BLTP, JINR

Gauged Baby Skyrmions and Merons

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Outline

- Baby Skyrme model and multisoliton solutions
- Gauged baby Skyrmions
- Gauged merons
- Maxwell-Chern-Simons baby Skyrmions
- Gauged Hopfions

Skyrme family

- ***(2+1)-dim***: **Baby Skyrme model**

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{4}\left(\partial_\mu \phi \times \partial_\nu \phi\right)^2 - V(\phi)$$

$$\phi : S^2 \to S^2; \quad \phi_\infty = (0,0,1)$$

Standard choice: $V(\phi) = \mu^2(1-\phi_3)$

$$Q \in \mathbb{Z} = \pi_2(S^2)$$

$$Q = \frac{1}{4\pi}\int_{\mathbb{R}^2} \phi \cdot (\partial_1 \phi \times \partial_2 \phi) d^2x$$

- ***(3+1)-dim***: **Skyrme model**

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{4}\left(\partial_\mu \phi \times \partial_\nu \phi\right)^2 - V(\phi)$$

$$\phi : S^3 \to S^3; \quad \phi_\infty = (0,0,0,1)$$

$$R_\mu = \partial_\mu U U^\dagger; \quad U = \phi_0 \mathbb{I} + i\sigma^a \cdot \phi^a$$

$$\mathcal{L} = -\,\mathrm{Tr}\left\{\frac{1}{2}(R_\mu R^\mu) + \frac{1}{16}([R_\mu, R_\nu][R^\mu, R^\nu]) + \mu^2(U - \mathbb{I})\right\}$$

↓

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \phi^a)^2 - \tfrac{1}{4}[(\partial_\mu \phi^a \partial_\nu \phi^a)^2 - (\partial_\mu \phi^a)^4] + \mu^2\ (1-\phi^3)$$

$$Q \in \mathbb{Z} = \pi_3(S^3)$$

$$Q = \frac{1}{24\pi^2}\ \mathrm{Tr}\int_{\mathbb{R}^3} \varepsilon_{ijk} R_i R_j R_k d^3x$$

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Storage

Durham University bags £7r explore 'magnetic skyrmion storage

Quantum mechanics could dramatically improve d capacities

Graeme Bu

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Aug 9, 2013

Skyrmion spin control cou electronics

Researchers at the University of succeeded in controlling tiny m as "skyrmions" for the first time important for future high-densi and nanodigital electronic devic transfer speeds and processing

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Research

Nanosize magnetic whirlpools could be the future of data storage

(2 August 2016)

Could magnetic skyrmions hold the answer to better data storage?

The use of nanoscale magnetic whirlpools, known as magnetic skyrmions, to create novel and efficient ways to store data will be explored in a new £7M research programme led by Durham University.

Skyrmions, which are a new quantum mechanical state of matter, could be used to make our day-to-day gadgets, such as mobile phones and laptops, much smaller and cheaper whilst using less energy and generating less heat.

It is hoped better and more in-depth knowledge of skyrmions could address society's ever-increasing demands for processing and storing large amounts of data and improve current hard drive technology.

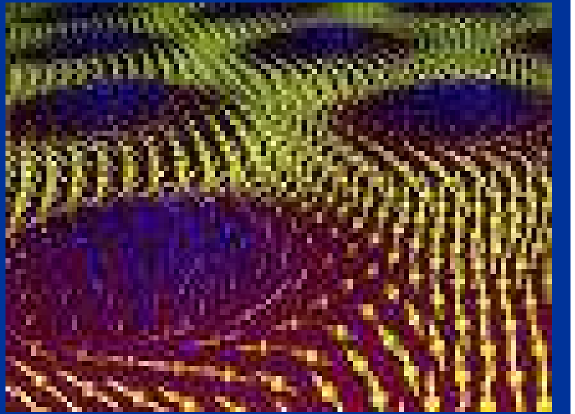
Revolutionise data storage

Scientists first predicted the existence of skyrmions in 1962 but they were only discovered experimentally in magnetic materials in 2009.

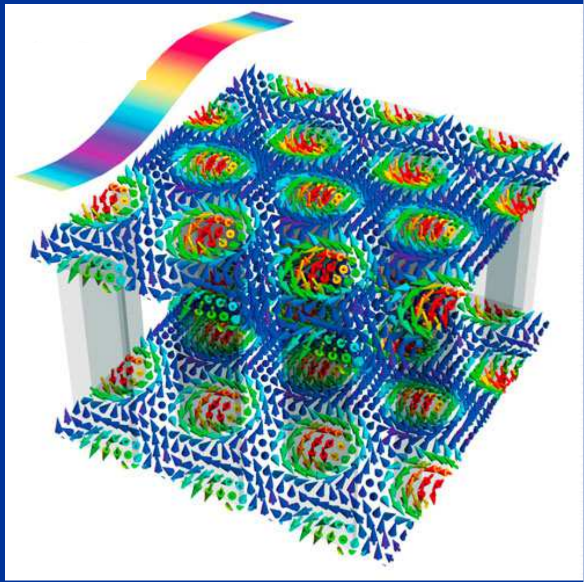
The UK team, funded by the Engineering and Physical Sciences Research Council (EPSRC), now aims to make a step change in our understanding of skyrmions with the goal of producing a new type of demonstrator device in partnership with industry.

Skyrmions, tiny swirling patterns in magnetic fields, can be created, manipulated and controlled in certain magnetic materials. Inside a skyrmion, magnetic moments point in different directions in a self-organised vortex.

Baby Skyrme model: Applications

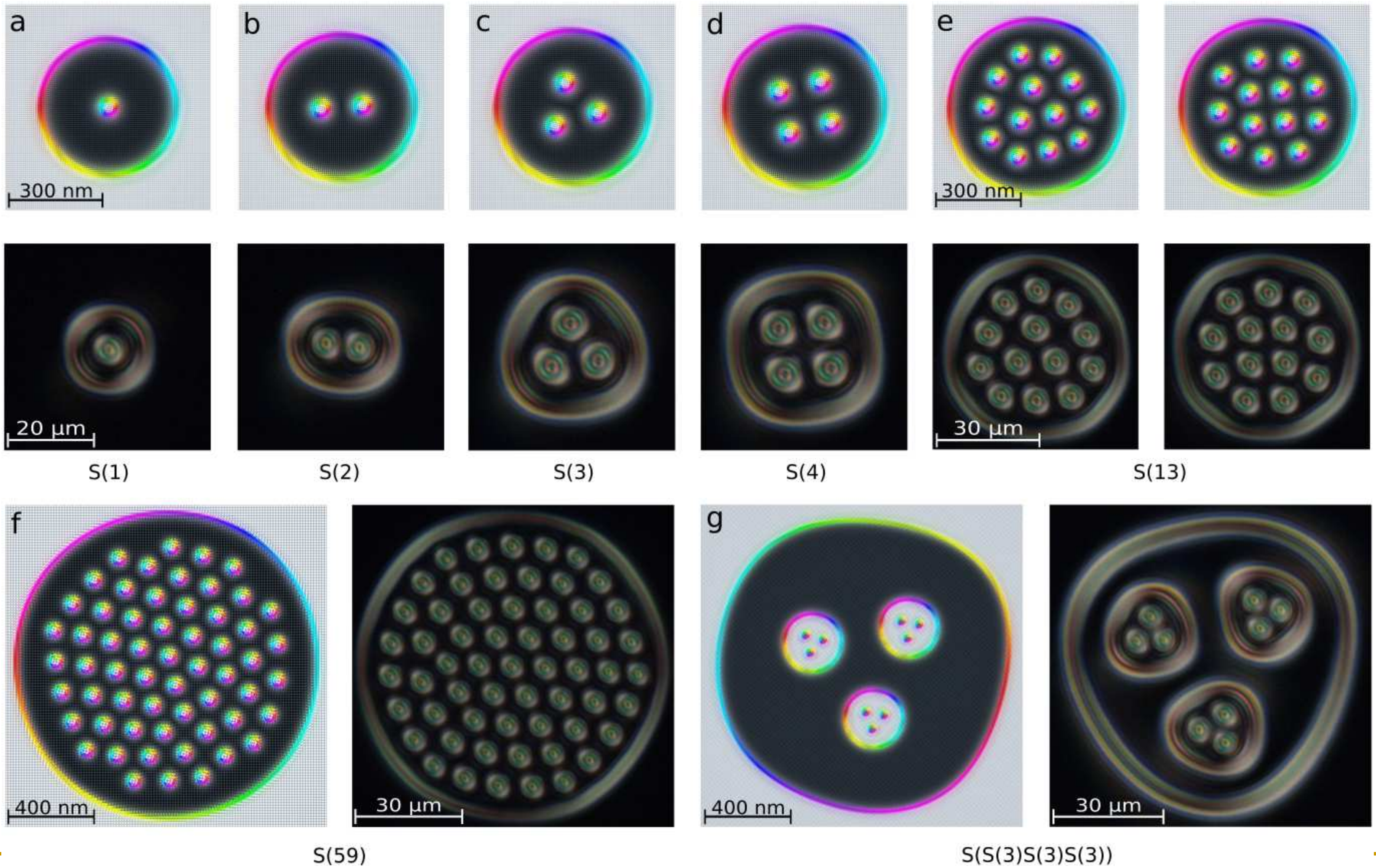


- A Heisenberg-type model of interacting spins
- A model of the topological quantum Hall effect
- Elementary excitations in quantum Hall magnets
- Chiral magnetic structures
- A model of ferromagnetic planar structures
- Applications in future development of data storage technologies
- Models of condensed matter systems with intrinsic and induced chirality



Baby Skyrmions bags

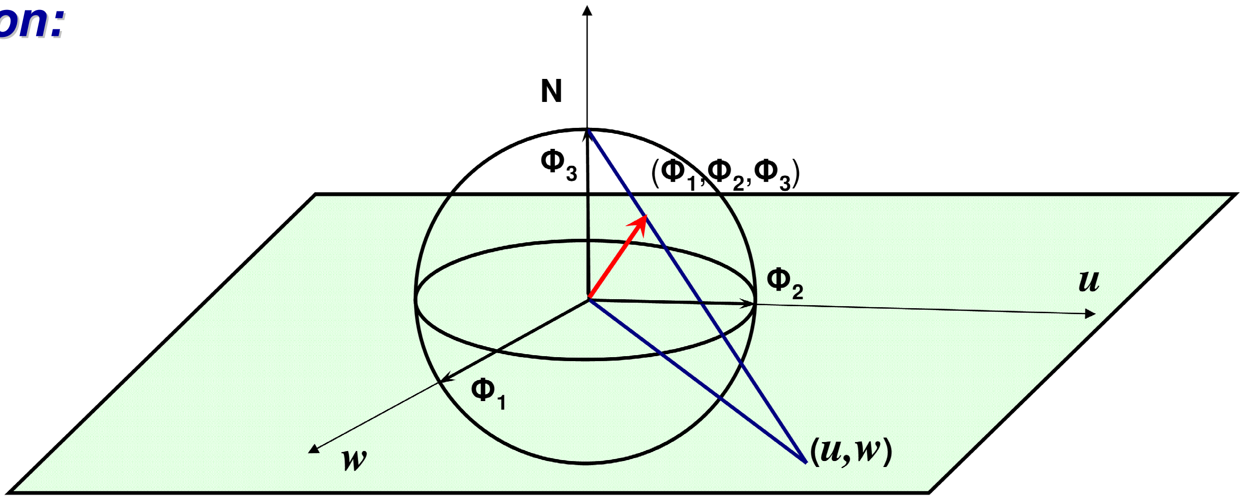
D. Foster et al **arXiv:1806.02576** (2018)



O(3) sigma-model vs $\mathbb{CP}^1$ model

- ***Stereographic projection:***

Coordinates on the projective space $\mathbb{CP}^1$



$$(u,w) = \left(\frac{\phi_1}{1-\phi_3}, \frac{\phi_2}{1-\phi_3}\right)$$

$$Z = u + iw = \frac{\phi_1 + i\phi_2}{1-\phi^3}$$

Target space:

$z = x + iy$ ← Domain space:

Inverse transformation onto $\mathbb{S}^2$

$$(\phi_1,\phi_2,\phi_3) = \left(\frac{2u}{1+u^2+w^2}, \frac{2w}{1+u^2+w^2}, \frac{1-u^2-w^2}{1+u^2+w^2}\right)$$

$$= \left(\frac{Z+\bar{Z}}{1+Z\bar{Z}}, i\frac{\bar{Z}-Z}{1+Z\bar{Z}}, \frac{1-Z\bar{Z}}{1+Z\bar{Z}}\right)$$

