Three two-component particles with zero-range interactions under one-dimensional confinement

O. I. Kartavtsev, A. V. Malykh,

JINR, Dubna, Russia

S. A. Sofianos

UNISA, Pretoria, South Africa

Introduction

- Experimentally available are
  - Many-component systems
  - Systems consisting of particles of different symmetry (fermions and bosons)
  - Particles confined within a trap of arbitrary dimension (1D, 2D, quasi-1D, quasi-2D)
  - Examples ^{6}Li - ^{40}K , ^{6}Li -^{133}Cs, ^{7}Li - ^{87}Sr
- The role of the admixture in ultracold gases
- First step is to study a three-body system with two identical particles and a different third particle
- Aim: two-component systems in 1D, 2D, 3D, in harmonic traps
  - Spectra, some scattering characteristics
- Contact interaction allows one to obtain the universal description of the few-body system at low energy

Problem

- Two identical particles of mass **m**, and distinct one of mass $\mathbf{m_1}$
- Hamiltonian reads as

$$H = -\sum_i \frac{\hbar^2}{2m_i}\frac{\partial^2}{\partial x_i^2} + \lambda_1\delta(x_1) + \lambda\delta(x_2) + \lambda\delta(x_3)$$

  - where strength of the attraction between different particles (1 и 2), (1 и 3) $\boldsymbol{\lambda<0}$;
  - strength of the interaction between identical particles $\boldsymbol{\lambda_1}$

- Units ħ=|λ|=m=1
- Aim
  - Dependence of the bound state energies and scattering lengths *A* (of the third particle off a bound pair of different particles) and A_1 (of the third particle off a bound pair of identical particles) on the parameters of the system

Parameters : m/m_1; λ_1/λ, parity

$$E_{\rm th} = -1/[2(1+m/m_1)]$$

$$E'_{\rm th} = -\lambda_1^2/4$$

Analytical results

Bosonic system with $\lambda_1=\infty$ correspond to fermionic system with $\lambda_1=0$

✓ Two light particles and heavy one $m/m_1=0$

- Noninteracting identical bosons $\lambda_1=0$
  - one bound state with E=-1
  - scattering at threshold energy E_{th} = -1/2 with A=1
- Infinite repulsion of identical bosons $\lambda_1=\infty$ (fermions for $\lambda_1=0$)
  - no bound states,
  - scattering at threshold energy E_{th} = -1/2 with A=0

✓ Particles with identical masses $m/m_1=1$

- Three identical partilces $\lambda_1 = \lambda$ (analytical solution is known for N particles [1,2])

[1] E. H. Lieb and W. Liniger **Phys. Rev. 130 1605 (1963)**

[2] J. B. McGuire **J. Math. Phys. 5, 622 (1964)**

  - one bound state with E=-1
  - scattering at threshold energy E'_{th} = -1/4 with A=∞
- Noninteracting identical bosons $\lambda_1=0$
  - The transcendental equation for energy of the ground state is found in [M. Girardeau **J. Math. Phys. 1, 516 (1960)**]
- Infinite repulsion of identical bosons $\lambda_1=\infty$ (fermions for $\lambda_1=0$)
  - scattering at threshold energy E_{th} = -1/4 with A=∞

✓ For $m/m_1 \geq 1$ at least one bound state exists

✓ For $m/m_1 = 1$ exactly one bound state exists for any λ_1/λ and symmetry of the system

Asymptotic dependencies

Even states

✓Infinite attraction strength of identical bosons $\lambda_1 \to -\infty$

$$\varepsilon \approx 4/(1 + 2m/m_1)$$

$$A_1 \approx (1 + 2m/m_1)/4$$

✓Two heavy bosons and one light particle $m/m_1 \to \infty$

$$m/m_1 \approx C(N + \delta)^2$$

$$C = \frac{\pi^2}{2}\left[\int_0^1 \sqrt{2t + t^2}\frac{1 + (1 - \ln t)t}{2t(1 + t)^2}dt\right]^{-2} \approx 2.59$$

The interpolation of the calculated critical values m/m_1, at which the N-th bound state appears, gives $C \approx 2.60$ for $\lambda_1 \to \infty$ and $\lambda_1 = 0$;

$\delta = 0.73$ for $\lambda_1 \to \infty$

$\delta = 0.22$ for $\lambda_1 = 0$.

