# AdS7 solutions and their holographic duals 

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based on
1506.05462 with A. Passias, A. Rota;
1502.06620 with F. Apruzzi, M. Fazzi, A. Passias; 1502.06622 with A. Rota I309.2949 with F. Apruzzi, M. Fazzi, D. Rosa; I4O4.07II with D. Gaiotto [ı407.6359 with M. del Zotto, J.Heckman, C.Vafa]

+ work in progress with S. Cremonesi
- 

运DEGLI STUDI

## Introduction

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- Mothers of interesting theories in $d \leq 4$
[Gaiotto 'o9, Alday,
Gaiotto, Tachikawa '09...]
- Harder to define.
e.g. $\operatorname{Tr}\left(F_{\mu \nu}\right)^{2}$ relevant in $d>4$. Similar problem to $\sqrt{-g} R$ in $d>2$
- They might allow us to get a handle on the elusive ( 2,0 ) theory living on $\mathrm{M}_{5}$-brane stacks
- number of degrees of freedom $\sim N^{3}$


## crucial features:

- 'chiral tensors': $b_{\mu \nu}$ such that $h_{\mu \nu \rho}$ is self-dual
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This talk: Holographic approach

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- Classification of $\mathrm{AdS}_{7}$ solutions in type II sugra
- infinitely many; analytical


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- natural structure: linear quiver
- in string theory, they appear from NS5-D6-D8 brane constructions


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- Match of Weyl anomaly!


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- $\operatorname{AdS}_{7} \times M_{4}$ in 11 d sugra:
cone over $M_{4}$ should have reduced holonomy
$\Rightarrow \quad M_{4}=S^{4} / \Gamma_{\mathrm{ADE}}$
- $\operatorname{AdS}_{7} \times M_{3}$ in type II: 'pure spinor' methods [Apruzzi, Fazzi, Rosa, AT 'ı3] originally applied to $\mathrm{AdS}_{4} \times M_{6}$ in type II
- IIB: no solutions! $\left[\begin{array}{c}\text { this doesn't include } \\ \text { F-theory }\end{array}\right]$
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This $S^{2}$ realizes the $\operatorname{SU}(2)$ R-symmetry of a $(1,0) 6 d$ theory.
$A(r), \phi(r), v(r)$ determined by ODEs
solved at first numerically [Apruzzi, Fazzi, Rosa, AT $\left.{ }^{\prime}{ }^{3}\right]$ ] then analytically with the help of $\mathrm{AdS}_{4}$ and $\mathrm{AdS}_{5}$

- Warm-up: $F_{0}=0$
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local solutions also in [Blåbäck, Danielsson, Junghans, Van Riet, Wrase, Zagermann 'ır〕 susy-breaking? in [Junghans, Schmidt, Zagermann 'I4]

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## we can make <br> one of the poles regular:

$$
d s_{M_{3}}^{2}=\frac{n_{\mathrm{D} 6}}{F_{0}}\left(\frac{d y^{2}}{4 \sqrt{y+2}(1-y)}+\frac{1}{3} \frac{(1-y)(y+2)^{3 / 2}}{8-4 y-y^{2}} d s_{S^{2}}^{2}\right)
$$


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more generally we can have two unequal D6 stacks

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these solutions are also analytic, but a bit more complicated.

metric: gluing of two pieces of earlier metric

## we can also

include D8's:
actually, 'magnetized' D8's
D8-D6 bound states

D8-D6 stack
metric: gluing of two pieces of earlier metric

## intuitively: D8's don't slip off because of electric attraction

stacks with opposite D6 charge
metric: gluing of two pieces of metric in prev. slide

+ central region from two slides ago
and so on...



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D-brane engineering:


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coincident NS;s = strong coupling point; CFT?
the branes can also be arranged differently...

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## Conjecture: near-horizon limit gives our $\mathrm{AdS}_{7}$ solutions


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$$
N=\# \text { NSj's \# D6's ending on a D8 }
$$


flux integer $\int_{M_{3}} H \quad$ D6 charge of the D8

## These theories can be labeled by two Young diagrams

[combinatorics well-known in other dimensions]

## (4)-(8)-(1)-(1)-(1)-(10)-(1)-(8)-(4)



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$$
\begin{gathered}
d s^{2}=8 \sqrt{-\frac{\ddot{\alpha}}{\alpha}} d s_{\mathrm{AdS}_{7}}^{2}+\sqrt{-\frac{\alpha}{\ddot{\alpha}}} d z^{2} \\
+\frac{\alpha^{3 / 2}(-\ddot{\alpha})^{1 / 2}}{\sqrt{2 \alpha \ddot{\alpha}-\dot{\alpha}^{2}}} d s_{S^{2}}^{2}
\end{gathered}
$$



## Some notable examples:


reduction of
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$k$

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- Cancel gauge anomalies [Green,Schwarz,West'86, Sagnotti '92]
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- Compute global $\mathrm{SU}(2)_{\mathrm{R}}$ and gravitational anomaly

[Intriligator 'ı4, Ohmori, Shimizu, Tachikawa, Yonekura 'i4]
[Cordova, Dumitrescu, Intriligator ' 15 § $\left\langle T_{\mu}^{\mu}\right\rangle \sim a$ Euler + Weyl comb.