$\mathbb{CP}^1$ model: Solitons

$$E = \int \frac{|Z_z|^2 + |Z_{\bar{z}}|^2}{(1+|Z|^2)^2} dz d\bar{z}$$

The energy is minimal if $Z_{\bar{z}} = 0$

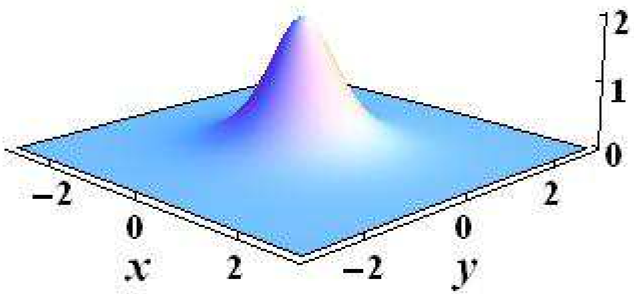
Cauchy-Riemann conditions for Z

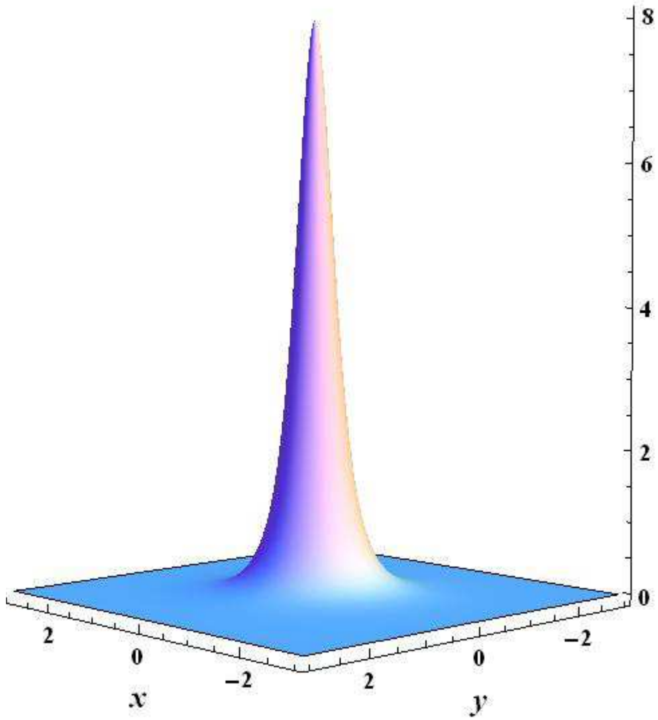
Simplest holomorphic solution: $Z = \lambda z; \quad \lambda \in \mathbb{C} = a e^{i\delta}$

Rational map holomorphic solution of degree 1:

$$Z = \frac{P(z)}{Q(z)} = \frac{\lambda(z-a)}{z-b}$$

Q=1:





Belavin-Polyakov instantons

Simplest rotationally invariant ansatz:

$$\phi^\alpha = n^\alpha \sin f(r), \quad \phi^3 = \cos f(r)$$
$$n^\alpha = (\cos\varphi; \quad \sin\varphi)$$

Q=1

$$Z_{\bar z} = 0 \;\Rightarrow\; f' = \frac{1}{r}\sin f$$

$$f = 2\arctan\frac{r}{r_0}$$

Toy model of the SU(2) Yang-Mills instantons

$$L = \frac{1}{2g^2}\mathrm{Tr}\; F_{\mu\nu}F^{\mu\nu} \;\Rightarrow\; F_{\mu\nu} = \pm\tilde F_{\mu\nu}$$

$$Q = \frac{1}{16\pi^2}\mathrm{Tr}\int d^4x \tilde F_{\mu\nu}F^{\mu\nu}$$

One instanton solution:

$$A^a_\mu = -\bar\eta^a_{\mu\nu}\partial^\nu \ln\left(1 + \frac{\rho^2}{(r-r_0)^2}\right)$$

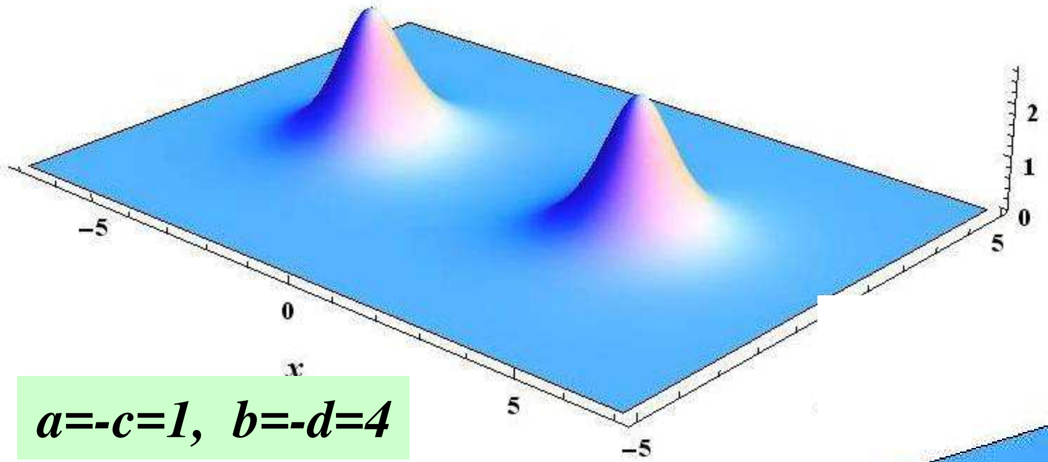
$\mathbb{CP}^1$ model: Solitons

Rational map holomorphic solution of degree Q:

$$Z = \frac{P(z)}{Q(z)}$$

Q=2:

$$Z = \frac{(z-a)(z-c)}{(z-b)(z-d)}$$

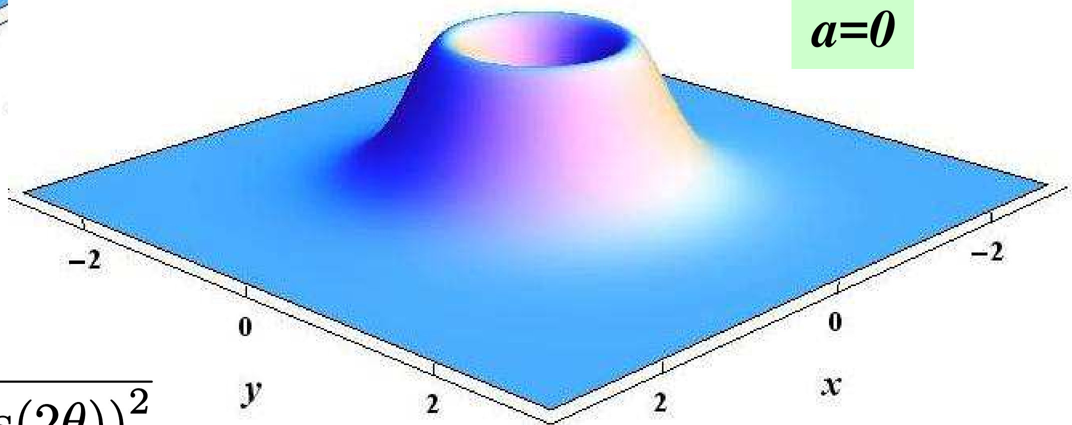


a=-c=1, b=-d=4

Symmetry restrictions:

$$Z = \lambda(z^2 - a^2)$$

a=0



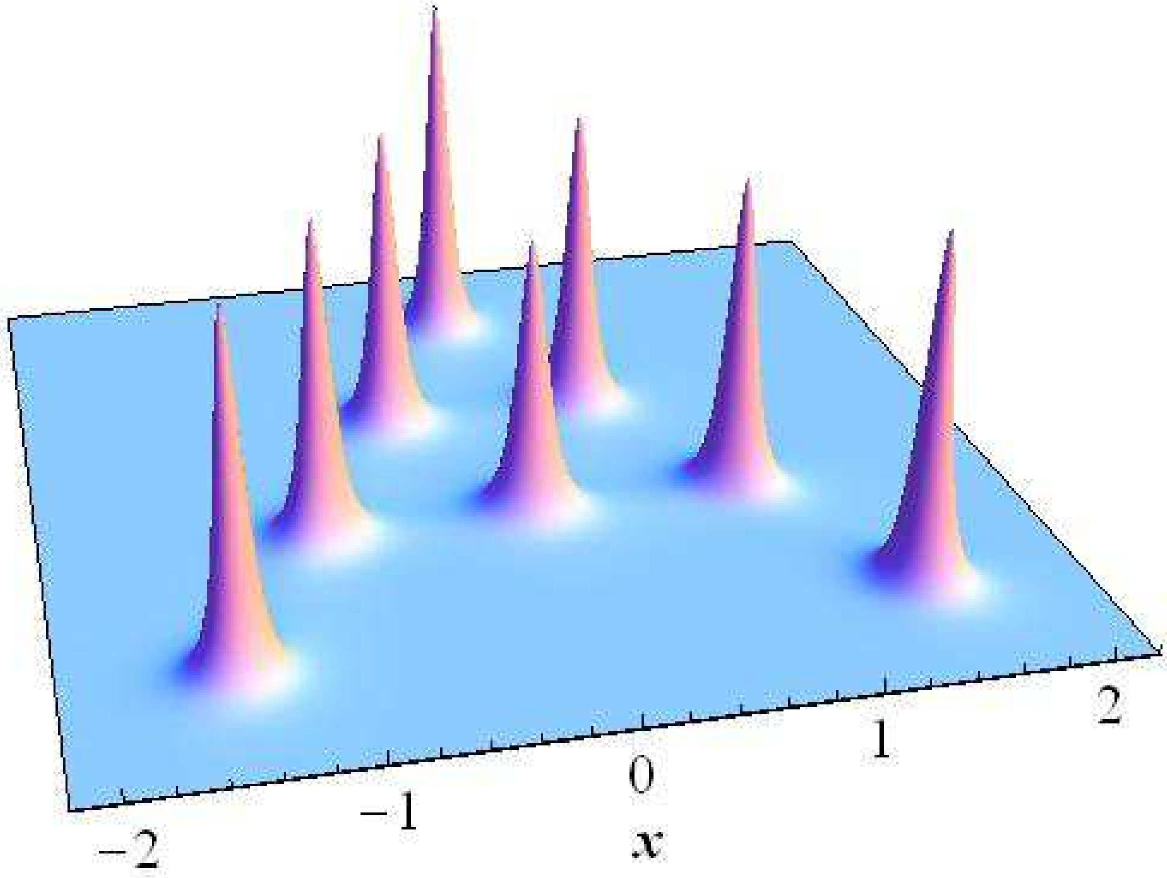
$$E(r,\theta) = \frac{4r^2}{\left(1 + a^4 + r^4 - 2a^2r^2\cos(2\theta)\right)^2}$$

$\mathbb{CP}^1$ model: Solitons

Rational map holomorphic solution of degree 8:

$$Z = \frac{P(z)}{Q(z)}$$

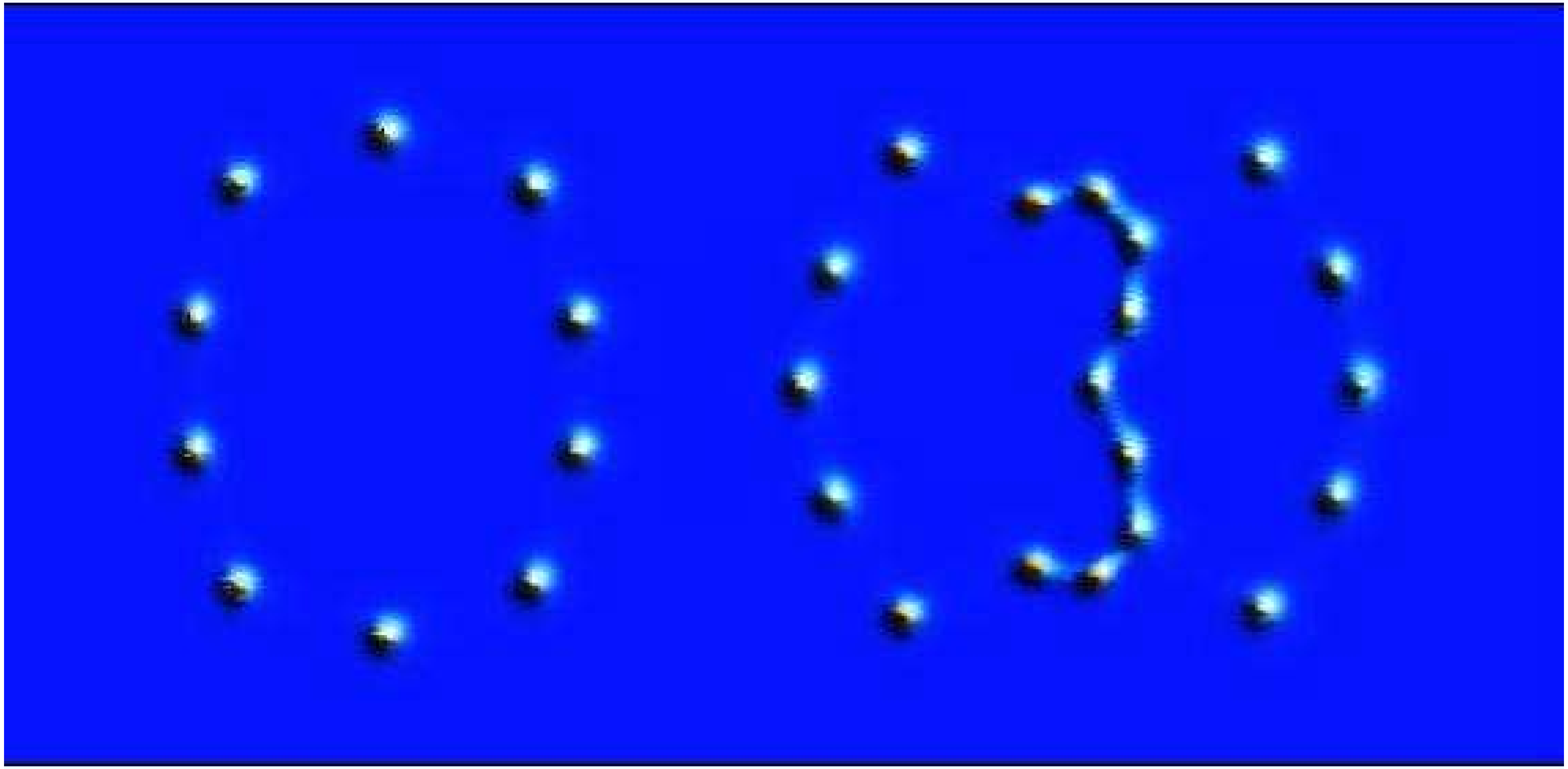
$$Z(z) = \frac{4}{\frac{1}{z} + \frac{1}{z+\frac{1}{2}-i} + \frac{1}{z-\frac{1}{2}-i} + \frac{1}{z-1} + \frac{1}{z-1} + \frac{1}{z+\frac{3}{2}+i} + \frac{1}{z-\frac{3}{2}+i} + \frac{1}{z-2i}}$$



