Covariant relativistic separable kernel approach for the electrodisintegration of the deuteron at high momentum transfer

E.P. Rogochaya

BLTP JINR, Dubna

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Plan

- Bethe-Salpeter formalism
- Description of the deuteron
- Electrodisintegration of the deuteron
- Conclusion

Bethe-Salpeter equation for scattered states:

$$T(p',p;P) = V(p',p;P) + \frac{\imath}{4\pi^3}\int d^4k\, V(p',k;P)\, S_2(k;P)\, T(k,p;P)$$

V - interaction kernel, T - scattering matrix

Two-particle Green function:

$$S_2^{-1}(k;P) = \left(\tfrac{1}{2}\, P\cdot\gamma + k\cdot\gamma - m\right)^{(1)} \left(\tfrac{1}{2}\, P\cdot\gamma - k\cdot\gamma - m\right)^{(2)}$$

Bethe-Salpeter equation for bound states:

$$\Phi^{JM}(p;P) = \frac{\imath}{(2\pi)^4} S_2(p;P) \int d^4k V(p,k;P)\Phi^{JM}(k;P)$$

Φ^{JM} - Bethe-Salpeter amplitude

$$\Phi^{JM}(p;P) = S_2(p;P)\Gamma^{JM}(p;P)$$

Γ^{JM} - vertex function

Partial-wave decomposition:

$$T_{\alpha\beta,\gamma\delta}(p',p;P_{(0)}) \qquad s = P_{(0)}^2$$
$$= \sum_{JMab} (\mathcal{Y}_{aM}(-\boldsymbol{p}')U_C)_{\alpha\beta} \otimes (U_C \mathcal{Y}^{\dagger}_{bM}(\boldsymbol{p}))_{\delta\gamma}\ T_{ab}(p'_0,|\boldsymbol{p}'|;p_0,|\boldsymbol{p}|;s)$$

Spin-angle functions:

$$\mathcal{Y}_{JM:LS\rho}(\boldsymbol{p})U_C$$
$$= \imath^L \sum_{m_L m_S m_1 m_2 \rho_1 \rho_2} C^{S_\rho \rho}_{\frac{1}{2}\rho_1 \frac{1}{2}\rho_2} C^{JM}_{Lm_L Sm_S} C^{Sm_S}_{\frac{1}{2}m_1 \frac{1}{2}m_2} Y_{Lm_L}(\boldsymbol{p}) u^{\rho_1}_{m_1}{}^{(1)}(\boldsymbol{p}) u^{\rho_2}_{m_2}{}^{(2)\,T}(-\boldsymbol{p})$$

Normalization:

$$\int d\varphi_{\boldsymbol{p}}\, d(\cos\theta_{\boldsymbol{p}})\, \mathrm{Sp}\left\{\mathcal{Y}^{\dagger}_{aM}(\boldsymbol{p})\mathcal{Y}_{a'M'}(\boldsymbol{p})\right\}$$
$$\equiv \int d\varphi_{\boldsymbol{p}}\, d(\cos\theta_{\boldsymbol{p}}) (\mathcal{Y}^{\dagger}_{aM}(\boldsymbol{p}))_{\beta\alpha} (\mathcal{Y}_{a'M'}(\boldsymbol{p}))_{\alpha\beta} = \delta_{aa'}\delta_{MM'}$$

Bethe-Salpeter equation for partial-wave components:

$$T_{ab}(p'_0,|\boldsymbol{p}'|;p_0,|\boldsymbol{p}|;s) = V_{ab}(p'_0,|\boldsymbol{p}'|;p_0,|\boldsymbol{p}|;s)$$
$$+ \frac{\imath}{4\pi^3} \sum_{cd} \int_{-\infty}^{+\infty} dk_0 \int_0^{\infty} \boldsymbol{k}^2 d|\boldsymbol{k}|\, V_{ac}(p'_0,|\boldsymbol{p}'|;k_0,|\boldsymbol{k}|;s) S_{cd}(k_0,|\boldsymbol{k}|;s)\, T_{db}(k_0,|\boldsymbol{k}|;p_0,|\boldsymbol{p}|;s)$$