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$$
(\# \text { gauge groups })^{3}
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This reproduces the famous cubic scaling.

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$$
a=\frac{16}{7} k^{2} N^{3}+\ldots \quad \text { [Ohmori, Shimizu, Tachikawa, Yonekura ’‘4] }
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## Example:

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\begin{aligned}
& r_{i}=k(1,1, \ldots, 1) \\
& \sum_{i, j} C_{i j}^{-1} r_{i} r_{j}=\frac{k^{2}}{12}\left(N^{3}-N\right) \\
& a=\frac{16}{7} k^{2} N^{3}+\ldots \quad \text { "Freudenthal-de Vries } \\
& \text { strange formula" }
\end{aligned}
$$

## It matches with holographic computation:

$$
a=\frac{R_{\mathrm{AdS}}^{5}}{G_{\mathrm{N}, \mathrm{~d} \mathrm{~d}}}
$$

[Henningson, Skenderis '98]
in IIA
[string frame]

$$
a=\frac{3}{56 \pi^{4}} \int_{M_{3}} e^{5 A-2 \phi} \operatorname{vol}_{3}
$$

## Another example:



$$
\sum_{i, j} C_{i j}^{-1} r_{i} r_{j}=\frac{1}{180} N\left(4 N^{2}-1\right)\left(N^{2}-1\right) \sim \frac{1}{45} N^{5}+\ldots
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[because $k=N$ in this case]

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## (1)-(2)-(3) $\cdots-)^{-N}$

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& N \\
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- On the other hand: Cartan is "discrete double derivative"

$$
C=\left(\begin{array}{cccc}
2 & -1 & 0 & \cdots \\
-1 & 2 & -1 & \cdots \\
0 & -1 & 2 & \ddots \\
\vdots & \vdots & \ddots & \ddots
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\ddot{\alpha} \alpha
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## Conclusions \& Extensions

- Classification of type II $\mathrm{AdS}_{7}$ solutions
infinitely many new ones!
- Dual field theories: strong coupling points in linear $\mathrm{U}(k)$ quivers

-There are also extensions involving exceptional gauge groups example:

['fractional M5-branes']
[del Zotto, Heckman, AT, Vafa 'ı4]
- One can also 'compactify'

$$
\text { so } \infty \text { new } \mathrm{CFT}_{4}, \mathrm{CFT}_{3} \ldots
$$

[Apruzzi, Fazzi, Passias, AT '15; Rota, AT '15]
in fact there is a
consistent truncation to 7 d
[Passias, Rota, AT ' ${ }^{5}$ ]

these are also interesting
flux compactifications

Backup Slides

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- $\mu_{i} \quad$ positive and growing for $F_{0}>0$ negative and growing for $F_{0}<0$
$\ldots$ - $_{\ldots}$ - Young diagrams $\rho_{\mathrm{L}}, \rho_{\mathrm{R}}$

$$
\left.\rho_{\mathrm{L}} \square\right\}^{\square} \mu_{1}^{\mathrm{L}}
$$



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$\xrightarrow{-\rightarrow-\rightarrow}$ Young diagrams $\rho_{\mathrm{L}}, \rho_{\mathrm{R}}$
- $N \geq\left|\mu_{1}^{\mathrm{L}}\right|+\left|\mu_{1}^{\mathrm{R}}\right|$
bordering
$F_{0}=0$ region.



## Consistent truncations.

For any $\mathrm{AdS}_{7}$ solution in IIA there is a consistent truncation to 'minimal gauged 7d sugra' [Passias, Rota, AT' ${ }_{55}$ ]

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$$
e^{2 A} d s_{7}^{2}+d r^{2}+\frac{v^{2}}{1+16\left(X^{5}-1\right) v^{2}} e^{2 A} d s_{S^{2}}^{2}
$$

$$
\text { scalar } X \cong \text { an internal 'distortion' }
$$

$$
e^{2 A} d s_{\mathrm{AdS}_{7}}^{2}+d r^{2}+v^{2} d s_{S^{2}}^{2}
$$

7d minimal gauged sugra

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\text { fields: } g_{\mu \nu}^{(7)}, A_{\mu}^{i}, X
$$

## Many solutions that one can lift:

- $\mathrm{AdS}_{5} \times \Sigma_{2}, \mathrm{AdS}_{4} \times \Sigma_{3}$ solutions dual to CFT5's and CFT4's
actually done earlier:
[Apruzzi, Fazzi, Passias, AT '15; Rota, AT 'I5]


