutron removal cross sections (mb) at 790 MeV/A**

A + ^{12}C	σ_I	σ_{-2n}	σ_{-4n}
^{9}Li	796 ±6		
^{11}Li	1060 ±10	220 ±40	
^{4}He	503 ±5		
^{6}He	722 ±5	189 ±14	
^{8}He	817 ±6	202 ±17	95 ±5

$$\sigma_I(\mathbf{a}) = \sigma_I(\mathbf{c}) + \sigma_{\mathbf{-xn}}$$

Tanihata I. et al.
PRL, 55 (1987) 2670;
PL, B289 (1992) 263

- **charge – changing cross sections**

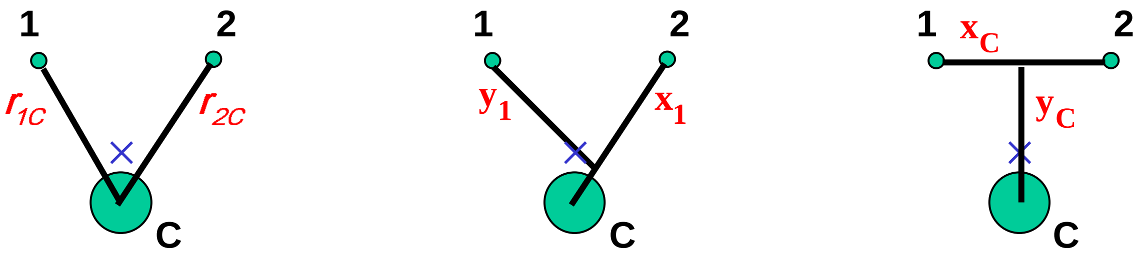


Blank B. et al.,
Z. Phys. A343 (1992) 375

- **static properties :**

	^{9}Li	^{11}Li
quadrupole moments	-27.4 ±1.0 mb	-31.2 ± 4.5 mb
magnetic moments	3.4391 ±0.0006 n.m.	3.6678 ±0.0025 n.m.

Schmidt limit : 3.71 n.m.



V - basis   **Y - basis**   **T - basis**

The normalised *T*-set of Jacobi coordinates ($A_i = m_i / m$)

$$\bar{x}_C = \sqrt{A_{12}}\,(\bar{r}_1 - \bar{r}_2), \qquad A_{12} = \frac{A_1 A_2}{A_1 + A_2}$$

$$\bar{y}_C = \sqrt{A_{(12)C}}\left(\bar{r}_C - \frac{A_1 \bar{r}_1 + A_2 \bar{r}_2}{A_1 + A_2}\right), \qquad A_{(12)C} = \frac{(A_1 + A_2) A_C}{A_1 + A_2 + A_C}$$

$$\bar{R} = \frac{1}{A}\left(A_1 \bar{r}_1 + A_2 \bar{r}_2 + A_C \bar{r}_C\right), \qquad A = (A_1 + A_2 + A_C)$$

The hyperspherical coordnates : $\rho,\ \alpha_C,\ \theta_{x_C},\ \varphi_{x_C},\ \theta_{y_C},\ \varphi_{y_C}$

$$\rho = \sqrt{x_C^2 + y_C^2} = \sqrt{\sum_{i=1}^{3} A_i \left(\bar{r}_i - \bar{R}\right)^2} = \sqrt{\frac{1}{A}\sum_{i>j=1}^{3} A_i\, A_j \left(\bar{r}_i - \bar{r}_j\right)^2}$$

$$\alpha_C = \arctan\left(\frac{x_C}{y_C}\right), \qquad 0 \le \alpha_C \le \frac{\pi}{2}$$

ρ is the rotation, translation and permutation invariant variable

$$\rho^2 = x_1^2 + y_1^2 = x_2^2 + y_2^2 = x_C^2 + y_C^2 \qquad \begin{cases} x_i = \rho \sin \alpha_i \\ y_i = \rho \cos \alpha_i \end{cases}$$

Volume element in the 6-dimensional space

$$d\bar{x}_i\, d\bar{y}_i = x_i^2\, dx_i\ \ y_i^2 dy_i\ d\Omega_{x_i}\ d\Omega_{y_i}$$

$$= \rho^5\, d\rho\ \sin^2\alpha_i \cos^2\alpha_i\, d\alpha_i\ d\Omega_{x_i}\ d\Omega_{y_i} = \rho^5\, d\rho\, d\Omega_5^i$$

The kinetic energy operator T has the separable form

$$T = -\frac{\hbar^2}{2m}\left(\frac{\hbar^2}{A_{12}}\Delta_x + \frac{\hbar^2}{A_{(12)C}}\Delta_y\right) = -\frac{\hbar^2}{2m}\Delta_6$$

$$= -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial\rho^2} + \frac{5}{\rho}\frac{\partial}{\partial\rho} - \frac{1}{\rho^2}\widehat{K}^2\left(\Omega_5^{\mathrm{i}}\right)\right)$$

