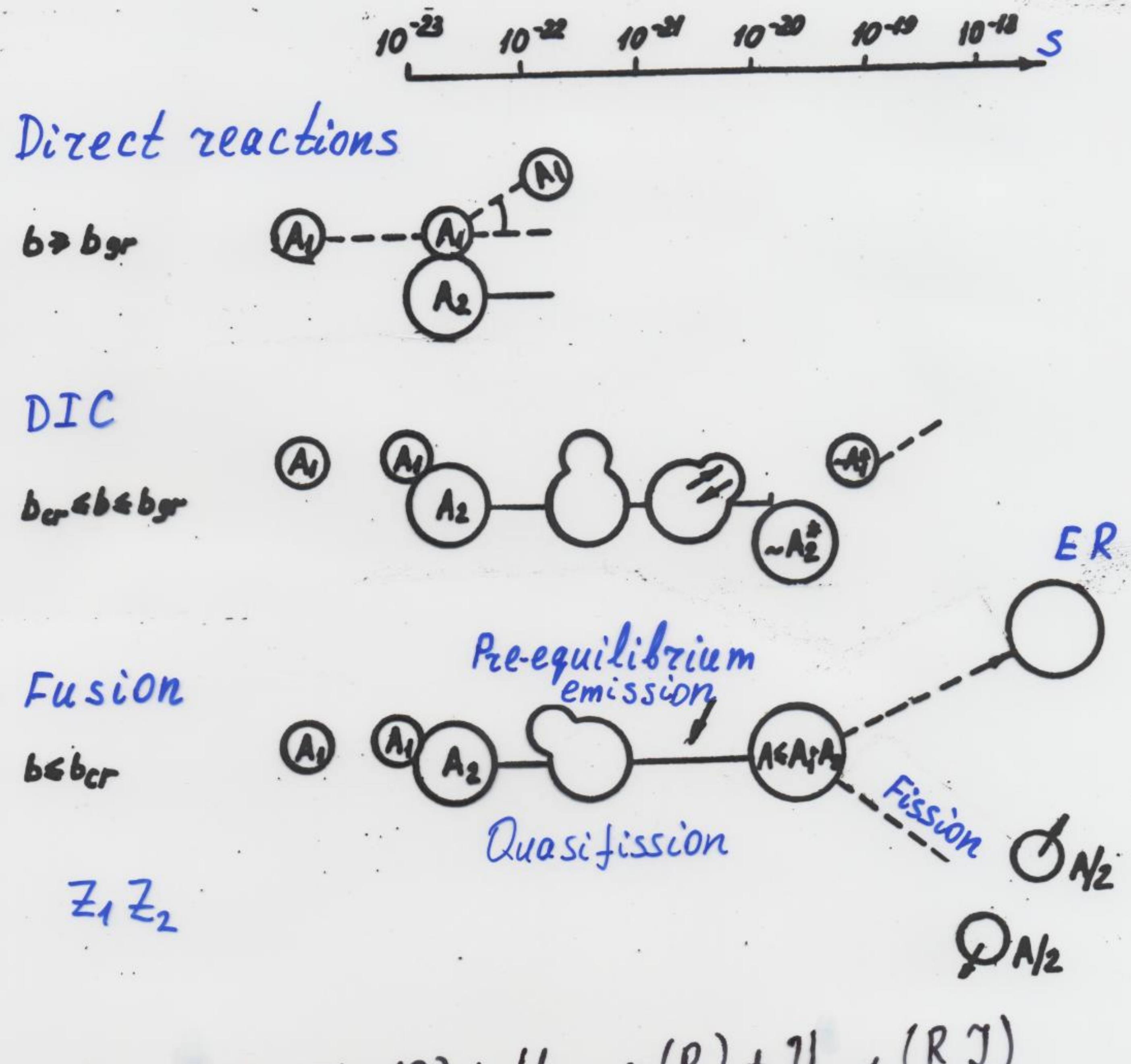
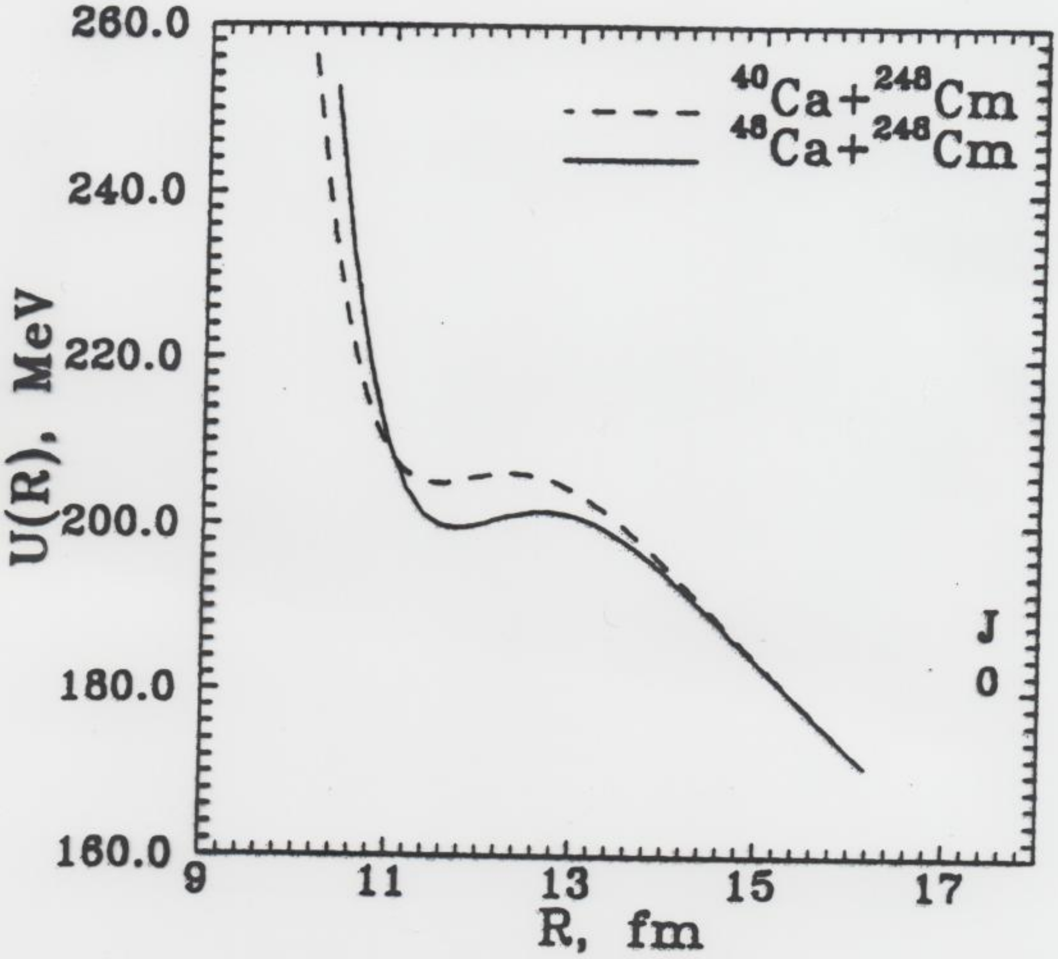
N. Antonenko

