Positivity domains for pairs or triples of spin observables

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Polarized $1/2 + 1/2 \rightarrow 1/2 + 1/2$ reaction

$\boldsymbol{S}$=polarization *3-vector.* $|\boldsymbol{S}| \leq 1$

Polarization *4-vector* : $S_\mu = (S_0\, , \boldsymbol{S}\,)$ μ=0,1,2,3 $S_0 = 1$

Cross section:

$d\sigma/\ d\Omega_{pol} = \ I_0\ F\{\boldsymbol{S}(A), \boldsymbol{S}(B), \boldsymbol{S}(C), \boldsymbol{S}(D)\}$

I_0 = 1/4 the unpolarized cross section.

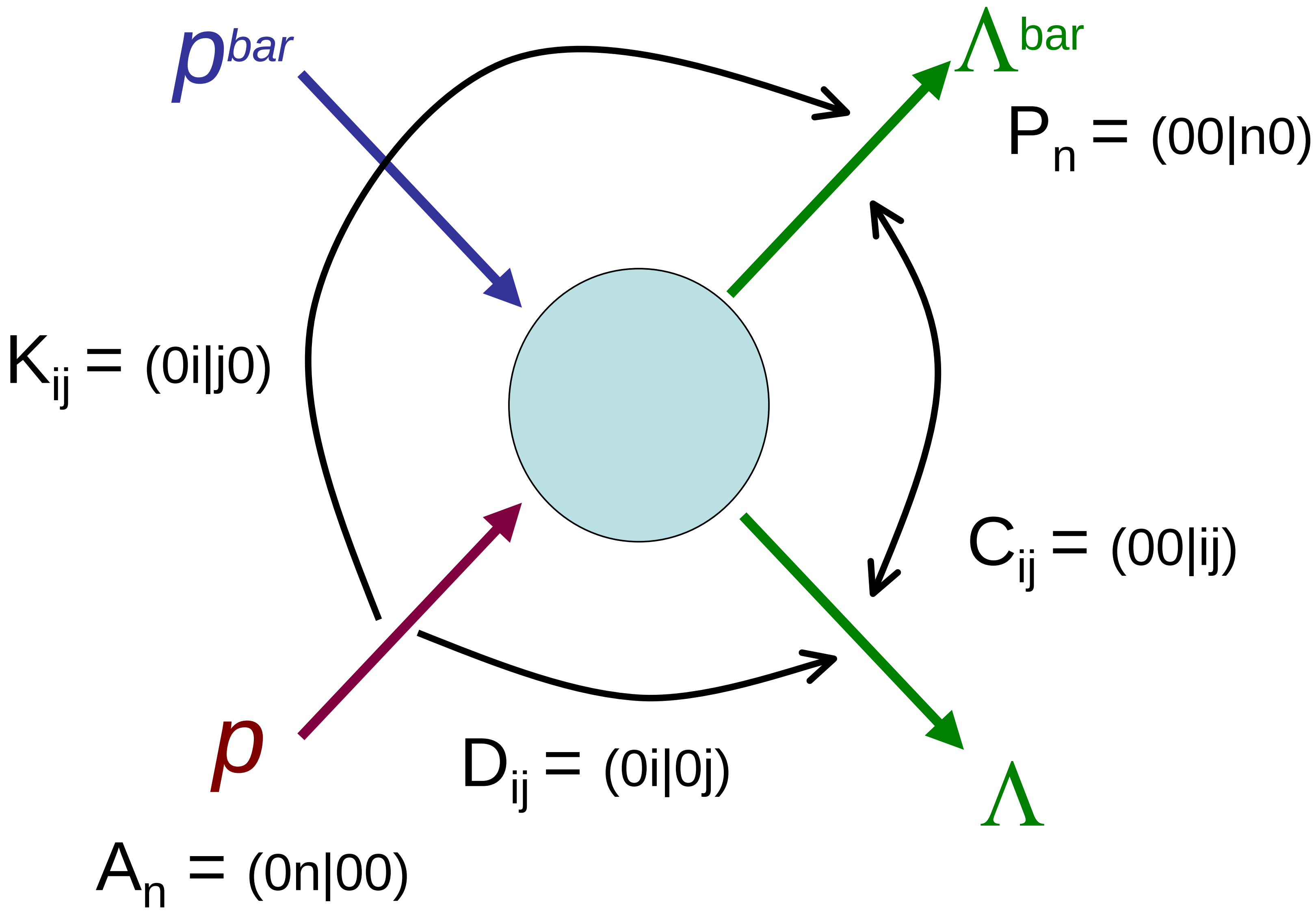
$F\{\boldsymbol{S}(A),\boldsymbol{S}(B),\boldsymbol{S}(C),\boldsymbol{S}(D)\}$

$$= S_\lambda(A)\ S_\mu(B)\ (\lambda\mu|\nu\tau)\ S_\nu(C)\ S_\tau(D) \qquad (0|0) = 1$$

$(\lambda\mu|\nu\tau)$ are the *Cartesian reaction parameters* (or "*correlation*" parameters).