$\widehat{K}^2\left(\Omega_5^{\mathrm{i}}\right)$ is a square of the 6-dimensional hyperorbital momentum

$$\widehat{K}^2\left(\Omega_5^{\mathrm{i}}\right) = -\frac{\partial^2}{\partial\alpha_{\mathrm{i}}^2} - 4\cot\left(2\alpha_{\mathrm{i}}\right)\frac{\partial}{\partial\alpha_{\mathrm{i}}} + \frac{1}{\sin^2\alpha_{\mathrm{i}}}\hat{l}^2\left(\hat{x}_{\mathrm{i}}\right) + \frac{1}{\cos^2\alpha_{\mathrm{i}}}\hat{l}^2\left(\hat{y}_{\mathrm{i}}\right)$$

Eigenfunctions of Δ_6 are the homogeneous harmonic polynomials

$$\Delta_6\, P_K\left(\bar{x}, \bar{y}\right) = \Delta_6\, \rho^K\, \Phi_K^{l_{x_{\mathrm{i}}} m_{x_{\mathrm{i}}}, l_{y_{\mathrm{i}}} m_{y_{\mathrm{i}}}}\left(\Omega_5^{\mathrm{i}}\right) = 0$$

$$\left\{\widehat{K}^2\left(\Omega_5^{\mathrm{i}}\right) - K\left(K+4\right)\right\}\Phi_K^{l_{x_{\mathrm{i}}} m_{x_{\mathrm{i}}}, l_{y_{\mathrm{i}}} m_{y_{\mathrm{i}}}}\left(\Omega_5^{\mathrm{i}}\right) = 0$$

$\Phi_K^{l_{x_{\mathrm{i}}} m_{x_{\mathrm{i}}}, l_{y_{\mathrm{i}}} m_{y_{\mathrm{i}}}}\left(\Omega_5^{\mathrm{i}}\right)$ are hyperspherical harmonics or K-harmonics.

They give a complete set of orthogonal functions in
the 6-dimensional space on unit hypersphere $\left(K = l_{x_{\mathrm{i}}} + l_{y_{\mathrm{i}}} + 2n\right)$

$$\Phi_K^{l_{x_{\mathrm{i}}} m_{x_{\mathrm{i}}}, l_{y_{\mathrm{i}}} m_{y_{\mathrm{i}}}}\left(\Omega_5^{\mathrm{i}}\right) = N_K^{l_{x_{\mathrm{i}}} l_{y_{\mathrm{i}}}}\, Y_{l_{x_{\mathrm{i}}} m_{x_{\mathrm{i}}}}\left(\hat{x}_{\mathrm{i}}\right) Y_{l_{y_{\mathrm{i}}} m_{y_{\mathrm{i}}}}\left(\hat{y}_{\mathrm{i}}\right)$$

$$*\left(\sin\alpha_{\mathrm{i}}\right)^{l_{x_{\mathrm{i}}}}\left(\cos\alpha_{\mathrm{i}}\right)^{l_{y_{\mathrm{i}}}} P_{\frac{1}{2}\left(K - l_{x_{\mathrm{i}}} - l_{y_{\mathrm{i}}}\right)}^{\left(l_{x_{\mathrm{i}}}+\frac{1}{2},\, l_{y_{\mathrm{i}}}+\frac{1}{2}\right)}\left(\cos 2\alpha_{\mathrm{i}}\right)$$

$P_n^{(\alpha,\beta)}\left(z\right)$ are the Jacobi polynomials

$Y_{lm}\left(\hat{x}\right)$ are the spherical harmonics

The functions with fixed total orbital moment $\bar{L} = \bar{l}_x + \bar{l}_y$

and its projection M_L **are linear combination of HH .**

$$\Phi_{KLM}^{l_x, l_y}\left(\Omega_5^i\right) = \sum_{m_x, m_y} \left(l_x m_x, l_y m_y \middle| LM\right) \Phi_K^{l_x m_x, l_y m_y}\left(\Omega_5^i\right)$$

a normalizing coefficient $N_K^{l_{x_i} l_{y_i}}$ **is defined by the relation**

$$\int d\Omega_5^i \, \Phi_{K'L'M'}^{*l'_x, l'_y}\left(\Omega_5^i\right) \Phi_{KLM}^{l_x, l_y}\left(\Omega_5^i\right) = \delta_{KK'} \delta_{LL'} \delta_{MM'} \delta_{l_x l_{x'}} \delta_{l_y l_{y'}}$$

The parity of HH depends only on $K = l_x + l_y + 2n$

$$\text{parity is} \begin{cases} + (\text{positive}), \text{ if } K \text{ - even} \\ - (\text{negative}), \text{ if } K \text{ - odd} \end{cases}$$