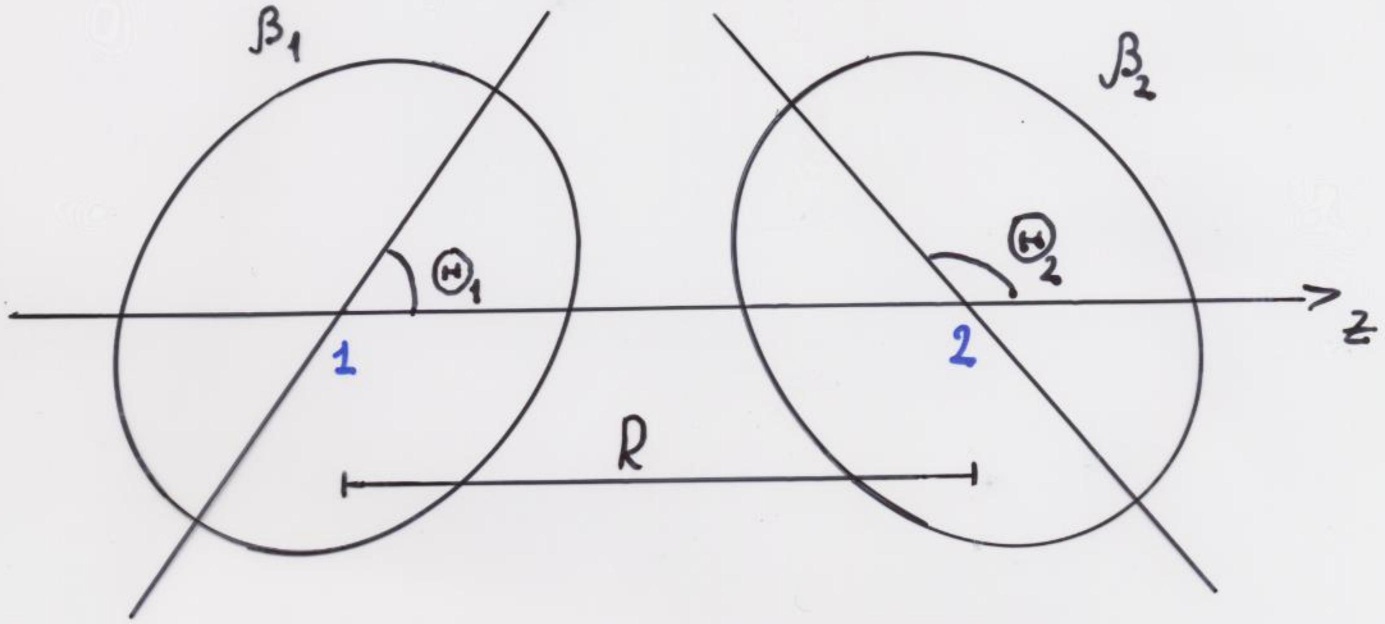
Systematic of heavy ion reactions at low energies



$$\mathcal{U}(R, J) = \mathcal{U}_N(R) + \mathcal{U}_{coul}(R) + \mathcal{U}_{rot}(R, J)$$



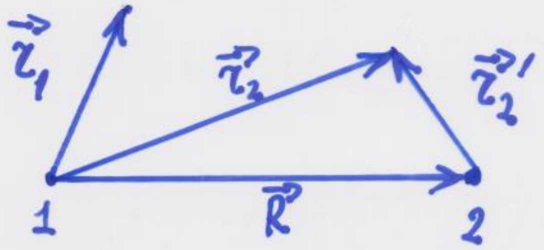
260.0
240.0
220.0
200.0
180.0
160.0
U(R), MeV
9
11
13
15
17
R, fm
^{40}Ca+^{248}Cm
^{48}Ca+^{248}Cm
J
0