Odd states

✓Two heavy bosons and one light particle $m/m_1 \to \infty$

$$A = \frac{m}{m_1}\sqrt{1 + \frac{m_1}{2m}}\left(\ln\frac{m}{m_1} + 2\gamma\right)$$

Numerical calculation

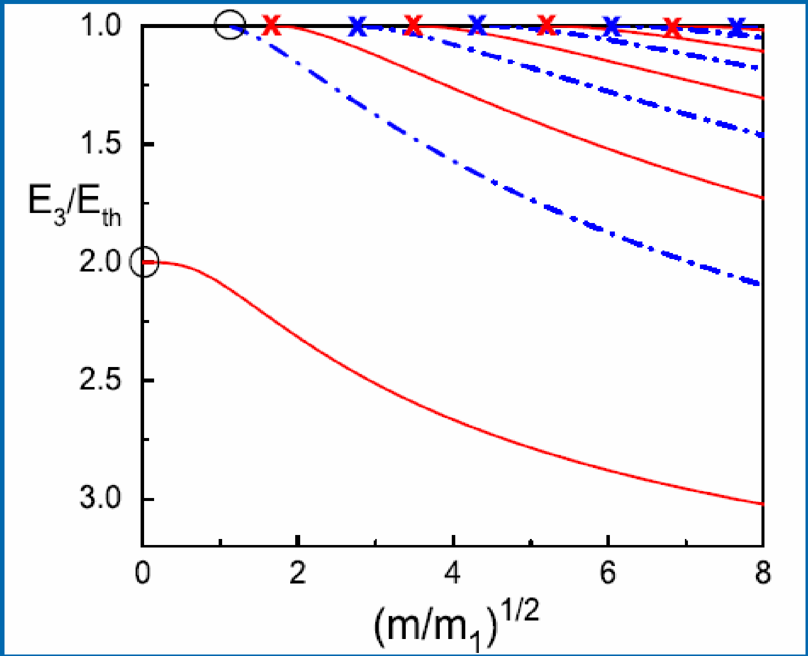
- for bosonic system with $\lambda_1 \to \infty$ and $\lambda_1 = 0$
- for fermionic system with $\lambda_1 = 0$

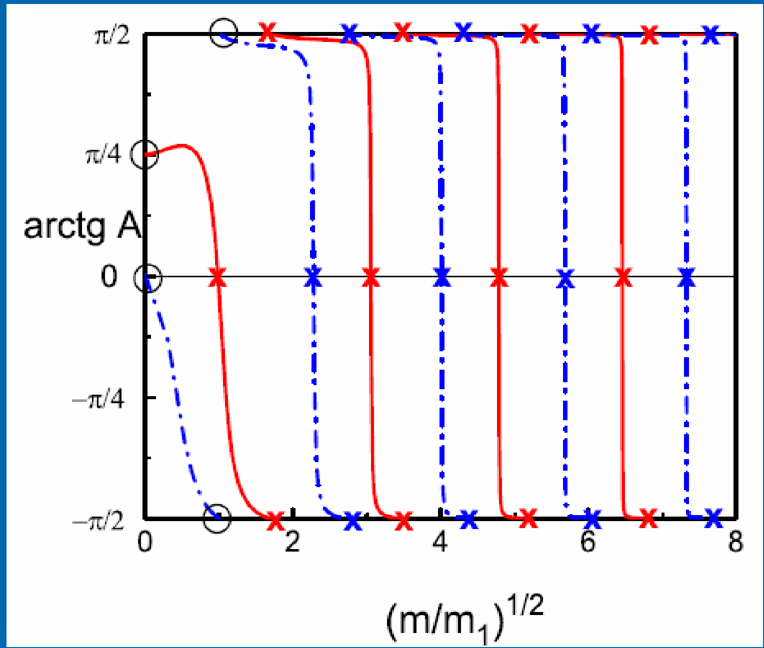
	$\lambda_1 = 0$		$\lambda_1 \to \infty$	
N	$m/m_1(A=0)$	$m/m_1(\|A\| \to \infty)$	$m/m_1(A=0)$	$m/m_1(\|A\| \to \infty)$
1	-	-	0^*	1^*
2	0.971	2.86954	5.2107	7.3791
3	9.365	11.9510	16.1197	19.0289
4	22.951	26.218	32.298	35.879
5	41.762	45.673	53.709	57.923
6	65.791	70.317	80.339	85.159
7	95.032	100.151	112.179	117.583
8	129.477	135.170	149.222	155.193
9	169.120	175.374	191.463	197.989
10	213.964	220.765	238.904	245.973

