Black holes with AdS asymptotics and holographic RG flows

Anastasia Golubtsova[1]

based on work with
Irina Aref'eva (MI RAS, Moscow) and Giuseppe Policastro (ENS, Paris)
arXiv:1803.06764

(1) BLTP JINR, Dubna

Supersymmetry in Integrable Systems (SIS'18)
August 13-16

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Outline

1. Introduction
   - Holographic dictionary
2. Exact holographic RG flows
   - Set up
   - How to integrate
   - Vacuum solutions
   - Non-vacuum solutions, black holes
3. RG equations at $T=0$
4. RG-flow at finite T
5. Outlook

DW/CFT dualities Itzhaki et. al.'98, Boonstra et. al.'98;Skenderis'99

- $AdS \Leftrightarrow DW$, CFT$\Leftrightarrow$ QFT,
- AdS isometry group $\Leftrightarrow$ Poincaré isometry group of DW
- a restoration of the conformal symmetry only at UV and/or IR fixed points

$$S = M_p^{d-1} \int d^d x \int dr \sqrt{-g} \left[R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right] + S_{YH}.$$

The domain wall solution

$$ds^2 = e^{2A(r)} \eta_{ij} dx^i dx^j + dr^2, \quad \phi = \phi(r)$$

- The scale factor e^A – measures the field theory energy scale
- The scalar field e^ϕ – the running coupling λ
- The β-function

$$\beta = \frac{d\lambda}{d \log E} = \frac{d\phi}{dA}$$

Possibilities for the potential

Improved holographic QCD Gursoy,Kiritsis' 07, Gubser'08

For asymptotically AdS UV	$\lambda \to 0$	$V(\lambda)$	$=$	$V_0 + v_1\lambda + v_2\lambda^2 + \ldots$
For confinement in the IR	$\lambda \to \infty$	$V(\lambda)$	$\sim$	$\lambda^Q (\log \lambda)^P$

The auxiliary scalar function $W(\phi)$ (aka superpotential)

$$W(\phi(u)) = -2(d-1)\frac{dA}{dr}, \quad -\frac{d}{4(d-1)}W^2 + \frac{1}{2}(\partial_\phi W)^2 = V.$$

- $V(\phi)$ from IHQCD model, Kiritsis et al'07'11'14'17'18

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  - :(no exact solutions for the model

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  - :) has good behaviour in the IR-limit (can study conformal anomalies, apply to deconfined phase of QCD)Policastro'15
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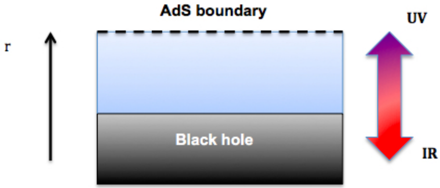
  - :(UV-fixed point is not the AdS
- $V = \sum C_i e^{k_i\phi}$, in particular, $V(\phi) = C_1 e^{k_1\phi} + C_2 e^{k_2\phi}$ – ?

RG flow at finite temperature

Thermal gas solution

$$ds^2 = e^{2A(r)}\eta_{ij}dx^i dx^j + dr^2, \quad \phi = \phi(r).$$



The black hole

$$ds^2 = e^{2A(r)}\left(-f(r)dt^2 + \delta_{ij}dx^i dx^j\right) + \frac{dr^2}{f(r)}, \quad f(r) = 1 - C_2\lambda^{-\frac{4(1-X^2)}{3X}}.$$

Gubser's bound for singular solutions (2000)

$$V(\phi_h) < 0, \quad V(\phi_h) \leq V(\phi_{UV}).$$

Outline

Set up

The action reads

$$S = \frac{1}{2\kappa^2}\int d^4x \int du\sqrt{-g}\left(R - \frac{4}{3}(\partial\phi)^2 + V(\phi)\right) - \frac{1}{\kappa^2}\int_\partial d^4x\sqrt{-\gamma},$$

$V(\phi) = C_1 e^{2k_1\phi} + C_2 e^{2k_2\phi}$, C_i, k_i, $i = 1, 2$ are some constants.

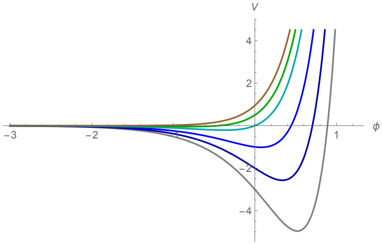


Figure: The behaviour of the potential $V(\phi)$ for $C_1 < 0$, $C_2 > 0$.

