Chaotic spin precession in anisotropic Universe

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Main Topics

- Description of spin-gravity interactions: Dirac eqn / EMT matrix elements
- Equivalence principle with spin and its manifestations
- Dirac eq. and FW transformation
- Precession in Bianchi-1 and 9 Universe
- Gravity induced transitions to sterile Dirac neutrinos and dark matter

Equivalence principle

- Newtonian – "Falling elevator" – well known and checked (also for elementary particles)
- Post-Newtonian – gravity action on (quantum!) SPIN – known since 1962 (Kobzarev and Okun'); rederived from conservarion laws - Kobzarev and Zakharov
- Anomalous gravitomagnetic (and electric-CP-odd) moment iz ZERO or
- Classical and QUANTUM rotators behave in the SAME way
- - not checked on purpose but in fact checked in atomic spins experiments at % level (Silenko,OT'07)

Spin-gravity interactions

- 1. Dirac equation
- Gauge structure of gravity manifested; limit of classical gravity - FW transformation
- 2. Matrix elements of Energy- Momentum Tensor
- May be studied in non-gravitational experiments/theory
- Simple interpretation in comparison to EM field case

Gravitational Formfactors

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity

Generalized Parton Diistributions (related to matrix elements of non local operators) – models for both EM and Gravitational Formfactors (Selyugin,OT '09)

- Smaller mass square radius (attraction vs repulsion!?)

$$\rho(b) = \sum_q e_q \int dx q(x,b) \qquad = \int d^2q F_1(Q^2 = q^2) e^{i\vec{q}\,\vec{b}}$$

$$= \int_0^\infty \frac{q dq}{2\pi} J_0(qb) \frac{G_E(q^2) + \tau G_M(q^2)}{1+\tau}$$

$$\rho_0^{\rm Gr}(b) = \frac{1}{2\pi} \int_\infty^0 dq q J_0(qb) A(q^2)$$

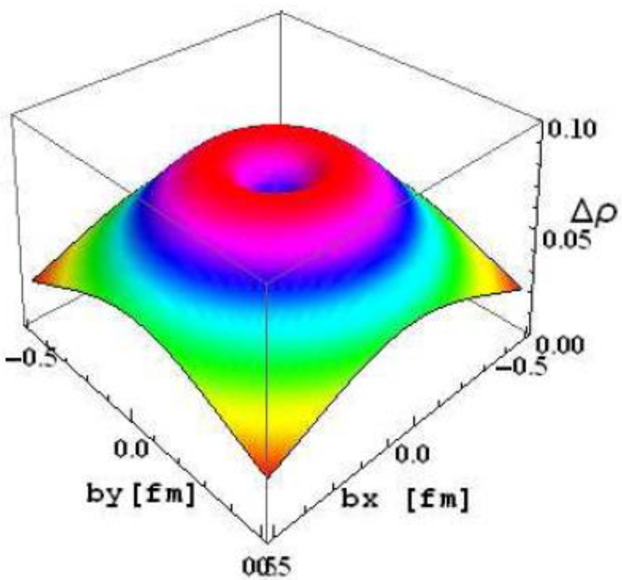

FIG. 17: Difference in the forms of charge density F_1^P and "matter" density (A)

Electromagnetism vs Gravity

- Interaction – field vs metric deviation

$$M = \langle P' | J_q^\mu | P \rangle A_\mu(q)$$

$$M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P | J_q^\mu | P \rangle = 2e_q P^\mu$$

$$\sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle = 2P^\mu P^\nu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P | J_q^\mu | P \rangle A_\mu = 2e_q M \phi(q)$$

$$M_0 = \frac{1}{2} \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle h_{\mu\nu} = 2M \cdot M \phi(q)$$

- Mass as charge – equivalence principle

Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from $M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

$$\vec{H}_J = \frac{1}{2} rot\vec{g};\ \vec{g}_i \equiv g_{0i}$$

spin dragging twice smaller than EM

- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$ Larmor frequency same as EM

$$\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L\ \vec{H}_L = rot\vec{g}$$

- Orbital and Spin momenta dragging – the same - Equivalence principle

Manifestation of equivalence principle (cf with EM)

- Classical and quantum rotators rotate with the same frequency (EM: spin ½ – twice faster)
- Velocity rotates twice faster than classical rotator- helicity changes
(EM – helicity of Dirac fermion conserved – used for AMM measurement) –BUT conserved in the rotating comoving frame

Experimental test of PNEP

- Reinterpretation of the data on G(EDM) search

PHYSICAL REVIEW LETTERS

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Search for a Coupling of the Earth's Gravitational Field to Nuclear Spins in Atomic Mercury

B. J. Venema, P. K. Majumder, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson
Physics Department, FM-15, University of Washington, Seattle, Washington 98195
(Received 25 September 1991)