β1
β2
Θ1
Θ2
1
2
R
z

COULOMB POTENTIAL

$$U_{Coul}(R) = e^2 Z_1 Z_2 \int \frac{\rho_1^z(\mathbf{r}_1)\rho_2^z(\mathbf{r}_2)}{|\mathbf{r}_1 - \mathbf{r}_2|} d\mathbf{r}_1 d\mathbf{r}_2$$



$$\frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|} = 4\pi \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{1}{2l+1} \frac{r_1^l}{r_2^{l+1}} Y_{lm}(\theta_1,\varphi_1) Y_{lm}^*(\theta_2,\varphi_2)$$

at $r_1 < r_2$

$$Y_{lm}(\theta,\varphi) = (-1)^m \sqrt{\frac{(2l+1)(l-m)!}{2(l+m)!}}\, P_l^m(\cos\theta) \frac{1}{\sqrt{2\pi}} e^{im\varphi}$$

$$\frac{1}{r_2^{l+1}} Y_{lm}^*(\theta_2,\varphi_2) = \frac{1}{|\mathbf{R} + \mathbf{r}_2'|^{l+1}} Y_{lm}^*(\theta_2,\varphi_2)$$

$$= \sqrt{\frac{1}{(2l)!}} \sum_{\substack{l_1,l_2=0 \\ l_2-l_1=l}} (-1)^{l_1+l_2} \sqrt{\frac{(2l_2+1)!}{(2l_1+1)!}}\, C_{l_1m,l_20}^{lm} \frac{r_2'^{\,l_1}}{R^{l_2+1}} Y_{lm}^*(\theta_2',\varphi_2')$$

$r_2' < R$

$$U_{Coul}(R) = e^2 Z_1 Z_2 4\pi \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \frac{1}{2l+1} \int r_1^l Y_{lm}(\theta_1,\varphi_1)\rho_1^z(\mathbf{r}_1)d\mathbf{r}_1$$

$$\times \sqrt{\frac{1}{(2l)!}} \sum_{\substack{l_1,l_2=0 \\ l_2-l_1=l}} (-1)^{l_1+l_2} \sqrt{\frac{(2l_2+1)!}{(2l_1+1)!}} C^{lm}_{l_1 m, l_2 0} \frac{1}{R^{l_2+1}} \int r_2'^{l_1} \rho_2^z(\mathbf{r}_2') Y^*_{l_1 m}(\theta_2',\varphi_2')d\mathbf{r}_2'$$

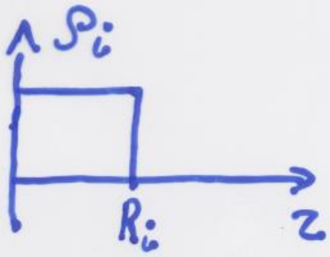
$l = l_1 = l_2 = m = 0$; $l=0,\ l_1 = l_2 = 2,\ m=0$; $l = l_2 = 2,\ l_1 = 0,$
$m = 0$

shape parametrization

$$R_i(\theta_{i0}) = R_{0i}(1 + \beta_i Y_{20}(\theta_{i0}))$$

$$Y_{20}(\theta_{i0}) = \sqrt{\frac{5}{16\pi}}\,(3\cos^2\theta_{i0} - 1)$$

$$Y_{20}(\theta_{i0}) = \sqrt{\frac{4\pi}{5}} \sum_m (-1)^m Y_{2m}(\theta_i,\varphi_i) Y_{2m}(\Theta_i,\Phi_i)$$



$$\rho_i^z(\mathbf{r_i}) = \rho_0^z S(r - R_i(\theta_{i0}))$$
$$= \rho_0^z[S(r - R_{0i}) + R_{0i}\beta_i Y_{20}(\theta_{i0})\delta(r - R_{0i})$$
$$-\frac{1}{2}(R_{0i}\beta_i Y_{20}(\theta_{i0}))^2 \delta'(r - R_{0i})]$$

$$(Y_{20}(\theta_{i0}))^2 = \frac{4\pi}{5}\sum_{m_1,m_2}(-1)^{m_1+m_2}Y_{2m_1}(\Theta_i,\Phi_i)Y_{2m_2}(\Theta_i,\Phi_i)Y_{2m_1}(\theta_i,\varphi_i)Y_{2m_2}(\theta_i,\varphi_i)$$

$$=\frac{4\pi}{5}\sum_{m_1,m_2}(-1)^{m_1+m_2}Y_{2m_1}(\Theta_i,\Phi_i)Y_{2m_2}(\Theta_i,\Phi_i)\sum_L\sqrt{\frac{25}{4\pi(2L+1)}}C^{L0}_{2020}C^{LM}_{2m_1 2m_2}Y_{LM}(\theta_i,\varphi_i)$$

$$\times Y_{20}(\theta_i)\cdots\int d\Omega_i \quad ,$$

$$\int Y_{LM}(\theta_i,\varphi_i)Y_{20}(\theta_i)\,d\Omega_i = \delta_{L,2}\,\delta_{M,0}$$

$$\frac{4\pi}{5}\sqrt{\frac{5}{4\pi}}C^{20}_{2020}\sum_{m_1,m_2}C^{20}_{2m_1 2m_2}Y_{2m_1}(\Theta_i,\Phi_i)Y_{2m_2}(\Theta_i,\Phi_i)=[C^{20}_{2020}]^2Y_{20}(\Theta_i)$$

$$\int r_i^l Y_{lm}(\theta_i,\varphi_i)\rho_i^z(\mathbf{r}_i)d\mathbf{r}_i =$$

$$|\, l=0, m=0\,|$$

$$= Z_i / \sqrt{4\pi}$$

$$|\, l=2, m=0\,|$$

$$= Z_i\sqrt{\frac{4\pi}{5}}\left(\frac{3}{4\pi}R_{0i}^2\beta_i Y_{20}(\Theta_i) + \frac{3}{7\pi}\sqrt{\frac{5}{4\pi}}[R_{0i}\beta_i]^2 Y_{20}(\Theta_i)\right)$$

$$C^{20}_{2020} = -\sqrt{\frac{2}{7}} \quad , \quad C^{00}_{2020} = \frac{1}{\sqrt{5}} \quad , \quad C^{20}_{0020} = 1$$

$$U_{Coul}(R) = \frac{e^2 Z_1 Z_2}{R} + \frac{3}{5}\frac{e^2 Z_1 Z_2}{R^3}\sum_{i=1,2} R_{0i}^2 \beta_i Y_{20}(\Theta_i)$$

$$+\frac{12}{35}\sqrt{\frac{5}{4\pi}}\frac{e^2 Z_1 Z_2}{R^3}\sum_{i=1,2}[R_{0i}\beta_i]^2 Y_{20}(\Theta_i)$$

$$f(\vec{x}) = \frac{1}{(2\pi)^3} \int d\vec{p} \; \tilde{f}(\vec{p}) \, e^{-i\vec{p}\vec{x}}$$