$\mathbb{CP}^1$ model: Solitons

Rational map holomorphic solution of degree 29:

$$Z = \frac{P(z)}{Q(z)}$$



O(3) sigma-model: Merons

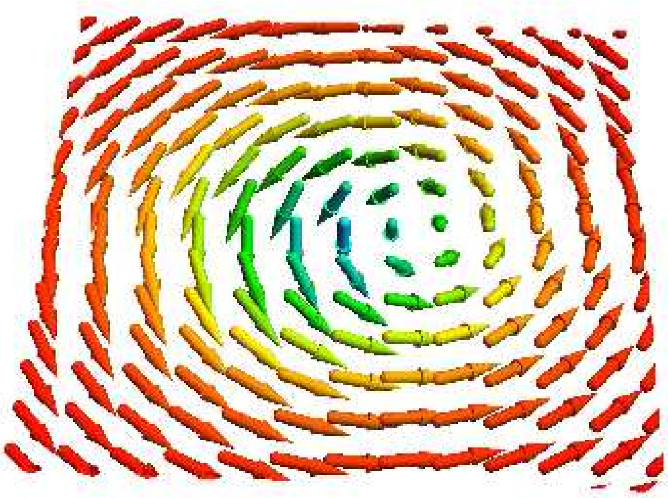
Singular solution:

(μέρος = part)

D. J. Gross, *Nucl. Phys. B* **132**, 439 (1978);
V. de Alfaro, S. Fubini, and G. Furlan, *Nuovo Cim. A* **48**, 485 (1978).

$$\phi_1 = \frac{x}{r}, \quad \phi_2 = \frac{y}{r}, \quad \phi_3 = 0$$

$$E = \frac{1}{2r^2}, \qquad Q = \frac{1}{2}\int d^2x\ \delta^2(r) = \frac{1}{2}$$



2-merons rational map:

$$Z = \sqrt{\frac{(z-a)(\bar{z}-\bar{b})}{(z-b)(\bar{z}-\bar{a})}} \quad \Longrightarrow \quad E = \left|\frac{1}{z-a} + \frac{1}{z-b}\right|^2$$

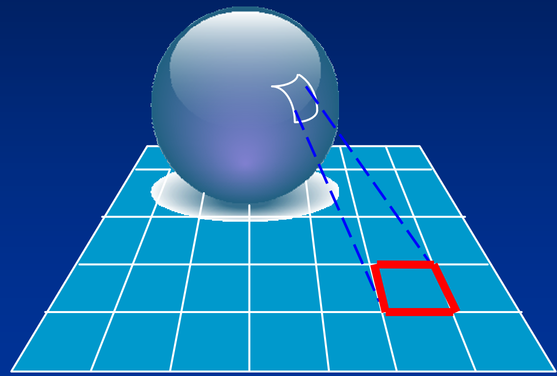
Yang-Mills merons – half-instanton solutions with finite energy and infinite action

Baby Skyrme model

(Bogolubskaya, Bogolubsky (1989)
R.A. Leese et al (1990)

$$\phi = (\phi^1, \phi^2, \phi^3); \qquad \phi^a \cdot \phi^a = 1; \qquad \phi: \; S^2 \to S^2$$

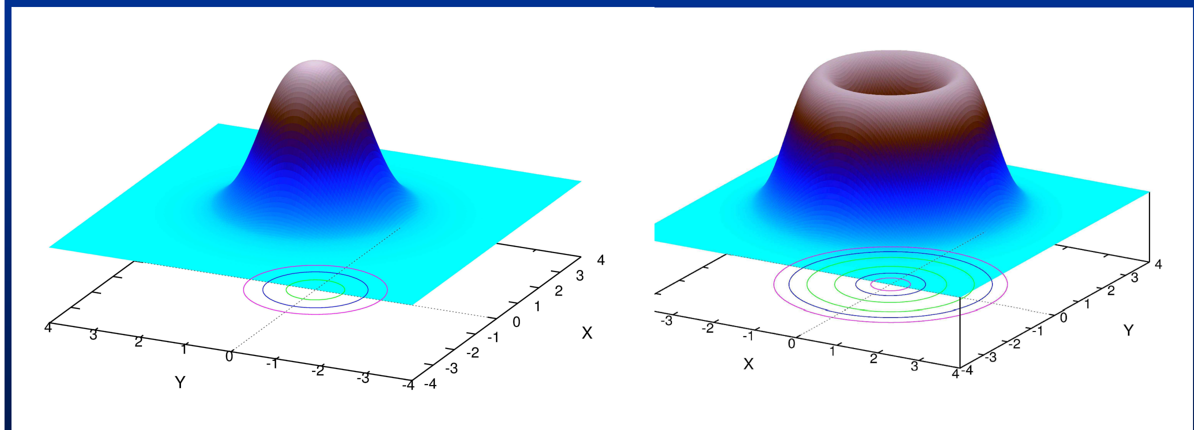
$$Q = \frac{1}{4\pi} \int d^2x \; \varepsilon_{abc}\varepsilon_{ij}\phi^a \partial_i \phi^b \partial_j \phi^c = 1$$



Derrick's scaling theorem: Skyrme term provides a scale but cannot stabilise the soliton: potential term is necessary

$$L = \frac{1}{4}(\partial_\mu \phi^a)^2 - \frac{\kappa}{8}\left[(\partial_\mu \phi^a \partial_\mu \phi^a)^2 - (\partial_\mu \phi^a \partial_\nu \phi^a)(\partial^\mu \phi^a \partial^\nu \phi^a)\right] + m^2(1 - \phi^3)$$

$E \geq \pm 4\pi Q$ equality is possible if $\kappa = 0$ and $m = 0$



Q=1 Q=2

Axially symmetric ansatz:

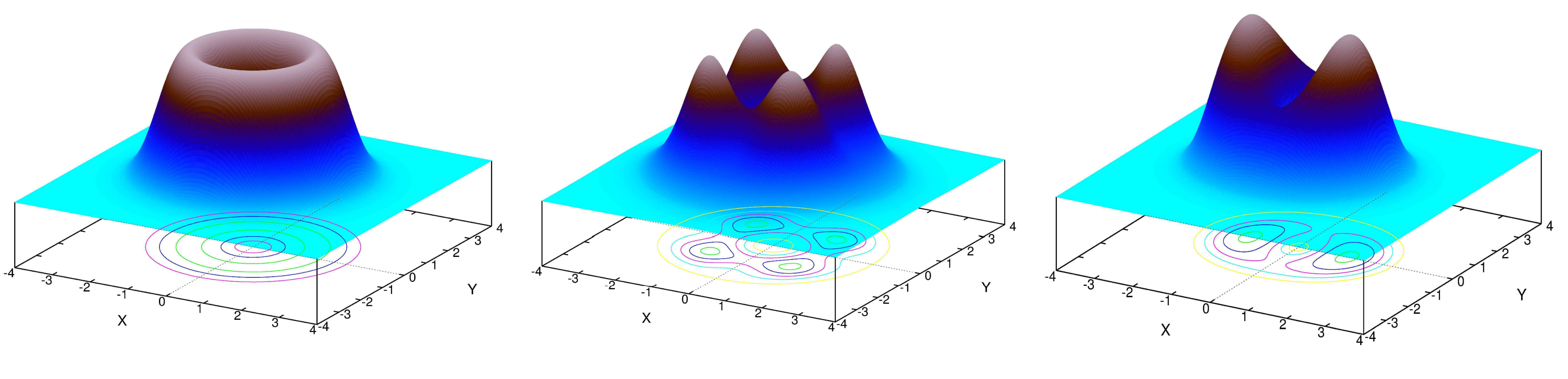
$$\phi^1 = \sin f(r) \cos(Q\varphi - \delta);$$
$$\phi^1 = \sin f(r) \sin(Q\varphi - \delta);$$
$$\phi^3 = \cos f(r)$$

Baby Skyrme model

Potential of the baby Skyrme model: potential term U(ϕ) may be chosen almost arbitrarily, however must vanish at infinity for a given vacuum field value in order to ensure existance of the finite energy solutions: $\phi^a_{(0)} = (0,0,1)$

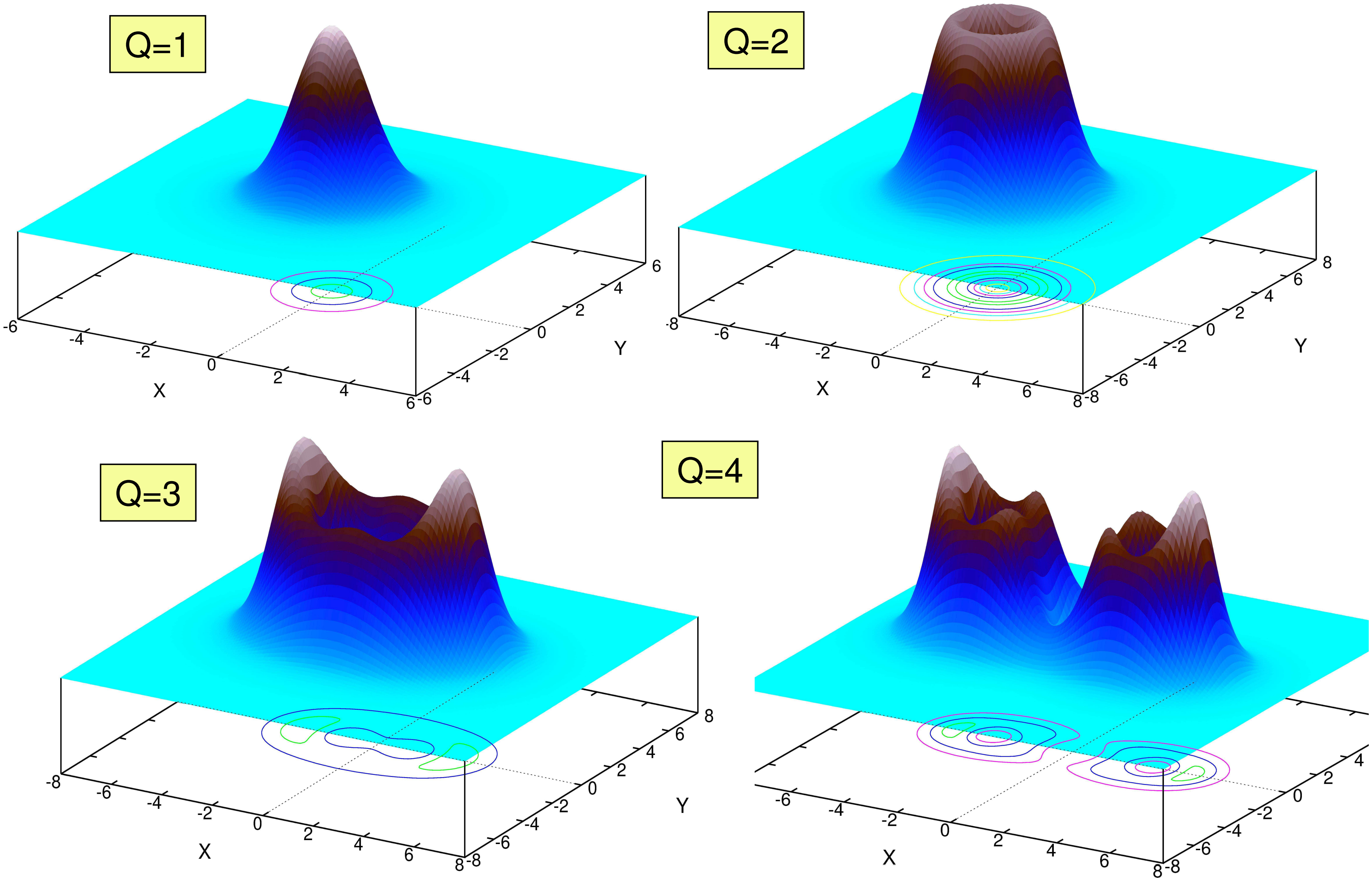
Several potential terms have been studied in great detail:

- "Old" model, with $U(\phi) = m^2(1-\phi_3)$
- Holomorphic model, with $U(\phi) = m^2(1-\phi_3)^4$
- "Double vacuum" model, with $U(\phi) = m^2(1-\phi_3^2)$



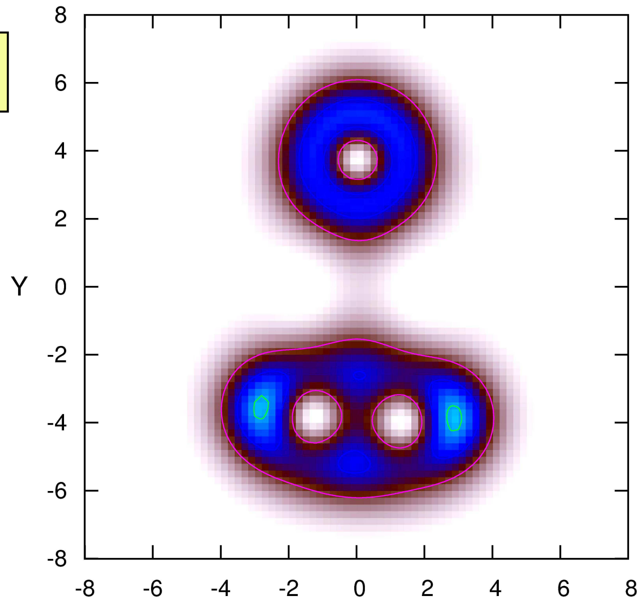
Karliner, Hen (2007) $U(\phi) = m^\alpha(1-\phi_3^\beta)$