The ansatz for the metric

$$ds^2 = -e^{2A(u)}dt^2 + e^{2B(u)}\sum_{i=1}^{3} dy_i^2 + e^{2C(u)}du^2,$$

The gauge $C = A + 3B.$

The sigma-model

$$L = \frac{1}{2}G_{MN}\dot{x}^M\dot{x}^N - V, \quad V = -\frac{1}{2}\sum_{s=1}^{2} C_s e^{2(x^1+3x^2+k_s x^3)}, \dot{} \equiv \frac{d}{du}.$$

$x^1 = A,\ x^2 = B,\ x^3 = \phi,\ x = C.$

$$(G_{MN}) = \begin{pmatrix} 0 & -3 & 0 \\ -3 & -6 & 0 \\ 0 & 0 & \frac{4}{3} \end{pmatrix}, \quad M, N = 1, 2, 3.$$

(G_{MN}) – minisuperspace metric on the target space $\mathcal{M}$

$$L = \frac{1}{2}\langle \dot{x}, \dot{x}\rangle + \frac{C_1}{2} e^{\langle V, x\rangle} + \frac{C_2}{2} e^{\langle W, x\rangle}.$$

V – time-like, W - spacelike vectors on $\mathcal{M}$ (the basis is (e_1, e_2, e_3))

$$\langle V, V\rangle = 3\left(k_1^2 - \frac{16}{9}\right), \langle W, W\rangle = 3\left(k_2^2 - \frac{16}{9}\right), \langle V, W\rangle = 3\left(k_1 k_2 - \frac{16}{9}\right).$$

LET $\langle V, W\rangle = 0 \Leftrightarrow k_1 k_2 = \frac{16}{9}, \quad k_1 = k, \quad k_2 = \frac{16}{9k}, \quad 0 < k < 4/3.$

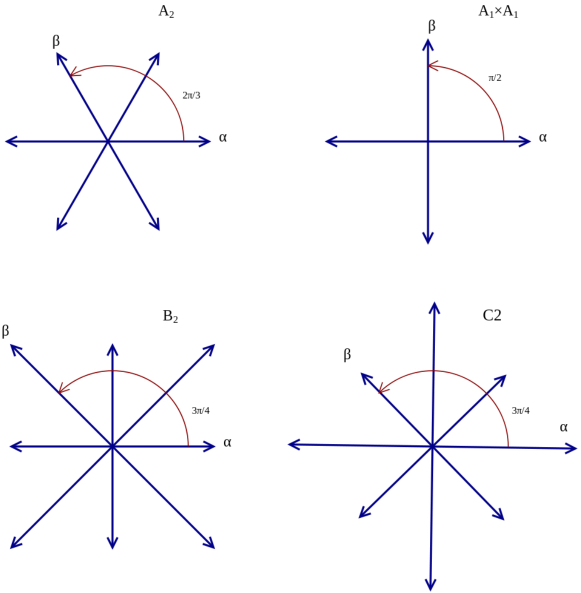
The new basis

$$e_1^{'} = \frac{V}{||V||}, \quad e_2^{'} = \frac{W}{||W||}, \quad \left\langle e_i^{'}, e_j^{'}\right\rangle = \eta_{ij}, \quad (\eta_{ij}) = diag\,(-1, 1, 1)\,.$$

$$X^i = \eta_{ii}\left\langle e_i^{'}, x\right\rangle, \quad x^i = \sum_{j=1}^{3} S_j^i X^j, \quad e_j^{'} = \sum_{i=1}^{3} S_j^i e_i.$$

S_j^i – components of general Lorentz transformations.

Root systems



The $A_1 \times A_1$-mechanical model

Let V and W vectors to root vectors of $su(2) \oplus su(2)$ Lie algebra

$$L = \frac{1}{2}\sum_{i,j=1}^{3} \eta_{ij}\dot{X}^i\dot{X}^j + \frac{C_1}{2}e^{\eta_{11}|\langle V,V\rangle|^{1/2}X^1} + \frac{C_2}{2}e^{\eta_{22}|\langle W,W\rangle|^{1/2}X^2},$$

$$E_0 = \frac{1}{2}\sum_{i,j=1}^{3} \eta_{ij}\dot{X}^i\dot{X}^j - \frac{C_1}{2}e^{\eta_{11}|\langle V,V\rangle|^{1/2}X^1} - \frac{C_2}{2}e^{\eta_{22}|\langle W,W\rangle|^{1/2}X^2}.$$

Liouville equations for $sl(2)$-Toda chains ($sl(2) \cong su(2)$)

$$\begin{aligned}\ddot{X}^s &= -\sqrt{|\langle R_s, R_s\rangle|}\tilde{C}_s e^{\eta_{ss}|\langle R_s,R_s\rangle|^{1/2}X^s}, \quad s = 1,2,\\ \ddot{X}^3 &= 0, \quad \text{with} \quad \langle R_1, R_1\rangle = \langle V, V\rangle, \quad \langle R_2, R_2\rangle = \langle W, W\rangle.\end{aligned}$$