$\tilde{f}(\vec{p}) = \int d\vec{x} \, e^{i\vec{p}\vec{x}} f(\vec{x})$ — the Fourier transform of $f(\vec{x})$

$$\mathcal{U}(R) = \int d\vec{r}_1 \, d\vec{r}_2 \; \rho_1(\vec{r}_1) \, \mathcal{F}(\vec{r}_{12} = \vec{R} + \vec{r}_2 - \vec{r}_1) \, \rho_2(\vec{r}_2) =$$

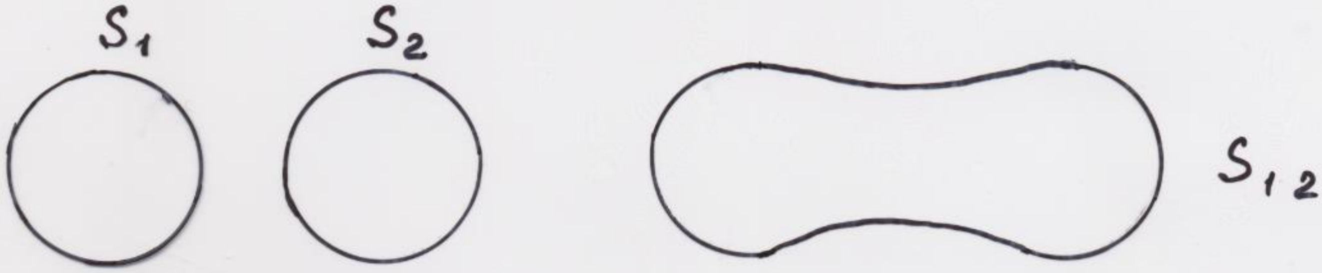
$$= \frac{1}{(2\pi)^3} \int d\vec{p} \, d\vec{r}_1 \, d\vec{r}_2 \; \tilde{\mathcal{F}}(\vec{p}) \, e^{-i\vec{p}(R + \vec{r}_2 - \vec{r}_1)} \rho_1(\vec{r}_1) \rho_2(\vec{r}_2) =$$

$$= \frac{1}{(2\pi)^3} \int d\vec{p} \; \tilde{\mathcal{F}}(\vec{p}) \, \rho_1(\vec{p}) \, \rho_2(-\vec{p}) \, e^{-i\vec{p}\vec{R}}$$

$$\mathcal{U}_{coul}(R) = \frac{2e^2 Z_1 Z_2}{(2\pi)^2} \int d\vec{p} \, e^{-i\vec{p}\vec{R}} \rho_1^z(\vec{p}) \, \rho_2^z(-\vec{p}) \frac{1}{p^2}$$

<u>Adiabatic approach</u>: the smooth change of internal structure of approaching nuclei, equilibrium $\rho(\vec{r})$ at each R

$$U_N(R) = \sigma\,\underbrace{(S_{12} - S_1 - S_2)}_{\text{the change of surface}}$$



$$\sigma \approx 0{,}95\ \mathrm{MeV}\cdot\mathrm{fm}^{-2}$$

<u>Sudden approximation</u> remains the structures of interacting nuclei

$$\rho(\vec{r}) = \rho_1(\vec{r}) + \rho_2(\vec{r})$$

small compressibility of nuclear matter $\rightarrow$ repulsive core

Energy density approach

$$\langle \Psi(R) | \hat{H} | \Psi(R) \rangle = \int d\vec{r}\, \varepsilon(\rho)$$

$$\mathcal{U}_N(R) = \int d\vec{r} \left\{ \varepsilon(\rho_1 + \rho_2) - \varepsilon(\rho_1) - \varepsilon(\rho_2) \right\}$$

parametrization of $\varepsilon(\rho)$

V. N. Bragin, M.V Zhukov, Part. Nucl. 15(1984)725

$$\mathcal{U}_N(R) = \bar{C} \begin{cases} -34\, e^{-0{,}27 S^2}, & S > -1{,}6 \text{ fm} \\ -34 + 5{,}4\,(S+16)^2, & S < -1{,}6 \text{ fm} \end{cases}$$