- If (CP-odd!) GEDM=0 -> constraint for AGM (Silenko, OT'07) from Earth rotation – was considered as obvious (but it is just EP!) background

$$\mathcal{H} = -g\mu_N \boldsymbol{B}\cdot\boldsymbol{S} - \zeta\hbar\boldsymbol{\omega}\cdot\boldsymbol{S}, \quad \zeta = 1+\chi$$

$$|\chi(^{201}\mathrm{Hg}) + 0.369\chi(^{199}\mathrm{Hg})| < 0.042 \quad (95\%\mathrm{C.L.})$$

Equivalence principle for moving particles

- Compare gravity and acceleration: gravity provides EXTRA space components of metrics $h_{zz} = h_{xx} = h_{yy} = h_{00}$
- Matrix elements DIFFER

$$\mathcal{M}_g = (\epsilon^2 + p^2) h_{00}(q), \qquad \mathcal{M}_a = \epsilon^2 h_{00}(q)$$

- Ratio of accelerations: $R = \frac{\epsilon^2 + p^2}{\epsilon^2}$ - confirmed by explicit solution of Dirac equation (Silenko, OT, '05)
- Arbitrary fields – Obukhov, Silenko, OT '09,'11,'13

Dirac equation and Foldy - Wouthausen transformation in arbitrary gravitational field (Obukhov, Silenko, OT, PRD88 (2013) 084814)

- Metric of the type

$$ds^2 = V^2c^2dt^2 - \delta_{\hat{a}\hat{b}}W^{\hat{a}}{}_cW^{\hat{b}}{}_d(dx^c - K^c cdt)(dx^d - K^d cdt).$$

- Tetrads in Schwinger gauge

$$e_i^{\hat{0}} = V\delta_i^0, \qquad e_i^{\hat{a}} = W^{\hat{a}}{}_b(\delta_i^b - cK^b\delta_i^0),$$

$$e_{\hat{0}}^i = \frac{1}{V}(\delta^i{}_0 + \delta^i{}_a cK^a), \qquad e_{\hat{a}}^i = \delta^i{}_b W^b{}_{\hat{a}}, \qquad a = 1, 2, 3,$$

- Dirac eq $(i\hbar\gamma^\alpha D_\alpha - mc)\Psi = 0, \qquad \alpha = 0, 1, 2, 3.$

$$D_\alpha = e_\alpha^i D_i, \qquad D_i = \partial_i + \frac{iq}{\hbar}A_i + \frac{i}{4}\sigma^{\alpha\beta}\Gamma_{i\alpha\beta}.$$

Dirac Hamiltonian

- Connection

$$\Gamma_{i\hat{a}\hat{0}} = \frac{c^2}{V} W^b{}_{\hat{a}} \partial_b V e_i{}^{\hat{0}} - \frac{c}{V} \mathcal{Q}_{(\hat{a}\hat{b})} e_i{}^{\hat{b}},$$

$$\Gamma_{i\hat{a}\hat{b}} = \frac{c}{V} \mathcal{Q}_{[\hat{a}\hat{b}]} e_i{}^{\hat{0}} + (\mathcal{C}_{\hat{a}\hat{b}\hat{c}} + \mathcal{C}_{\hat{a}\hat{c}\hat{b}} + \mathcal{C}_{\hat{c}\hat{b}\hat{a}}) e_i{}^{\hat{c}}.$$

$$\mathcal{Q}_{\hat{a}\hat{b}} = g_{\hat{a}\hat{c}} W^d{}_{\hat{b}} \left(\frac{1}{c} \dot{W}^{\hat{c}}{}_d + K^e \partial_e W^{\hat{c}}{}_d + W^{\hat{c}}{}_e \partial_d K^e \right),$$

$$\mathcal{C}_{\hat{a}\hat{b}}{}^{\hat{c}} = W^d{}_{\hat{a}} W^e{}_{\hat{b}} \partial_{[d} W^{\hat{c}}{}_{e]}, \qquad \mathcal{C}_{\hat{a}\hat{b}\hat{c}} = g_{\hat{c}\hat{d}} \mathcal{C}_{\hat{a}\hat{b}}{}^{\hat{d}}.$$