Gavrilov, Ivashchuk, Melnikov'9407019

Lü, Pope,9607027, 9604058

Lü, Yang, 1307.2305

The solution to the $A_1 \times A_1$- mechanical model

The solution reads

$$\begin{aligned} X^1 &= |\langle V,V\rangle|^{-1/2}\ln\left(F_1^2(u-u_{01})\right),\\ X^2 &= -|\langle W,W\rangle|^{-1/2}\ln\left(F_2^2(u-u_{02})\right),\\ X^3 &= p^3u+q^3, \end{aligned}$$

with

$$F_s(u-u_{0s}) = \begin{cases} \sqrt{\left|\frac{C_s}{2E_s}\right|}\sinh\left[\sqrt{\frac{|E_s\langle R_s,R_s\rangle|}{2}}(u-u_{0s})\right], & \eta_{ss}C_s>0, \eta_{ss}E_s>0,\\ \sqrt{\left|\frac{C_s}{2E_s}\right|}\sin\left[\sqrt{\frac{|E_s\langle R_s,R_s\rangle|}{2}}(u-u_{0s})\right], & \eta_{ss}C_s>0, \eta_{ss}E_s<0,\\ \sqrt{\frac{|\langle R_s,R_s\rangle\tilde{C}_s|}{2}}(u-u_{0s}), & \eta_{ss}C_s>0, E_s=0,\\ \sqrt{\left|\frac{C_s}{2E_s}\right|}\cosh\left[\sqrt{\frac{|E_s\langle R_s,R_s\rangle|}{2}}(u-u_{0s})\right], & \eta_{ss}C_s<0, \eta_{ss}E_s>0, \end{cases}$$

u_{0s}, E_s, E_s, p^3, q^3 are constants of integration.
Lorenz transformations

$$S_1^i = \frac{V^i}{|\langle V,V\rangle|^{1/2}},\quad S_2^i = \frac{W^i}{\langle W,W\rangle^{1/2}},\quad \alpha^i = S_3^i p^3,\quad \beta^i = S_3^i q^3$$

The general solution

$$
\begin{aligned}
ds^2 &= F_1^{\frac{8}{9k^2-16}} F_2^{\frac{9k^2}{2(16-9k^2)}} \left(-e^{2\alpha^1 u} dt^2 + e^{-\frac{2}{3}\alpha^1 u} d\vec{y}^{\,2}\right) + F_1^{\frac{32}{9k^2-16}} F_2^{\frac{18k^2}{16-9k^2}} du^2 \\
\phi &= -\frac{9k}{9k^2-16} \ln F_1 + \frac{9k}{9k^2-16} \ln F_2
\end{aligned}
$$

$$
F_s(u-u_{0s}) = \begin{cases}
\sqrt{\frac{|C_s|}{2|E_s|}} \sinh\left[\mu_s(u-u_{0s})\right], & \text{if} \quad \eta_{ss}C_s > 0, \eta_{ss}E_s > 0, \\
\sqrt{\frac{|C_s|}{2|E_s|}} \sin\left[\mu_s(u-u_{0s})\right], & \text{if} \quad \eta_{ss}C_s > 0, \eta_{ss}E_s < 0, \\
\sqrt{\frac{C_s}{2}} |\mu_s(u-u_{0s})|, & \text{if} \quad \eta_{ss}C_s > 0, E_s = 0, \\
\sqrt{\frac{|C_s|}{2|E_s|}} \cosh\left[\mu_s(u-u_{0s})\right], & \text{if} \quad \eta_{ss}C_s < 0, \eta_{ss}E_s > 0,
\end{cases}
$$

$$
s = 1, 2, \quad \mu_1 = \sqrt{\left|\frac{3E_1}{2}(k^2 - \frac{16}{9})\right|}, \quad \mu_2 = \sqrt{\left|\frac{3E_2}{2}\left(\left(\frac{16}{9}\right)^2 \frac{1}{k^2} - \frac{16}{9}\right)\right|}.
$$

Constraints

$$E_1 + E_2 + \frac{2(\alpha^1)^2}{3} = 0.$$

1. $\alpha^1 = 0$ Vacuum solutions, Poincaré invariant, $|E_1| = |E_2|$
2. $\alpha^1 \neq 0$ Non-vacuum ones, no Poincaré invariance $|E_1| \neq |E_2|$