$$\bar{C} = C_1 C_2 / (C_1 + C_2)$$

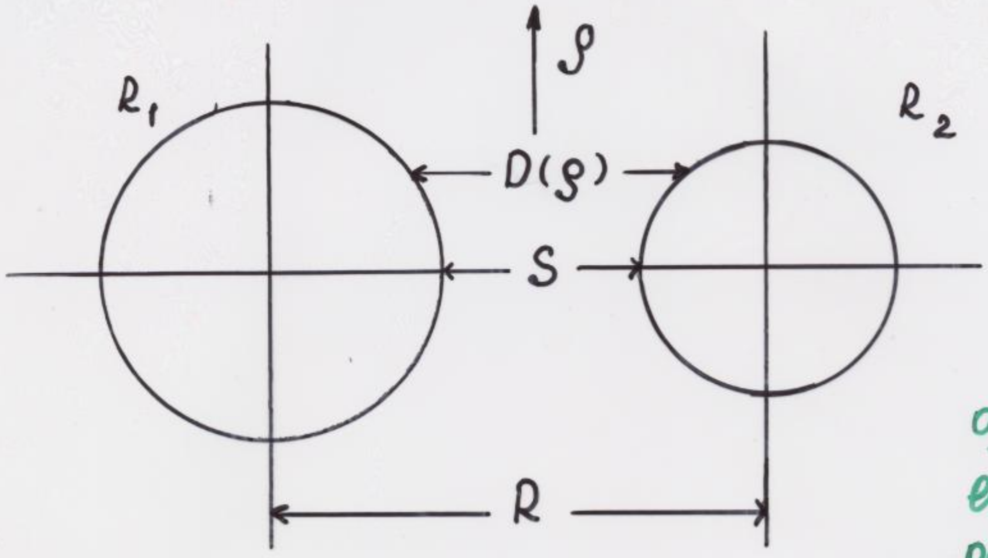
$$S = R - C_1 - C_2, \quad R_i = 1{,}16\, A_i^{1/3} \text{ fm}, \quad C_i = R_i - 1/R_i$$

repulsive core due to the condition of saturation of nuclear forces in $\varepsilon(\rho)$

$$\varepsilon(\rho) = \tau + \rho\, v(\rho, \alpha) + \frac{\hbar^2}{8m} \eta (\nabla \rho)^2$$

$$\alpha = \frac{\rho^N - \rho^Z}{\rho^N + \rho^Z}, \quad \tau \sim \rho^{5/3}$$

Proximity potential



surface density of interaction energy of two plane layer

$$U_N(R) = \int dS\, e(D) = 2\pi \bar{R}_{12} \int_{D=S}^{\infty} dD\, e(D) =$$

$$= 4\pi \sigma b \bar{R}_{12}\, \Phi(\zeta)$$

$\zeta = S/b$, $b \approx 1 fm$, $\bar{R}_{12} = R_1 R_2/(R_1+R_2)$ - the reduced curvature radius,

$$\Phi(\zeta) = \int_{\zeta}^{\infty} d\zeta'\, \varphi(\zeta)$$

$$\varphi(\zeta) = e(b\zeta')/(2\sigma)$$

$$\Phi(\zeta)=\begin{cases} -1{,}7817+\zeta\,, & \zeta<0 \text{ (adiabatic limit)} \\ -1{,}7817+0{,}927\zeta+0{,}143\zeta^2-0{,}09\zeta^3\,, & \zeta<0 \text{ (sudden limit)} \\ -1{,}7817+0{,}927\zeta+0{,}1696\,\zeta^2-0{,}05148\,\zeta^3\,, & 0<\zeta<1{,}9475 \\ -4{,}41\,e^{-\zeta/0{,}7176}\,, & \zeta\geq 1{,}9475 \end{cases}$$

DOUBLE FOLDING POTENTIAL

$$U_N(R) = \int \rho_1(\mathbf{r}_1)\rho_2(\mathbf{R} - \mathbf{r}_2)F(\mathbf{r}_1 - \mathbf{r}_2)d\mathbf{r}_1 d\mathbf{r}_2$$

The method allows us to take into account the finite size of interacting nuclei by their densities. However, there is a question of the choice of the nucleon-nucleon interaction. The microscopic theories were developed together with the phenomenological approaches.

With the density-independent nucleon-nucleon interaction U_N is deep and does not take into account the exchange effects connected with antisymmetrization. These effects are separately treated excluding the forbidden states of the deep potential well from consideration.

The density dependence of the nucleon-nucleon interaction allows one take into account the exchange and saturation effects phenomenologically. Among that kind of interactions, the Skyrme-type interactions are often used due to their simple structure. Without momentum dependence the expression for the Skyrme interaction reduces to the expression for local interaction

$$F(\mathbf{r}_1-\mathbf{r}_2)=C_0\left(F_{in}\frac{\rho(\mathbf{r}_1)}{\rho_{00}}+F_{ex}\left(1-\frac{\rho(\mathbf{r}_1)}{\rho_{00}}\right)\right)\delta(\mathbf{r}_1-\mathbf{r}_2)$$

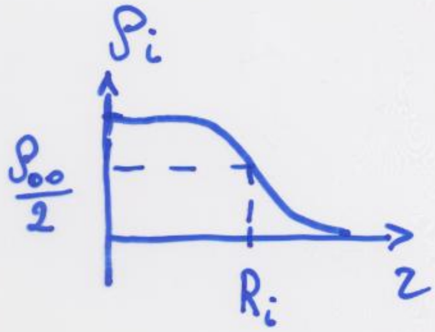
$$\rho(\mathbf{r}) = \rho_1(\mathbf{r}) + \rho_2(\mathbf{r})$$

$$U_N(R)=C_0\{\frac{F_{in}-F_{ex}}{\rho_{00}}\left(\int\rho_1^2(\mathbf{r})\rho_2(\mathbf{R}-\mathbf{r})d\mathbf{r}+\int\rho_1(\mathbf{r})\rho_2^2(\mathbf{R}-\mathbf{r})d\mathbf{r}\right)$$
$$+F_{ex}\int\rho_1(\mathbf{r})\rho_2(\mathbf{R}-\mathbf{r})d\mathbf{r}\}$$

$C_0 = 300$ MeV fm^3, $\rho_{00} = 0.17\ fm^{-3}$

$F_{in} \simeq 0.1$ $F_{ex} \simeq -2.6$

Two-parameter Woods-Saxon function



$$\rho_i(\mathbf{r})=\frac{\rho_{00}}{1+\exp[(r-R_i(\theta_i,\varphi_i))/a_i]}$$

or symmetrized Woods-Saxon function

$$\rho_i(\mathbf{r}) = \frac{\rho_{00}\sinh[R_i(\theta_i,\varphi_i)/a_i]}{\cosh[R_i(\theta_i,\varphi_i)/a_i] + \cosh[r/a_i]}$$