The three equivalent sets of Jacobi coordinates are connected by transformation (kinematic rotation)

$$\begin{cases} \bar{x}_j = -\cos\varphi_{ji} \, \bar{x}_i - \sin\varphi_{ji} \, \bar{y}_i \\ \bar{y}_j = \sin\varphi_{ji} \, \bar{x}_i - \cos\varphi_{ji} \, \bar{y}_i \end{cases} \qquad \varphi_{ji} = \varphi_{ji}\left(A_1, A_2, A_C\right)$$

Quantum numbers *K, L, M* don't change under a kinematic rotation. HH are transformed in a simple way and the parity is also conserved.

$$\Phi_{KLM}^{l_{x_i}, l_{y_i}}\left(\Omega_5^i\right) = \sum_{l_{x_k}, l_{y_k}} \underbrace{\left\langle l_{x_k}, l_{y_k} \middle| l_{x_i}, l_{y_i} \right\rangle_{KL}} \Phi_{KLM}^{l_{x_k}, l_{y_k}}\left(\Omega_5^k\right)$$

$\Downarrow$

Reynal-Revai coefficients

The three-body bound-state and continuum wave functions (within cluster representation)

$$\Psi_{JM} = \phi_C(\xi_C)\,\Phi_{JM}(\bar{\mathbf{x}},\bar{\mathbf{y}})\frac{1}{(2\pi)^{3/2}}\exp\{i(\bar{P}\circ\bar{R})\}$$

The Schrodinger 3-body equation for the wave function $\Phi_{JM}(\bar{\mathbf{x}},\bar{\mathbf{y}})$

$$(T+V-E)\,\Phi_{JM}(\bar{\mathbf{x}},\bar{\mathbf{y}})=0$$

where the kinetic energy operator : $T=-\frac{\hbar^2}{2\mu_x}\Delta_x-\frac{\hbar^2}{2\mu_y}\Delta_y$

and the interaction : $V=V_{12}(\bar{r}_{12})+V_{1C}(\bar{r}_{1C})+V_{2C}(\bar{r}_{1C})$

The bound state wave function (E < 0)

$$\Phi_{JM}(\bar{\mathbf{x}},\bar{\mathbf{y}})=\rho^{-5/2}\sum_{LSKl_Xl_Y}\chi^{LS}_{Kl_Xl_Y}(\rho)\left[\Phi^{l_x,l_y}_{KL}(\Omega^{\mathrm{i}}_5)\otimes\chi_S\right]_{JM}$$

$\chi_{SM_S}=\left[\,|1/2\rangle_1\otimes|1/2\rangle_2\right]_{SM_S}$ **- spin function of two nucleons**

The continuum wave function (E > 0)

$$\Phi_{S'M'_S}(\bar{k}_x,\bar{k}_y,\bar{\mathbf{x}},\bar{\mathbf{y}})=(\kappa\rho)^{-5/2}\sum_{\gamma,\gamma'}\chi^{LS,L'S'}_{Kl_Xl_Y,K'l'_Xl'_Y}(\kappa,\rho)$$

$$*\left[\Phi^{l_x,l_y}_{KL}(\Omega^{\mathrm{i}}_5)\otimes\chi_S\right]_{JM} i^{K'}(L'M'_L\,S'M'_S|J\,M)\,\Phi^{*l'_x,l'_y}_{K'L'}(\Omega^{\kappa}_5)$$

$\kappa=\sqrt{k_x^2+k_y^2}=\frac{1}{\hbar}\sqrt{2m|E|}$ **is the momentum conjugated to ρ**

The HH expansion of the 6-dimensional plane wave

$$\exp\left\{i\left(\bar{k}_x \circ \bar{\mathbf{x}} + \bar{k}_y \circ \bar{\mathbf{y}}\right)\right\} = \frac{(2\pi)^3}{(\kappa\rho)^2} \sum_{\gamma} i^K J_{K+2}(\kappa\rho)\, \Phi_{KLM}^{l_x,l_y}\left(\Omega_5^i\right) \Phi_{KLM}^{*l_x,l_y}\left(\Omega_5^\kappa\right)$$

Normalization condition for bound state wave function

$$\int d\bar{\mathbf{x}}\, d\bar{\mathbf{y}}\; \Phi_{J'M'}^{*}\left(\bar{\mathbf{x}},\bar{\mathbf{y}}\right) \Phi_{JM}\left(\bar{\mathbf{x}},\bar{\mathbf{y}}\right) = \delta_{JJ'}\,\delta_{MM'}$$

