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Weyl Invariance and Higher Spin Gauge Theory

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Plan:

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Motivation

- Conformal gravity has attracted considerable attention during the last thirty years , parallel to the development of higher spin gauge field theories. It remained an intriguing task to combine these two developments and to construct an interacting higher spin conformal gauge field theory

- The interest in these intriguing topics intensified during the last decade with new applications of conformal higher spin theories in the context of the AdS/CFT correspondence. Furthermore, the remarkable trivialization of the partition function in flat space could be explained by the high level of gauge symmetry. The possibility to obtain the exact partition function in some conformal higher spin field configurations could be useful for future nontrivial checks of the AdS/CFT conjecture.
- *Does AdS/CFT works correctly on the level of loop diagrams in the general case and is it possible to use this correspondence for real reconstruction of unknown local interacting theories on the bulk from more or less well known conformal field theories on the boundary side?*

All these complicated physical tasks necessitate *quantum loop* calculations for HSF field theory and therefore information about manifest, off-shell and Lagrangian formulation of possible interactions for HSF and conformal HSF.

Introduction

- We construct all possible Weyl invariant actions in d=4 for linearized spin three field in a general gravitational background.
- The first action is obtained as the square of the generalized Weyl tensor for a spin three gauge field in nonlinear gravitational background. It is, however, not invariant under spin three gauge transformations.
- We then construct two other nontrivial Weyl but not gauge invariant actions which are linear in the Weyl tensor of the background geometry.
- We then discuss existence and uniqueness of a possible linear combination of these three actions which is gauge invariant. We do this at the linear order in the background curvature for Ricci flat backgrounds.

Introduction

The main result of our paper is the derivation of a second nontrivial Weyl invariant . The existence of this additional primary, quadratic in the generalized spin 3 Christoffel symbols and linear in the gravitational Weyl tensor, opens up the possibility to construct a unique Lagrangian which, besides being invariant under spin 2 and spin 3 Weyl transformations, is also invariant under spin-3 gauge transformations.

Spin Two Example

- We show that possible Weyl invariant expressions can be combined into a unique gauge invariant action.
- To realize this idea we concentrate on the construction of possible primary fields with weight Δ

$$\delta\mathcal{L}_{\Delta}(g_{\alpha\beta},\nabla_{\lambda},h_{\mu\nu}) = \Delta\sigma(x)\mathcal{L}_{\Delta}(g_{\alpha\beta},\nabla_{\lambda},h_{\mu\nu}),$$

The Weyl transformations of background metric and the linearized spin two field are defined in a similar way

$$\delta h_{\mu\nu}(x) = 2\sigma(x)h_{\mu\nu}(x),$$

$$\delta g_{\mu\nu}(x) = 2\sigma(x)g_{\mu\nu}(x)$$

- The most interesting primaries are scalars with conformal dimension -4 ,

$$S^{Weyl\,inv} = \int d^4x\sqrt{g}\,\mathcal{L}_{-4}(g_{\alpha\beta},\nabla_{\lambda},h_{\mu\nu}).$$

$$\delta\Gamma^{\lambda}_{\mu\nu} = \sigma_{(\mu}\delta^{\lambda}_{\nu)} - g_{\mu\nu}\sigma^{\lambda} \qquad \text{where} \qquad \sigma_{\mu} \equiv \partial_{\mu}\sigma$$

$$\mathcal{R}_{\alpha\mu,\beta\nu} = \frac{1}{4}[\{\nabla_{[\alpha},\nabla_{\{\beta}\}h_{\mu]\nu\}} - 2K_{[\alpha\{\beta}h_{\mu]\nu\}} - K^{\tau}_{[\alpha}g_{\mu][\beta}h_{\nu]\tau} - K^{\tau}_{[\beta}g_{\nu][\alpha}h_{\mu]\tau}],$$

$$\begin{aligned}\delta\mathcal{R}_{\alpha\mu,\beta\nu} &= 2\sigma\mathcal{R}_{\alpha\mu,\beta\nu} + \frac{1}{2}g_{[\alpha\{\beta}(\sigma^{\tau}\nabla_{\tau}h_{\mu]\nu\}} - \nabla_{(\mu]}h_{\nu\})\tau}\sigma^{\tau}) \\ &= 2\sigma\mathcal{R}_{\alpha\mu,\beta\nu} - g_{[\alpha\{\beta}\sigma^{\tau}\mathcal{G}_{\tau;\mu]\nu\}},\end{aligned}$$