For light spherical nuclei,

$$\rho_i(\mathbf{r}) = A_i(\gamma^2/\pi)^{3/2}\exp[-\gamma^2 r^2]$$

$$\rho_i^2(r) = -\rho_{00}a_i\sinh\frac{R_{0i}}{a_i}\frac{d}{dR_{0i}}\frac{\rho_i(r)}{\sinh\frac{R_{0i}}{a_i}}$$

$$\int\rho_1^2(\mathbf{r})\rho_2(\mathbf{R}-\mathbf{r})d\mathbf{r}$$

$$= -4\pi\rho_{00}a_i\sinh\frac{R_{0i}}{a_i}\frac{d}{dR_{0i}}\frac{1}{\sinh\frac{R_{0i}}{a_i}}\int_0^\infty\rho_1(p)\rho_2(p)j_0(pR)p^2dp$$

$$\rho_i(p) = \frac{\sqrt{2\pi} a_i R_{0i} \rho_{00}}{p \sinh(\pi a_i p)} \left(\frac{\pi a_i}{R_{0i}} \sin(pR_{0i}) \coth(\pi a_i p) - \cos(pR_{0i}) \right)$$

$a_1 = a_2 = a$, poles at $p = in/a$, $n = 1, 2 \ldots$

$$\int \rho_1^2(\mathbf{r}) \rho_2(\mathbf{R}-\mathbf{r}) d\mathbf{r}$$

$$= -\frac{4\pi}{3} \rho_{00}^3 \frac{a^2}{R} \sinh\frac{R_{01}}{a} \frac{d}{dR_{01}} \frac{1}{\sinh\frac{R_{01}}{a}} \sum_{n=1}^{\infty} \frac{1}{n} \exp[-\frac{nR}{a}]$$

$$\times \left\{ \left[R^3 + \frac{3a}{n}\left(R^2 + \frac{2Ra}{n} + \frac{2a^2}{n^2} \right) - 3a^2 (R + \frac{n}{a}) \left(\frac{2\pi^2}{3} + \frac{R_{01}^2 + R_{02}^2}{a^2} \right) \right] \right.$$

$$\times \sinh\frac{nR_{01}}{a} \sinh\frac{nR_{02}}{a} + 2R_{01}(\pi^2 a^2 + R_{01}^2) \cosh\frac{nR_{01}}{a} \sinh\frac{nR_{02}}{a}$$

$$\left. + 2R_{02}(\pi^2 a^2 + R_{02}^2) \cosh\frac{nR_{02}}{a} \sinh\frac{nR_{01}}{a} \right\}$$

$R > R_{01} + R_{02}$

two light nuclei

$$\int \rho_1^2(\mathbf{r})\rho_2(\mathbf{R}-\mathbf{r})d\mathbf{r}$$

$$= \pi A_1^2 A_2 \left(\frac{\gamma_1^2}{\pi}\right)^3 \left(\frac{\gamma_2^2}{\pi}\right)^{3/2} \frac{\sqrt{\pi}}{(2\gamma_1^2+\gamma_2^2)^{3/2}} \exp\left[-\frac{2\gamma_1^2\gamma_2^2}{2\gamma_1^2+\gamma_2^2}R^2\right]$$

spherical light-spherical heavy nuclei

$$U_N(R) = 2C_0 A_1 \left(\frac{\gamma_1^2}{\pi}\right)^{1/2} \exp[-\gamma_1^2 R^2]\frac{1}{R}$$

$$\times \int_0^\infty \exp[-\gamma_1^2 r^2]\frac{\rho_2(r)}{\rho_{00}}\left[(F_{in}-F_{ex})\big(\rho_2(r)\sinh(2\gamma_1^2 Rr)\right.$$

$$+\frac{A_1}{4}\left(\frac{\gamma_1^2}{\pi}\right)^{3/2} \exp[-\gamma_1^2(r^2+R^2)]\sinh(4\gamma_1^2 Rr)\big)$$

$$\left.+\rho_{00}F_{ex}\sinh(2\gamma_1^2 Rr)\right]rdr$$

Relationship of Double Folding Potential and Proximity Potential

$$U_N(R) \approx C_0 \left\{ (F_{in} - F_{ex}) \left(2 - a_1 \frac{\partial}{\partial R_{01}} - a_2 \frac{\partial}{\partial R_{02}} \right) + F_{ex} \right\}$$

$$\times \int \rho_1(\vec{r}) \rho_2(\vec{r} - \vec{R}) \, d\vec{r}$$

$$\left(1 - a_i \frac{\partial}{\partial R_i} \right) \rho_i = \left(1 - \frac{e^{\frac{r - R_i}{a_i}}}{1 + e^{\frac{r - R_i}{a_i}}} \right) \rho_i = \frac{\rho_i^2}{\rho_{00}}$$

$$a_1 = a_2 = a$$

$$U_N(R) \approx 2\pi \rho_{00}^2 C_0 a^2 \frac{R_{01} R_{02}}{R_0}$$

$$\times \left\{ \underbrace{\sum_{n=1}^{\infty} e^{-n\delta} \left[\frac{2F_{in} - F_{ex}}{n^2} (1 + n\delta) - 2(F_{in} - F_{ex})\delta \right]}_{\varphi_0(\delta)} \right.$$

$$\left. + \frac{R_0^2}{2R_{01}R_{02}} \frac{a}{R_0} \varphi_1(\delta) + \frac{R_0^2}{6R_{01}R_{02}} \left(\frac{a}{R_0} \right)^2 \varphi_2(\delta) \right\}$$