Normalization condition for continuum wave function

$$\int d\bar{\mathbf{x}}\, d\bar{\mathbf{y}}\; \Phi_{S'M'_S}^{*}\left(\bar{k}'_x, \bar{k}'_y, \bar{\mathbf{x}}, \bar{\mathbf{y}}\right) \Phi_{SM_S}\left(\bar{k}_x, \bar{k}_y, \bar{\mathbf{x}}, \bar{\mathbf{y}}\right) =$$

$$\delta_{SS'}\,\delta_{M_S M'_S}\,\delta\left(\bar{k}'_x - \bar{k}_x\right)\delta\left(\bar{k}'_y - \bar{k}_y\right) = \delta_{SS'}\,\delta_{M_S M'_S}\,\frac{1}{\kappa^5}\,\delta\left(\kappa' - \kappa\right)\delta\left(\Omega_5'^{\kappa} - \Omega_5^{\kappa}\right)$$

After projecting onto the hyperangular part of the wave function the Schrodinger equation is reduced to a set of coupled equations

$$\left\{-\frac{\hbar^2}{2m}\left[\frac{d^2}{d\rho^2} + \frac{\Lambda(\Lambda+1)}{\rho^2}\right] + V_{K\gamma,K\gamma}(\rho) - E\right\} \chi_{K\gamma}(\rho)$$

$$= -\sum_{K'\gamma' \neq K\gamma} V_{K\gamma,K'\gamma'}(\rho)\, \chi_{K'\gamma'}(\rho)$$

where $\Lambda = K + 3/2$ and partial-wave coupling interactions

$$V_{K\gamma,K'\gamma'}(\rho) = \left\langle \Phi_{K\gamma}\left(\Omega_5^i\right) \middle| V_{12}\left(\bar{r}_{12}\right) + V_{1C}\left(\bar{r}_{1C}\right) + V_{2C}\left(\bar{r}_{1C}\right) \middle| \Phi_{K'\gamma'}\left(\Omega_5^i\right) \right\rangle$$

the boundary conditions

$$\chi_{K\gamma}(\rho \Rightarrow 0) \;\sim\; \rho^{\Lambda+1} = \rho^{K+5/2}$$

The asymptotic hyperradial behaviour of $V_{K\gamma,K'\gamma'}(\rho)$

The simplest case : $K=K',\ \gamma=\gamma',\ K=0,\ l_x=0,\ l_y=0$

two-body potentials : $V_{ij}=V_{jk}=V_{ki} \Rightarrow$ a square well, radius R

$$V_{00}(\rho)=3\int d\Omega_5^i\ \Phi_{000}^{00}(\Omega_5^i)\, V_{jk}(\bar{x}_i)\, \Phi_{000}^{00}(\Omega_5^i)$$

$$=3\int_0^{\pi/2} d\alpha \sin^2\alpha\cos^2\alpha\, V_{jk}(\rho\sin\alpha) \xrightarrow[\rho\to\infty]{} \int_0^{R/\rho} d\alpha\,\alpha^2 \sim \frac{1}{\rho^3}$$

$\frac{1}{\rho^3} \Rightarrow$ a general behaviour of three-body effective potential if the two-body potentials are short-range potentials

At $\rho\to\infty$ the system of differential equations is decoupled since effective potentials can be neglected.

$$\left\{-\frac{\hbar^2}{2m}\left[\frac{d^2}{d\rho^2}+\frac{\Lambda(\Lambda+1)}{\rho^2}\right]-E\right\}\chi_{K\gamma}(\rho)=0$$

if E < 0

$$\chi_{K\gamma}(\rho\to\infty)\sim\exp(-\kappa\rho) \Rightarrow \Phi_{JM}(\bar{x},\bar{y})\sim\frac{1}{\rho^{5/2}}\exp(-\kappa\rho)$$

if E > 0

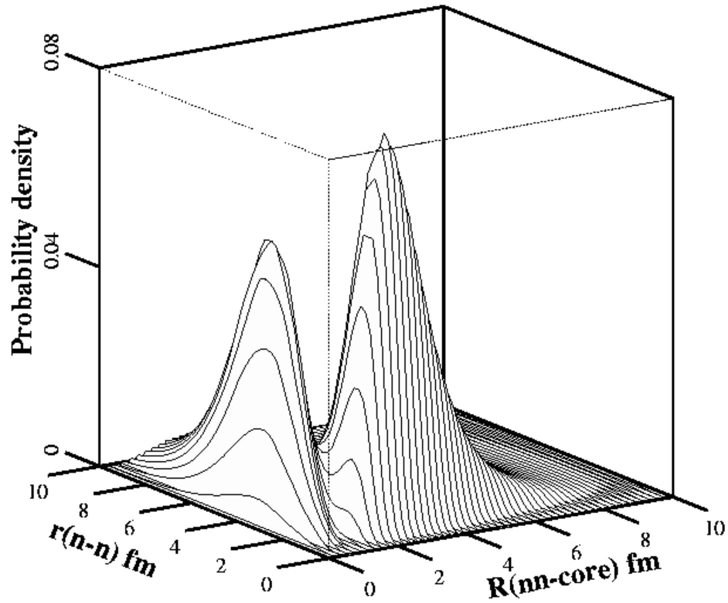
$$\chi_{K\gamma,K'\gamma'}(\rho\to\infty)\sim\sqrt{\kappa\rho}\left[H_{K+2}^{(-)}(\kappa\rho)\,\delta_{K\gamma,K'\gamma'}-S_{K\gamma,K'\gamma'}H_{K+2}^{(+)}(\kappa\rho)\right]$$