Baby Skyrme model: solitons

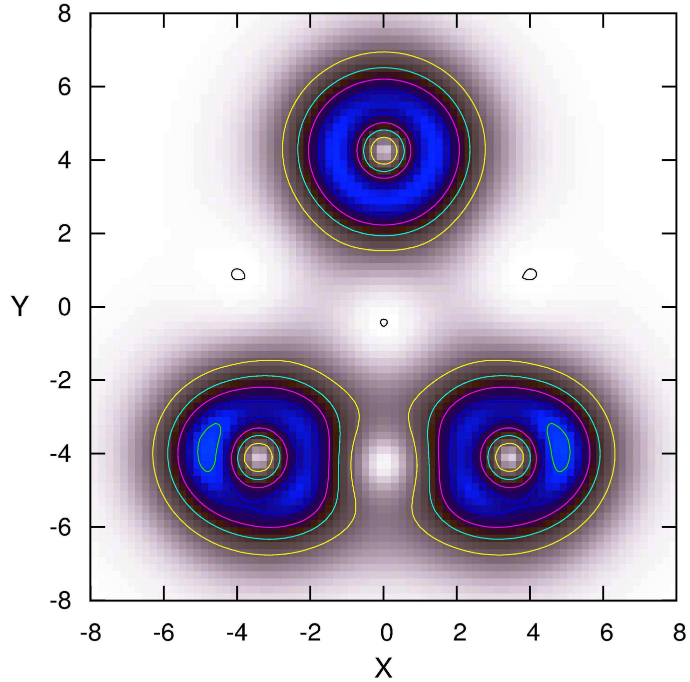


Baby Skyrme model: solitons

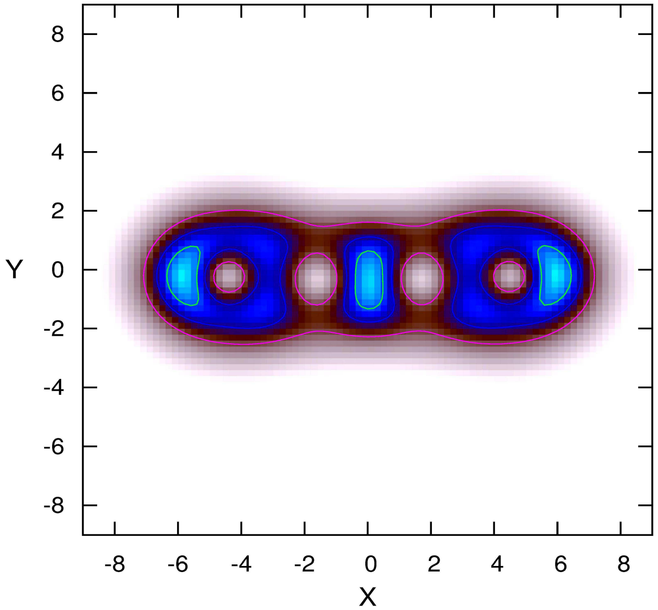
Q=5



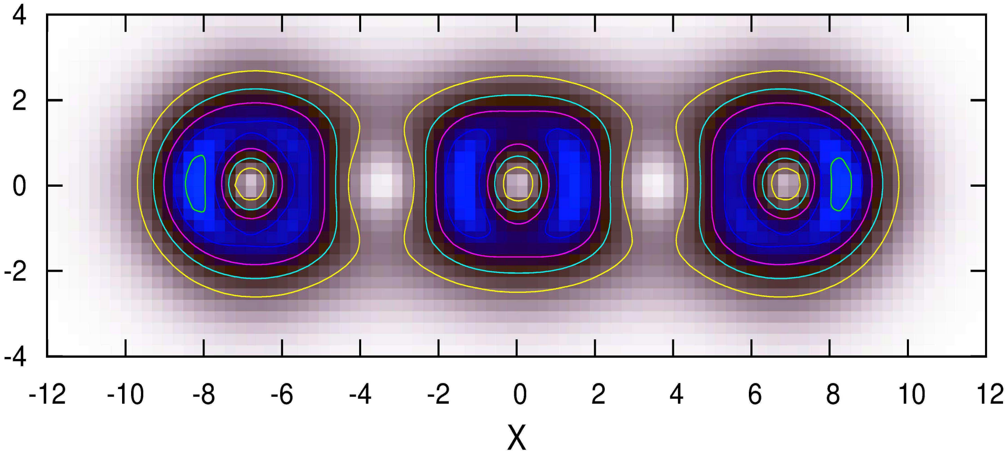
Q=6



Q=5

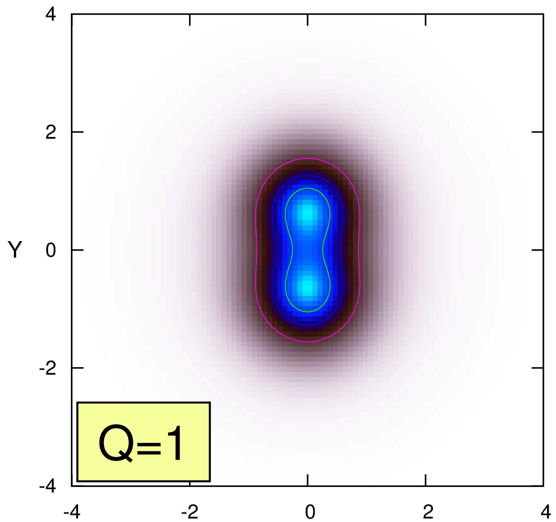


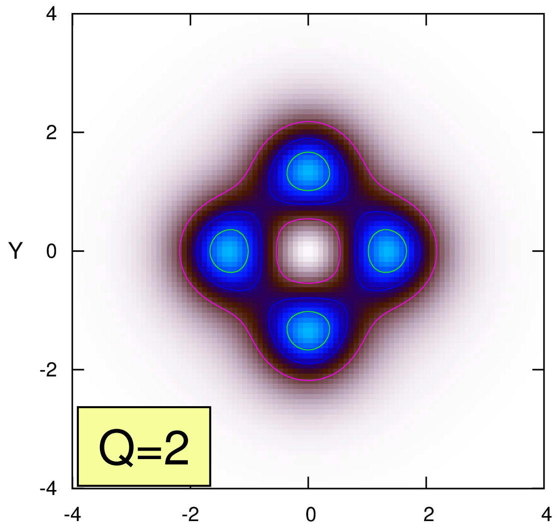
Q=6

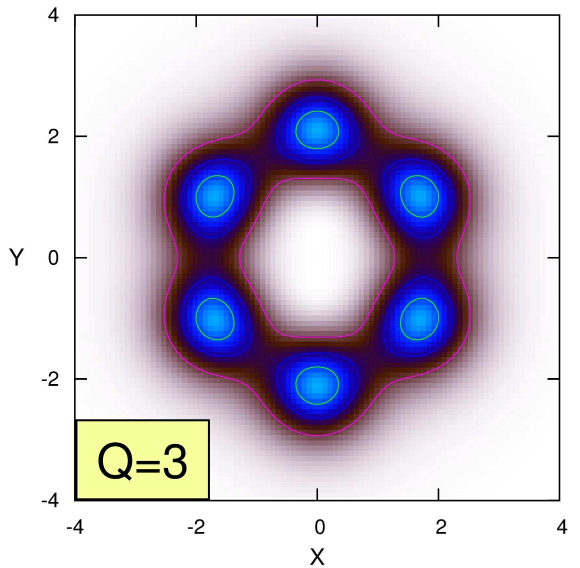


Baby Skyrme model: solitons

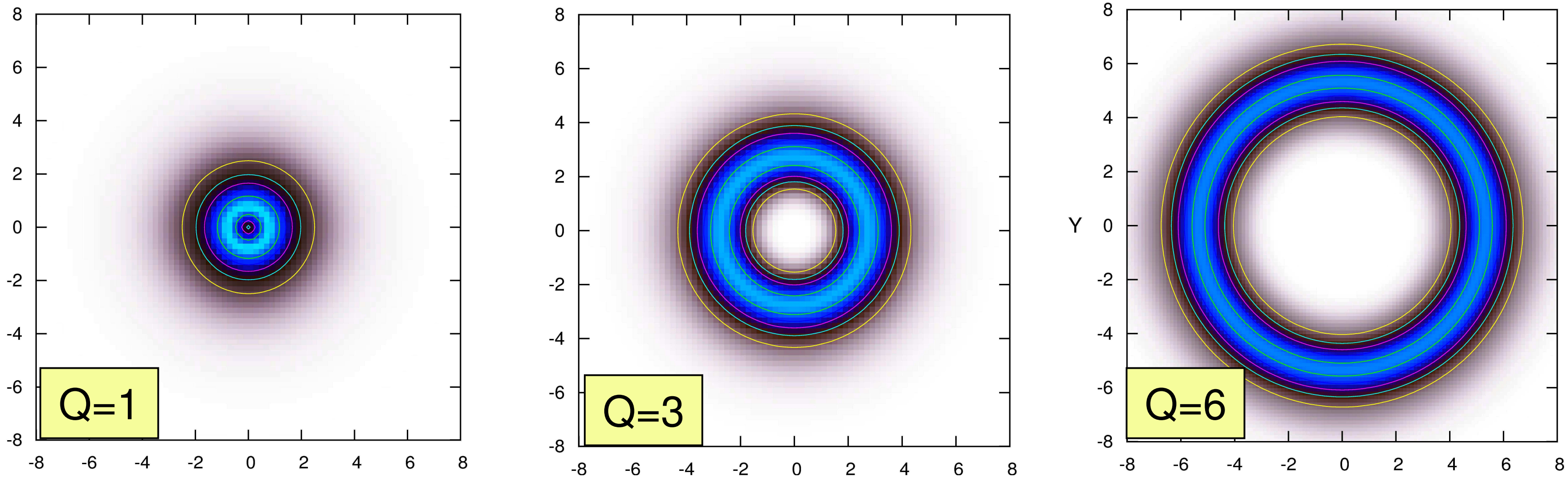
- **Easy plane potential** $U(\phi) = \mu^2 \phi_1^2$





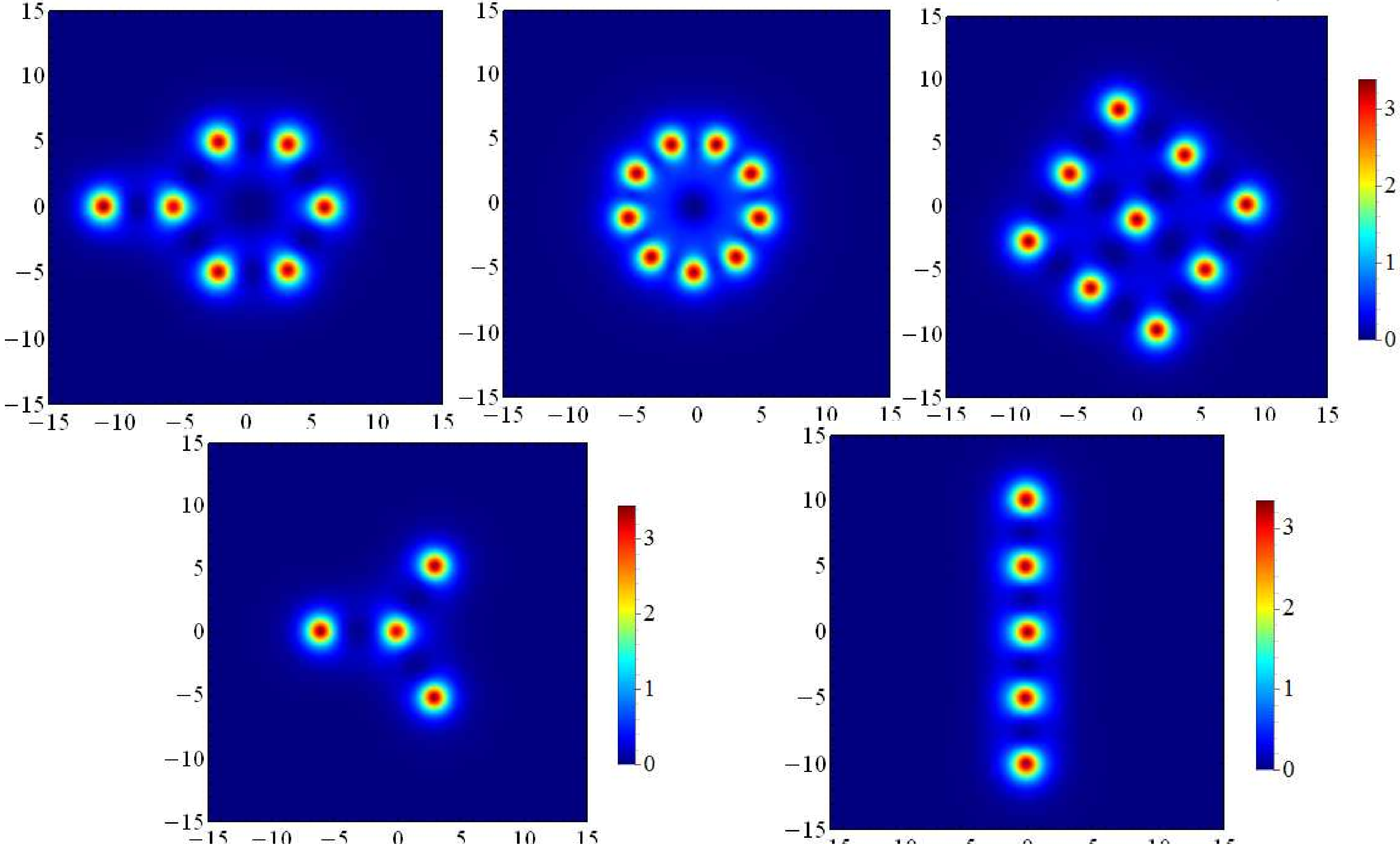


- **Double vacuum potential** $U(\phi) = \mu^2(1 - \phi_3^2)$



Baby Skyrme model: solitons

- **Weakly bounding potential** $U(\phi) = \mu^2\left(\alpha(1-\phi_3) + (1-\alpha)(1-\phi_3)^4\right)$