where $$\mathcal{G}_{\tau;\mu\nu} = \frac{1}{2}(\nabla_{(\mu}h_{\nu)\tau} - \nabla_{\tau}h_{\mu\nu})$$

$$\delta\mathcal{K}_{\mu\nu} = \delta\frac{1}{2}(\mathcal{R}_{\mu\nu} - \frac{g_{\mu\nu}}{6}\mathcal{R}) = -\sigma^{\tau}\mathcal{G}_{\tau;\mu\nu}.$$

$$(\delta - 2\sigma)\mathcal{R}_{\alpha\mu,\beta\nu} = g_{[\alpha\{\beta}\delta\mathcal{K}_{\mu]\nu\}} = (\delta - 2\sigma)(g_{[\alpha\{\beta}\mathcal{K}_{\mu]\nu\}})$$

$$(\delta - 2\sigma)\mathcal{W}_{\alpha\mu,\beta\nu} = (\delta - 2\sigma)(\mathcal{R}_{\alpha\mu,\beta\nu} - g_{[\alpha\{\beta}\mathcal{K}_{\mu]\nu\}}) = 0$$

- We have thus constructed invariant linearized Weyl tensor

$$\mathcal{W}_{\alpha\mu,\beta\nu} = \mathcal{R}_{\alpha\mu,\beta\nu} - g_{[\alpha\{\beta}\mathcal{K}_{\mu]\nu\}}.$$

$$\mathcal{W}_{\alpha\mu,\beta\nu} = \frac{1}{4}[\{\nabla_{[\alpha}, \nabla_{\{\beta}\}h_{\mu]\nu\}} - 2K_{[\alpha\{\beta}h_{\mu]\nu\}}] - traces,$$

and it is a conformal primary:

$$\delta\mathcal{W}_{\alpha\mu,\beta\nu} = 2\sigma(x)\mathcal{W}_{\alpha\mu,\beta\nu}.$$

The background Weyl tensor is also $\Delta = 2$

primary but without dependence on $h_{\mu\nu}$

Having these two primaries we can construct several relevant ($\Delta = -4$) primaries:

1) $\mathcal{L}_{-4}^{lin} = W^{\alpha\mu,\beta\nu}\mathcal{W}_{\alpha\mu,\beta\nu} = 2W^{\alpha\mu,\beta\nu}(\nabla_\alpha\nabla_\beta - K_{\alpha\beta})h_{\mu\nu},$

2) $\mathcal{L}_{-4}^{\mathcal{W}^2} = \frac{1}{2}\mathcal{W}^{\alpha\mu,\beta\nu}\mathcal{W}_{\alpha\mu,\beta\nu},$

3) $\mathcal{L}_{-4}^{\mathcal{W}W} = \mathcal{W}^{\alpha\mu,\beta\nu}W_{\alpha\mu,\beta\nu}h_\rho^\rho,$

4) $\mathcal{L}_{-4}^{(1)W^2} = W^{\alpha\mu,\beta\nu}W_{\alpha\mu,\beta\nu}h_{\rho\tau}h^{\rho\tau},$

5) $\mathcal{L}_{-4}^{(2)W^2} = W_{\alpha\mu,\beta\nu}W_{\alpha\mu,\beta\nu}h_\rho^\rho h_\tau^\tau.$

and corresponding invariant action produces correct equation of motion with Bach tensor for background metric:

$$B^{\mu\nu} = (\nabla_\alpha\nabla_\beta - K_{\alpha\beta})W^{\alpha\mu,\beta\nu} = 0.$$

But it is not the whole story

We now turn to the Weyl variation of the linearized Christoffel symbol

$$(\delta - 2\sigma)\mathcal{G}_{\tau;\mu\nu} = -\sigma_\tau h_{\mu\nu} + g_{\mu\nu} h_{\tau\lambda}\sigma^\lambda .$$

we can guess the last nontrivial primary with four derivatives and second order on spin three gauge field:

$$\mathcal{L}_{-4}^{W\mathcal{G}^2} = \frac{1}{2} W^{\alpha\mu,\beta\nu}(\mathcal{G}_{\tau;\alpha\beta}\mathcal{G}^{\tau}{}_{;\mu\nu} - 2h_{\alpha\beta}\mathcal{K}_{\mu\nu}),$$

with conformal weight -4: $\delta\mathcal{L}_{-4}^{W\mathcal{G}^2} = -4\sigma(x)\mathcal{L}_{-4}^{W\mathcal{G}^2}$