Interaction kernel:

$$V_{l'l}(p'_0, |\boldsymbol{p'}|; p_0, |\boldsymbol{p}|; s) = \sum_{i,j=1}^{N} \lambda_{ij}(s) g_i^{[l']}(p'_0, |\boldsymbol{p'}|) g_j^{[l]}(p_0, |\boldsymbol{p}|)$$

T **matrix:**

$$T_{l'l}(p'_0, |\boldsymbol{p'}|; p_0, |\boldsymbol{p}|; s) = \sum_{i,j=1}^{N} \tau_{ij}(s) g_i^{[l']}(p'_0, |\boldsymbol{p'}|) g_j^{[l]}(p_0, |\boldsymbol{p}|)$$

$$\tau_{ij}(s) = 1/(\lambda_{ij}^{-1}(s) + h_{ij}(s))$$

$$h_{ij}(s) = -\frac{\imath}{4\pi^3} \sum_l \int dk_0 \int \boldsymbol{k}^2 d|\boldsymbol{k}| \frac{g_i^{[l]}(k_0, |\boldsymbol{k}|) g_j^{[l]}(k_0, |\boldsymbol{k}|)}{(\sqrt{s}/2 - E_{\mathbf{k}} + \imath\epsilon)^2 - k_0^2},$$

$$E_{\mathbf{k}} = \sqrt{\boldsymbol{k}^2 + m^2}$$

Bethe-Salpeter equation for the deuteron:

$$\Phi^{JM}(p;P) = \frac{\imath}{(2\pi)^4} S_2(p;P) \int d^4k V(p,k;P) \Phi^{JM}(k;P)$$

Radial part of the deuteron vertex function:

$$g_l(p) = \sum_{i,j=1}^{N} \lambda_{ij}(s) g_i^{[l]}(p_0, |\boldsymbol{p}|) c_j(s), \quad l \to {}^3S_1^+ - {}^3D_1^+$$

System of linear homogeneous equations:

$$c_i(s) - \sum_{k,j=1}^{N} h_{ik}(s) \lambda_{kj}(s) c_j(s) = 0$$

Normalization:

$$p_S + p_D = 1: \quad p_l = \frac{\imath}{2M_d(2\pi)^4} \int dp_0 \int \boldsymbol{p}^2 d|\boldsymbol{p}| \frac{(E_{\mathbf{p}} - M_d/2)[g_l(p_0, |\boldsymbol{p}|)]^2}{\left((M_d/2 - E_{\mathbf{p}} + \imath\epsilon)^2 - p_0^2\right)^2}$$

Rank-six kernel for $^3S_1^+$-$^3D_1^+$ state MY6:

$$g_1^{[S]}(p) = \frac{(p_{c1} - p_0^2 + \boldsymbol{p}^2)}{(p_0^2 - \boldsymbol{p}^2 - \beta_1^2)^2 + \alpha_1^4}$$

$$g_2^{[S]}(p) = \frac{(p_0^2 - \boldsymbol{p}^2)(p_{c2} - p_0^2 + \boldsymbol{p}^2)^2}{((p_0^2 - \boldsymbol{p}^2 - \beta_2^2)^2 + \alpha_2^4)^2}$$

$$g_3^{[S]}(p) = \frac{(p_0^2 - \boldsymbol{p}^2)^3(p_{c3} - p_0^2 + \boldsymbol{p}^2)^2}{((p_0^2 - \boldsymbol{p}^2 - \beta_3^2)^2 + \alpha_3^4)^3}$$

$$g_4^{[D]}(p) = \frac{-(p_0^2 - \boldsymbol{p}^2)(p_{c4} - p_0^2 + \boldsymbol{p}^2)^2}{((p_0^2 - \boldsymbol{p}^2 - \beta_{41}^2)^2 + \alpha_{41}^4)((p_0^2 - \boldsymbol{p}^2 - \beta_{42}^2)^2 + \alpha_{42}^4)}$$