Gauged baby Skyrme model

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}D_\mu\vec{\phi}\cdot D^\mu\vec{\phi} - \frac{1}{4}\left(D_\mu\vec{\phi}\times D_\nu\vec{\phi}\right)^2 - V(\phi)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu; \qquad D_\mu\vec{\phi} = \partial_\mu\vec{\phi} + gA_\mu\vec{\phi}\times\phi_\infty$$

$\phi : S^2 \to S^2; \quad \phi_\infty = (0,0,1)$ → SO(2) ≃ U(1) unbroken symmetry group

$$(\phi_1 + i\phi_2) = \phi_\perp \to \phi'_\perp = U\phi_\perp; \quad U = e^{ig\alpha} \qquad A_\mu \to A'_\mu = A_\mu + \frac{i}{g}U\partial_\mu U^{-1}$$

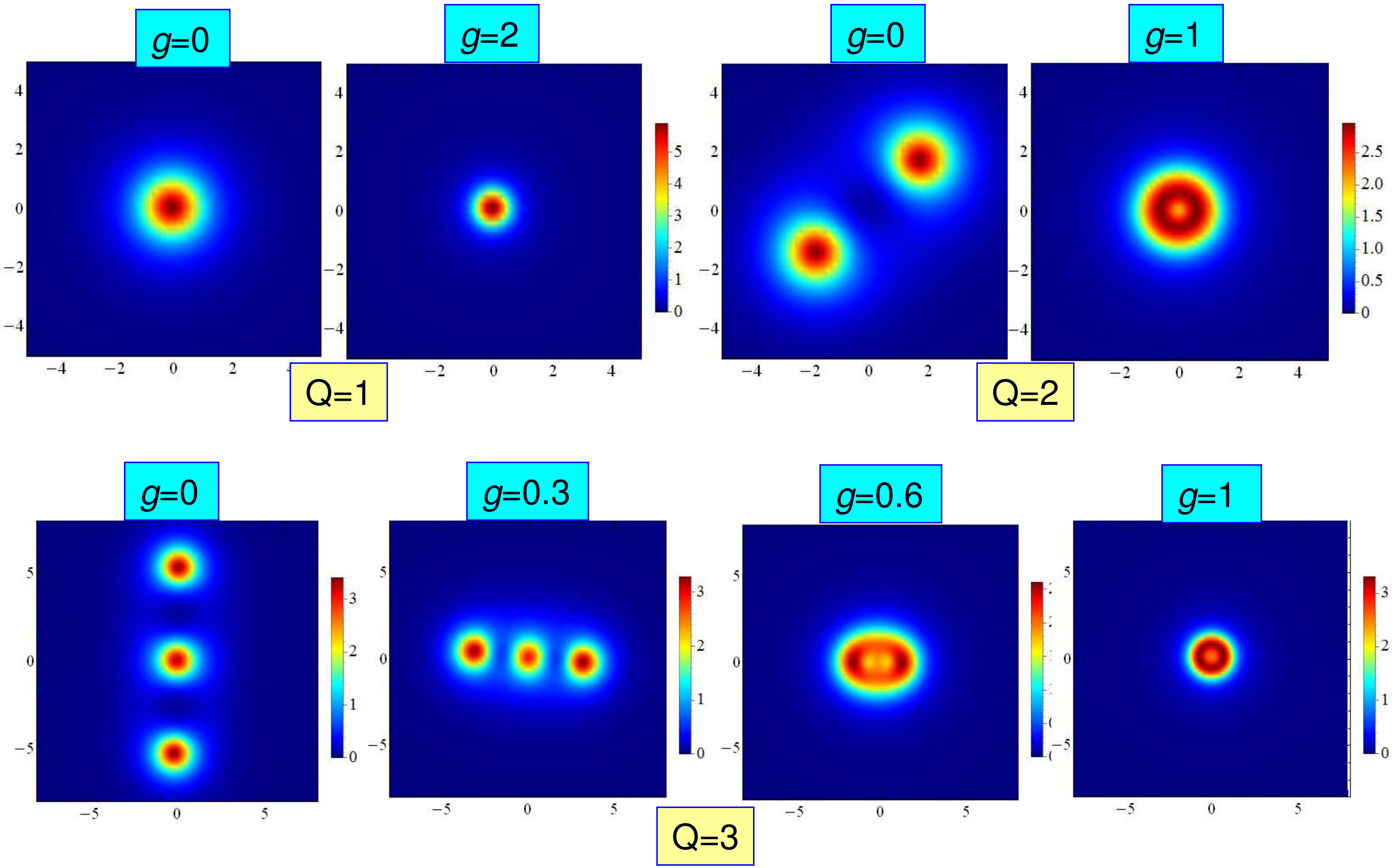
- ***Field equations:***

$$D_\mu\vec{J}^\mu = \frac{V}{\vec{\phi}}\times\vec{\phi}$$

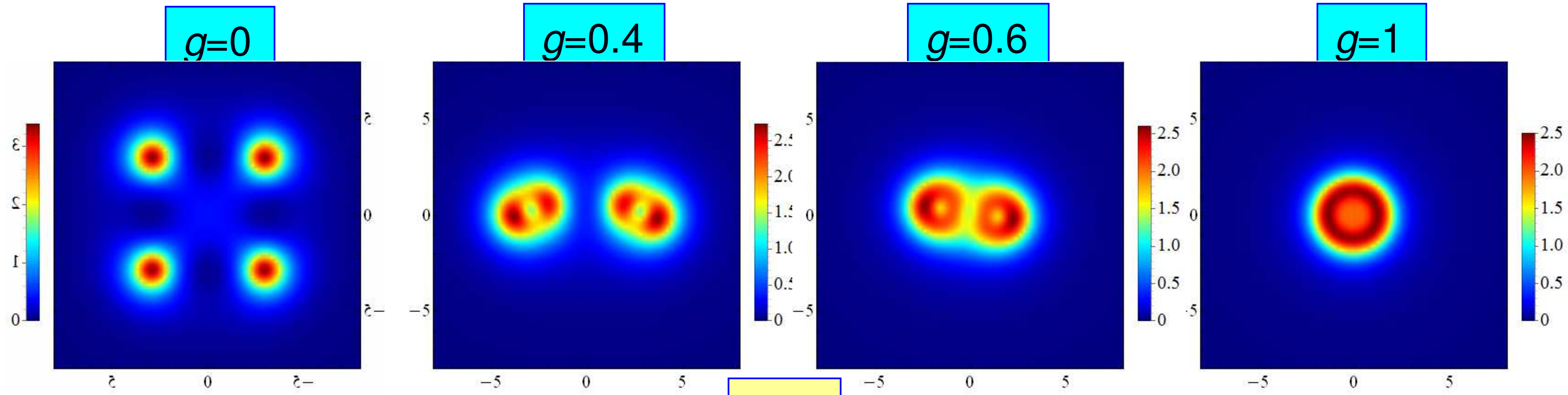
$$\partial_\mu F^{\mu\nu} = g\vec{\phi}_\infty\cdot\vec{J}^\nu$$

- ***Current:*** $\vec{J}^\mu = \vec{\phi}\times D^\mu\vec{\phi} - D_\nu\vec{\phi}(D^\nu\vec{\phi}\cdot\vec{\phi}\times D^\mu\vec{\phi})$

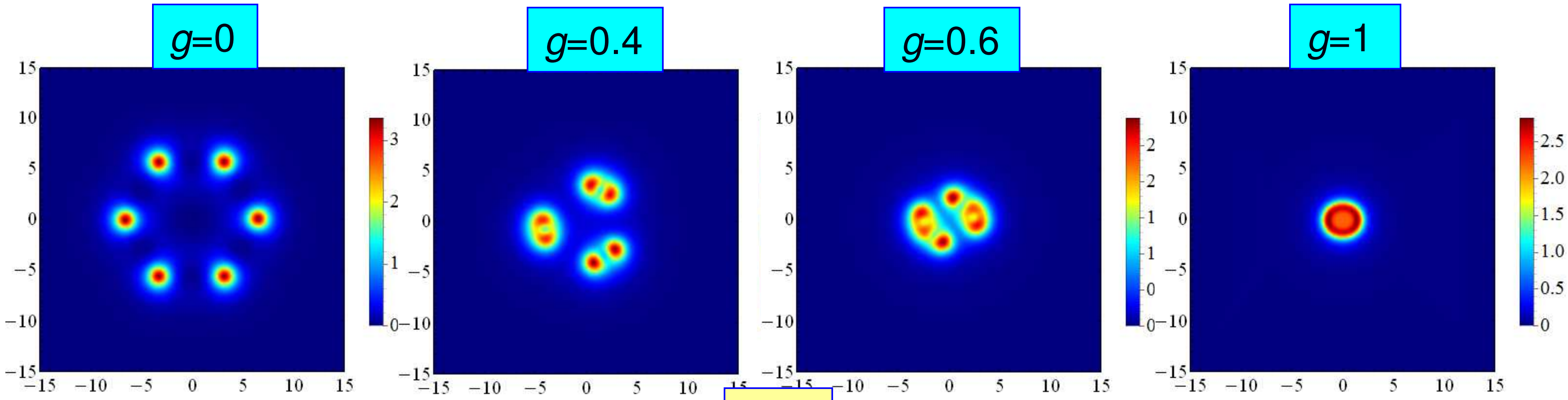
Weakly bounding potential: $U(\phi) = \mu^2 \left[\alpha(1-\phi_3) + (1-\alpha)(1-\phi_3)^4\right]$



Weakly bounding potential: $U(\phi) = \mu^2 \left[\alpha(1-\phi_3) + (1-\alpha)(1-\phi_3)^4\right]$