- Conditions from the $V(\phi)$: $C_1 < 0, C_2 > 0, 0 < k < 4/3$.
- Constants of integration $u_{02} < u_{01}$

$$\begin{aligned} \text{left:} \quad & u < u_{02} \\ \text{middle:} \quad & u_{02} < u < u_{01} \\ \text{right:} \quad & u > u_{01} \end{aligned}$$

- The degenerate case with $u_{01} = u_{02} = u_0$,

$$\begin{aligned} \text{left:} \quad & u < u_0 \\ \text{right:} \quad & u > u_0. \end{aligned}$$

Behaviour of solutions $u_{01} \neq u_{02}$, $\alpha^1 = 0$

$$ds^2 = F_1^{\frac{8}{9k^2-16}} F_2^{\frac{9k^2}{2(16-9k^2)}} (-dt^2 + dy_1^2 + dy_2^2 + dy_3^2) + F_1^{\frac{32}{9k^2-16}} F_2^{\frac{18k^2}{16-9k^2}} du^2,$$

$$F_1 = \sqrt{\left|\frac{C_1}{2E_1}\right|} \sinh(\mu_1 |u - u_{01}|), \quad F_2 = \sqrt{\left|\frac{C_2}{2E_2}\right|} \sinh(\mu_2 |u - u_{02}|),$$

$$E_1 = -E_2, \quad E_1 < 0, \quad E_2 > 0, \quad \mu_2 = \frac{4}{3k}\mu_1.$$

The dilaton

$$\phi = \frac{9k}{9k^2 - 16} \log \frac{F_2}{F_1}$$

and its potential

$$V = C_1 e^{2k\phi} + C_2 e^{32\phi/(9k)} = C_1 \left(\frac{F_2}{F_1}\right)^{\frac{18k^2}{9k^2 16}} + C_2 \left(\frac{F_2}{F_1}\right)^{\frac{32}{9k^2-16}} .$$

Boundaries for $u_{01} \neq u_{02} \neq 0$

The left solution $u < u_{02}$ (conformally flat)

- $u \to -\infty$ $\quad ds^2 \sim z^{2/3}\left(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dz^2\right),$
  $z \sim e^{-\frac{3\mu_1 u}{4+3k}}, \quad \phi \sim \frac{9k}{16-9k^2}(\mu_2 - \mu_1)u \sim \log z \to -\infty$
- $u \to u_{02} - \epsilon$ $\; ds^2 \sim z^{\frac{18k^2}{64-9k^2}}\left(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dz^2\right),$
  $z \sim \frac{64-9k^2}{4(16-9k^2)}(u - u_{02})^{\frac{64-9k^2}{4(16-9k^2)}}, \; \phi \sim -\frac{36k}{64-9k^2}\log z \to +\infty.$

The middle solution $u_{02} < u < u_{01}$ (conformally flat)

- $u \to u_{02} + \epsilon$ the same as at $u \to u_{02} - \epsilon$
- $u \to u_{01} - \epsilon$ $\; ds^2 \sim z^{\frac{8}{9k^2-4}}\left(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dz^2\right),$
  $\phi \sim \frac{9k}{4-9k^2}\log z \to -\infty, \; z \sim \frac{16-9k^2}{9k^2-4}(u - u_{01})^{\frac{4-9k^2}{16-9k^2}}.$

The right solution $u > u_{01}$ (conformally flat)

- $u \to u_{01} + \epsilon$ the same as at $u \to u_{01} + \epsilon$
- $u \to +\infty$ $\; ds^2 \sim z^{2/3}\left(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dz^2\right),$
  $\phi \sim \log z \to -\infty$

Boundaries: $u_{01} = u_{02} = u_0$, $\alpha^1 = 0$

- In the UV $u \to u_0$ we obtain the AdS-spacetime

$$ds^2 \sim \frac{1}{z^2}(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dz^2), \quad z \sim 4u^{1/4}.$$

The dilaton is constant in the UV

$$\phi = \frac{9k}{16 - 9k^2} \log \frac{3k}{4} + \frac{9k}{2(16 - 9k^2)} \log \left| \frac{C_1}{C_2} \right|.$$

- In the IR $u \to +\infty$ we obtain the conformally flat spacetime

$$ds^2 \sim z^{2/3}(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dz^2), \quad z \sim e^{-\frac{3\mu_1 u}{4+3k}}.$$

The dilaton in the IR

$$\phi \sim \log z \to -\infty$$

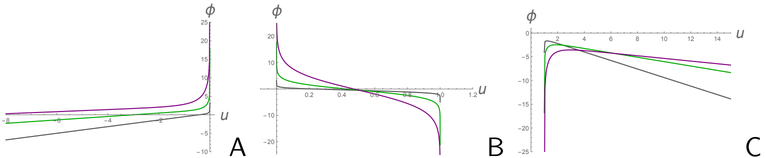


Figure: Dilaton as a function of u: A) $u < u_{02}$, B) $u_{02} < u < u_{01}$, C) the dilaton for $u > u_{01}$, $u_{01} = 1$. For all $u_{01} = 1$, $u_{02} = 0, E_1 = -E_2 = -1$, $C_1 = -C_2 = -1$, $k = 0.4, 1, 1.2$.