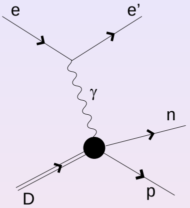
$$g_5^{[D]}(p) = \frac{-(p_0^2 - \boldsymbol{p}^2)}{(p_0^2 - \boldsymbol{p}^2 - \beta_5^2)^2 + \alpha_5^4}$$

$$g_6^{[D]}(p) = \frac{(p_0^2 - \boldsymbol{p}^2)^4(p_{c6} - p_0^2 + \boldsymbol{p}^2)}{((p_0^2 - \boldsymbol{p}^2 - \beta_{61}^2)^2 + \alpha_{61}^4)^2((p_0^2 - \boldsymbol{p}^2 - \beta_{62}^2)^2 + \alpha_{62}^4)}$$

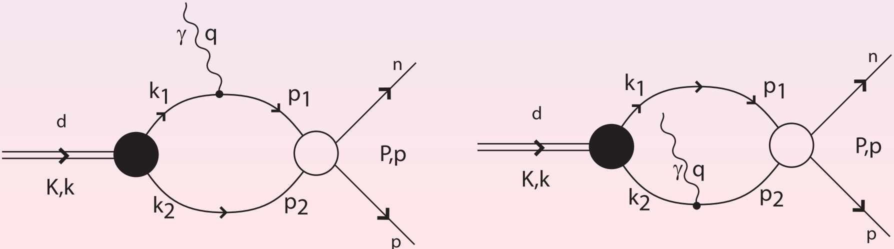
$$g_4^{[S]} = g_5^{[S]} = g_6^{[S]} = 0 = g_1^{[D]} = g_2^{[D]} = g_3^{[D]} = 0$$

S.G. Bondarenko, V.V. Burov, W.-Y.Pauchy Hwang, E.P. Rogochaya, Nucl. Phys. **A**, doi:10.1016/j.nuclphysa.2010.08.007, 17 pp. [arXiv:1002.0487[nucl-th]]

One-photon approximation:



Impulse approximation:



Hadron current:

$$< np : SM_S|j_\mu|d : 1M >^{\mathrm{LS}} = \imath \sum_l \int \frac{d^4p}{(2\pi)^4} \mathrm{Sp}\,\{\bar\psi_{SM_S}(P,p)\times$$

$$\Lambda^{\mathrm{LS}\to\mathrm{CM}}(\mathcal{L})\Gamma^{(l)}_\mu(q)S^{(l)}\left(\frac{K}{2}-(-1)^l p-\frac{q}{2}\right)\Gamma_M\left(p+(-1)^l\frac{q}{2}\right)\Lambda^{\mathrm{LS}\to\mathrm{CM}}(\mathcal{L}^{-1})\Big\}$$

np **pair wave-function in the plane-wave approximation:**

$$\bar\psi_{SM_S}(P,p;p^*) \ \to\ \bar\psi^{(0)}_{SM_S}(P,p;p^*) = (2\pi)^4\bar\chi_{SM_S}\delta(p-p^*),\quad p^* = \left(0,\sqrt{\frac{s}{4}-m^2}\right)$$

$$< np : SM_S|j_\mu|d : 1M >^{\mathrm{LS}} = \imath \sum_{l=1,2}$$

$$\mathrm{Sp}\left\{\bar\chi_{SM_S}(P,p^*)\Lambda(\mathcal{L})\Gamma^{(l)}_\mu(q)S^{(l)}\left(\frac{K}{2}-(-1)^l p^*-\frac{q}{2}\right)\Gamma_M\left(p^*+(-1)^l\frac{q}{2}\right)\Lambda(\mathcal{L}^{-1})\right\}$$

Interaction vertex:

$$\Gamma_\mu(q) = \gamma_\mu F_1(q^2) - \frac{1}{4m}\left(\gamma_\mu\hat q - \hat q\gamma_\mu\right)F_2(q^2)$$

$F_1(q^2)$ - Dirac form factor, $F_2(q^2)$ - Pauli form factor

Electromagnetic form factors:

1. **Dipole fit**: H. Pietschmann, H. Stremnitzer, Lett. Nuovo Cim. **2**, 841 (1969)
2. **Modified dipole fit**: neutron electric form factor - S. Galster *et al.*, Nucl. Phys. **B32**, 221 (1971), proton electric form factor - O. Gayou *et al.*, Phys. Rev. Lett. **88**, 092301-1 (2002)

