Self-consistent models of nuclear clustering

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Abstract. In recent years a phenomenological core-cluster has been constructed with a Saxon-Woods plus cubic Saxon-Woods term successfully predicts a reasonable number of observable phenomenon which is related to alpha clustering. This model, however successful, lacks a microscopic description of clustering phenomenon in nuclear systems. A fully microscopic formalism is presented, where the core and cluster baryon densities are derived from a relativistic mean field approach. The Lorentz covariant IA1 representation of the nucleon-nucleon interaction is folded with the derived core and cluster densities. Theoretical predictions of the ground-state decay half-life and positive parity energy band of ^{212}Po are obtained with the relativistic mean field formalism and compared to predictions made with the phenomenological Saxon-Woods plus cubic Saxon-Wood core-cluster potential.

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INTRODUCTION

Clustering phenomenon is one of the essential features of nuclear matter which has been studied in great detail [1] in nuclear physics. In the physics of unstable nuclei, clustering is one of the central areas of study. The cluster-core interaction lies central to the identification of clustering in the nuclear matter and the description of clustering phenomenon in various nuclei. During the last decade the modified phenomenological Saxon-Woods plus Cubic Saxon-Woods cluster potential has successfully described various phenomenon related to alpha clustering in light as well as even-even heavy nuclei. In order to fully describe clustering in nuclear systems one would have to develop a microscopic model of the phenomenon at the nucleon-nucleon scale.

At a more microscopic level the core-cluster interaction may be constructed from a nucleon-nucleon interaction. Prior to the development of the Saxon-Wood plus Saxon-Wood cubed potential form, such a microscopic interaction had been employed in various forms to describe α cluster bound states in light nuclei [2] and the exotic decays in heavy nuclei [3]. In recent years the microscopic M3Y-type potential model has been extended to describe the alpha decay half-lives and the structure of heavy nuclei [4], [5], and [6]. An application of the interaction to ^{94}Mo and ^{212}Po in particular suggests a good amount of α clustering in these nuclei [7].

Relativistic mean field theory (RMFT) [8] has proven to be very successful in describing various properties of nuclear structure [9]. In this work a RMFT description of clustering is presented and a comparison is made between the experimental ground-state decay half-lives and band energy spectral of ^{212}Po and cluster model predictions of these quantities which were obtained from the Saxon-Woods + cubic Saxon-Woods potential, double folded M3Y nucleon-nucleon interaction [10] and the microscopic RMFT based core-cluster interaction.

THE BINARY CLUSTER MODEL

This model is based on the preformed binary cluster model for which the decay half life is given by

$$T_{1/2} = \hbar \frac{\ln 2}{\Gamma}, \tag{1}$$

where Γ represents the cluster decay width. For the breakup of a nucleus into the core and cluster the decay width is defined by the relationship

$$\Gamma = P \frac{\hbar^2}{2\mu} \frac{exp(-2\int_{r_2}^{r_3} k(r)dr)}{\int_{r_1}^{r_2} [k^{-1}(r)]dr} \tag{2}$$

with P being the core-cluster preformation probability in the parent nucleus, μ is reduced mass of the core-cluster system and $k(r)$ is cluster wavenumber. The wavenumber depends on both the decay energy (E) and the core-cluster potential $V(r)$, and is given by

$$k(r) = \sqrt{\frac{2\mu}{\hbar^2}|E - V(r)|}. \tag{3}$$

The energy band structure of the quasi-boundstates can be obtained from a combination of the Bohr-Sommerfeld (BS) quantization integral

$$\int_{r_1}^{r2} \sqrt{\frac{2\mu}{\hbar^2}\left[E_l - V(r)\right] dr} = (2n+1)\frac{\pi}{2} \tag{4}$$

and the Wildermuth condition $G = 2n + l$, where n is the number of nodes of the radial wavefunction and l is the orbital angular momentum of the cluster state. G defines the global quantum number of the core-cluster relative motion. The interaction between the core and cluster, $V(r)$, is described by the sum of the attractive nuclear cluster-core potential $U(r)$, the Coulomb potential between the two charged centres, and the centrifugal potential.