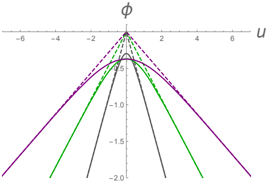


Figure: The behaviour of the dilaton (solid lines) and its asymptotics at infinity (dashed lines) for $u_{01} = u_{02} = 0$, $C_1 = -C_2 = -1$, $E_1 = -E_2 = -1$ and different values of k. From bottom to top $k = 0.4, 1, 1.2$.

Non-vacuum solutions, black holes

The metric

$$ds^2 = F_1^{\frac{8}{9k^2-16}} F_2^{\frac{9k^2}{2(16-9k^2)}} \left(-e^{2\alpha^1 u} dt^2 + e^{-\frac{2}{3}\alpha^1 u} \sum_{i=1}^{3} dy_i^2\right) + F_1^{\frac{32}{9k^2-16}} F_2^{\frac{18k^2}{16-9k^2}} du^2.$$

The dilaton reads

$$\phi = -\frac{9k}{9k^2-16} \ln F_1 + \frac{9k}{9k^2-16} \ln F_2,$$

$$F_1 = \sqrt{\left|\frac{C_1}{2E_1}\right|} \sinh(\mu_1 |u-u_{01}|), \; F_2 = \sqrt{\left|\frac{C_2}{2E_2}\right|} \sinh(\mu_2 |u-u_{02}|),$$

$$\mu_1 = \sqrt{\left|\frac{3E_1}{2}\right|} \sqrt{\frac{16}{9} - k^2}, \quad \mu_2 = \sqrt{\left|\frac{3E_2}{2}\right|} \frac{4}{3k} \sqrt{\frac{16}{9} - k^2} = \frac{4}{3k} \sqrt{\frac{E_2}{E_1}} \mu_1.$$

$$E_1 + E_2 + \frac{2}{3}(\alpha^1)^2 = 0.$$

Dilaton at boundaries $u_{01} \neq u_{02}$, $\alpha^1 \neq 0$

- The left solution $u < u_{02}$
  - $u \to -\infty$ $\phi_{u\to-\infty} \sim \frac{9k}{16-9k^2}\left[(\mu_2 - \mu_1)\, u + \frac{1}{2}\log\left|\frac{C_2 E_1}{C_1 E_2}\right|\right]$
  - $u \to u_{02} - \epsilon$
    $\phi_{u\to u_{02}-\epsilon} \sim -\frac{9k}{16-9k^2}\log\left[\sqrt{\frac{C_2 E_1}{C_1 E_2}}\frac{\mu_2\,\epsilon}{\sinh(\mu_1\,(u_{01}-u_{02}))}\right] \to +\infty.$
- The middle solution $u_{02} < u < u_{01}$
  - $u \to u_{02} + \epsilon$ the same as for the left solution at $u \to u_{02} - \epsilon$
  - $u \to u_{01} - \epsilon$
    $\phi_{u\to u_{01}-\epsilon} \sim -\frac{9k}{16-9k^2}\log\left[\sqrt{\frac{C_2 E_1}{C_1 E_2}}\frac{\sinh(\mu_2\,(u_{01}-u_{02}))}{\mu_1\,\epsilon}\right] \to -\infty.$
- The right solution $u > u_{01}$
  - $u \to u_{01} + \epsilon$ the same as for the middle solution at $u \to u_{01} + \epsilon$
  - $u \to +\infty$ $\phi_{u\to\infty} \sim -\frac{9k}{16-9k^2}\left[(\mu_2 - \mu_1)\, u + \frac{1}{2}\log\left|\frac{C_2 E_1}{C_1 E_2}\right|\right].$

- $\mu_1 = \mu_2, \quad E_2 = \dfrac{6k^2(\alpha^1)^2}{16-9k^2}.$
- $\mu_1 > \mu_2$

Dilaton at boundaries $u_{01} = u_{02}$, $\alpha^1 \neq 0$

$$\phi|_{u\to\pm\infty} \sim \frac{9k}{9k^2-16}\Big[\pm(\mu_2-\mu_1)\,u + \frac{1}{2}\log|\frac{C_2E_1}{C_1E_2}|\Big]$$

$$\phi|_{u\to u_0} \sim \frac{9k}{16-9k^2}\Big[\log\left(\frac{\mu_2}{\mu_1}\right) + \frac{1}{2}\log|\frac{C_1E_2}{C_2E_1}|\Big].$$

