D-branes as a single object

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based on works with

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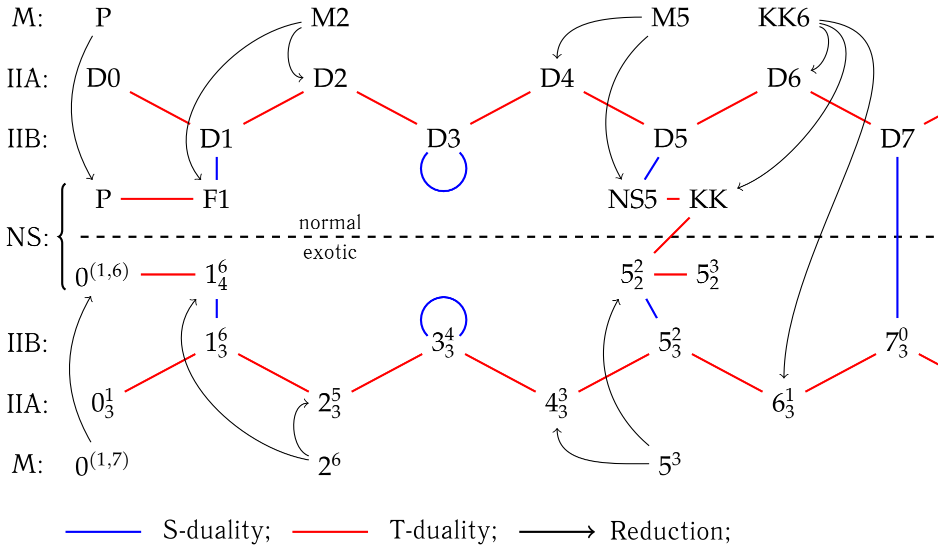
Web of (some) branes



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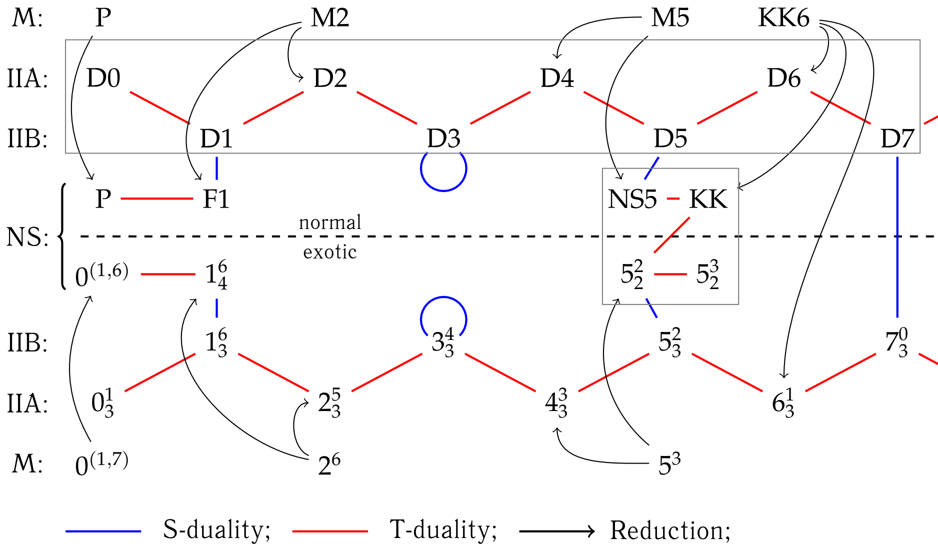


Why bother?

D-branes

- AdS/CFT correspondence: different limits of a single D-brane effective action;
- phenomenological constructions: braneworld models, interesection of branes;
- black hole entropy counting;

NS-branes

- Little String Theories: $\mathcal{N} = (2,0)$ (for IIA) and $\mathcal{N} = (1,1)$ (for IIB) 6D theories;
- non-conformal field theory holographies;
- cosmological moduli stabilization via (non-geometric) NS-NS fluxes;

The results

T-duality orbits

$$\mathrm{D0} - \mathrm{D1} - \mathrm{D2} - \mathrm{D3} - \mathrm{D4} - \mathrm{D5} - \mathrm{D6} - \mathrm{D7} - \mathrm{D8} - \mathrm{D9}$$

$$\mathrm{NS5}(5^0_2) \;—\; \mathrm{KK5}(5^1_2) \;——\; \mathrm{Q}(5^2_2) \;——\; \mathrm{R}(5^3_2) \;——\; \mathrm{R}'(5^4_2)$$