- Hermitian Hamiltonian $i\hbar \frac{\partial \psi}{\partial t} = \mathcal{H} \psi. \qquad \psi = (\sqrt{-g} e_{\hat{0}}^0)^{\frac{1}{2}} \Psi.$

$$\mathcal{H} = \beta m c^2 V + q\Phi + \frac{c}{2} (\pi_b \mathcal{F}^b{}_a \alpha^a + \alpha^a \mathcal{F}^b{}_a \pi_b) + \frac{c}{2} (\boldsymbol{K} \cdot \boldsymbol{\pi} + \boldsymbol{\pi} \cdot \boldsymbol{K}) + \frac{\hbar c}{4} (\boldsymbol{\Xi} \cdot \boldsymbol{\Sigma} - \Upsilon \gamma_5).$$

$$\Upsilon = V \epsilon^{\hat{a}\hat{b}\hat{c}} \Gamma_{\hat{a}\hat{b}\hat{c}} = -V \epsilon^{\hat{a}\hat{b}\hat{c}} \mathcal{C}_{\hat{a}\hat{b}\hat{c}},$$

$$\Xi_{\hat{a}} = \frac{V}{c} \epsilon_{\hat{a}\hat{b}\hat{c}} \Gamma_{\hat{0}}{}^{\hat{b}\hat{c}} = \epsilon_{\hat{a}\hat{b}\hat{c}} \mathcal{Q}^{\hat{b}\hat{c}}.$$

Foldy-Wouthuysen transformation

- Even and odd parts

$$\mathcal{H} = \beta\mathcal{M} + \mathcal{E} + \mathcal{O}, \qquad \beta\mathcal{M} = \mathcal{M}\beta,$$
$$\beta\mathcal{E} = \mathcal{E}\beta, \qquad \beta\mathcal{O} = -\mathcal{O}\beta.$$

- FW transformation (Silenko '08)

$$U = \frac{\beta\epsilon + \beta\mathcal{M} - \mathcal{O}}{\sqrt{(\beta\epsilon + \beta\mathcal{M} - \mathcal{O})^2}}\beta, \qquad \psi_{\text{FW}} = U\psi, \qquad \mathcal{H}_{\text{FW}} = U\mathcal{H}U^{-1} - i\hbar U\partial_t U^{-1}.$$

$$U^{-1} = \beta\frac{\beta\epsilon + \beta\mathcal{M} - \mathcal{O}}{\sqrt{(\beta\epsilon + \beta\mathcal{M} - \mathcal{O})^2}}. \qquad \epsilon = \sqrt{\mathcal{M}^2 + \mathcal{O}^2}$$

$$\begin{aligned}\mathcal{H}' = \beta\epsilon + \mathcal{E} + \frac{1}{2T}([T,[T,(\beta\epsilon + Z)]] \\ + \beta[\mathcal{O},[\mathcal{O},\mathcal{M}]] - [\mathcal{O},[\mathcal{O},Z]] \\ - [(\epsilon + \mathcal{M}),[(\epsilon + \mathcal{M}),Z]] - [(\epsilon + \mathcal{M}),[\mathcal{M},\mathcal{O}]] \\ - \beta\{\mathcal{O},[(\epsilon + \mathcal{M}),Z]\} + \beta\{(\epsilon + \mathcal{M}),[\mathcal{O},Z]\})\frac{1}{T},\end{aligned}$$

$$T = \sqrt{(\beta\epsilon + \beta\mathcal{M} - \mathcal{O})^2}$$

$$Z = \mathcal{E} - i\hbar\frac{\partial}{\partial t}$$

$$\mathcal{H}' = \beta\epsilon + \mathcal{E}' + \mathcal{O}', \qquad \beta\mathcal{E}' = \mathcal{E}'\beta, \qquad \beta\mathcal{O}' = -\mathcal{O}'\beta.$$

$$\mathcal{H}_{\text{FW}} = \beta\epsilon + \mathcal{E}' + \frac{1}{4}\beta\left\{\mathcal{O}'^2, \frac{1}{\epsilon}\right\}.$$

FW for arbitrary gravitational field

- Result

$$\mathcal{H}_{\rm FW} = \mathcal{H}^{(1)}_{\rm FW} + \mathcal{H}^{(2)}_{\rm FW}.$$

$$\epsilon' = \sqrt{m^2c^4V^2 + \frac{c^2}{4}\delta^{ac}\{p_b, \mathcal{F}^b{}_a\}\{p_d, \mathcal{F}^d{}_c\}},$$
$$\mathcal{T} = 2\epsilon'^2 + \{\epsilon', mc^2V\}.$$

$$\mathcal{M} = mc^2V,$$

$$\mathcal{E} = q\Phi + \frac{c}{2}(\boldsymbol{K}\cdot\boldsymbol{\pi} + \boldsymbol{\pi}\cdot\boldsymbol{K}) + \frac{\hbar c}{4}\boldsymbol{\Xi}\cdot\boldsymbol{\Sigma},$$

$$\mathcal{O} = \frac{c}{2}(\pi_b\mathcal{F}^b{}_a\alpha^a + \alpha^a\mathcal{F}^b{}_a\pi_b) - \frac{\hbar c}{4}\Upsilon\gamma_5.$$