Cross section (for unpolarized particles):

$$\frac{d^3\sigma}{dQ^2 d|\boldsymbol{p}_n| d\Omega_n} = \frac{\sigma_{\rm Mott}\pi \boldsymbol{p}_n^2}{(2\pi)^3 2M_d E_e E'_e} \times$$
$$\times \left[l^0_{00} W_{00} + l^0_{++}(W_{++} + W_{--}) + l^0_{+-}\cos 2\phi\ 2{\rm Re}W_{+-} - l^0_{+-}\sin 2\phi\ 2{\rm Im}W_{+-}\right.$$
$$\left. -l^0_{0+}\cos\phi\ 2{\rm Re}(W_{0+} - W_{0-}) - l^0_{0+}\sin\phi\ 2{\rm Im}(W_{0+} + W_{0-})\right].$$

$$\sigma_{\sf Mott} = (\alpha\cos\frac{\theta_e}{2}/2E_e\sin^2\frac{\theta_e}{2})^2$$

$\alpha = e^2/(4\pi)$ - fine structure constant

$$\textbf{Lepton tensor}: \qquad l^0_{00} = \frac{Q^2}{\boldsymbol{q}^2},\quad l^0_{0+} = \frac{Q}{|\boldsymbol{q}|\sqrt{2}}\sqrt{\frac{Q^2}{\boldsymbol{q}^2} + \tan^2\frac{\theta_e}{2}},$$
$$l^0_{++} = \tan^2\frac{\theta_e}{2} + \frac{Q^2}{2\boldsymbol{q}^2},\quad l^0_{+-} = -\frac{Q^2}{2\boldsymbol{q}^2}.$$

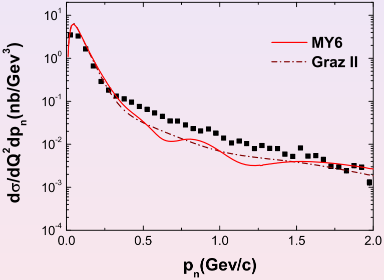
Hadron tensor:

$$W_{\mu\nu} = \frac{1}{3}\sum_{s_d s_n s_p} < np : SM_S|j_\mu|d : 1M >< np : SM_S|j_\nu|d : 1M >^*$$

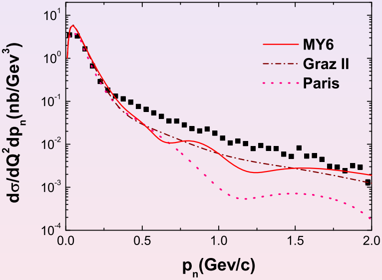
K.S. Egiyan, Phys. Rev. Lett. **98**, 262502 (2007): $\frac{d^2\sigma}{dQ^2 d|\boldsymbol{p}_n|} = \int\limits_{0^\circ}^{180^\circ} d\phi \int\limits_{20^\circ}^{160^\circ} d\theta_n \frac{d^3\sigma}{dQ^2 d|\boldsymbol{p}_n| d\Omega_n}$

$$Q^2 = 2 \pm 0.25\,\text{GeV}^2$$

dipole FF



modified dipole FF



Experimental data were taken from K.Sh. Egiyan et al., CLAS Note No. 2006-025, http://www1.jlab.org/ul/Physics/Hall-B/clas/
Graz II: G. Rupp, J.A. Tjon, Phys. Rev. **C41**, 472 (1990)
Paris: M. Lacombe *et al.*, Phys. Rev. **C21**, 861 (1980)

Cross section: Comparison with experiment, $Q^2 = 2\text{GeV}^2$

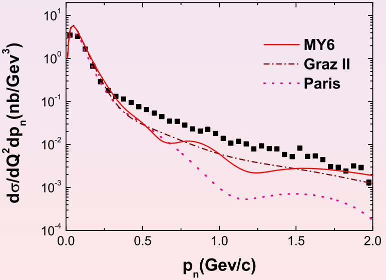
S.G. Bondarenko, V.V. Burov, M. Beyer, S.M. Dorkin, Phys. Rev. **C58**, 3143 (1998)