CORE CLUSTER POTENTIALS

Phenomenological core-cluster interaction

The recently developed modified Saxon-Wood with an additional cubic Saxon-Woods core-cluster phenomenological potential

$$U(r) = U_0 \left[\frac{x}{1+exp\left(\frac{r-R}{a}\right)} + \frac{1-x}{1+exp\left(\frac{r-R}{3a}\right)^3} \right] \quad (5)$$

is found to consistently reproduce not only the alpha and exotic decay half-lives, but also correctly predict the level properties of nuclei in the rare earth and the actinide region. This potential is parameterized in terms of the potential depth (U_0), nuclear radius (R), diffuseness (a), and x is a mixing parameter. Despite its success this potential model tells us very little about the microscopic nature of clustering in closed shell nuclei.

Relativistic mean field construction of the cluster-core potential

In the IA1 representation of the nucleon-nucleon scattering amplitude [12]

$$F = F^S I^a I_b + F^V \gamma_a^\mu \gamma_{\mu b} + F^{PS} \gamma_a^5 \gamma_{5b} + F^T \sigma_a^{\mu\nu} \sigma_{\mu\nu b} + F^A \gamma_a^5 \gamma_a^\mu \gamma_b^5 \gamma_{\mu b}, \quad (6)$$

Lorentz covariance, parity conservation, isospin invariance, and the constraint that the free nucleons are on the mass shell imply that the invariant NN scattering operator F be written in terms of the five complex functions for pp and five for pn scattering. The quantities $\lambda_i^L = (I, \gamma^\mu, \gamma^5, \sigma^{\mu\nu}, \gamma^5\gamma^\mu)$ represent the five Dirac gamma matrices [13], and the index $(i = a, b)$ labels the two interacting nucleons. The index L labels the scalar, vector, pseudo-scalar, tensor and axial terms.

Out of the Lorentz covariant McNeil, Ray and Wallace (MRW) construction of the optical potential for nucleon-nucleus scattering [11], arises the double folded MRW form which describes the cluster-core potential

$$U^L(r,\varepsilon) = -\frac{4\pi i p}{Mc^2} \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} F^L(q,\varepsilon) \int d^3r' e^{-i\mathbf{q}\cdot\,\mathbf{r}'} \rho_1^L(r') \int d^3r'' e^{-i\mathbf{q}\cdot\,\mathbf{r}''} \rho_2^L(r''), \quad (7)$$

where r represents the separation distance between the cluster (1)and core (2)center, and ε is the laboratory energy of the nucleons in the cluster. The momentum of the nucleons in the nucleon-nucleon (NN)center of mass system is given by p while M represents the nucleon mass. Equation (7) contains the Lorentz covariant nucleon-nucleon scattering amplitudes $F^L(q,\varepsilon)$, which are functions of the NN centre of mass momentum transfer ($\vec{q}$) and nucleon laboratory energy (ε), as well as the respective cluster and core densities ρ_1^L and ρ_2^L.

The Walecka model is based on a relativistic mean field theory with an effective Lagrangian which describes the NN interaction via the electromagnetic interaction and the effective meson fields [8]. The dynamical equation which results from the Lagrangian is given by

$$\hat{H}\psi(\mathbf{r}) = \left(i\alpha\cdot\nabla - g_v\gamma^0 V^0(r) + \beta[M - g_s\phi(r)]\right)\psi(\mathbf{r}) = E\psi(\mathbf{r}) \quad (8)$$

with the Dirac Hamiltonian operator ($\hat{H} = i\alpha\cdot\nabla - g_v\gamma^0 V^0(r) + \beta[M - g_s\phi(r)]$), vector and scalar fields g_v and g_s respectively, as well as the zeroth component vector field (V_0)

and scalar field (ϕ). Equation (8) has both positive and negative solutions $U(\mathbf{r})$ and $V(\mathbf{r})$, and thus the field operator can be expanded as

$$\hat{\psi}(\mathbf{r}) = \sum_{\Lambda} \left[A_{\Lambda} U_{\Lambda}(\mathbf{r}) + B_{\Lambda}^{\dagger} V_{\Lambda}(\mathbf{r}) \right]. \tag{9}$$