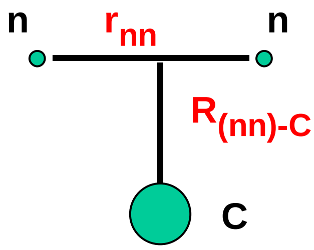
$$H_{K+2}^{(\pm)}(\kappa\rho)\sim\frac{1}{\sqrt{\kappa\rho}}\exp(\pm i\kappa\rho)$$

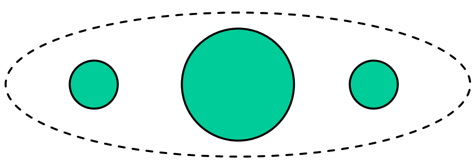
$$\Phi_{SM_S}(\bar{k}_x,\bar{k}_y,\bar{x},\bar{y})\sim\frac{1}{\rho^{5/2}}\left(A\sin(\kappa\rho)+B\cos(\kappa\rho)\right)$$

Correlation density for the ground state of ^{6}He

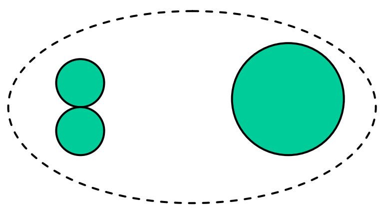
$$P\left(r_{nn},R_{nn\text{-}C}\right)=r_{nn}^{2}\,R_{nn\text{-}C}^{2}\,\frac{1}{2J+1}\sum_{M}\int d\hat{r}_{nn}\,d\hat{R}_{nn\text{-}C}\left|\Psi_{JM}^{T}\left(\bar{r}_{nn},\bar{R}_{nn\text{-}C}\right)\right|^{2}$$





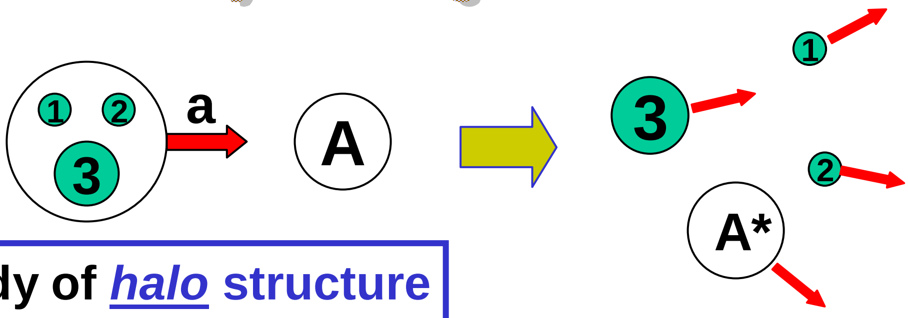


cigar-like configuration



di-neutron configuration

Three-body halo fragmentation reactions



Study of *halo* structure ⇓ **events with *undestroyed* core** ⇒ ***peripheral* reactions**

***complex* constituents**

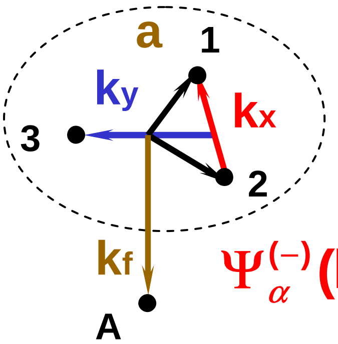
$$a + A \to \begin{cases} 1 + 2 + 3 + A_{gr}, & \text{elastic (4-body)} \\ 1 + 2 + 3 + A^*, & \text{inelastic (} \geq \text{ 4-bodies)} \end{cases}$$

Cross section

$$\sigma = \frac{(2\pi)^4}{\hbar v_i} \sum_{\alpha} \int dk_1 dk_2 dk_3 dk_{A^*} \delta(E_i - E_f) \delta(\vec{P}_i - \vec{P}_f) \left| T_{fi} \right|^2$$

