# AdS7 solutions and their holographic duals

#### Alessandro Tomasiello

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based on

1506.05462 with A. Passias, A. Rota; 1502.06620 with F. Apruzzi, M. Fazzi, A. Passias; 1502.06622 with A. Rota 1309.2949 with F. Apruzzi, M. Fazzi, D. Rosa; 1404.0711 with D. Gaiotto [1407.6359 with M. del Zotto, J.Heckman, C.Vafa] + work in progress with S. Cremonesi







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• Mothers of interesting theories in  $d \le 4$ 

[Gaiotto '09, Alday, Gaiotto, Tachikawa '09...]

• Harder to define.

e.g.  $\operatorname{Tr}(F_{\mu\nu})^2$  relevant in d > 4. Similar problem to  $\sqrt{-g}R$  in d > 2

• They might allow us to get a handle on the elusive (2,0) theory living on M5-brane stacks

• number of degrees of freedom  $\sim N^3$ crucial features:

• 'chiral tensors':  $b_{\mu\nu}$  such that  $h_{\mu\nu\rho}$  is self-dual

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This talk: Holographic approach

# Plan

• Classification of AdS<sub>7</sub> solutions in type II sugra

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- Their CFT<sub>6</sub> duals: NS5-D6-D8 brane constructions
  - natural structure: linear quiver
  - in string theory, they appear from NS5-D6-D8 brane constructions



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  - natural structure: linear quiver
  - in string theory, they appear from NS5-D6-D8 brane constructions
- •Match of Weyl anomaly!



### **AdS7 classification**



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 $\Box > M_4 = S^4 / \Gamma_{\rm ADE}$ 

### **AdS7 classification**

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• AdS<sub>7</sub> ×  $M_3$  in type II: 'pure spinor' methods [Apruzzi, Fazzi, Rosa, AT'13] originally applied to AdS<sub>4</sub> ×  $M_6$  in type II [Graña, Minasian, Petrini, AT'05] later extended to any 10d solution in type II [AT'11] we will later see a similar classification for AdS<sub>5</sub> ×  $M_5$  in IIA [Apruzzi, Fazzi, Passias, AT'15]

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[no Ansatz necessary]

of a (1,0) 6d theory.

•IIB: no solutions! this doesn't include F-theory

•IIA: internal  $M_3$  is locally  $S^2$ -fibration over interval

$$\begin{array}{ll} \mbox{[no Ansatz necessary]} & ds^2 \sim e^{2A(r)} ds^2_{AdS_7} + dr^2 + v^2(r) ds^2_{S^2} \\ & & \\ \mbox{Fluxes:} \ F_0, F_2 \sim \mathrm{vol}_{S^2}, H \sim dr \wedge \mathrm{vol}_{S^2} \end{array} \begin{array}{ll} & & \\ \mbox{This } S^2 \ \mathrm{realizes} \\ & & \\ \mbox{the } \mathrm{SU}(2) \ \mathrm{R-symmetry} \\ & & \\ \mbox{of a } (1,0) \ \mathrm{6d \ theory.} \end{array} \end{array}$$

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 $A(r), \phi(r), v(r)$  determined by ODEs

solved at first numerically [Apruzzi, Fazzi, Rosa, AT '13] then analytically with the help of AdS4 and AdS5 [Rota, AT '15] [Apruzzi, Fazzi, Passias, AT '15]



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$$F_0 = 0$$

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we can make one of the poles regular:

> local solutions also in [Blåbäck, Danielsson, Junghans, Van Riet, Wrase, Zagermann '11] susy-breaking? in [Junghans, Schmidt, Zagermann '14]

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#### or also an O6 and a D6 stack



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#### these solutions are also analytic, but a bit more complicated.

we can also include D8's:

actually, 'magnetized' D8's

D8-D6 bound states

D8–D6 stack

metric: gluing of two pieces of earlier metric

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#### actually, 'magnetized' D8's || D8-D6 bound states

metric: gluing of two pieces of earlier metric



intuitively: D8's don't slip off because of electric attraction

stacks with opposite D6 charge

metric: gluing of two pieces of metric in prev. slide + central region from two slides ago and so on...



## Holographic duals



Natural class: linear quivers

At each node,  $n_F = 2n_c$ 



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coincident NS5s = strong coupling point; CFT?





Hanany-Witten brane-creation effect





brane supergravity solution not known, but...