$$\mathcal{H}^{(1)}_{\rm FW} = \beta\epsilon' + \frac{\hbar c^2}{16}\Big\{\frac{1}{\epsilon'}, (2\epsilon^{cae}\Pi_e\{p_b, \mathcal{F}^d{}_c\partial_d\mathcal{F}^b{}_a\} + \Pi^a\{p_b, \mathcal{F}^b{}_a\Upsilon\})\Big\} + \frac{\hbar mc^4}{4}\epsilon^{cae}\Pi_e\Big\{\frac{1}{\mathcal{T}}, \{p_d, \mathcal{F}^d{}_c\mathcal{F}^b{}_a\partial_bV\}\Big\}, \quad ($$

$$\mathcal{H}^{(2)}_{\rm FW} = \frac{c}{2}(K^ap_a + p_aK^a) + \frac{\hbar c}{4}\Sigma_a\Xi^a + \frac{\hbar c^2}{16}\Big\{\frac{1}{\mathcal{T}}, \Big\{\Sigma_a\{p_e, \mathcal{F}^e{}_b\}, \Big\{p_f, \Big[\epsilon^{abc}\Big(\frac{1}{c}\dot{\mathcal{F}}^f{}_c - \mathcal{F}^d{}_c\partial_dK^f + K^d\partial_d\mathcal{F}^f{}_c\Big) - \frac{1}{2}\mathcal{F}^f{}_d(\delta^{db}\Xi^a - \delta^{da}\Xi^b)\Big]\Big\}\Big\}\Big\}, \quad ($$

Operator EOM

- Polarization operator $\mathbf{\Pi} = \beta \mathbf{\Sigma}$

$$\frac{d\mathbf{\Pi}}{dt} = \frac{i}{\hbar}[\mathcal{H}_{\mathrm{FW}}, \mathbf{\Pi}] = \mathbf{\Omega}_{(1)} \times \mathbf{\Sigma} + \mathbf{\Omega}_{(2)} \times \mathbf{\Pi}.$$

- Angular velocities

$$\Omega^a_{(1)} = \frac{mc^4}{2}\left\{\frac{1}{\mathcal{T}}, \{p_e, \epsilon^{abc}\mathcal{F}^e{}_b\mathcal{F}^d{}_c\partial_d V\}\right\} + \frac{c^2}{8}\left\{\frac{1}{\epsilon'}, \{p_e, (2\epsilon^{abc}\mathcal{F}^d{}_b\partial_d\mathcal{F}^e{}_c + \delta^{ab}\mathcal{F}^e{}_b\Upsilon)\}\right\},$$

$$\Omega^a_{(2)} = \frac{\hbar c^2}{8}\left\{\frac{1}{\mathcal{T}}, \left\{\{p_e, \mathcal{F}^e{}_b\}, \left\{p_f, \left[\epsilon^{abc}\left(\frac{1}{c}\dot{\mathcal{F}}^f{}_c - \mathcal{F}^d{}_c\partial_d K^f + K^d\partial_d\mathcal{F}^f{}_c\right) - \frac{1}{2}\mathcal{F}^f{}_d(\delta^{db}\Xi^a - \delta^{da}\Xi^b)\right]\right\}\right\}\right\} + \frac{c}{2}\Xi^a.$$

Semi-classical limit

- Average spin

$$\frac{ds}{dt} = \mathbf{\Omega} \times \mathbf{s} = (\mathbf{\Omega}_{(1)} + \mathbf{\Omega}_{(2)}) \times \mathbf{s},$$

$$\Omega^a_{(1)} = \frac{c^2}{\epsilon'} \mathcal{F}^d{}_c p_d \Big(\frac{1}{2} \Upsilon \delta^{ac} - \epsilon^{aef} V C_{ef}{}^c + \frac{\epsilon'}{\epsilon' + mc^2 V} \epsilon^{abc} W^e{}_{\hat{b}} \partial_e V \Big),$$

$$\Omega^a_{(2)} = \frac{c}{2} \Xi^a - \frac{c^3}{\epsilon'(\epsilon' + mc^2 V)} \epsilon^{abc} Q_{(bd)} \delta^{dn} \mathcal{F}^k{}_n p_k \mathcal{F}^l{}_c p_l,$$

Spin in anisotropic universe (Kamenshchik, OT, 1510.8523)

- Bianchi-1 Universe

$$ds^2 = dt^2 - a^2(t)(dx^1)^2 - b^2(t)(dx^2)^2 - c^2(t)(dx^3)^2.$$

- Particular case $W^{\hat{1}}_1 = a(t),\ W^{\hat{2}}_2 = b(t),\ W^{\hat{3}}_3 = c(t).$

$$W^1_{\hat{1}} = \frac{1}{a(t)},\ W^2_{\hat{2}} = \frac{1}{b(t)},\ W^3_{\hat{3}} = \frac{1}{c(t)}.$$