Now we consider the linearized gauge invariance:

$\bar{\delta} h_{\mu\nu} = \nabla_{(\mu}\varepsilon_{\nu)}.$ hiding long calculation we arrive to the following Unique gauge invariant combination of our primaries

$$\mathcal{L}^{GI}_{-4} = \mathcal{L}^{W\mathcal{G}^2}_{-4} + \frac{1}{4}\mathcal{L}^{\mathcal{W}^2}_{-4} - \frac{1}{16}\mathcal{L}^{\mathcal{W}W}_{-4} + \frac{1}{32}\mathcal{L}^{(1)W^2}_{-4} - \frac{1}{64}\mathcal{L}^{(2)W^2}_{-4}$$

With the property

$$\bar{\delta}\int d^4x\sqrt{g}\,\mathcal{L}^{GI}_{-4} = \int d^4x\sqrt{g}\{-\frac{1}{2}B^{\alpha\beta}\mathcal{L}_{\varepsilon}h_{\alpha\beta}\}.$$

Therefore in the background with zero Bach tensor gauge and Weyl invariant action is:

$$S_{GI} = \frac{1}{8}\int d^4x\sqrt{g}\,\mathcal{W}^{\alpha\mu,\beta\nu}\mathcal{W}_{\alpha\mu,\beta\nu} + \frac{1}{2}\int d^4x\sqrt{g}\,W^{\alpha\mu,\beta\nu}(\mathcal{G}_{\tau;\alpha\beta}\mathcal{G}^{\tau}{}_{;\mu\nu} - 2h_{\alpha\beta}\mathcal{K}_{\mu\nu})$$
$$-\frac{1}{16}\int d^4x\sqrt{g}\{\mathcal{W}^{\alpha\mu,\beta\nu}W_{\alpha\mu,\beta\nu}h^{\rho}_{\rho} - \frac{1}{2}W^{\alpha\mu,\beta\nu}W_{\alpha\mu,\beta\nu}[h_{\rho\tau}h^{\rho\tau} - \frac{1}{4}h^{\rho}_{\rho}h^{\tau}_{\tau}]\}$$

Of course this action can be obtained from expansion of the action for conformal gravity:

$$S_{W(G)} = \frac{1}{2}\int d^4x\sqrt{G}\,W^{\alpha\mu,\beta\nu}(G)W_{\alpha\mu,\beta\nu}(G)$$ where $$G_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}.$$

Linearized Spin 3 Weyl Tensor

$$\delta h_{\mu\nu\lambda}(x) = 4\sigma(x) h_{\mu\nu\lambda}(x).$$

$$H_{\alpha\beta\gamma,\mu\nu\lambda} = \frac{1}{6}\nabla_{(\alpha}\nabla_{\beta}\nabla_{\gamma)}h_{\mu\nu\lambda} - \frac{2}{3}\nabla_{(\alpha}K_{\beta\gamma)}h_{\mu\nu\lambda} + \frac{4}{3}K_{\alpha\beta}\nabla_{\lambda}h_{\mu\nu\gamma} + \frac{4}{3}K_{\beta\gamma}\nabla_{\mu}h_{\nu\lambda\alpha} + \frac{4}{3}K_{\gamma\alpha}\nabla_{\nu}h_{\lambda\mu\beta}.$$

$$R_{\alpha\mu,\beta\nu,\gamma\lambda} = H_{\alpha\beta\gamma,\mu\nu\lambda} - H_{\mu\beta\gamma,\alpha\nu\lambda} - H_{\alpha\nu\gamma,\mu\beta\lambda} + H_{\mu\nu\gamma,\alpha\beta\lambda} - H_{\alpha\beta\lambda,\mu\nu\gamma} + H_{\mu\beta\lambda,\alpha\nu\gamma} + H_{\alpha\nu\lambda,\mu\beta\gamma} - H_{\mu\nu\lambda,\alpha\beta\gamma},$$