* Exact

Critical mass ratios m/m_1 at which 2+1 scattering length becomes zero $(A = 0)$ and at which the N-th three-body bound state appears $(A \to \infty)$. Calculation was done for even states with zero $\lambda_1 = 0$ and infinite $\lambda_1 \to \infty$ interaction strength between identical bosons.

Even parity states





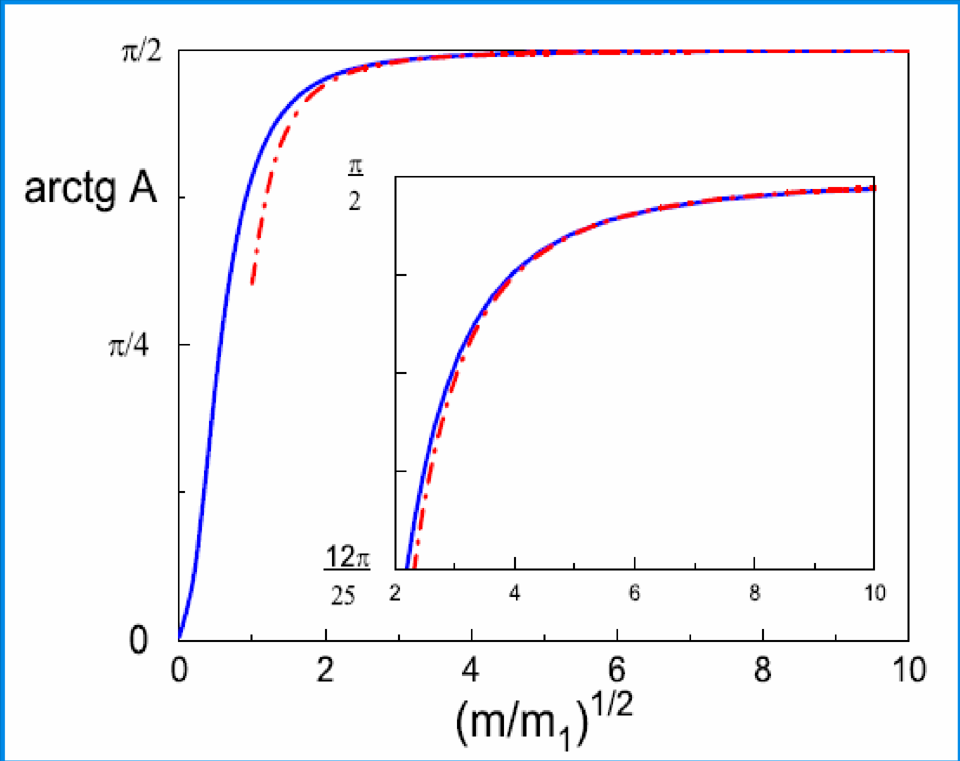
Dependence of the three-body binding energies E_3 (left) and arctg 2 + 1 scattering lengths A (right) of mass-ratio for even parity states. Analytical solutions are marked by circles