- No anholonomity $\Upsilon = 0$

$$\Omega^{\hat{1}}_{(2)} = \frac{\gamma}{\gamma+1} v_{\hat{2}} v_{\hat{3}} \left(\frac{\dot{b}}{b} - \frac{\dot{c}}{c}\right).$$

$$Q_{\hat{1}\hat{1}} = -\frac{\dot{a}}{a},\ Q_{\hat{2}\hat{2}} = -\frac{\dot{b}}{b},\ Q_{\hat{3}\hat{3}} = -\frac{\dot{c}}{c}.$$

Kasner solution

- t-dependence

$$a(t) = a_0 t^{p_1},\ b(t) = b_0 t^{p_2},\ c(t) = c_0 t^{p_3},$$

$$p_1 + p_2 + p_3 = 1,\ \ p_1^2 + p_2^2 + p_3^2 = 1.$$

- Euler-type expressions

$$\Omega^{\bar{1}}_{(2)} = \frac{\gamma}{\gamma + 1} v_{\bar{2}} v_{\bar{3}} \left(\frac{p_2 - p_3}{t} \right)$$

Heckmann-Schucking solution

- Dust admixture

$$a(t) = a_0 t^{p_1}(t_0+t)^{\frac{2}{3}-p_1},\ b(t) = b_0 t^{p_2}(t_0+t)^{\frac{2}{3}-p_2},$$
$$c(t) = c_0 t^{p_3}(t_0+t)^{\frac{2}{3}-p_3}.$$

- Modification:

$$\Omega^1_{(2)} = \frac{\gamma}{\gamma+1} v_2 v_3 \frac{(p_2-p_3)t_0}{t(t_0+t)}$$

$$= \frac{\gamma}{\gamma+1} v_2 v_3 \frac{(p_2-p_3)t_0}{t^2}\left(1+o\left(\frac{t_0}{t}\right)\right)$$

Biancki-IX Universe

- Metric $$W^{\hat{b}}{}_{a} = \begin{pmatrix} -a\sin x^3 & a\sin x^1 \cos x^3 & 0 \\ b\cos x^3 & b\sin x^1 \sin x^3 & 0 \\ 0 & c\cos x^1 & c \end{pmatrix} \quad W^{c}{}_{\hat{b}} = \begin{pmatrix} -\frac{1}{a}\sin x^3 & \frac{1}{b}\cos x^3 & 0 \\ \frac{1}{a}\frac{\cos x^3}{\sin x^1} & \frac{1}{b}\frac{\sin x^3}{\sin x^1} & 0 \\ -\frac{1}{a}\frac{\cos x^1 \cos x^3}{\sin x^1} & -\frac{1}{b}\frac{\sin x^3 \cos x^1}{\sin x^1} & \frac{1}{c} \end{pmatrix}$$
- Anholonomity coefficients
- $C^{\hat{3}}_{\hat{1}\hat{2}} = \frac{c}{ab}$ + cyclic permutations
- -> non-zero $$\Upsilon = 2\left(\frac{c}{ab} + \frac{b}{ac} + \frac{a}{bc}\right)$$

$$\Omega^{\hat{1}}_{(1)} = v^{\hat{1}}\left(\frac{c}{ab} + \frac{b}{ac} - \frac{a}{bc}\right)$$

Approach to singularity

- Chaotic oscillations – sequence of Kasner regimes

$$p_1=-\frac{u}{1+u+u^2},\ p_2=\frac{1+u}{1+u+u^2},\ p_3=\frac{u(1+u)}{1+u+u^2}$$

- If Lifshitz-Khalatnıkov parameter u >1 – "epochs"

$$p_1'=p_2(u-1),\ p_2'=p_1(u-1),\ p_3'=p_3(u-1)$$

$$p_1'=p_1\left(\frac{1}{u}\right),\ p_2'=p_3\left(\frac{1}{u}\right),\ p_3'=p_2\left(\frac{1}{u}\right)$$

- If u< 1 – "eras"
- Change of eras – chaotic mapping of [0,1]interrval

$$Tx=\left\{\frac{1}{x}\right\},\quad x_{s+1}=\left\{\frac{1}{x_s}\right\}$$

Angular velocities

- New epoch: u -> -u
- New era – changed sign

$$\Omega_{(2)}^{\hat{1}} = \frac{\gamma}{(\gamma+1)t} v_{\hat{2}} v_{\hat{3}} \cdot \frac{1-u^2}{1+u+u^2},$$
$$\Omega_{(2)}^{\hat{2}} = \frac{\gamma}{(\gamma+1)t} v_{\hat{1}} v_{\hat{3}} \cdot \frac{2u+u^2}{1+u+u^2},$$
$$\Omega_{(2)}^{\hat{3}} = -\frac{\gamma}{(\gamma+1)t} v_{\hat{1}} v_{\hat{2}} \cdot \frac{1+2u}{1+u+u^2}.$$