To substract traces from spin three curvature we introduce analog of gravitational Schouten tensor in spin three case (d=4):

$$K_{\mu\nu;\gamma\lambda} = \frac{1}{4}[R_{\mu\nu;\gamma\lambda} - \frac{1}{10}\left(g_{\mu\nu}R^{(1)}_{\gamma\lambda} + g_{[\gamma(\mu}R^{(2)}_{\nu)\lambda]}\right)],$$

$$\mathcal{W}_{\alpha\mu,\beta\nu,\gamma\lambda} = R_{\alpha\mu,\beta\nu,\gamma\lambda} - g_{[\alpha\{\beta}K_{\mu]\nu\};\gamma\lambda} - g_{[\alpha\{\gamma}K_{\mu]\lambda\};\beta\nu} - g_{[\gamma\{\beta}K_{\lambda]\nu\};\alpha\mu},$$

$$\delta\mathcal{W}_{\alpha\mu,\beta\nu,\gamma\lambda} = 4\sigma(x)\mathcal{W}_{\alpha\mu,\beta\nu,\gamma\lambda}.$$

$$\boxed{L^{\mathcal{W}^2}_{-4} = \mathcal{W}^{\alpha\mu,\beta\nu,\gamma\lambda}\mathcal{W}_{\alpha\mu,\beta\nu,\gamma\lambda}.}$$

first and second Christoffel symbols with one and two covariant derivatives.

$$\Gamma^{(1)}_{\gamma;\mu\nu\lambda} = \nabla_\gamma h_{\mu\nu\lambda} - \nabla_{(\mu} h_{\nu\lambda)\gamma},$$

$$\Gamma^{(2)}_{\beta\gamma;\mu\nu\lambda} = \nabla_{(\beta}\Gamma^{(1)}_{\gamma);\mu\nu\lambda} - \frac{1}{2}\nabla_{(\mu}\Gamma^{(1)}_{<\beta;\gamma>\nu\lambda)} - 8K_{\beta\gamma}h_{\mu\nu\lambda} + 2K_{(\mu<\beta}h_{\gamma>\nu\lambda)}$$
$$+2g_{(\mu\nu}h_{\lambda)\tau<\beta}K^{\tau}_{\gamma>} - 2g_{(\mu\nu}K^{\tau}_{\lambda)}h_{\tau\beta\gamma} - g_{(<\beta(\mu}K^{\tau}_{\nu}h_{\lambda)\gamma>\tau}.$$

Looking at the traces of transformations of first Christoffel symbol we can define the following primary:

$$\boxed{L^{W\mathcal{W}}_{-4} = \frac{1}{4}\Gamma^{(1)\ \rho}_{[\gamma,\lambda]\rho}\mathcal{W}_{\alpha\mu,\beta\nu}{}^{\gamma\lambda}W^{\alpha\mu,\beta\nu} - h^{\rho\lambda}_{\rho}\nabla^{\gamma}\mathcal{W}_{\gamma\lambda,\alpha\mu,\beta\nu}W^{\alpha\mu,\beta\nu},}$$

Traceless parts of our spin three field and Christoffel symbols

$$h^T_{\mu\nu\lambda} = h_{\mu\nu\lambda} - \frac{1}{6} g_{(\mu\nu} h_{\lambda)\alpha}{}^{\alpha}, \qquad \Gamma^T_{\mu\nu\lambda} = g^{\alpha\beta} \Gamma^{(2)T}_{\alpha\beta;\mu\nu\lambda}.$$

$$\Gamma^{(1)T}_{\gamma;\mu\nu\lambda} = \Gamma^{(1)}_{\gamma;\mu\nu\lambda} - \frac{1}{6} \Gamma^{(1)\alpha}_{\gamma;\alpha\,(\lambda} g_{\mu\nu)}, \qquad \Gamma^{(2)T}_{\beta\gamma;\lambda\mu\nu} = \Gamma^{(2)}_{\beta\gamma;\lambda\mu\nu} - \frac{1}{6} g_{(\mu\nu} \Gamma^{(2)}_{\beta\gamma;\lambda)\alpha}{}^{\alpha},$$