Where $\delta = (R - R_{01} - R_{02})/a$ and $R_0 = R_{01} + R_{02}$

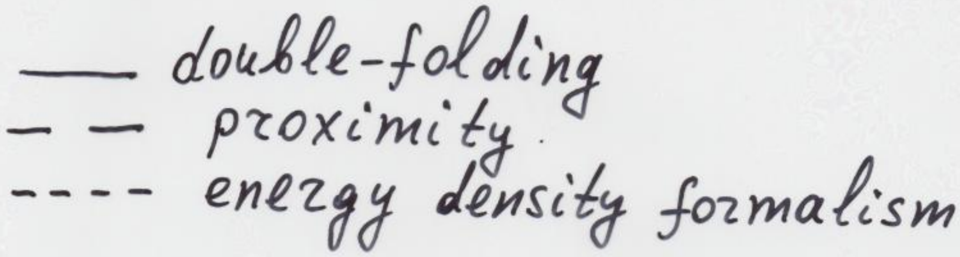
$$U_{rot}(R) = \frac{\hbar^2 \varphi J(\varphi J+1)}{2(J_1+J_2+\mu R^2)} + \frac{\hbar^2 (1-\varphi)J((1-\varphi)J+1)}{2\mu R^2}$$

the parameter φ characterizes the contribution of the rolling

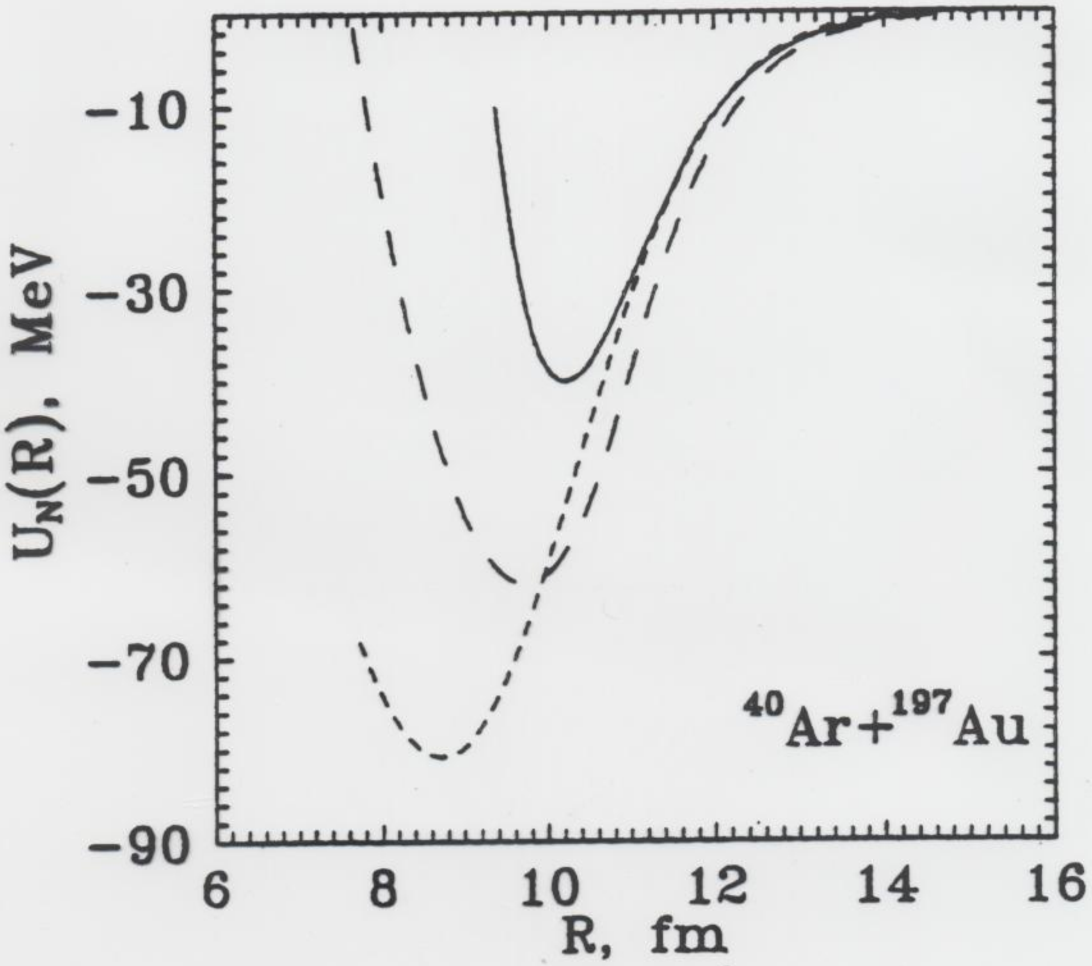
$\varphi=0$: $$\frac{\hbar^2 J(J+1)}{2\mu R^2}$$