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actually done earlier:
[Apruzzi, Fazzi, Passias, AT '15;
Rota, AT 'I5]
- RG flows from $\mathrm{AdS}_{7}$ to $\mathrm{AdS}_{5} \times \Sigma_{2}$ and $\mathrm{AdS}_{4} \times \Sigma_{3}$
- $\mathrm{AdS}_{3}$ to $\mathrm{AdS}_{3} \times \Sigma_{4}$ solutions
- non-susy AdS $_{7}$ solution

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- All is determined by a single function $\beta(y)$

$$
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where $\left(\frac{y^{2} \beta}{\beta^{\prime 2}}\right)^{\prime}=\frac{F_{0}}{72}$
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$$

$-\beta$ has single zero $\Rightarrow$ regular point; double zero $\Rightarrow \mathrm{D} 6$ stack
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$$

[it's easy to solve]
$-\beta$ has single zero $\Rightarrow$ regular point; double zero $\Rightarrow \mathrm{D} 6$ stack

$$
F_{0}=0, \text { two D6 stacks } \beta \propto\left(y^{2}-y_{0}^{2}\right)^{2}
$$

examples:

## If you're curious about the

 analytic expressions:- All is determined by a single function $\beta(y)$ where $\left(\frac{y^{2} \beta}{\beta^{\prime 2}}\right)^{\prime}=\frac{F_{0}}{72}$

$$
d s^{2}=\frac{4}{9} \sqrt{-\frac{\beta^{\prime}}{y}}\left[d s_{\mathrm{AdS}_{7}}^{2}-\frac{1}{16} \frac{\beta^{\prime}}{y \beta} d y^{2}+\frac{\beta / 4}{4 \beta-y \beta^{\prime}} d s_{S^{2}}^{2}\right]
$$

[it's easy to solve]
$-\beta$ has single zero $\Rightarrow$ regular point; double zero $\Rightarrow \mathrm{D} 6$ stack

$$
\begin{array}{ll} 
& F_{0}=0, \text { two D6 stacks } \beta \propto\left(y^{2}-y_{0}^{2}\right)^{2} \\
\text { examples: } & F_{0} \neq 0, \text { one D6 stack } \beta \propto\left(y-y_{0}\right)\left(y+2 y_{0}\right)^{2}
\end{array}
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## If you're curious about the analytic expressions:

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& \\
& F_{0} \neq 0, \text { most general: } \beta \propto(\sqrt{\hat{y}}-6)^{2}\left(\hat{y}+6 \sqrt{\hat{y}}+6 b_{2}-72\right)^{2} \\
& \\
& \\
&
\end{aligned}
$$

## More general CFT6 from F-theory

So far we have seen chains of $S U(N)$ gauge groups

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- F-theory allows to include more general gauge groups
- The D8's should be dual in F-theory to an object called "T-brane"
- First generalization: SO/Sp gauge groups
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known IIA phenomenon: an NS5 can 'fractionate' on an O6

[Evans, Johnson,Shapere '97]
〔Elitzur,Giveon, Kutasov, Tsabar '98]
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In F-theory this is reproduced geometrically:

fibre deg.

- First generalization: SO/Sp gauge groups


## known IIA phenomenon:

an NS5 can 'fractionate' on an O6


In F-theory this is reproduced geometrically:

"blow-up"

fibre deg.

- There is also an analogue for exceptional gauge groups
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Final result: the $\left(E_{8}, E_{8}\right)$ theory
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$\underbrace{}_{2}=\begin{gathered}\text { (wo tensors } \\ \text { (no gauge group) }\end{gathered}$
- There is also an analogue for exceptional gauge groups $\begin{gathered}\text { now we need } \\ \text { several blowups... }\end{gathered} \underbrace{\longrightarrow}_{E_{8}} \rightarrow \cdots$$\rightarrow \cdots$



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Final result: the $\left(E_{8}, E_{8}\right)$ theory tensor multiplets pattern also appeared in [Berhadsky, Johansen '96] [Aspinwall, Morrison '97] [Intriligator'97]...





Final result: the $\left(E_{8}, E_{8}\right)$ theory


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In M-theory:


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In M-theory:


Conjecture: 12 fractional M5's

$\mathbb{R} \times \mathbb{R}^{4} / \Gamma_{E_{8}}$ sing.

Final result: the $\left(E_{8}, E_{8}\right)$ theory


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a 'discrete flux' is created whenever a fractional $\mathrm{M}_{5}$ is crossed

Final result: the $\left(E_{8}, E_{8}\right)$ theory


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Conjecture: i2 fractional M5's

a 'discrete flux' is created whenever a fractional $\mathrm{M}_{5}$ is crossed