They satisfy *symmetry constraints*, *quadratic identities* and *positivity conditions.*

Spin parameters in $p^{bar}\ p \to \Lambda^{bar}\ \Lambda$

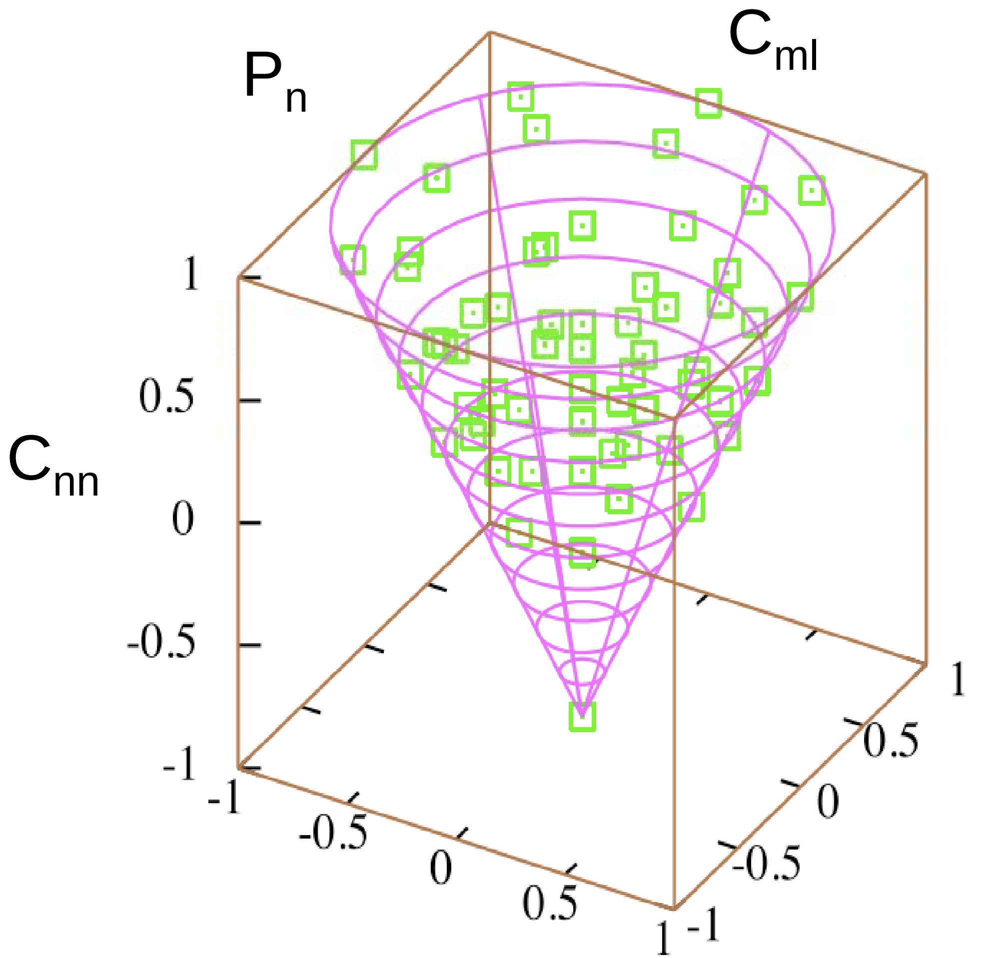


Positivity domain for a *pair* of parameters

	A_n	C_{nn}	D_{nn}	K_{nn}	C_{ml}	D_{mm}	C_{mm}	C_{ll}	D_{ml}	K_{mm}	K_{ml}	C_{nlm}	C_{nml}	C_{nmm}	C_{mnl}	C_{mln}	C_{mnm}	C_{mmn}
P_n	□	◁	□	□	○	⊗	○	○	⊗	○	○	○	○	○	○	○	○	○
A_n		◁	□	□	□	○	□	□	○	○	○	□	□	□	○	○	○	○
C_{nn}			□	□	▽	○	□	□	○	○	○	□	□	▽	○	○	○	○
D_{nn}				□	○	□	○	○	□	○	○	○	○	○	□	○	□	○
K_{nn}					○	○	○	○	○	□	□	○	○	○	○	□	○	□
C_{ml}						○	○	○	⊗	⊗	○	□	□	○	○	○	○	○
D_{mm}							□	○	○	□	⊗	○	□	○	○	○	○	○
C_{mm}								□	○	□	○	○	○	□	□	□	○	○
C_{ll}									□	○	□	○	○	□	○	○	□	□
D_{ml}										⊗	□	□	○	⊗	○	○	○	○
K_{mm}											○	□	○	○	○	○	○	○
K_{ml}												○	□	⊗	○	○	○	○
C_{nlm}													□	○	○	□	□	○
C_{nml}														○	□	○	○	□
C_{nmm}															○	○	⊗	⊗
C_{mnl}																□	○	⊗
C_{mln}																	⊗	○
C_{mnm}																		□

Domains for triples. Exemple:

simulation with random amplitudes



Other 3-dimensional domain shapes

QuickTime™ et un
décompresseur TIFF (LZW)
sont requis pour visionner cette image.