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<u>Conjecture</u>: near-horizon limit gives our AdS7 solutions



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N = # NS5's # D6's ending on a D8 flux integer  $\int_{M_3} H$  D6 charge of the D8  $f_i \qquad f_{i+1}$ 

#### These theories can be labeled by two Young diagrams

s

[combinatorics well-known in other dimensions]





R = 6

L = 5

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Some notable examples:



an orbifold of the (2,0) theory

#### Some notable examples:

N



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1

[Cremonesi, AT, to appear]

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• Cancel gauge anomalies [Green, Schwarz, West'86, Sagnotti '92]

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• Compute global  $SU(2)_R$  and gravitational anomaly conformal anomaly a

[Cordova, Dumitrescu, Intriligator '15]

 $\langle T^{\mu}_{\mu} \rangle \sim a$  Euler+ Weyl comb.

[Cremonesi, AT, to appear]



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#### This reproduces the famous cubic scaling.

$$k k k \cdots k r_i = k(1, 1, \dots, 1)$$

 $\sum_{i,j} C_{ij}^{-1} r_i r_j = \frac{k^2}{12} (N^3 - N)$ 

"Freudenthal-de Vries strange formula"

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#### It matches with holographic computation:

$$a = rac{R_{
m AdS}^5}{G_{
m N,7d}}$$

[Henningson, Skenderis '98]

in IIA [string frame]

k

$$a = \frac{3}{56\pi^{4}} \int_{M_{3}} e^{5A - 2\phi} \operatorname{vol}_{3}$$

### Another example:

 $1 - 2 - 3 - \cdots - N$ 

$$\sum_{i,j} C_{ij}^{-1} r_i r_j = \frac{1}{180} N (4N^2 - 1) (N^2 - 1) \sim \frac{1}{45} N^5 + \dots$$

[because k = N in this case]

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 $a \sim \frac{192}{7} \sum_{i,j} C_{ij}^{-1} r_i r_j + \ldots = \frac{16}{7} k^2 \left( N^3 - 4Nk^2 + \frac{16}{5}k^3 \right) + \ldots$ all large: overall degree 3 in N, k



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Heuristic argument:



R = 6

L = 5



• On the other hand: Cartan is "discrete double derivative" L=5

R = 6

 $C = \begin{pmatrix} 2 & -1 & 0 & \dots \\ -1 & 2 & -1 & \dots \\ 0 & -1 & 2 & \ddots \\ \vdots & \vdots & \ddots & \ddots \end{pmatrix} \qquad C_{ij} = 2\delta_{ij} - \delta_{i-1,j} - \delta_{i+1,j}$ 



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• hence 
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 $\ddot{\alpha} \quad \alpha$
#### We have proven that this always works



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• hence 
$$a \sim \frac{192}{7} \sum r_i C_{ij}^{-1} r_j \longrightarrow \frac{192}{7} \int \ddot{\alpha} \alpha$$

## **Conclusions & Extensions**

• Classification of type II AdS7 solutions

infinitely many new ones!



 $\bullet$  Dual field theories: strong coupling points in linear  $\mathrm{U}(k)$  quivers



#### • There are also extensions involving exceptional gauge groups

example:

 $E_8$  $E_8$ Sp(1)

['fractional M5-branes']

[del Zotto, Heckman, AT, Vafa '14]

One can also 'compactify'

so ∞ new CFT4, CFT3... [Apruzzi, Fazzi, Passias, AT'15; Rota, AT'15]

in fact there is a consistent truncation to 7d

[Passias, Rota, AT'15]





[Apruzzi, Fazzi, Rosa, AT'13; Gaiotto, AT'14]

• numbers  $N_i$  of D8's, and their D6 charges  $\mu_i$ 



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• 
$$\mu_i$$
 positive and growing for  $F_0 > 0$   
negative and growing for  $F_0 < 0$  ----> Young diagrams  $\rho_L$ ,  $\rho_R$ 



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For any AdS7 solution in IIA there is a consistent truncation to 'minimal gauged 7d sugra'

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 $e^{2A}ds_7^2 + dr^2 + \frac{v^2}{1+16(X^5-1)v^2}e^{2A}ds_{S^2}^2$ 





fields:  $g^{(7)}_{\mu
u}, A^i_{\mu}, X$ 

Many solutions that one can lift:

•  $AdS_5 \times \Sigma_2$ ,  $AdS_4 \times \Sigma_3$  solutions

dual to CFT5's and CFT4's

actually done earlier: [Apruzzi, Fazzi, Passias, AT '15; Rota, AT '15] Many solutions that one can lift:

•  $AdS_5 \times \Sigma_2$ ,  $AdS_4 \times \Sigma_3$  solutions dual to CFT5's and CFT4's

actually done earlier: [Apruzzi, Fazzi, Passias, AT'15; Rota, AT'15]

- RG flows from  $AdS_7$  to  $AdS_5 \times \Sigma_2$  and  $AdS_4 \times \Sigma_3$
- $AdS_3$  to  $AdS_3 \times \Sigma_4$  solutions
- non-susy AdS<sub>7</sub> solution

• All is determined by a single function  $\beta(y)$ 

$$ds^{2} = \frac{4}{9}\sqrt{-\frac{\beta'}{y}} \left[ ds^{2}_{\text{AdS}_{7}} - \frac{1}{16} \frac{\beta'}{y\beta} dy^{2} + \frac{\beta/4}{4\beta - y\beta'} ds^{2}_{S^{2}} \right]$$

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$$\left(\frac{y^2\beta}{\beta'^2}\right)' = \frac{F_0}{72}$$

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examples:

 $F_0 \neq 0$ , one D6 stack  $\beta \propto (y - y_0)(y + 2y_0)^2$  $F_0 \neq 0$ , most general:  $\beta \propto (\sqrt{\hat{y}} - 6)^2 (\hat{y} + 6\sqrt{\hat{y}} + 6b_2 - 72)^2$ 

$$\hat{y} \equiv 2b_2 \left(\frac{y}{y_0} - 1\right) + 36$$

So far we have seen chains of SU(N) gauge groups

simplest example:

IIA













So far we have seen chains of SU(N) gauge groups



• F-theory allows to include more general gauge groups

So far we have seen chains of SU(N) gauge groups



- F-theory allows to include more general gauge groups
- The D8's should be dual in F-theory to an object called "T-brane"

[del Zotto, Heckman, AT, Vafa '14]

known IIA phenomenon: an NS5 can 'fractionate' on an O6



[Evans, Johnson, Shapere '97] [Elitzur, Giveon, Kutasov, Tsabar '98]

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• First generalization: SO/Sp gauge groups



SO(2n+8)

 $\operatorname{Sp}(n)$ 

SO(2n+8)

In F-theory this is reproduced geometrically:



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In F-theory this is reproduced geometrically:







now we need several blowups...  $E_8$  $E_8$ this pattern also appeared in [Berhadsky, Johansen '96] [Aspinwall, Morrison '97] Final result: the  $(E_8, E_8)$  theory tensor multiplets [Intriligator'97]...  $G_2$  $G_2$  $F_4$  $E_8$ Sp(1) $E_8$ Sp(1) $\mathbf{2}$  $\mathbf{2}$ 1 

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$$- \bigcirc_{2}^{+} = \underset{\text{(no gauge group)}}{\text{two tensors}}$$









In M-theory:



$$\underbrace{E_8}_{1} \underbrace{\bigcirc}_{2} \underbrace{(\operatorname{Sp}(1))}_{2} \underbrace{G_2}_{1} \underbrace{\bigcirc}_{1} \underbrace{(F_4)}_{1} \underbrace{\bigcirc}_{2} \underbrace{(\operatorname{Sp}(1))}_{2} \underbrace{\bigcirc}_{2} \underbrace{\bigcirc}_{1} \underbrace{E_8}_{1} \underbrace{(F_4)}_{2} \underbrace{(\operatorname{Sp}(1))}_{2} \underbrace{\bigcirc}_{2} \underbrace{(\operatorname{Sp}(1))}_{2} \underbrace{\bigcirc}_{2} \underbrace{(\operatorname{Sp}(1))}_{2} \underbrace{(\operatorname{Sp}(1))}$$



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In M-theory:

M5

Conjecture: 12 fractional M5's

 $\mathbb{R} \times \mathbb{R}^4 / \Gamma_{E_8} \text{ sing.}$ 

a 'discrete flux' is created whenever a fractional M5 is crossed

> for a nice alternative explanation [Ohmori, Shimizu, Tachikawa, Yonekura '15]