- Effective actions for these T-duality orbits has been constructed.
- Depending on orientation these project down to actions for normal branes.
- One observes non-geometric effects for D-branes

D-branes: geometry and dynamics

In supergravity D-branes

- are RN black-hole-like solutions of SUGRA equations of motion,
- preserve $\frac{1}{2}$SUSY
- are described by non-zero metric $G_{\mu\nu}$ and gauge field $C_{\mu_1 \dots \mu_p}$

As fundamental objects D-branes

- are described by effective action

$$S_{DBI} = \int_\Sigma d^{p+1}\xi \sqrt{\det\left(G_{\mu\nu}\partial_a x^\mu \partial_b x^\nu + \mathcal{F}_{ab}\right)} + \int_\Sigma \mathcal{C}_{p+1} \tag{1}$$

- carry worldvolume fields $x^\mu = x^\mu(\xi)$ and $\mathcal{F}_{ab}$

T-duality

- Mass spectrum of a string on a torus $\mathbb{T}^d$ is invariant under $O(d,d)$ group
- SUGRA solutions transform into solutions
- String does not feel the change in backgrounds
- T-duality is performed along isometries

T-duality orbit of NS branes

$$
\begin{array}{lcccccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\text{NS5}: & \multicolumn{6}{c}{\underbrace{\times \quad \times \quad \times \quad \times \quad \times \quad \times}_{\text{world-volume}}} & \multicolumn{4}{c}{\underbrace{\bullet \quad \bullet \quad \bullet \quad \bullet}_{\text{transverse}}} \\
 & & & & & & & & & \multicolumn{2}{r}{\Big\updownarrow\, T_9} \\
\text{KK5}: & \multicolumn{6}{c}{\underbrace{\times \quad \times \quad \times \quad \times \quad \times \quad \times}_{\text{world-volume}}} & \multicolumn{3}{c}{\underbrace{\bullet \quad \bullet \quad \bullet}_{\text{transverse}}} & \underset{\text{special}}{\odot} \\
 & & & & & & & & & \multicolumn{2}{r}{\Big\updownarrow\, T_8} \\
Q: & \times & \times & \times & \times & \times & \times & \bullet & \bullet & \odot & \odot \\
R: & \times & \times & \times & \times & \times & \times & \bullet & \odot & \odot & \odot \\
R': & \times & \times & \times & \times & \times & \times & \odot & \odot & \odot & \odot
\end{array}
\tag{2}
$$

[deBoer, Shigemori]

T-duality orbit of D-branes

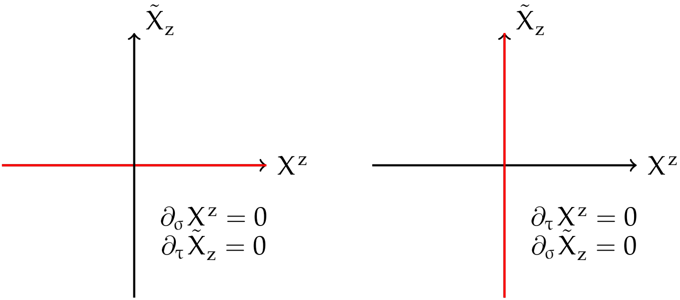
$$
\begin{array}{rcccccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\mathrm{D0}: & \times & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\mathrm{D1}: & \times & \times & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\mathrm{D2}: & \times & \times & \times & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\mathrm{D3}: & \times & \times & \times & \times & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\mathrm{D4}: & \times & \times & \times & \times & \times & \bullet & \bullet & \bullet & \bullet & \bullet \\
\mathrm{D5}: & \times & \times & \times & \times & \times & \times & \bullet & \bullet & \bullet & \bullet \\
\mathrm{D6}: & \times & \times & \times & \times & \times & \times & \times & \bullet & \bullet & \bullet \\
\mathrm{D7}: & \times & \times & \times & \times & \times & \times & \times & \times & \bullet & \bullet \\
\mathrm{D8}: & \times & \times & \times & \times & \times & \times & \times & \times & \times & \bullet \\
\mathrm{D9}: & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times
\end{array} \tag{3}
$$