Reaction amplitude T_{fi} (*prior* representation)

$$T_{fi} = \left\langle \Psi_{\alpha}^{(-)}(\vec{k}_f, \vec{k}_x, \vec{k}_y) \left| \sum_{p,t} V_{pt} - U_{aA} \right| \Phi_0, \Psi_{A_{gr}}, \chi_i^{(+)}(\vec{k}_i) \right\rangle$$



$$E_a^* = \frac{k_x^2}{2\mu_x} + \frac{k_y^2}{2\mu_y}$$

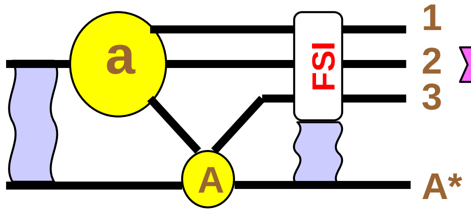
- Φ_0 - halo ground state wave function
- $\Psi_{A_{gr}}$ - target ground state wave function
- $\chi_i^{(+)}(\vec{k}_i)$ - distorted wave for relative projectile-target motion
- $\Psi_{\alpha}^{(-)}(\vec{k}_f, \vec{k}_x, \vec{k}_y)$ - exact scattering wave function
- V_{pt} - NN - interaction between projectile and target nucleons
- U_{aA} - optical potential in initial channel

Reaction amplitude T_{fi} (prior representation)

$$T_{fi} = \left\langle \Psi_{\alpha}^{(-)}(\vec{k}_f, \vec{k}_x, \vec{k}_y) \left| \sum_{p,t} V_{pt} - U_{aA} \right| \Phi_0, \Psi_{A_{gr}}, \chi_i^{(+)}(\vec{k}_i) \right\rangle$$

DW: *low-energy* **halo excitations** $\Rightarrow$ *small* k_x & k_y
(no spectators, three-body continuum, full scale FSI)

$$T_{fi} = \left\langle \chi_f^{(-)}(\vec{k}_f), \Psi_{A_{gr}}, \Phi^{(-)}(\vec{k}_x, \vec{k}_y) \left| \sum_{p,t} V_{pt} \right| \Phi_0, \Psi_{A_{gr}}, \chi_i^{(+)}(\vec{k}_i) \right\rangle$$

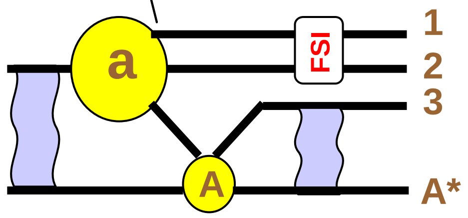


Kinematically complete experiments
- sensitivity to 3-body correlations (halo)
- selection of halo excitation energy
- variety of observables
- elastic & inelastic breakup

$$\vec{k}_x = \mu_x \left(\frac{\vec{k}_2}{m_2} - \frac{\vec{k}_1}{m_1} \right) \quad \vec{k}_y = \mu_y \left(\frac{\vec{k}_3}{m_3} - \frac{\vec{k}_1 + \vec{k}_2}{m_1 + m_2} \right) \quad \Rightarrow \vec{k}_3 \text{ (in the halo rest frame)}$$

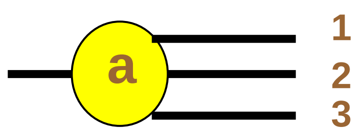
DW: *high-energy* **halo excitations** $\Rightarrow$ *small* k_x & *large* k_y
(spectators-participant, two-body continuum, part of FSI)

$$T_{fi} = \left\langle \chi_{1A^*}^{(-)}(\vec{k}_1 - \vec{k}_{A^*}), \Psi_{A_{gr}}, \Phi^{(-)}(\vec{k}_x) \left| \sum_{p,t} V_{pt} - U_{aA} \right| \Phi_0, \Psi_{A_{gr}}, \chi_i^{(+)}(\vec{k}_i) \right\rangle$$



two-particle coincidence
- sensitivity to 2-body correlations
- large cross sections
- integrated over *halo* excitations

Serber model $\Rightarrow$ *large* k_x & k_y
(spectator-participants, plane waves, no FSI)



$$T_{fi} \sim \left\langle e^{-i(\bar{k}_1 \circ \bar{r}_1)}, e^{-i(\bar{k}_2 \circ \bar{r}_2)}, e^{-i(\bar{k}_3 \circ \bar{r}_3)} \middle| \Phi_0 \right\rangle$$

Model assumptions

$$T_{fi} = \left\langle \Psi_{\alpha}^{(-)}(\vec{k}_f,\vec{k}_x,\vec{k}_y) \middle| \sum_{p,t} V_{pt} - U_{aA} \middle| \Phi_0, \Psi_{A_{gr}}, \chi_i^{(+)}(\vec{k}_i) \right\rangle$$

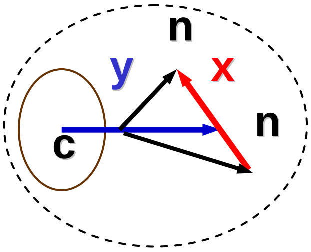
approximations

$$T_{fi} = \left\langle \chi_f^{(-)}(\vec{k}_f), \Psi_{A_{gr}}, \Phi^{(-)}(\vec{k}_x,\vec{k}_y) \middle| \sum_{p,t} V_{pt} \middle| \Phi_0, \Psi_{A_{gr}}, \chi_i^{(+)}(\vec{k}_i) \right\rangle$$