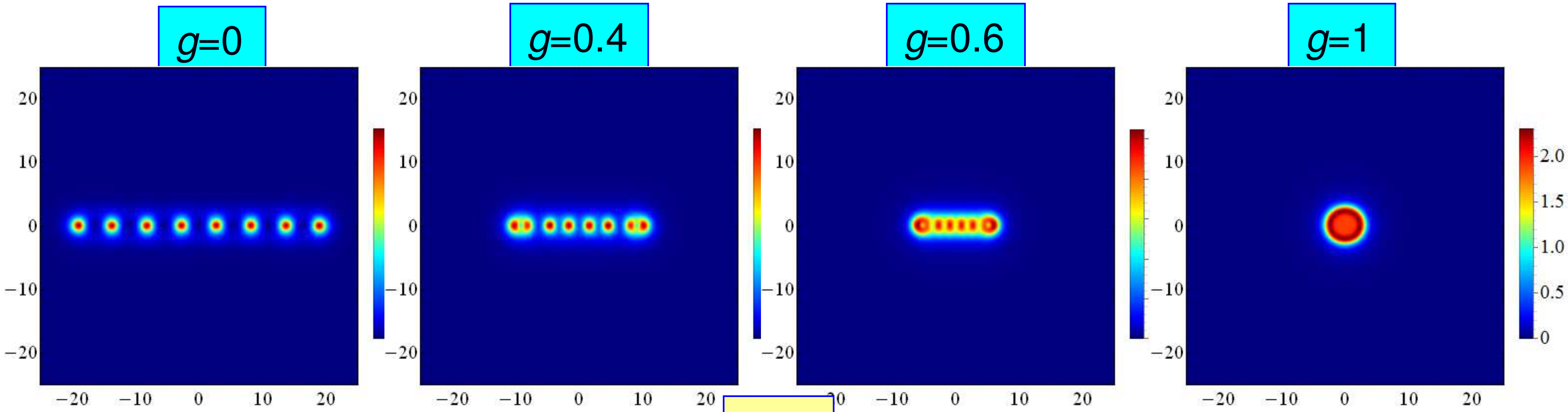


Q=3

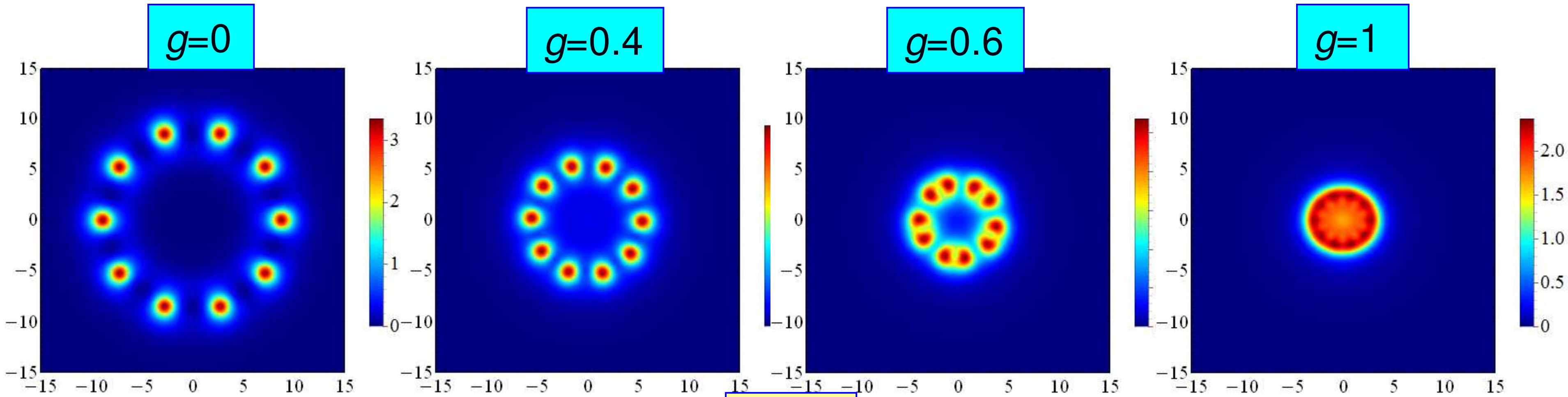


Q=6

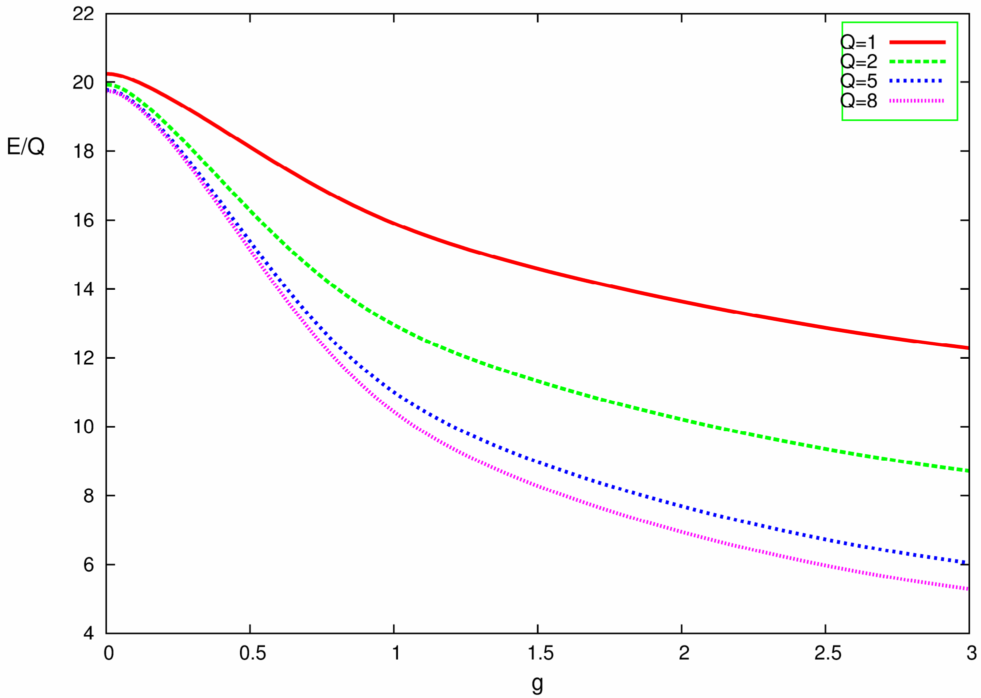
Weakly bounding potential: $U(\phi) = \mu^2 \left[\alpha(1-\phi_3) + (1-\alpha)(1-\phi_3)^4\right]$

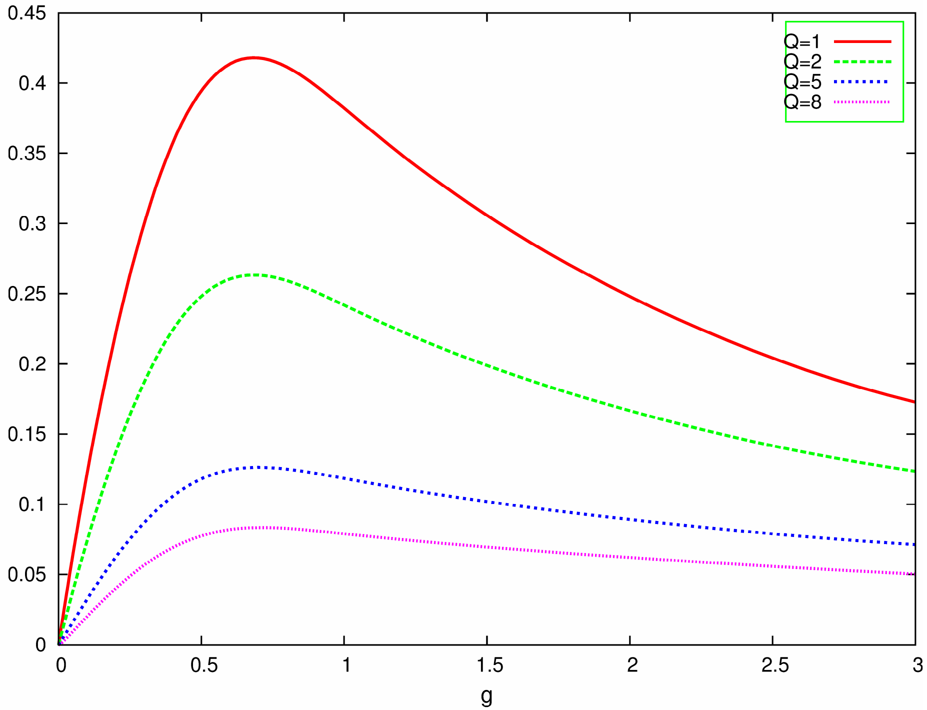


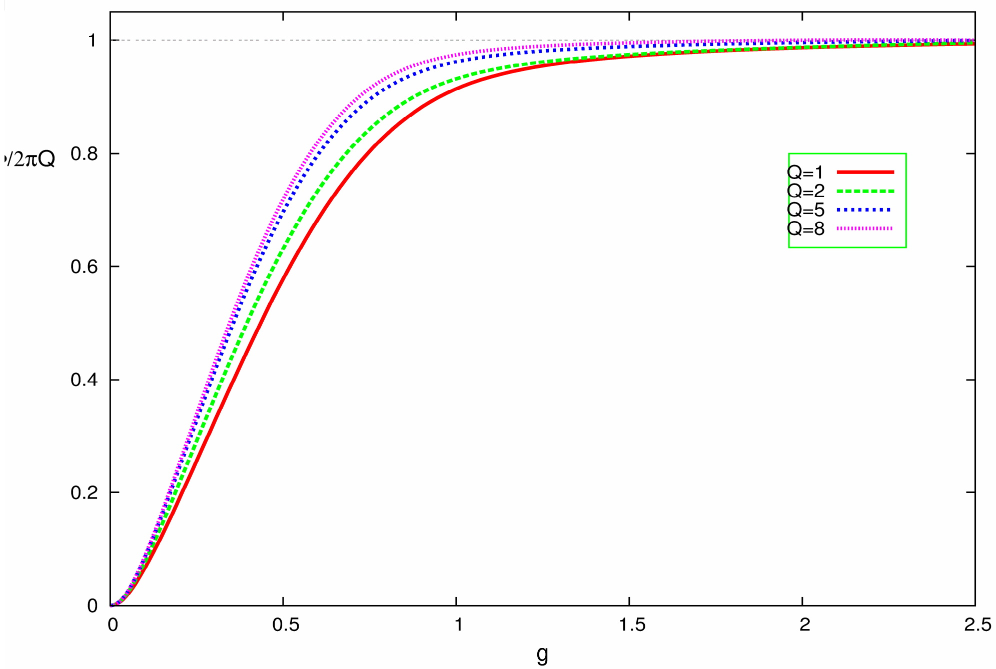
Q=8



Q=10







- ***There is no electric field in the usual gauged planar Skyrme model***
- ***In the strong coupling limit the total magnetic flux is quantized, gΦ=Q***
- ***The energy of the soliton is decreasing as g grows***
- ***As*** $g \to \infty$ ***the maxima of the energy density distribution are at***

$$\phi_3 \to -1, \ \phi_\perp \to 0$$

- ***Maxwell term alone cannot stabilize the soliton***

Chern-Simons-Maxwell baby Skyrme model

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{c}{4}\varepsilon^{\mu\nu\rho}F_{\mu\nu}A_\rho + \frac{1}{2}D_\mu\vec{\phi}\cdot D^\mu\vec{\phi} - \frac{1}{4}(D_\mu\vec{\phi}\times D_\nu\vec{\phi})^2 - V(\vec{\phi})$$

P-,T- violating Chern-Simons term

$$T_{\mu\nu} = -F_{\mu\lambda}F_\nu{}^\lambda + \frac{1}{4}g_{\mu\nu}F_{\lambda\rho}F^{\lambda\rho} + D_\mu\vec{\phi}\cdot D_\nu\vec{\phi} - (D_\mu\vec{\phi}\times D_\rho\vec{\phi})\cdot(D_\nu\vec{\phi}\times D^\rho\vec{\phi})$$
$$- g_{\mu\nu}\left[\frac{1}{2}D_\rho\vec{\phi}\cdot D^\rho\vec{\phi} - \frac{1}{4}(D_\rho\vec{\phi}\times D_\sigma\vec{\phi})\cdot(D^\rho\vec{\phi}\times D^\sigma\vec{\phi}) - V\right]$$

Field equations:

$$\begin{cases} D_\mu\vec{J}^\mu = \dfrac{V}{\vec{\phi}}\times\vec{\phi} \\ \partial_\mu F^{\mu\nu} + \dfrac{c}{2}\varepsilon^{\nu\alpha\beta}F_{\alpha\beta} = g\vec{\phi}_\infty\cdot\vec{J}^\nu \end{cases}$$

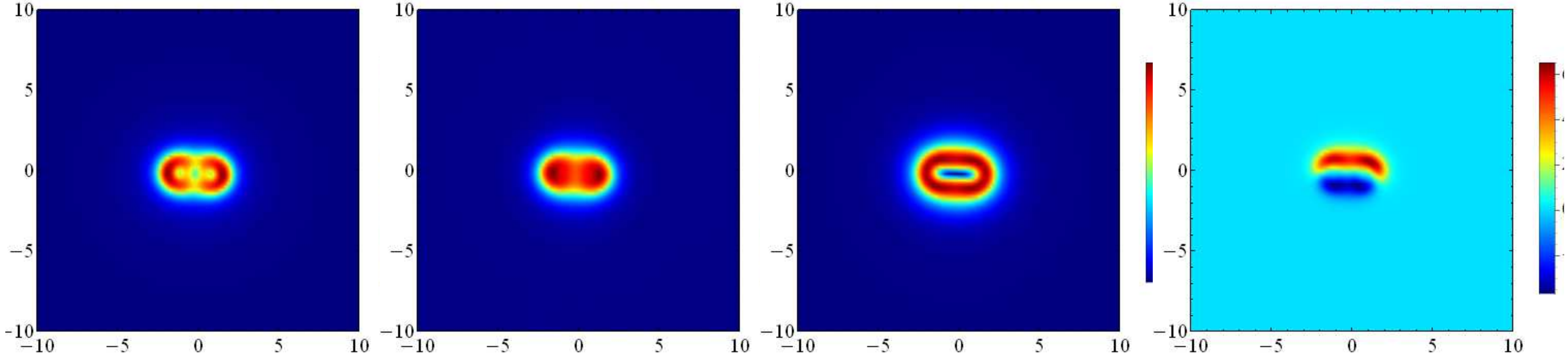
$$A_0(\infty)\to\omega$$

Gauss law: $\nabla\vec{E} + cB = g\rho$ ⟹ $\Phi = \int d^2x B \sim q$

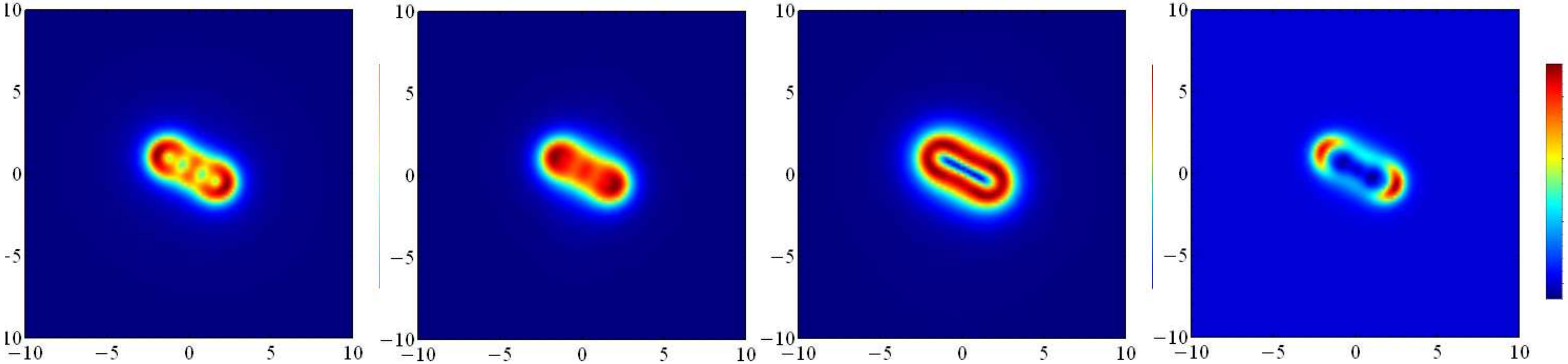
Angular momentum: $J = \int T_{\varphi 0}\, d^2x$