! T-duality changes dimension of a D-brane

T-duality orbit of D-branes

Boundary conditions for open string ends with $\omega^\mu = p_L^\mu - p_R^\mu$, $p^\mu = p_L^\mu + p_R^\mu$:

$$
\begin{aligned}
N: \quad & \partial_\sigma X^\mu \Big|_{\partial\Sigma} = 0, \quad && D: \quad \partial_\tau X^\mu \Big|_{\partial\Sigma} = 0, \\
N: \quad & \omega^\mu = 0, && D: \quad p^\mu = 0,
\end{aligned}
\tag{4}
$$



Covariant potentials

- D-brane potentials $C_{(p+1)}$ can be combined:

$$|\chi\rangle = \sum_{p=0}^{10} C_{m_1 \dots m_p} \Gamma^{m_1 \cdots m_p} |0\rangle \tag{5}$$

- $O(10,10)$ algebra that includes $GL(10)$ as $T^M = (T^m, T_m)$:

$$\begin{aligned} \{\Gamma_M, \Gamma_N\} &= 2\eta_{MN} \quad \Longrightarrow \quad \{\Gamma_m, \Gamma^n\} = \delta_m{}^n, \\ \text{Clifford vacuum:} & \qquad\qquad \Gamma_m|0\rangle = 0 \end{aligned} \tag{6}$$

- For each brane one defines **a charge** $\langle Q|$

Covariant potentials

$$
\begin{aligned}
&\text{for say } \langle Q| = \langle 0| \\
&\text{the only non-zero: } \langle Q|\Gamma_{m_1\dots m_{10}}|\chi\rangle = C_{m_1\dots m_{10}} \\
\\
&\text{for say } \langle Q| = \langle 0|\Gamma_{\hat{m}} \\
&\text{the only non-zero: } \langle Q|\Gamma_{m_1\dots m_9}{}^{\hat{m}}|\chi\rangle = C_{m_1\dots m_9}.
\end{aligned} \tag{7}
$$

Invariant interaction

$$
S_{wz} = \int d^{10}\xi\ \varepsilon^{a_1\dots a_{10}} \langle Q|\Gamma_{M_1\dots M_{10}}|\chi\rangle \partial_{a_1} X^{M_1} \cdots \partial_{a_{10}} X^{M_{10}}\,. \tag{8}
$$

Upon choice of $\langle Q|$ this reproduces smth like

$$
S_{wz}{}^{D} = \int C_{m_1\dots m_p} dX^{m_1} \wedge \dots \wedge dX^{m_p} \tag{9}
$$

Doubled geometry

- Doubled coordinates $X^M = (x^m, \tilde{x}_m)$
- section constraint for consistency of the theory, kills half of the coordinates
- T-duality: $T_x : x \longleftrightarrow \tilde{x}$
- Generalized metric is a T-duality covariant object

$$\mathcal{H}_{MN} = \begin{bmatrix} g - Bg^{-1}B & Bg^{-1} \\ g^{-1}B & g^{-1} \end{bmatrix}, \quad \text{in analogy with} \quad F_{\mu\nu} = \begin{bmatrix} 0 & \vec{E} \\ -\vec{E} & *_3\vec{B} \end{bmatrix}$$

$$\mathcal{H}_{MN} \in \frac{O(10,10)}{O(1,9) \times O(1,9)} \tag{10}$$

- There exists an action for $\mathcal{H}_{MN}$

[Berman, Cederwall, Coimbra, Godazgar[2], Grana, Hohm, Hull, EtM, Nicolai, Perry, Samtleben, Thompson, Waldram, Zwiebach ...]

Dynamics

Dynamics is ruled by DBI action

$$S_p = \int d^{p+1}\xi e^{-\phi}\sqrt{-\det\left(G_{mn}\partial_a x^m \partial_b x^n + \ldots\right)}, \tag{11}$$

$x^m = x^m(\xi)$ scalar fields

A T-invariant version then would be

$$S_D = \int d^{10}\xi e^{-d}\sqrt{-\det\left(\mathcal{H}_{MN}\partial_a X^M \partial_b X^N + \ldots\right)}, \tag{12}$$

Dynamics

Dynamics is ruled by DBI action

$$S_p = \int d^{p+1}\xi e^{-\phi}\sqrt{-\det\left(G_{mn}\partial_a x^m \partial_b x^n + \ldots\right)}, \tag{11}$$
$$x^m = x^m(\xi) \quad \text{scalar fields}$$