◆ ***Nuclear structure*** ➡ **Transition densities**

Three-body models

$$\langle \Phi^{(-)}(\vec{k}_x,\vec{k}_y) \| \sum_p \frac{\delta(r-r_p)}{r r_p} \left[\Upsilon_l(\hat{r}_p) \times \sigma_p^s \right]_j \| \Phi_0 \rangle$$



Method of hyperspherical harmonics
3-body *bound* and *continuum* states

$$|\Phi\rangle = \varphi_c(\xi)\psi_h(\vec{x},\vec{y})$$

binding energy
electromagnetic moments
electromagnetic formfactors
geometrical properties
density distributions
. . . .

effective interactions
(NN & N-core)

?

◆ ***Reaction mechanism*** ➡ **One-step process**

Distorted wave approach

***effective* NN interactions V_{pt}**

distorted waves χ (r)

self-consistency

complex, energy, density dependent

optical potential

free NN scattering

nucleus-nucleus elastic scattering and reaction cross sections

ELECTRON SCATTERING

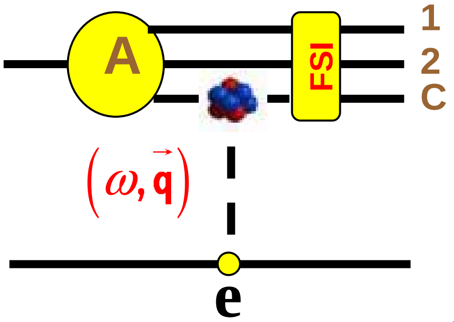
Electromagnetic forces are well known and weak

⇓

Reaction mechanism can be disentangled from nuclear structure

Maxwell equation & Continuity equation

$$\Box A_\mu(x) = 4\pi e \langle f | J_\mu(x) | i \rangle \qquad \partial_\mu J^\mu(x) = 0$$



Approximations:

- one photon exchange
- ultrarelativistic electrons
- small energy and momentum transfer

Inelastic cross-section

$$d\sigma = d\bar{k}_f \; d\bar{p}_1 \; d\bar{p}_2 \; d\bar{p}_C \; \delta^4(k_i + P_i - k_f - P_f) \frac{(\hbar c)^2}{\varepsilon_f^2} \sigma_M \sum V_{\alpha\beta} W_{\alpha\beta}$$