- Odd velocity

$$\Omega_{(1)}^{\hat{1}} \sim -v^{\hat{1}}(t)^{\left(-1-\frac{2u}{1+u+u^2}\right)},$$
$$\Omega_{(1)}^{\hat{b}} \sim v^{\hat{b}}(t)^{\left(-1-\frac{2u}{1+u+u^2}\right)},\ b = 2, 3.$$

$$\Omega_{(1)}^{\hat{2}} \sim -v^{\hat{2}}(t)^{\left(-1-\frac{2u-2}{1-u+u^2}\right)},$$
$$\Omega_{(1)}^{\hat{a}} \sim v^{\hat{a}}(t)^{\left(-1-\frac{2u-2}{1-u+u^2}\right)},\ a = 1, 3.$$

- New epoch
- New era - preserved

Possible applications

- Anisotropy (c.f. crystals) ~ magnetic field
- Spin precession + equivalence principle = helicity flip (~AMM effect)
- Dirac neutrino – transformed to sterile component
- Decouple much earlier
- Angular velocity ~ 1/t -> amount of decoupled ~ 1
- Possible new candidate for dark matter?!

CONCLUSIONS

- Gravity leads to spin effects controlled related Kobzarev-Okun equivalence principle
- Bianchi universe – spin precession
- Neutrino helicity flip – sterile components possibly contributiong to dark matter

Semi-classical limit

- Average spin precession

$$\frac{d\vec{s}}{dt} = \vec{\Omega} \times \vec{s} = (\vec{\Omega}_{(1)} + \vec{\Omega}_{(2)}) \times \vec{s}.$$

- Angular velocity contributions

$$\Omega^{\hat{a}}_{(1)} = \frac{1}{\varepsilon'} W^{d}_{\hat{c}} p_d \left(\frac{1}{2} \Upsilon \delta^{\hat{a}\hat{c}} - \varepsilon^{\hat{a}\hat{e}\hat{f}} C^{\hat{c}}_{\hat{e}\hat{f}} \right),$$

$$\Omega^{\hat{a}}_{(2)} = \frac{1}{2} \Xi^{\hat{a}} - \frac{1}{\varepsilon'(\varepsilon' + m)} \varepsilon^{\hat{a}\hat{b}\hat{c}} Q_{(\hat{b}\hat{d})} \delta^{\hat{d}\hat{n}} W^{k}_{\hat{n}} p_k W^{l}_{\hat{c}} p_l.$$

Torsion – acts only on spin

Dirac eq+FW transformation-Obukhov,Silenko,OT, in preparation

- Hermitian Dirac Hamiltonian

$$e_i^{\widehat{0}} = V\,\delta_i^0, \qquad e_i^{\widehat{a}} = W^{\widehat{a}}{}_b\left(\delta_i^b - cK^b\,\delta_i^0\right)$$

$$ds^2 = V^2c^2dt^2 - \delta_{\widehat{a}\widehat{b}}W^{\widehat{a}}{}_cW^{\widehat{b}}{}_d\,(dx^c - K^c cdt)\,(dx^d - K^d cdt)$$

$$\mathcal{H} = \beta mc^2V + q\Phi + \frac{c}{2}\left(\pi_b\,\mathcal{F}^b{}_a\alpha^a + \alpha^a\mathcal{F}^b{}_a\pi_b\right) + \frac{c}{2}\left(\boldsymbol{K}\cdot\boldsymbol{\pi} + \boldsymbol{\pi}\cdot\boldsymbol{K}\right) + \frac{\hbar c}{4}\left(\boldsymbol{\Xi}\cdot\boldsymbol{\Sigma} - \Upsilon\gamma_5\right),$$

$$\mathcal{F}^b{}_a = VW^b{}_{\widehat{a}}, \qquad \Upsilon = V\epsilon^{\widehat{a}\widehat{b}\widehat{c}}\Gamma_{\widehat{a}\widehat{b}\widehat{c}}, \qquad \Xi^a = \frac{V}{c}\epsilon^{\widehat{a}\widehat{b}\widehat{c}}\left(\Gamma_{\widehat{0}\widehat{b}\widehat{c}} + \Gamma_{\widehat{b}\widehat{c}\widehat{0}} + \Gamma_{\widehat{c}\widehat{0}\widehat{b}}\right)$$