- $\mu_1 = \mu_2, \quad E_2 = \dfrac{6k^2(\alpha^1)^2}{16-9k^2}.$
- $\mu_1 > \mu_2$

Black hole, solutions with $u \in [u_{01}, +\infty)$

$$
\begin{aligned}
ds^2 &= \mathcal{C}\,\mathcal{X}(u) e^{\kappa u - \frac{2}{3}\alpha^1 u}\left(-e^{\frac{8}{3}\alpha^1 u} dt^2 + d\vec{y}^2 + \mathcal{X}(u)^3 \mathcal{C}^3 e^{3\kappa + \frac{2}{3}\alpha^1 u} du^2\right)\\
\mathcal{X}(u) &= \left(1 - e^{-2\mu_1 (u-u_{01})}\right)^{-\frac{8}{16-9k^2}} \left(1 - e^{-2\mu_2 (u-u_{02})}\right)^{\frac{9k^2}{2(16-9k^2)}}\\
\kappa &\equiv \frac{8}{\sqrt{6\,(16-9k^2)}}\left(-\sqrt{E_2 + \frac{2}{3}(\alpha^1)^2} + \frac{3}{4}k\sqrt{E_2}\right),\\
\mathcal{C} &\equiv \left(\frac{1}{2}\sqrt{\left|\frac{C_1}{2E_1}\right|} e^{-\mu_1 u_{01}}\right)^{\frac{8}{9k^2-16}} \left(\frac{1}{2}\sqrt{\left|\frac{C_2}{2E_2}\right|} e^{-\mu_2 u_{02}}\right)^{\frac{9k^2}{2(16-9k^2)}}.
\end{aligned}
$$

The absence of conic singularity

- $\kappa - \frac{2}{3}\alpha^1 = 0$, $E_2 = \frac{6k^2(\alpha^1)^2}{16-9k^2}$, $\mu_2 = \mu_1$
- $\frac{4}{3\mathcal{C}^{3/2}}\alpha^1 \beta = 2\pi$

Null geodesics $ds^2 = 0$, for the light moving in the radial direction

$$
t - t_0 = \int_{u_0}^{u} d\bar{u}\,\mathcal{C}^{3/2}\left(1 + \ldots\right) \underset{u\to\infty}{\rightarrow} \infty.
$$

Black hole solution

$$ds^2 = \mathcal{C}\,\mathcal{X}\Big(- e^{\frac{8}{3}\alpha^1 u} dt^2 + d\vec{y}^2 \Big) + \mathcal{C}^4\,\mathcal{X}(u)^4 e^{\frac{8}{3}\alpha^1 u} du^2 ,$$

$$\mathcal{X} = \left(1 - e^{-2\mu(u-u_{01})}\right)^{-\frac{8}{16-9k^2}} \left(1 - e^{-2\mu(u-u_{02})}\right)^{\frac{9k^2}{2(16-9k^2)}} ,$$

$$\mathcal{C} \equiv \left(\frac{1}{2}\sqrt{\left|\frac{C_1}{2E_1}\right|} e^{-\mu u_{01}}\right)^{\frac{8}{9k^2-16}} \left(\frac{1}{2}\sqrt{\left|\frac{C_2}{2E_2}\right|} e^{-\mu u_{02}}\right)^{\frac{9k^2}{2(16-9k^2)}} .$$

$$\phi = \frac{9k}{9k^2-16} \log\left[\sqrt{\left|\frac{E_1 C_2}{E_2 C_1}\right|} \frac{\sinh(\mu(u-u_{02}))}{\sinh(\mu(u-u_{01}))}\right] .$$

and near horizon

$$\lim \phi_{u\to+\infty} = \frac{9k}{2(16-9k^2)} \log\left(\left|\frac{E_2 C_1}{E_1 C_2}\right|\right) .$$

The Hawking temperature

$$T = \frac{2}{3\pi} \frac{|\alpha^1|}{\mathcal{C}^{3/2}}$$

Special case: $u_{01}=u_{02}=u_0$

$$ds^2=\mathcal{C}\left(1-e^{-2\mu(u-u_0)}\right)^{-\frac{1}{2}}\left(-e^{-2\mu u}dt^2+d\vec{y}^2\right)+\mathcal{C}^4\left(1-e^{-2\mu(u-u_0)}\right)^{-2}e^{-2\mu u}du^2,$$

$$\mu=-\frac{4}{3}\alpha^1,\quad \mathcal{C}=(2\sqrt{2})^{1/2}e^{\mu u_0}\left(\frac{C_1}{E_1}\right)^{\frac{4}{9k^2-16}}\left(\frac{C_2}{E_2}\right)^{\frac{9k^2}{4(16-9k^2)}}.$$