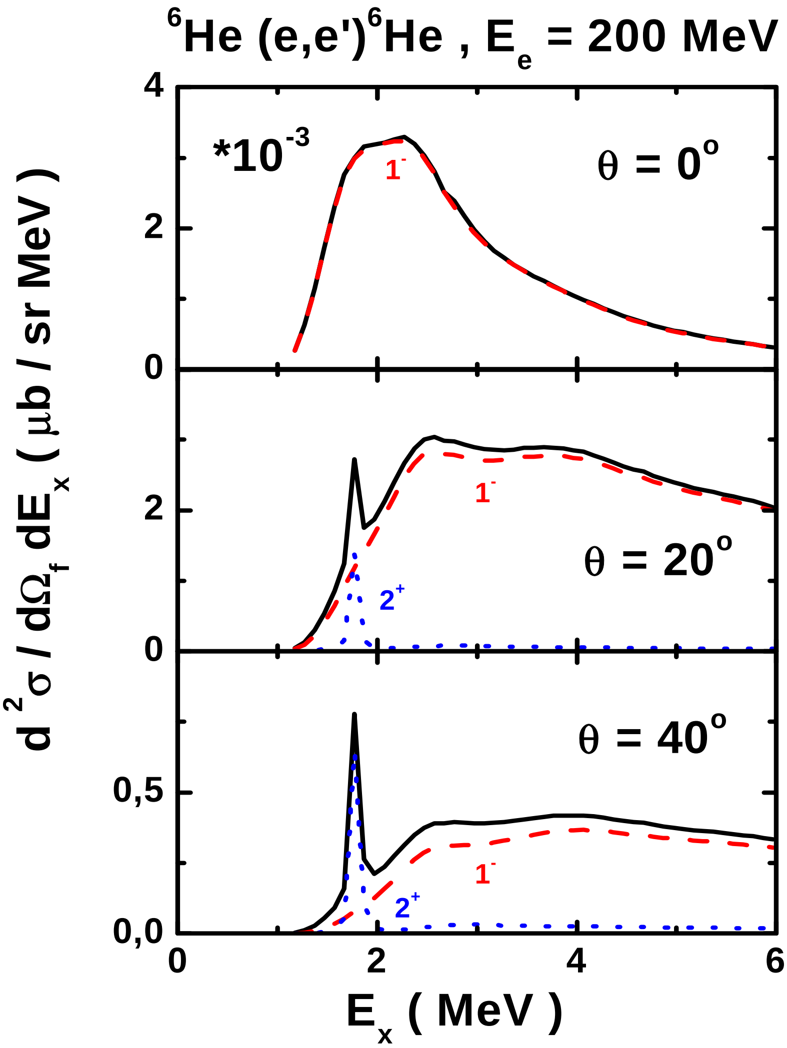
Coulomb contribution

$$W_{00} = \frac{1}{2J_A + 1} \sum \left| \left\langle \Psi_{m_i}^{(-)}(\bar{k}_x, \bar{k}_y) \right| \hat{\rho}(\bar{q}) \left| \Psi_{J_A M_A} \right\rangle \right|^2 \qquad V_{00} = \frac{\mathbf{Q}^4}{|\bar{\mathbf{q}}|^4}$$

Inclusive cross section

$$\frac{d^3\sigma}{d\hat{k}_f dE_x} = \frac{4\varepsilon_f^2 \alpha^2}{(\hbar c)^2} \frac{\cos^2\frac{\theta}{2}}{1 + \frac{\varepsilon_f}{M_A c^2}\left(1 - \frac{|\bar{k}_i| \cos\theta}{|\bar{k}_f|}\right)} \frac{2E_x^2}{|\bar{q}|^4} \frac{4\pi}{\hat{J}_A^2} \sum \left| \rho_{\gamma J_f J_A}^{101}(\bar{q}) \, \rho_C(\bar{q}) \right|^2$$

Inclusive electron scattering

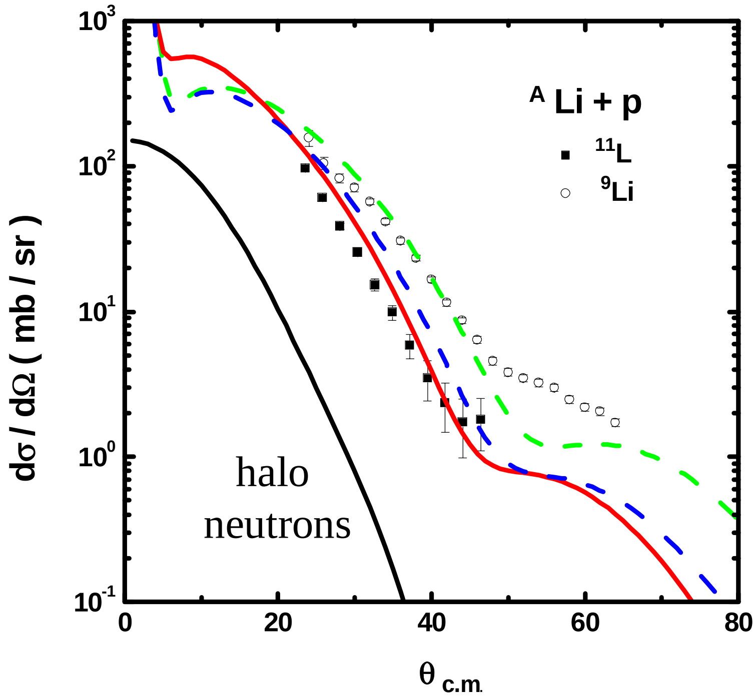


Elastic Scattering of Halo Nucleus on Proton

Experimental data :

^{11}Li + p, E/A = 68 MeV, A.A. Korsheninnikov et al., PRL, 78 (1997) 2317

^{9}Li + p, E/A = 60 MeV, C.B. Moon et al., PL, B297 (1992) 39



Reaction cross sections

$U_{^{11}Li}$	387 mb
U_{core}	214 mb
U_{2n}	231 mb
$U_{^{9}Li}$	219 mb

single folded optical potential : $U_{^{11}Li} = U_{core} + U_{2n}$

contribution from :

halo nucleons

$$U_{2n} = \int t_{NN}\, \rho_{2n}$$

t_{NN} ⟷ free NN t-matrix interaction

core nucleons

$$U_{core} = \int V_{NN}\, \rho_{core}$$

$$\rho_{core}(q) = \rho_{^{9}Li}(q)\, \rho_{c.m.}(q)$$

V_{NN} ⟷ density dependent GLM interaction

CONCLUSIONS

- **The remarkable discovery of new type of nuclear structure at driplines, *HALO,* have been made with radioactive nuclear beams.**

- **The theoretical description of dripline nuclei is an exciting challenge. The coupling between bound states and the continuum asks for a strong interplay between various aspects of nuclear structure and reaction theory.**

- **Development of new experimental techniques for production and /or detection of radioactive beams is the way to unexplored**

“ TERRA INCOGNITA “