- Spin-torsion coupling

$$-\frac{\hbar cV}{4}\left(\boldsymbol{\Sigma}\cdot\check{\boldsymbol{T}} + c\gamma_5\check{T}^{\hat{0}}\right)$$

$$\check{T}^\alpha = -\frac{1}{2}\eta^{\alpha\mu\nu\lambda}T_{\mu\nu\lambda}$$

- FW – semiclassical limit - precession

$$\Omega^{(T)} = -\frac{c}{2}\check{\boldsymbol{T}} + \beta\frac{c^3}{8}\left\{\frac{1}{\epsilon'},\left\{\boldsymbol{p},\check{T}^{\hat{0}}\right\}\right\} + \frac{c}{8}\left\{\frac{c^2}{\epsilon'(\epsilon'+mc^2)},\left(\{\boldsymbol{p}^2,\check{\boldsymbol{T}}\} - \{\boldsymbol{p},(\boldsymbol{p}\cdot\check{\boldsymbol{T}})\}\right)\right\}$$

Experimental bounds for torsion

- Magnetic field+rotation+torsion

$$H = -g_N \frac{\mu_N}{\hbar} \boldsymbol{B} \cdot \boldsymbol{s} - \boldsymbol{\omega} \cdot \boldsymbol{s} - \frac{c}{2} \check{\boldsymbol{T}} \cdot \boldsymbol{s}$$

- Same '92 EDM experiment

$$\frac{\hbar c}{4} |\check{\boldsymbol{T}}| \cdot |\cos\Theta| < 2.2 \times 10^{-21}\,\mathrm{eV}, \qquad |\check{\boldsymbol{T}}| \cdot |\cos\Theta| < 4.3 \times 10^{-14}\,\mathrm{m}^{-1}$$

- New(based on Gemmel et al '10)

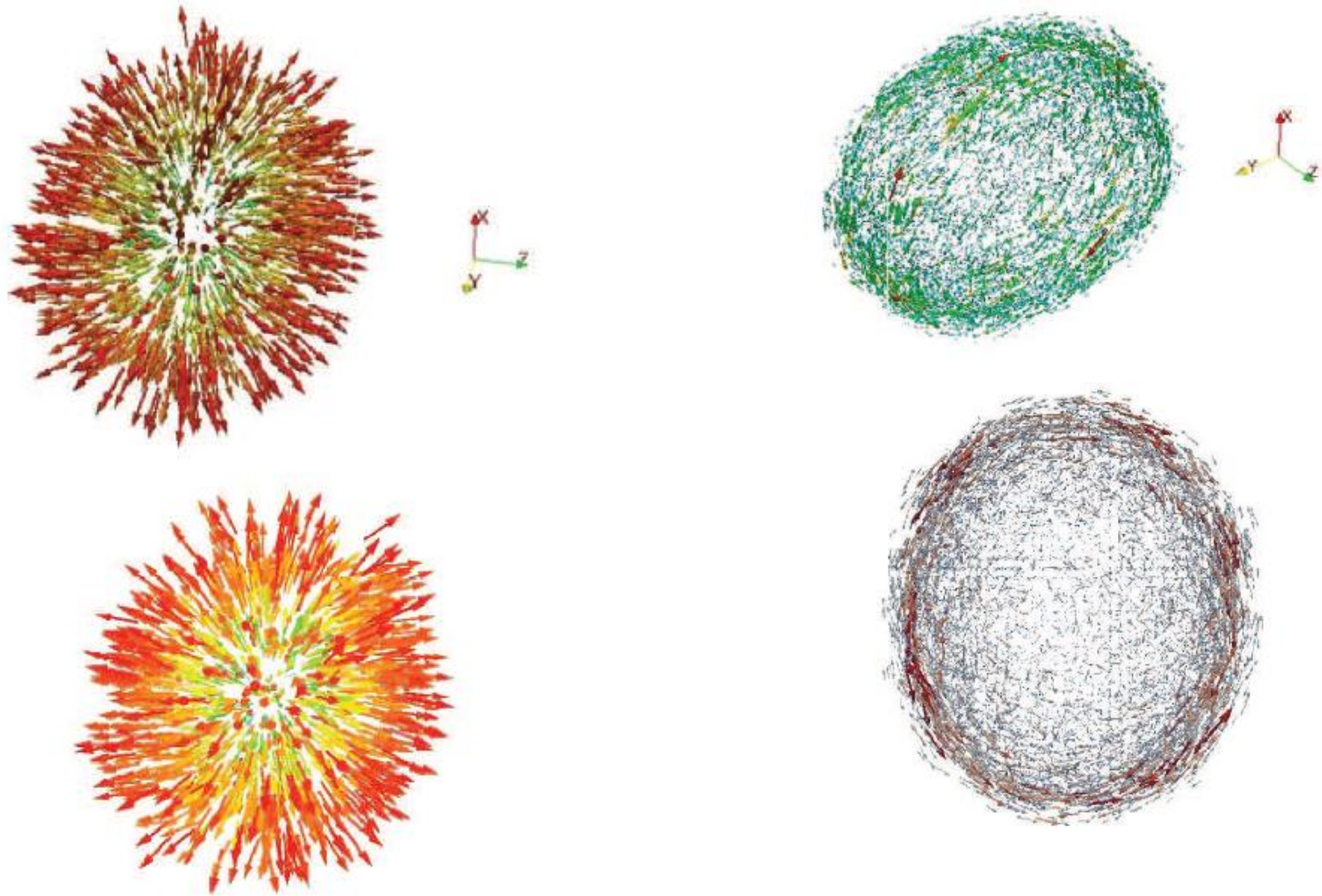
$$\frac{\hbar c}{2} |\check{\boldsymbol{T}}| \cdot |(1 - \mathcal{G}) \cos\Theta| < 4.1 \times 10^{-22}\,\mathrm{eV}, \qquad |\check{\boldsymbol{T}}| \cdot |\cos\Theta| < 2.4 \times 10^{-15}\,\mathrm{m}^{-1}$$