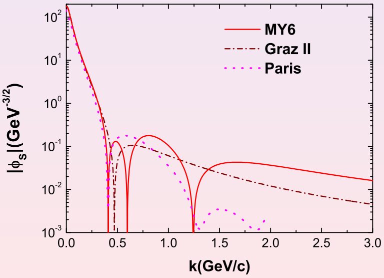
wave function for ${}^3S_1^+$ partial state

$$\phi_l(p_0, |\boldsymbol{p}|) = \frac{g_l(p_0, |\boldsymbol{p}|)}{(M_d/2 - E_{\mathbf{p}} + \imath\epsilon)^2 - p_0^2}$$

at $p_0 = M_d/2 - E_{\mathbf{p}}$

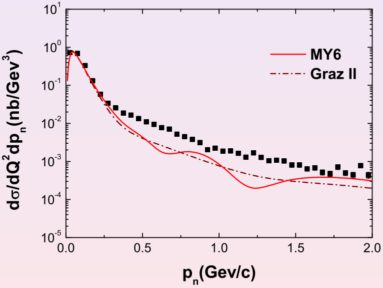
cross section



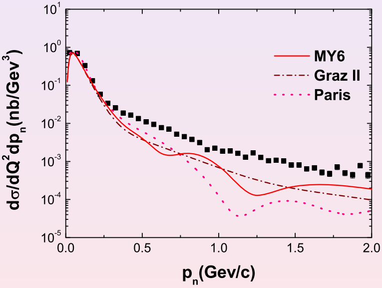


$$Q^2 = 3 \pm 0.5\,\text{GeV}^2$$

dipole FF



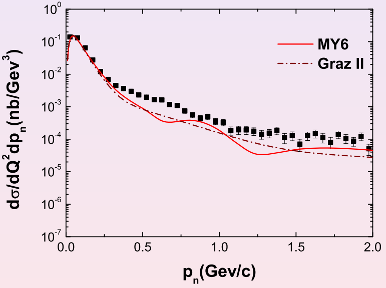
modified dipole FF



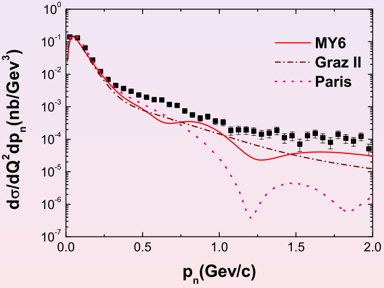
Cross section: Comparison with experiment, $Q^2 = 4\text{GeV}^2$

$$Q^2 = 4 \pm 0.5\,\text{GeV}^2$$

dipole FF

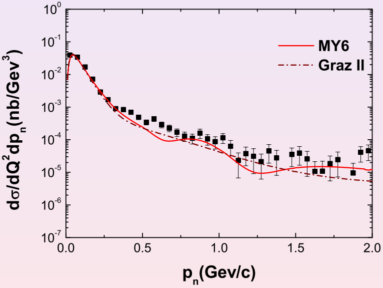


modified dipole FF

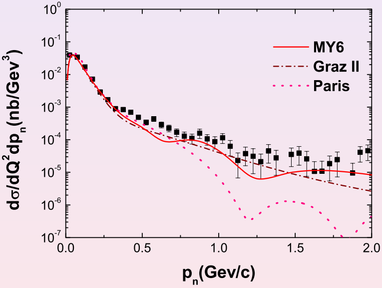


$$Q^2 = 5 \pm 0.5\,\text{GeV}^2$$

dipole FF



modified dipole FF



Conclusion

- The Bethe-Salpeter approximation with a covariant separable interaction kernel for the description of the nucleon-nucleon interactions was presented.
- Three different models of NN interactions and two models of nucleon electromagnetic form factors were considered for the description of the deuteron electrodisintegration at high momentum transfer.
- The considerable influence of relativistic effects with the increase of the momentum transfer and the outgoing neutron momentum was demonstrated.
- The investigation showed slight dependence of the calculated cross section on the used model for nucleon form factors. This conclusion should be tested in further calculations when final state interaction (possible only with MY6) and pair currents effects will be taken into account.