$$\Gamma^{(1)T;T}_{[\gamma;\lambda]\mu\nu} = \Gamma^{(1);T}_{[\gamma;\lambda]\mu\nu} - \frac{1}{4} g_{[\gamma(\mu} \Gamma^{(1);T\ \alpha}_{\alpha;\nu)\lambda]}, \qquad \Gamma^{(2)T;T}_{\beta[\gamma;\lambda]\mu\nu} = \Gamma^{(2);T}_{\beta[\gamma;\lambda]\mu\nu} - \frac{1}{4} g_{[\gamma(\mu} \Gamma^{(2);T\ \alpha}_{\beta\alpha;\nu)\lambda]},$$

So we see that traceless parts of some field, Christoffel symbols and spin three Schouten tensor

$$h^T_{\mu\nu\lambda}; \Gamma^{(1)T}_{\gamma;\mu\nu\lambda}; \Gamma^{(1)T;T}_{[\gamma;\lambda]\mu\nu}; \Gamma^{(2)T;T}_{\beta[\gamma;\lambda]\mu\nu}; \Gamma^T_{\mu\nu\lambda}; K^{(T)}_{\mu\nu;\gamma\lambda},$$

transform in close and more or less simple way in respect to Weyl transformations

For example

$$\delta K^{(T)}_{\mu\nu;\gamma\lambda} = \frac{1}{3} \sigma^{\beta} \Gamma^{(2)T;T}_{\beta[\gamma;\lambda]\mu\nu}.$$

Main Result- New Nontrivial Invariant

$$L_{-4}^{W\Gamma\Gamma}=\frac{2}{3}W_{\mu\ ,\nu}^{\ \tau\ \rho}\tilde{\Gamma}^{(2)T,T}_{\beta[\gamma,\lambda]\tau\rho}\tilde{\Gamma}^{\beta[\gamma,\lambda]\mu\nu}_{(2)T,T}+\frac{22}{9}W_{\gamma\ ,\mu}^{\ (\tau\ \rho)}\tilde{\Gamma}^{(2)T,T}_{\beta[\nu,\tau]\lambda\rho}\tilde{\Gamma}^{\beta[\gamma,\lambda]\mu\nu}_{(2)T,T}-\frac{1}{6}W_{\gamma\lambda,}^{\ \ \tau\rho}\tilde{\Gamma}^{(2)T,T}_{\beta[\tau,\rho]\mu\nu}\tilde{\Gamma}^{\beta[\gamma,\lambda]\mu\nu}_{(2)T,T}$$

$$-[\nabla_\gamma W_{\mu\ ,\nu}^{\ \tau\ \rho}-8\nabla_\mu W_{\nu\ ,\gamma}^{\ \tau\ \rho}+6C_{\mu\ ,\gamma}^{\ \rho}\delta_\nu^\tau]\left(\frac{4}{3}\Gamma^{(1)T}_{\beta,\lambda\tau\rho}\tilde{\Gamma}^{\beta[\gamma,\lambda]\mu\nu}_{(2)T,T}-\frac{1}{2}\Gamma^{T}_{\lambda\tau\rho}\Gamma^{[\gamma,\lambda]\mu\nu}_{(1)T;T}-16h^{T}_{\lambda\tau\rho}K^{\mu\nu;\gamma\lambda}_{(T)}\right)$$

$$-[12W_{\mu\ \ ,\nu}^{\ \ \tau\ \ \rho}\Gamma^{(1)T,T}_{[\gamma;\lambda]\tau\rho}+44W_{\gamma\ \ ,\mu}^{\ \ (\tau\ \ \rho)}\Gamma^{(1)T,T}_{[\nu;\tau]\lambda\rho}-3W_{\gamma\lambda,}^{\ \ \tau\rho}\Gamma^{(1)T,T}_{[\tau;\rho]\mu\nu}]K^{\mu\nu;\gamma\lambda}_{(T)}$$

$$-2[(\nabla^\sigma\nabla_\rho+4K^\sigma_\rho)W_{\alpha\ ,\beta}^{\ \mu\ \nu}]T^{\alpha\beta\rho}_{\mu\nu\sigma}+[4K^{\mu\tau}W_{\alpha\tau,\beta}^{\ \ \ \ \nu}-3(\Box+2J)W_{\alpha\ ,\beta}^{\ \mu\ \nu})]T^{\alpha\beta}_{\mu\nu},$$