The baryon and antibaryon creation operators $A_{\Lambda}^{\dagger}$ and $B_{\Lambda}^{\dagger}$ satisfy the standard anti-commutator relationships and the index Λ specifies the full set of single-particle quantum numbers, which for a spherically symmetric and parity conserving system, are the usual angular momentum and parity quantum numbers, as given by reference [13]. The positive-energy spinor can be written as

$$U_{\Lambda} \equiv U_{njlmt}(\mathbf{r}) = \begin{pmatrix} i\left[G_{njlt}(r)/r\right]\Phi_{jlm} \\ \left[F_{njlt}(r)/r\right]\Phi_{jl+1m} \end{pmatrix} \varsigma_t, \tag{10}$$

where Φ_{jlm} is the angular momentum and spin dependent part of the solution and n and ς_t represents the principal quantum number and two component isospinor which is labeled by the isospin projection t. The functions $G(r)$ and $F(r)$ represent the radial wave functions for the upper and lower components of the positive energy spinor U_{Λ} Neglecting the negative-energy spinors the local baryon (ρ_B) and scalar (ρ_s) densities can be derived from the positive-energy solutions

$$\left.\begin{matrix} \rho_B(\mathbf{r}) \\ \rho_s(\mathbf{r}) \end{matrix}\right\} = \sum_{\Lambda} \bar{U}_{\Lambda}(\mathbf{r}) \begin{pmatrix} \gamma^0 \\ I \end{pmatrix} U_{\Lambda}(\mathbf{r}). \tag{11}$$

MODEL PREDICTIONS AND CONCLUSIONS

For the BMP phenomenological form of the cluster-core potential prediction of the positive parity alpha band energy structure of ^{212}Po the parameters U_0 = 208 MeV, a = 0.66 fm, x = 0.30 and R = 6.784 were used with G = 18 [15].
The Walecka based RMFT prediction uses the experimental masses $M = 939$ MeV, $m_v = m_\omega = 738$ MeV, $m_\rho = 770$ MeV, $m_s = 520$ MeV, and $\alpha = e^2/4\pi = 1/137.36$ are used. The coupling constants for the scalar, vector, and ρ-meson are $g_s^2 = 109.6$, $g_v^2 = 190.4$, and $g_\rho^2 = 65.23$ respectively. We apply the Dirac-Hartree code Timora [14] to calulate the scalar and vector densities for both the protons and neutrons. The densities of the core and cluster systems are inturn used to calculate the core-cluster potential by means of the double folded MRW method.
The results of the calculated α-decay half-life of the ground-state as predicted by the phenomenological BMP, the microscopic M3Y with phenomenological core and cluster baryon densities [15], and MRW double folded relativistic mean field nucleon densities with Lorentz covariant NN scattering amplitudes are compared with experimental data [16] in Table I. Table II compares the predicted band energy structure from the BMP, M3Y and RMFT model calculations with available experimental data [16].
From the results in Table (1) and (2) one see that the Saxon-Woods plus cubic Saxon-Woods potential and the RMFT based model gives a reasonable prediction of the half-life of the 0^+, while the microscopic M3Y based model underpredict the ground-state

alpha decay half-life of ^{212}Po by a factor of approximately 2. Furturemore the energy spectra of the excited α states are predicted reasonably well both the Saxon-Woods plus cubic Saxon Woods core-cluster potential and the self-consistent RMFT core-cluster model where as the microscopic M3Y model potential results in a clear inversion of the energy spectra.

TABLE 1. The experimental ground state decay of ^{212}Po and the corresponding values obtained with the BMP (5), double folded M3Y, and the self-consistent RMFT potentials.

$T_{1/2}$(Exp) ns	$T_{1/2}$(BMP) (ns)	$T_{1/2}$(M3Y) (ns)	$T_{1/2}$(RMFT) (ns)
300	348.0	157.4	299.6

TABLE 2. The experimental energy level scheme of ^{212}Po and the calculated spectra obtained with the BMP (5), double folded M3Y and self-consistent RMFT potentials.

J^{π}	E_{exp} (MeV)	E_{BMP} MeV	E_{M3Y}	E_{RMFT} (MeV)
0^+	0.000	(0.495)	−0.004	0.203
2^+	0.727	0.659	−0.067	0.421
4^+	1.132	0.948	−0.229	0.699
6^+	1.355	1.318	−0.508	0.857
8^+	1.476	1.730	−0.930	1.085
10^+	1.834	2.145	−1.538	1.319
12^+	2.702	2.519	−2.358	1.553
14^+	2.885	2.805	−3.437	1.787
16^+	–	2.941	−4.800	2.021
18^+	2.921	2.841	−6.477	2.255

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