Red line: three-body system with two identical bosons and $\lambda_1 = 0$

Blue line: three-body system with two identical bosons and $\lambda_1 \to \infty$

with two identical fermions and $\lambda_1 = 0$

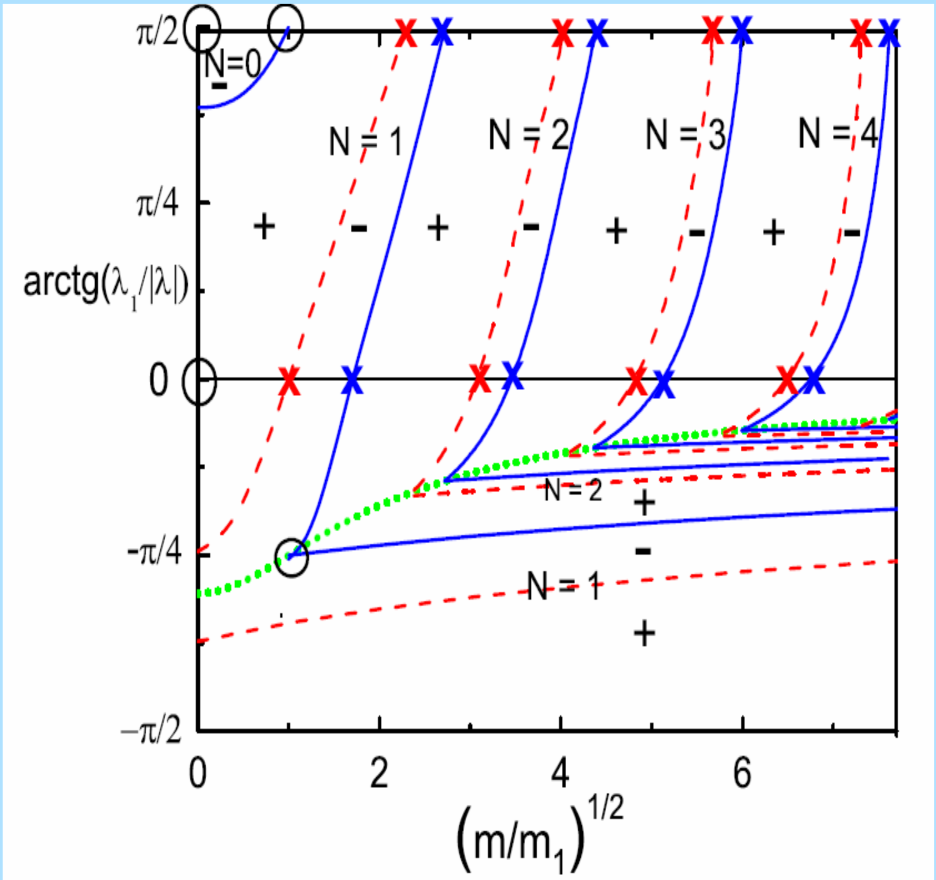
Odd parity states



There is no odd parity bound states for three-body system with two-identical noninteracting bosons ($\lambda_1 = 0$).

Shown are numerical calculation and asymptotic dependence at large m/m_1 of the 2 + 1 scattering lengths A of mass-ratio

Phase diagram (even parity)



Three-body system with two identical bosons. $\lambda<0$ and λ_1 are the strengths of the potential between different and identical particles.

A schematic diagram shows a number of the three-body bound states (marked by **N**) and a sign of the (2+1)-scattering length (marked by **±**) as a function of the mass m/m_1 and interaction-strength $\lambda_1/|\lambda|$ ratios (see figure).

Blue and **red** lines divide the areas with **different numbers of the bound states** and **sign of the 2+1 scattering length**, respectively.

O.I. Kartavtsev, A.V.Malykh S.A. Sofianos "Bound states and scattering of three one-dimensional particles with a contact interaction"‘, JETP 135, 2 (2009)

Peculiarities of nD-dimensional three-body systems at low energy

- 2D
  - there is no Efimov, no Tomas states
  - energy and the number of the bound states increase with increasing $\mathbf{m/m_1}$
  - Fermi statistics: the first trimer appears at $\mathbf{m/m_1} \simeq 3.33$ (p-wave)
  - Bose statistics: bound states appear at any $\mathbf{m/m_1}$

[L. Pricoupenko, P. Pedri, ***Universal three-body bound states in planar atomic wave guides,*** **arXive:0812.3718**] 1

- 3D: Efimov effect $a \to \infty$ (Tomas $R_e \to 0$) –

  infinite number of the bound states

  - requires an additional parameter, which determines the wave function in the vicinity of the triple-collision point