A T-invariant version then would be

$$S_D = \int d^{10}\xi e^{-d} \det|h_{\alpha\beta}|^{\frac{1}{4}}\sqrt{-\det\left(\mathcal{H}_{MN}\hat{\partial}_a X^M \hat{\partial}_b X^N + \ldots\right)}, \tag{12}$$

where one needs the projected derivatives

$$\begin{aligned} \hat{\partial}_a X^M &= \partial_a X^M - (h^{-1})^{\alpha\beta} k_\alpha{}^M k_\beta{}^N \mathcal{H}_{NK}\partial_a X^K, \\ h_{\alpha\beta} &= \boxed{k_\alpha{}^M k_\beta{}^N} \mathcal{H}_{MN}. \end{aligned} \tag{13}$$

Embedding of branes

	0	1	2	3	4	5	6	7	8	9	$\tilde{1}$	$\tilde{2}$	$\tilde{3}$	$\tilde{4}$	$\tilde{5}$	$\tilde{6}$	$\tilde{7}$	$\tilde{8}$	$\tilde{9}$
D0	k	•	•	•	•	•	•	•	•	•	k	k	k	k	k	k	k	k	k
D1	k	k	•	•	•	•	•	•	•	•	•	k	k	k	k	k	k	k	k
D2	k	k	k	•	•	•	•	•	•	•	•	•	k	k	k	k	k	k	k
D3	k	k	k	k	•	•	•	•	•	•	•	•	•	k	k	k	k	k	k
D4	k	k	k	k	k	•	•	•	•	•	•	•	•	•	k	k	k	k	k
D5	k	k	k	k	k	k	•	•	•	•	•	•	•	•	•	k	k	k	k
D6	k	k	k	k	k	k	k	•	•	•	•	•	•	•	•	•	k	k	k
D7	k	k	k	k	k	k	k	k	•	•	•	•	•	•	•	•	•	k	k
D8	k	k	k	k	k	k	k	k	k	•	•	•	•	•	•	•	•	•	k
D9	k	k	k	k	k	k	k	k	k	k	•	•	•	•	•	•	•	•	•

! Depending on the choice of **k**'s one gets different D-branes

!? D-branes can localize in dual space

Embedding of branes

NS five branes localize in dual space as well:

	world-volume						transverse directions							
	x^0	x^1	x^2	x^3	x^4	x^5	y^1	y^2	y^3	y^4	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$
NS5	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	k	k	k	k
KK5	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\bullet$	$\bullet$	$\bullet$	k	k	k	k	$\bullet$
Q	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\bullet$	$\bullet$	k	k	k	k	$\bullet$	$\bullet$
R	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\bullet$	k	k	k	k	$\bullet$	$\bullet$	$\bullet$
R'	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	k	k	k	k	$\bullet$	$\bullet$	$\bullet$	$\bullet$

(14)

$\bullet$ — localization direction, k — Killing direction

Non-perturbative instanton corrections on world-sheet of string localize KK-monopole in dual space. The same is true for 5^2_2-branes. [Jensen, Tong, Harvey, Kimura]

Other approaches

- 1206.6964 Asakawa, Sasa, Watamura: generalized geometry approach. D-brane is a Dirac structure on the doubled tangent bundle

$$\mathrm{L} \oplus \mathrm{L}^* \subset \mathrm{TM} \oplus \mathrm{T}^*\mathrm{M} \tag{15}$$

- 1107.0876 Albertsson, Dai, Kao, Lin: doubled formalism for open string ends. Dynamics of boundary terms $\Longrightarrow$ the doubled DBI action. Incapable to reproduce the conventional DBI action.

- unpublished Berman, Cederwall, Malek: DFT approach to a single D-brane. Worldvolume gauge fields are fluctuations along the winding coordinates.

The message

- One is able to construct a single action for several branes, related by T-duality
- For D-branes this suggests localization in dual space

Discussion

- Prove microscopically, that D-branes localize in dual space, calculate instanton corrections
- Field theories on worldvolume (especially for D-branes)
- Generalize stuff for exceptional field theories and U-dualities

What's the use of all that?

- Tadpole cancellation conditions for flux compactifications (Bianchi identities), support for internal space
- String behavior on such backgrounds: non-commutativity and non-associativity
- Little string theories from NS five-branes
- New stuff for AdS / CFT correspondence?

Thank you!