How to find the shape of the positivity domain of a set of Cartesian parameters ?

Various methods:

- Simulations with random helicity or transversity amplitudes.

- Derive some of the *quadratic identities* between the parameters (there are less amplitudes than parameters).

- Use positivity of the *cross section matrix* (second talk)

- Use anti-commutation relations (next slide)

Anti-commutation method

The Cartesian reaction parameters are *expectation values* of spin operators:

$(\lambda\mu|\nu\tau) = \langle \sigma_\mu(A) \otimes \sigma_\mu(B) \otimes \sigma_\nu(C) \otimes \sigma_\nu(D) \rangle \equiv \langle O \rangle$

- $O^2 = 1$
- two *O*'s are either commuting or anticommuting.

- If two operators *O* and *O*' anticommute, then
$\langle O \rangle^2 + \langle O' \rangle^2 \leq 1$. The allowed domain is contained in a disk.

Exemple:

$C_{ll} = \langle \sigma_z(C) \otimes \sigma_z(D) \rangle$ and $P_n = \langle \sigma_y(C) \rangle$ are anticommuting. Their domain is a *disk.*

Generalization to 3 or more observable : *sphere* , *etc.*

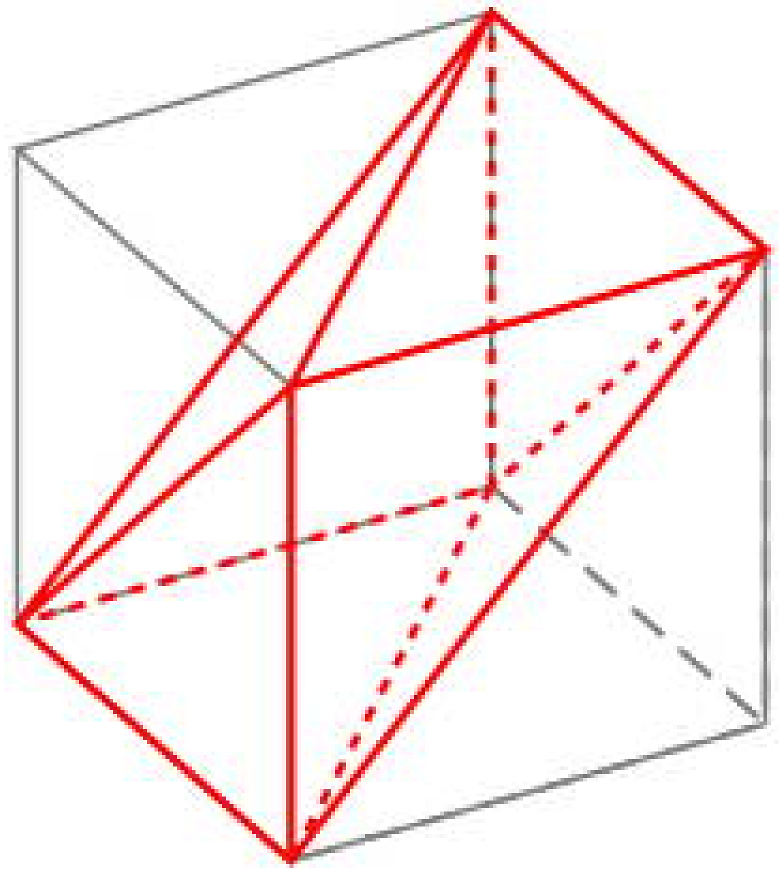
Can a positivity domain be the whole cube (or hyper-cube) ?

For a set of observables bearing on spin one-half particle,
at most N_A (hyper-) cube summits belong to the positivity domain,
where N_A = number of independent amplitudes

Exemple: $p^{bar}\, p \to \Lambda^{bar}\, \Lambda$

- 6 independent amplitudes

- No positivity domain can be the entire cube



Conclusion (of 1rst talk)

- Positivity domains for two observables are usually the *square* $[-1, +1]^2$, the *disk* $x^2 + y^2 \leq 1$ and the *triangle* $2|y| \leq 1+x$ (e.g., in the Soffer inequality).

- For 3 observables, one has the *cube* (if at least 8 independent amplitudes), the unit *ball*, *tetraedrons, prisms, cylinders* and more complicated domains.

- *Simulations* by random amplitudes and the *anti-commutation* relations help to find the domain.

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