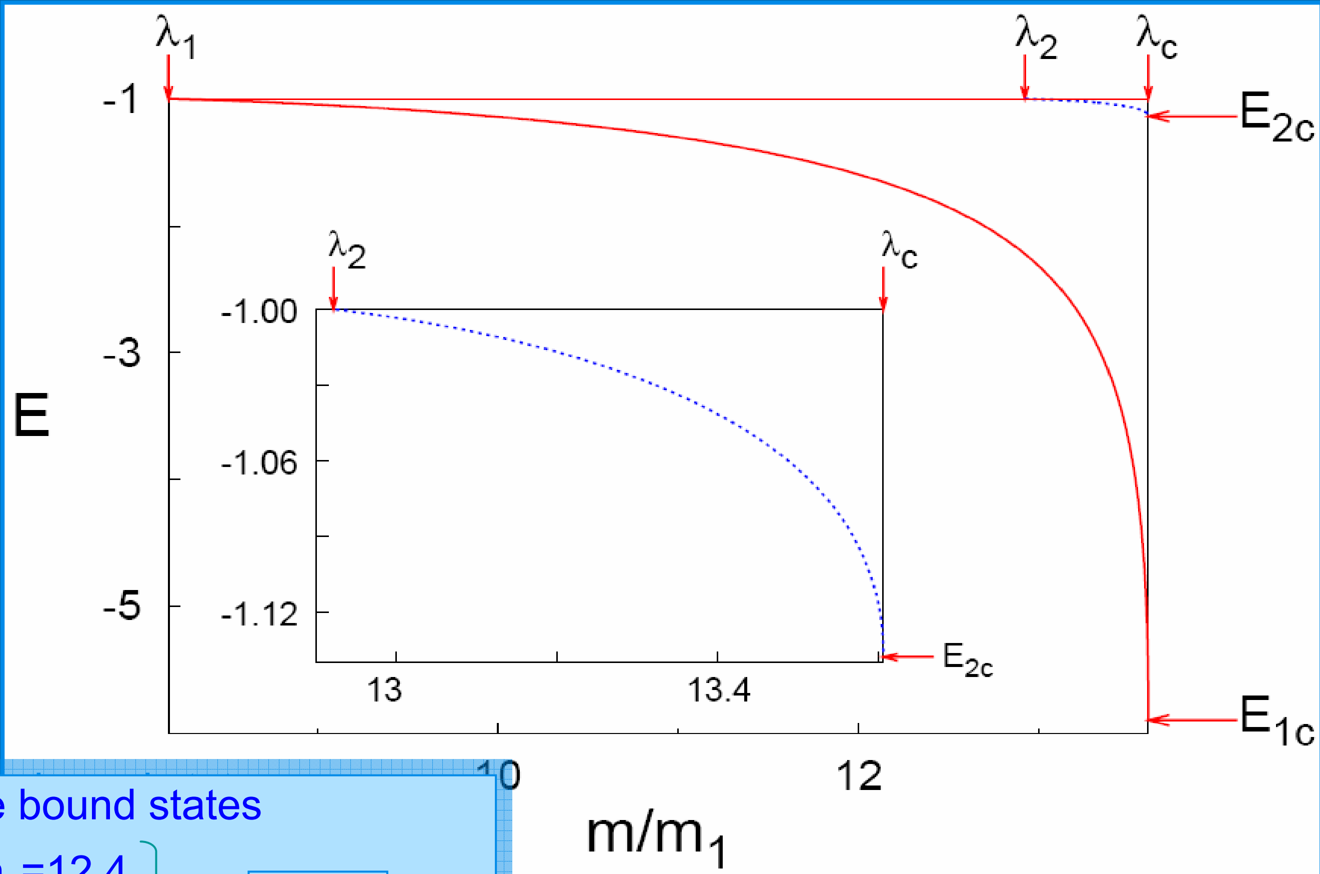
3D: Main properties of the spectrum

- L=0 infinite number of the bound states
- L>0
  - a<0 no bound states
  - Infinite number of the bound state appears at $m/m_1 > (m/m_1)_{CL}$
  - Finite number of the bound states exists for $m/m_1 \leq (m/m_1)_{CL}$
    - L=1,3,5,7,9,… if particles 2 and 3 – fermions
    - L=2,4,6,8,10…if particles 2 and 3 – bosons
  - How many bound states can appear at $m/m_1 \leq (m/m_1)_{CL}$?