where we introduced new notation:

$$T^{\mu\nu\lambda}_{\alpha\beta\gamma}=\Gamma^{\tau,\mu\nu\lambda}_{(1)T}\Gamma^{(1)T}_{\tau,\alpha\beta\gamma}-\frac{1}{2}(h^{\mu\nu\lambda}_{T}\Gamma^{T}_{\alpha\beta\gamma}+h^{T}_{\alpha\beta\gamma}\Gamma^{\mu\nu\lambda}_{T}),$$

$$T^{\mu\nu}_{\alpha\beta}=T^{\mu\nu\lambda}_{\alpha\beta\lambda},\qquad T^{\mu}_{\alpha}=T^{\mu\nu}_{\alpha\nu}.$$

$$\tilde{\Gamma}^{\beta[\gamma,\lambda]\mu\nu}_{(2)T,T}=\Gamma^{\beta[\gamma,\lambda]\mu\nu}_{(2)T,T}-\frac{3}{8}\left(g^{\beta[\gamma}\Gamma^{\lambda]\mu\nu}_{T}-\frac{1}{4}g^{[\gamma(\mu}\Gamma^{\nu)\lambda]\beta}_{T}\right),$$

On Gauge Invariant Action for Conformal Spin Three

- We should start from gauge variation

$$\delta_\varepsilon h_{\mu\nu\lambda} = \nabla_\mu \varepsilon_{\nu\lambda} + \nabla_\nu \varepsilon_{\lambda\mu} + \nabla_\lambda \varepsilon_{\mu\nu}$$

of actions

$$S_{\mathcal{W}^2} = \int d^4x \sqrt{g} L_{-4}^{\mathcal{W}^2}, \quad S_{W\Gamma\Gamma} = \int d^4x \sqrt{g} L_{-4}^{\mathcal{W}^2}, \qquad S_{\mathcal{W}W} = \int d^4x \sqrt{g} L_{-4}^{\mathcal{W}W}.$$

Result

$$\delta_\varepsilon [S_{\mathcal{W}^2} - \frac{8}{5} S_{W\Gamma\Gamma} + \frac{4}{3} S_{W\mathcal{W}}] = 0 + O(R^2, K^{\alpha\beta}, B)$$

We showed this, with the help of the computer, with the further assumption of a Ricci flat background

Another long check leads us to results that the parameter in gauge transformation could be non traceless as it should be in Fronsdal theory. The last important relation is the following: The action we proposed

$$S_{CGI} = S_{\mathcal{W}^2} - \frac{8}{5} S_{W\Gamma\Gamma} + \frac{4}{3} S_{W\mathcal{W}}$$

is not only conformal and gauge invariant in first order on background Weyl tensor, but invariant also in respect to spin 3 Weyl transformation (shifting of trace):

$$\delta_\alpha h_{\mu\nu\lambda} = g_{\mu\nu}\alpha_\lambda + g_{\nu\lambda}\alpha_\mu + g_{\lambda\mu}\alpha_\nu$$

in the following way:

$$\delta_\alpha S_{\mathcal{W}^2} = 0$$

$$\delta_\alpha [S_{W\mathcal{W}} - \frac{6}{5} S_{W\Gamma\Gamma}] = 0$$

5. Conclusions

- we investigated the structure of Weyl covariant primaries in d=4.
- This primaries are relevant for using as a Weyl invariant Lagrangians, expressed through the corresponding members of hierarchy of generalized Christoffel symbols and Weyl tensor for linearized spin 3 gauge field in general gravitational background.
- The main result is that in addition to the linearized spin 3 Weyl tensor corrected with background curvature terms we can construct additional nontrivial Weyl primary in full analogy with spin 2 case.
- This primary is linear in background Weyl tensor and quadratic in linearized second Christoffel symbol.
- A possible combination of these primaries, in principal, can be interpreted as a gauge and Weyl invariant action with corresponding restriction on background geometry.
- This could be investigated in the future.
- Here we only briefly discuss the possible combination of these invariants in linear on background Weyl tensor approximation using computer calculations.

Thank You for your attention!