The dilaton

$$\phi=\frac{9k}{2(16-9k^2)}\log\left|\frac{C_1E_2}{C_2E_1}\right|.$$

The curvature

$$R=-\frac{5\mu^2}{\mathcal{C}^4}.$$

$z=z_h\left(1-e^{-2\mu u}\right)^{\frac{1}{4}}$, $\mathcal{C}=z_h^{-2}$, $ds^2=\frac{1}{z^2}\left(-f(z)dt^2+d\vec{y}^2+\frac{dz^2}{f(z)}\right)$,
$f=1-\left(\frac{z}{z_h}\right)^4$.

The saturation of the Gubser's bound $V(\phi(u_h))=V_{UV}$.

Outline

1. Introduction
   - Holographic dictionary
2. Exact holographic RG flows
   - Set up
   - How to integrate
   - Vacuum solutions
   - Non-vacuum solutions, black holes
3. **RG equations at $T = 0$**
4. RG-flow at finite T
5. Outlook

Holographic RG equations

The solution in the domain wall coordinates

$$ds^2 = dw^2 + e^{2\mathcal{A}(w)}\left(-dt^2 + \eta_{ij}dx^i dx^j\right).$$

$\phi(w)$, $\lambda = e^\phi$ – the running coupling.
The β-function

$$\beta(\lambda) = \frac{d\lambda_{QFT}}{d\log E} = \frac{d\lambda}{d\mathcal{A}}$$

The β-function satisfies the holographic RG eqs. Kiritsis et al.'0812.0792

$$\frac{dX}{d\phi} = -\frac{4}{3}\left(1 - X^2\right)\left(1 + \frac{3}{8X}\frac{d\log V}{d\phi}\right),$$

where $X(\phi)$ is related with the β-function

$$X(\phi) = \frac{\beta(\lambda)}{3\lambda}$$

The energy scale

$$\mathsf{A} = e^{\mathcal{A}}$$

RG equations at $T = 0$

The domain wall coordinates $dw = F_1^{\frac{16}{9k^2-16}} F_2^{\frac{9k^2}{16-9k^2}} du$.
The running coupling

$$\lambda = e^{\phi} = \left(\frac{F_2}{F_1}\right)^{\frac{9k}{9k^2-16}}.$$

The energy scale

$$\mathrm{A} = e^{\mathcal{A}} = F_1^{\frac{4}{9k^2-16}} F^{\frac{9k^2}{4(16-9k^2)}}.$$

The X-function

$$X = \frac{1}{3}\left(\frac{F_2}{F_1}\right)^{\frac{9k}{16-9k^2}} \frac{\lambda'}{\mathcal{A}'}.$$

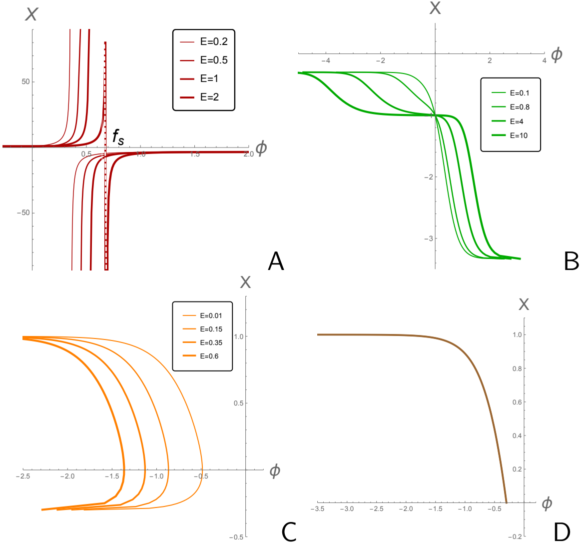


Figure: The behaviour of the X-function with the dependence on the dilaton plotted using the solutions for $\mathcal{A}$. A)left B) middle C)right D) $u_{01} = u_{02}$