$$N_{max} \approx 1.1 \sqrt{L(L+1)} + 1/2$$

L	λ_1	λ_2	λ_3	$(m/m_1)_{cL}$
1	7.930	12.789	-	13.6069657
2	22.34	31.285	37.657	38.6301583
3	42.98	55.766	67.012	75.9944943
4	69.88	86.420	101.92	125.764635
5	103.1	123.31	142.82	187.958355
6	142.5	166.45	189.86	262.582045
7	188.2	215.86	243.11	349.638445
8	240.3	271.56	302.59	449.128836
9	298.6	333.43	368.03	561.053956
10	363.2	401.79	440.35	685.414145
11	434.0	476.34	518.63	822.209692

3D: Mass-ratio dependence of energies L=1



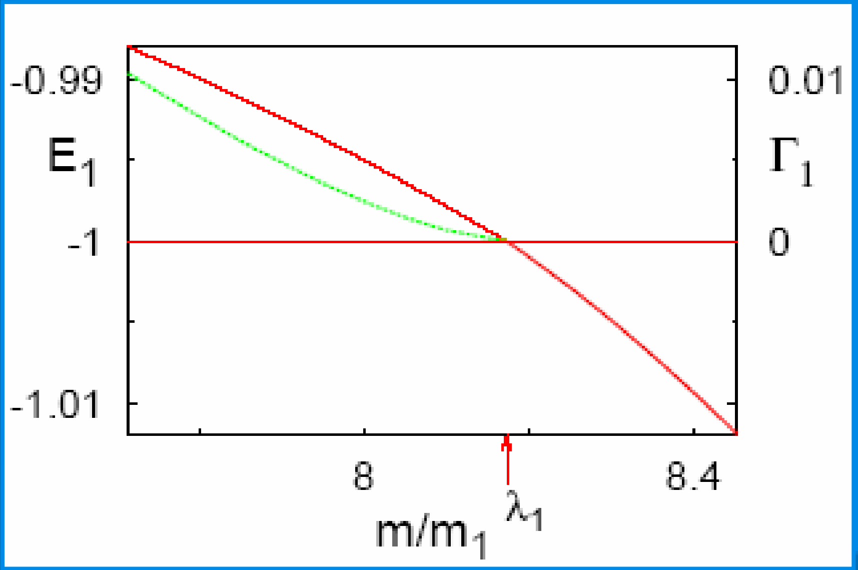
Examples of the bound states

$^{7}Li\ ^{87}Sr_2$: $m/m_1=12.4$
$^{6}Li\ ^{87}Sr_2$: $m/m_1=14.5$
L=1

$^{6}Li\ ^{133}Cs_2$: $m/m_1=22.16$
$^{6}Li\ ^{135}Cs_2$: $m/m_1=22.5$
$^{4}He^{133}Cs_2$: $m/m_1=33.25$
L=2

3D: Below critical values $m/m_1<\lambda_i$ bound states become a resonance

Example: p-wave resonance



$$|E_i+1| \propto m/m_1-\lambda_i$$

$$\Gamma_i \propto (\lambda_i - m/m_1)^2$$

As discussed in [Phys. Rev. Lett. **103**, 153202 (2009)] these resonances can become a bound state. As example, for ^{40}K- ^{6}Li system it is possible to put system into quasi-two-dimensional confinement. (Calculation with solution a integro-differential equation in the momentum space)

Comparison of results for 1D, 2D, 3D

- For fermionic system the first three-body bound state appears at $m/m_1=1$ in 1D, 3.33 in 2D and 8.17260 in 3D
- For bosonic system with noninteracting bosons excited state appears at $m/m_1=2.869539$ in 1D, 1.77 in 2D, Efimov effect in 3D
- Condition $\varepsilon_3<\varepsilon_2$ means stability of ultracold gases of two-atom molecule against appearance of the three-atom molecule
- For fermionic system $\varepsilon_3=\varepsilon_2$ for $m/m_1=49.8335$ in 1D, 18.3 in 2D and 12.69471 in 3D

References

- 3D
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- 1D

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Thank you for attention