Weakly bounding potential: $U(\phi) = \mu^2 \left[\alpha(1-\phi_3) + (1-\alpha)(1-\phi_3)^4\right]$

$\mathcal{E}$ | **B** | $|\vec{E}|$ | **J**



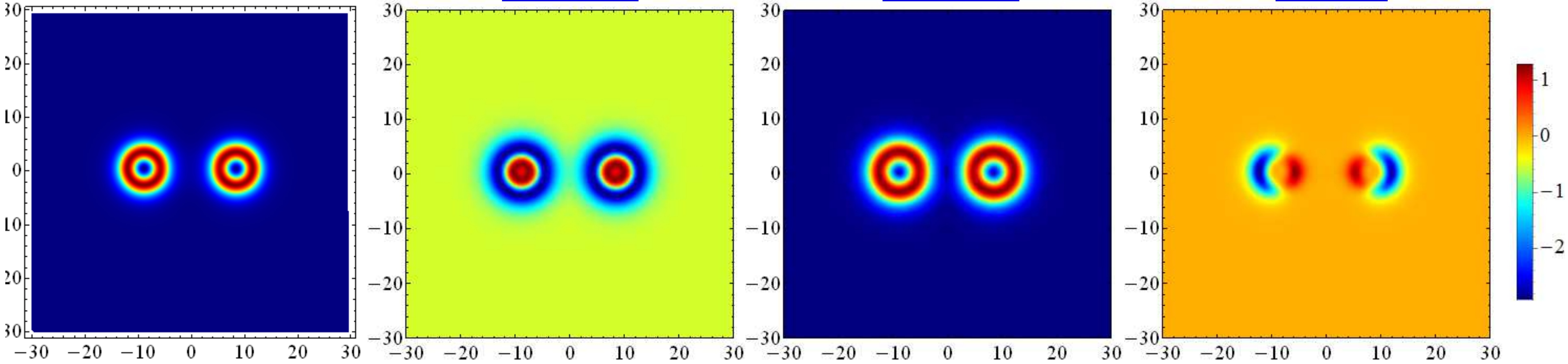
Q=4, g=1.5



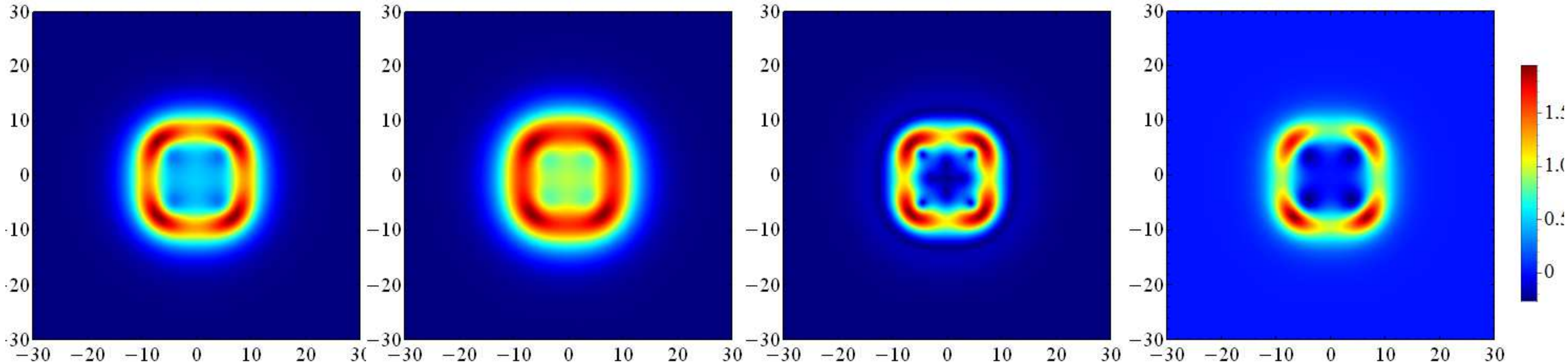
Q=5 , g=1.5

Double vacuum potential: $U(\phi) = \mu^2(1 - \phi_3^2)$

$\mathcal{E}$ | B | $|\vec{E}|$ | J



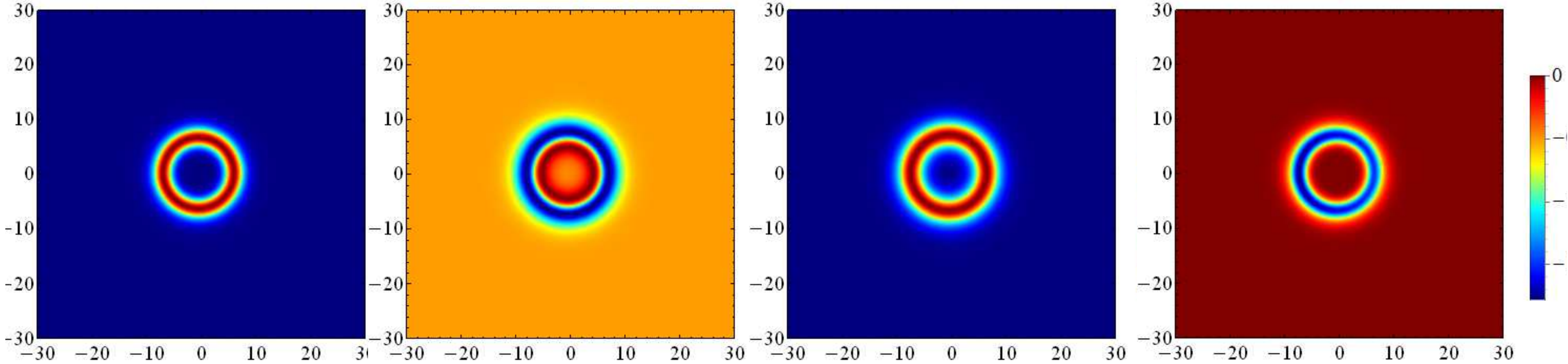
Q=4, g=0.3, A_0=0.9



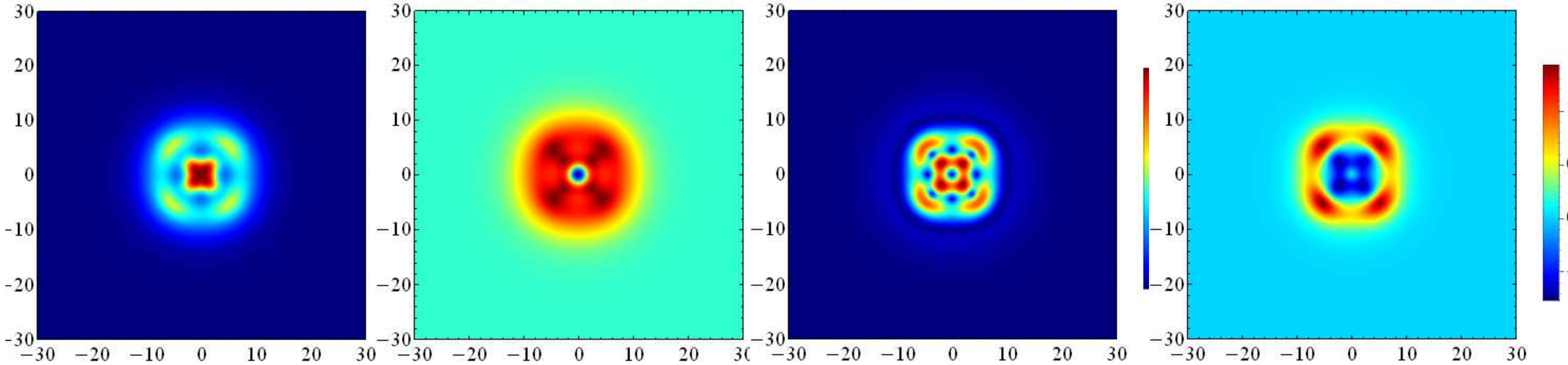
Q=4 , g=0.3, A_0=-0.9

Double vacuum potential: $U(\phi) = \mu^2(1 - \phi_3^2)$

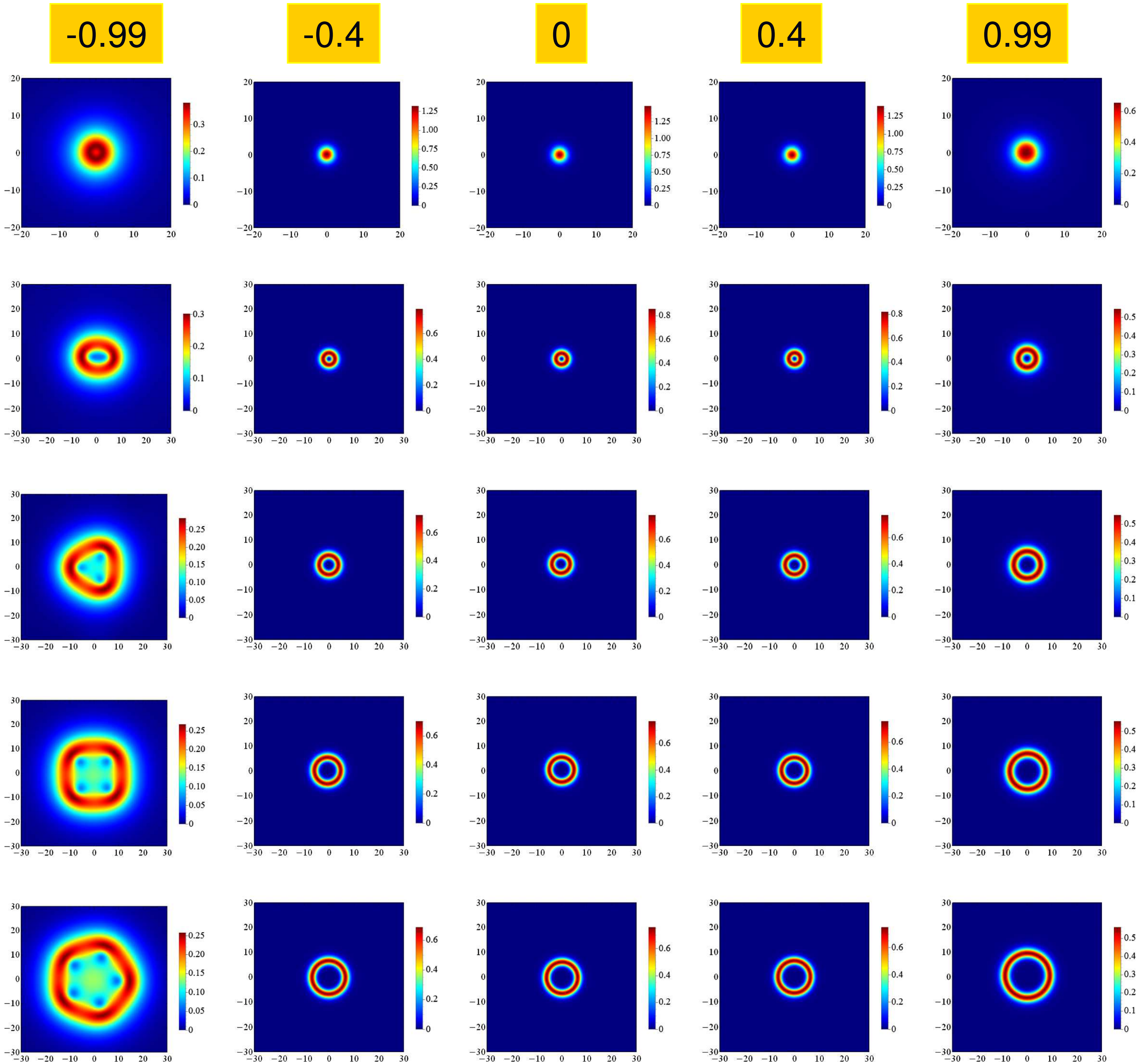
$\mathcal{E}$ | ***B*** | $|\vec{E}|$ | ***J***