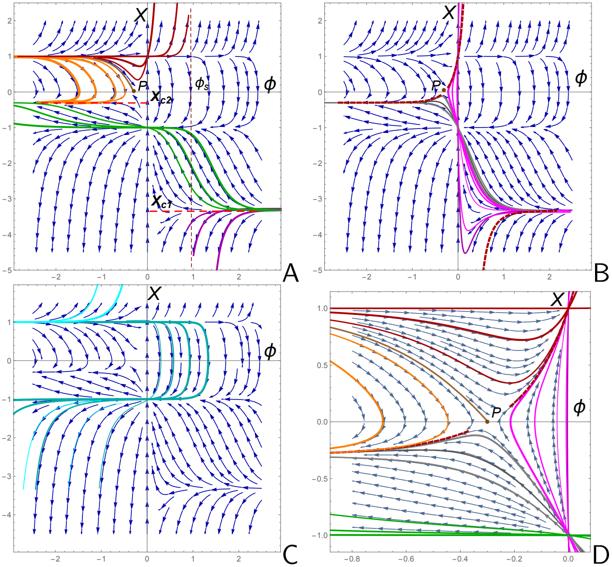


Figure: All solutions X with potential fixed as $C_1 = -C_2 = -2$ and $k = 0.4$

The behaviour of the running coupling λ on the energy scale

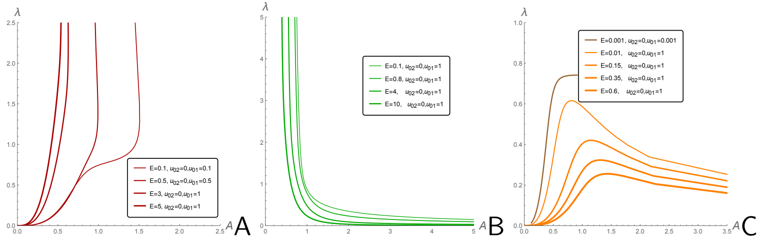


Figure: λ on the energy A on the dilaton plotted using the solutions for $\mathcal{A}$ and ϕ.A) the left branch with $u_{02} > u$, B) the middle branch $u_{02} < u < u_{01}$; C) the right branch $u > u_{01}$. For all plots $k = 0.4$, $C_1 = -2$, $C_2 = 2$, different curves on the same plot corresponds to the different values of $|E_1| = |E_2|$, labeled as E on the legends and different u_{01} and u_{01} also indicated on the legends.

Outline

RG flow at finite temperature

The black brane

$$ds^2 = e^{2\mathcal{A}(w)}\left(-f(w)dt^2 + \delta_{ij}dx^i dx^j\right) + \frac{dw^2}{f(w)},$$

Ex. The Chamblin-Reall solution $f = 1 - C_2\lambda^{-\frac{4(1-X^2)}{3X}}$, $\lambda = e^{\phi}$.
The Y-variable is defined through the function f

$$\begin{aligned}
Y(\phi) &= \frac{1}{4}\frac{g'}{\mathcal{A}'}, \quad g = \log f, \\
\frac{dX}{d\phi} &= -\frac{4}{3}\left(1 - X^2 + Y\right)\left(1 + \frac{3}{8X}\frac{d\log V}{d\phi}\right), \\
\frac{dY}{d\phi} &= -\frac{4}{3}\left(1 - X^2 + Y\right)\frac{Y}{X}.
\end{aligned}$$

The running coupling on the energy scale $(u_{01} \neq 0, u_{02} \neq 0)$

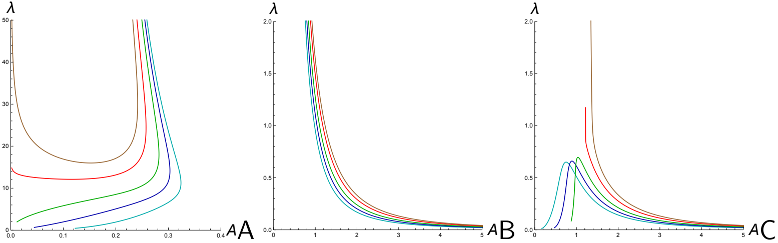


Figure: The dependence of λ on the energy scale A$= e^{\mathcal{A}}$ at the left solution A), the middle solution B) and the right one C). $\alpha^1 = 0$ (cyan), $\alpha^1 = -0.25$ (blue), $\alpha^1 = -0.5$ (green), $\alpha^1 = -0.8$ (red), $\alpha^1 = -1$ (brown).

Outline

The bottom line

Done

- Vacuum and non-vacuum holographic RG-flows were constructed
- Holographic running coupling mimic QCD
- Holographic RG flows can have AdS fixed points.
- A new solution with horizon was found
- Studies of the running coupling λ on the E scale not to deal with superpotential W

The bottom line

Done

- Vacuum and non-vacuum holographic RG-flows were constructed
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?

- Careful studies of the behaviour $\lambda = e^{\phi}$ on the energy scale at $T \neq 0$
- Analysis of confinement-deconfinement phase transition (Polchinski-Strassler model?).
- Holographic c-theorem?
- Full supergravity picture?

Thank you for attention!