$\varphi=1$: $$\frac{\hbar^2 J(J+1)}{2(J_1+J_2+\mu R^2)}$$

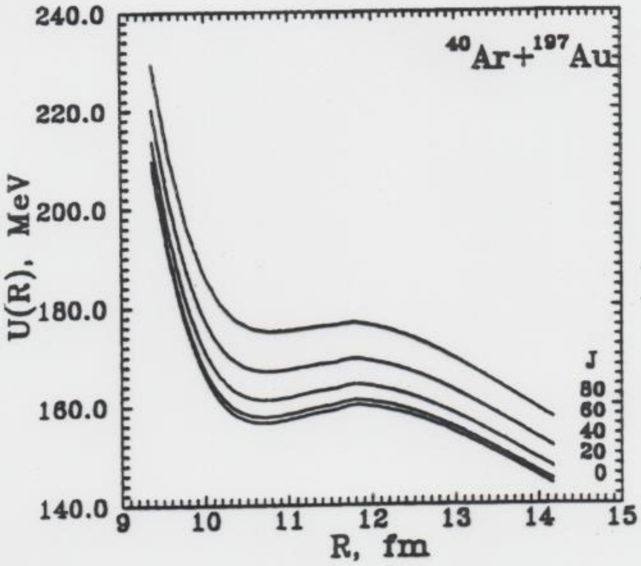
sticking condition



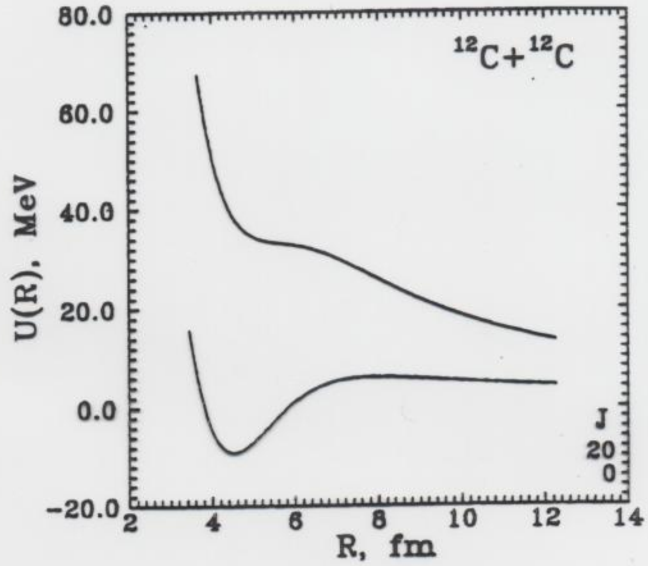
double-folding
proximity
energy density formalism



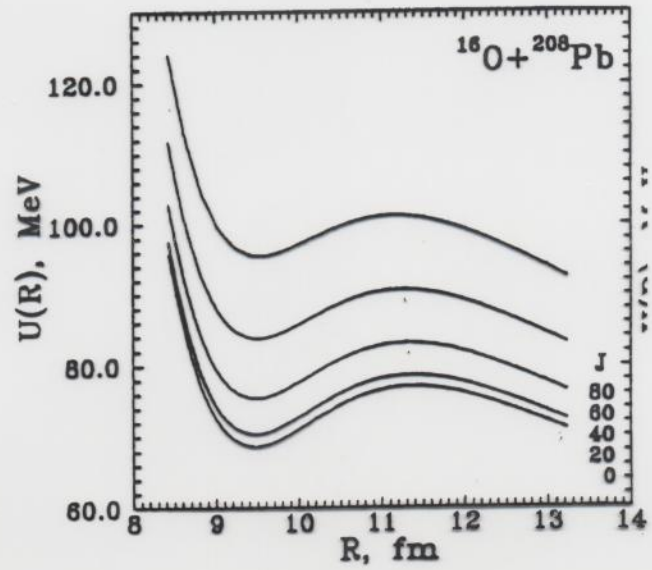
$U_N(R)$, MeV
−10
−30
−50
−70
−90
6
8
10
12
14
16
R, fm
$^{40}Ar+^{197}Au$



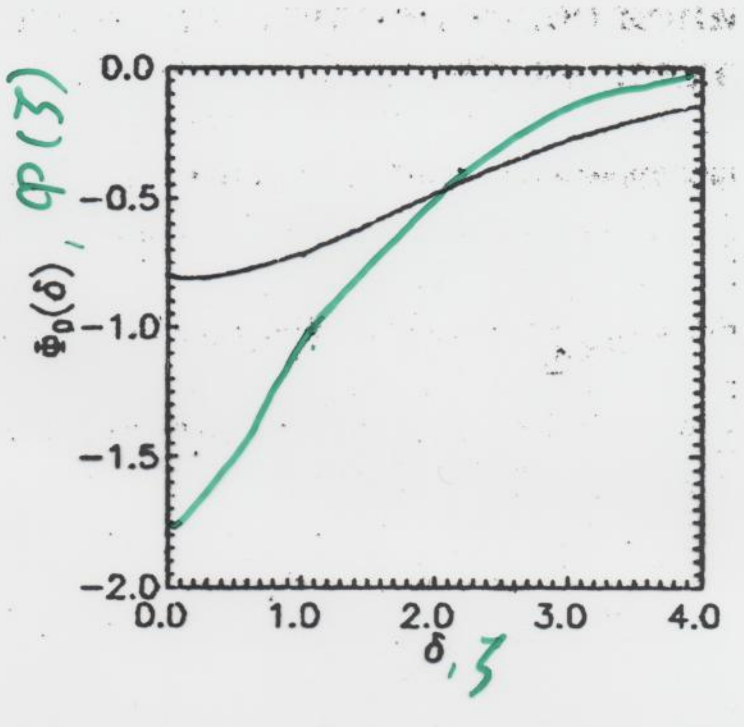
^{40}Ar+^{197}Au
U(R), MeV
R, fm
J
80
60
40
20
0



^{12}C+^{12}C
U(R), MeV
R, fm
J
20
0



^{16}O+^{208}Pb
U(R), MeV
R, fm
J
80
60
40
20
0


0.0
−0.5
−1.0
−1.5
−2.0
0.0
1.0
2.0
3.0
4.0
$\Phi_0(\delta)$, $\varphi(\zeta)$
δ, ζ

$$\beta_1 = \beta_2 = 0.26 \ , \ J = 0$$

$\Theta_1 = 0$

$\Theta_1 = \frac{\pi}{4}, \ \Theta_2 = \frac{3\pi}{4}$

$\Theta_1 = \Theta_2 = \frac{\pi}{2}$