Q=4, g=0.3, A_0=0.9

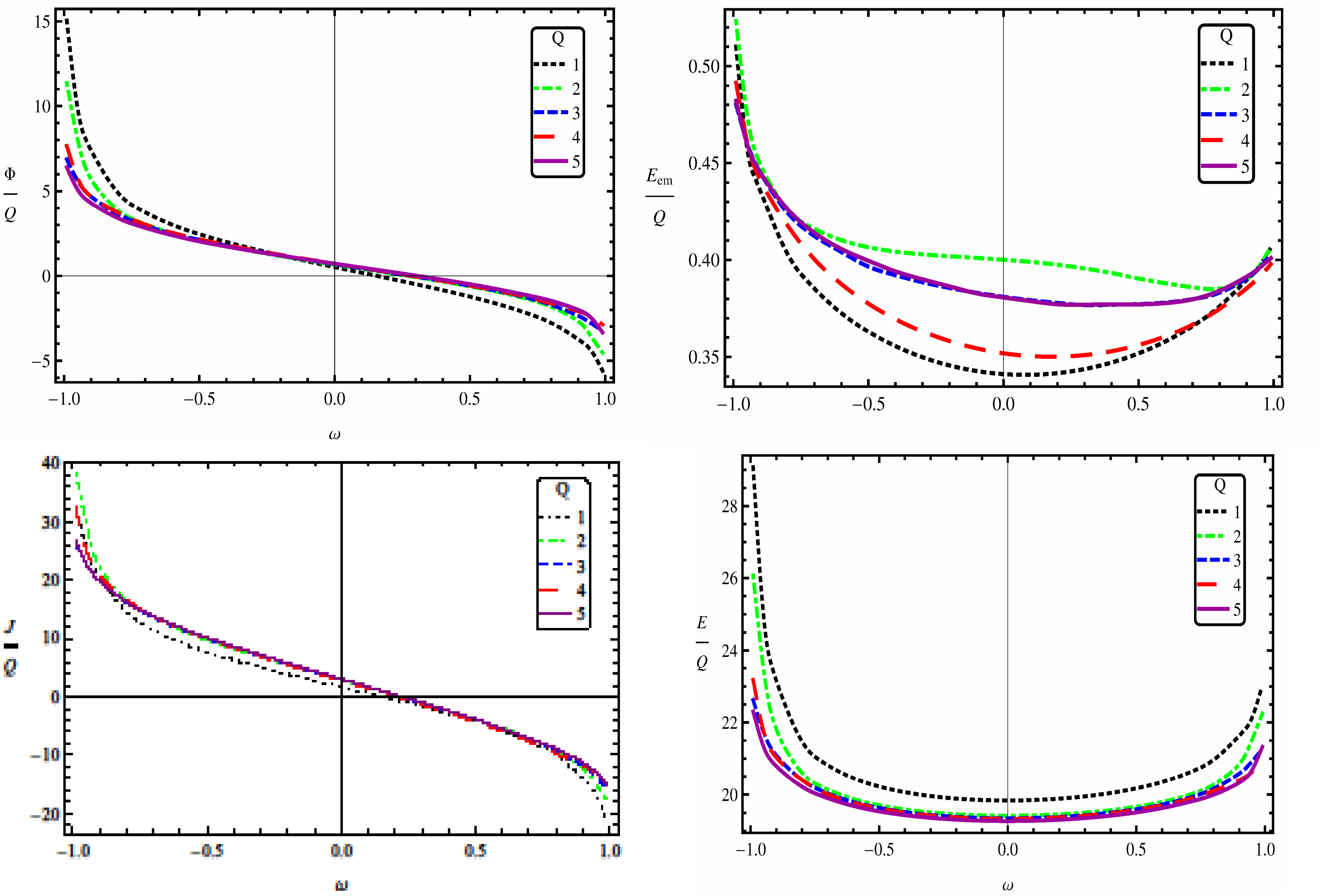


Q=4 , g=0.3, A_0=-0.9



-0.99
-0.4
0
0.4
0.99

g=0.3



Gauged merons

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}D_\mu\vec{\phi}\cdot D^\mu\vec{\phi} - \frac{1}{4}\left(D_\mu\vec{\phi}\times D_\nu\vec{\phi}\right)^2 - V(\phi)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu; \qquad D_\mu\vec{\phi} = \partial_\mu\vec{\phi} + gA_\mu\vec{\phi}\times\phi_\infty$$

$$V(\phi) = m^2(\phi_3 - c)^2$$

$\phi : S^2 \to S^2;\quad \phi_\infty = (0,0,c)$ ➡ SO(2) ≃ U(1) unbroken symmetry group

$$D_i\phi_\perp = \partial_i\phi_\perp - iA_i\phi_\perp \underset{r\to\infty}{\longrightarrow} 0, \quad ➡ \quad \phi_\perp \underset{r\to\infty}{\to} \sqrt{1-c^2}e^{i\Psi(\theta)}, \; A_i \underset{r\to\infty}{\to} \partial_i\alpha(\theta)$$

$$\Phi = \oint_{S^1} A_i dx^i = 2\pi n$$

$$Q = -\frac{1}{4\pi}\int d^2x\,\vec{\phi}\cdot(\partial_1\vec{\phi}\times\partial_2\vec{\phi})$$

Topological charge $\pi_1(S^1)$

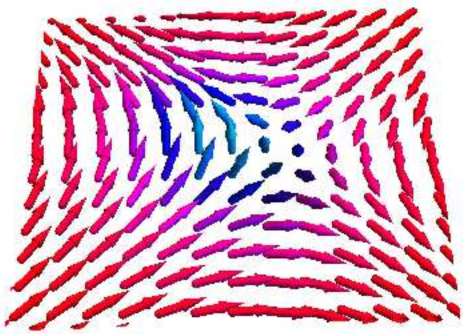
Topological charge $\pi_2(S^2)$

$$\vec{\phi}(0) = (0,0,-1) \longrightarrow Q = \frac{1+c}{2}$$

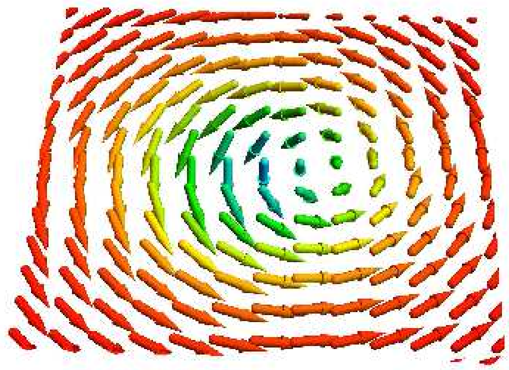
$$\vec{\phi}(0) = (0,0,1) \longrightarrow Q = \frac{1-c}{2}$$

Gauged merons

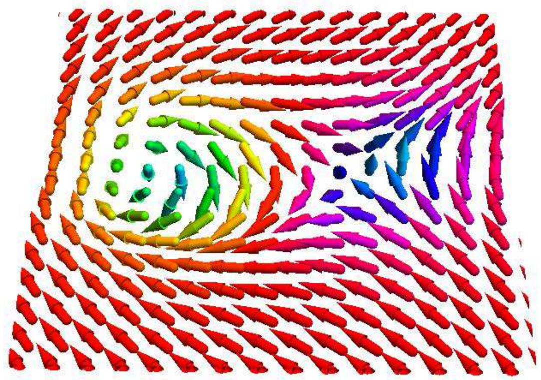
N Meron



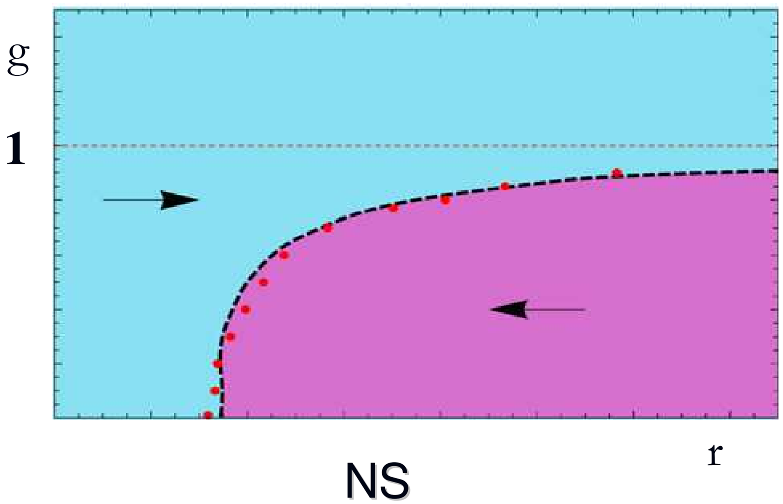
S Meron



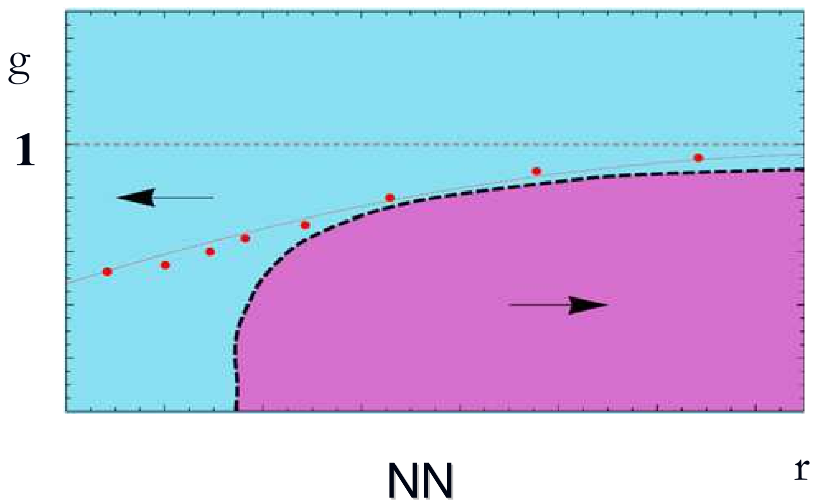
bounded NS Merons



SN



SS/NN



Interaction between the gauged merons

Linearized field eqs:

$$(\Delta - m^2)\phi_3 = 0$$

$$(\Delta - g^2)\delta A_i = 0; \qquad \partial_i \delta A_i = 0$$

$$\phi_3 \sim c_s K_0(mr), \qquad A_\theta \sim n + c_v r K_1(gr)$$

Interaction potential:

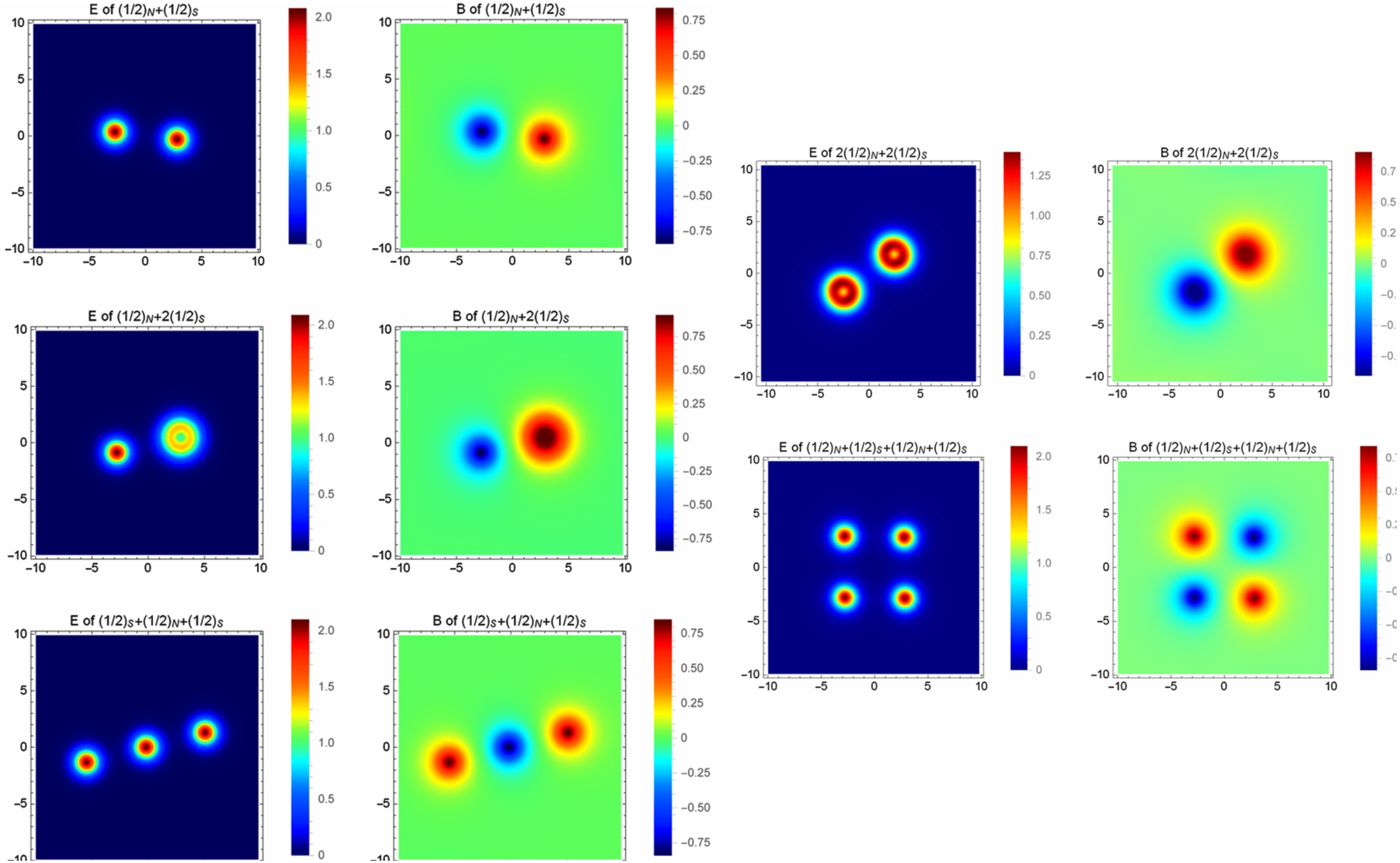
$$U(r) \sim c_v^{(1)} c_v^{(2)} K_0(gr) - c_s^{(1)} c_s^{(2)} K_0(mr)$$

Force between the merons:

$$F = \pm 2\pi \left[c_v^2 g K_1\left(gR\right) - c_s^2 m K_1(mR) \right]$$

There can be a stable equilibrium for the system of two merons of different types, N and S

Gauged merons



Summary and Outlook

- **Gauged planar Skyrmions are coupled to the magnetic fluxes, the quantization of the fluxes matches the topology of the scalar sector**
- **Rotational invariance of the multisoliton configurations is recovered in the strong coupling limit (without CS term)**
- **There is a compicated pattern of the P-, T- violating interactions between the CS-Maxwell baby Skyrmions**
- **Gauged multisolitons in the model with Dzyaloshinskii-Moriya interaction term?**
- **There is a new class of regular soliton solutions of the gauged planar Skyrme model – gauged merons**
- **Gauged CS BPS Skyrmions?**
- **Crystalline structures?**