$$\mathcal{G} = g_{He}/g_{Xe}$$

Microworld: where is the fastest possible rotation?

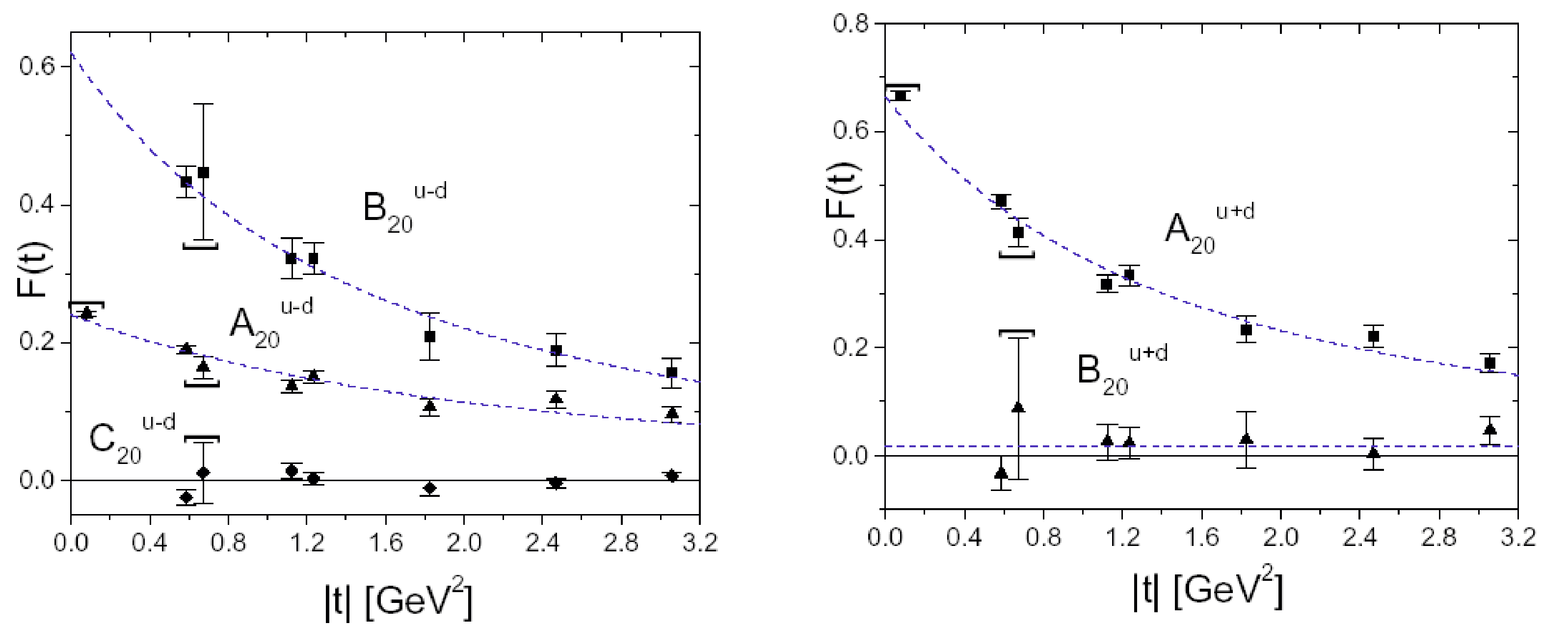
- Non-central heavy ion collisions (~c/Compton wavelength) – “small Bang”
- Differential rotation – vorticity
- Calculation in quark - gluon string model (Baznat,Gudima,Sorin,OT,PRC’13)

Structure of velocity and vorticity fields (NICA@JINR-5 GeV/c)



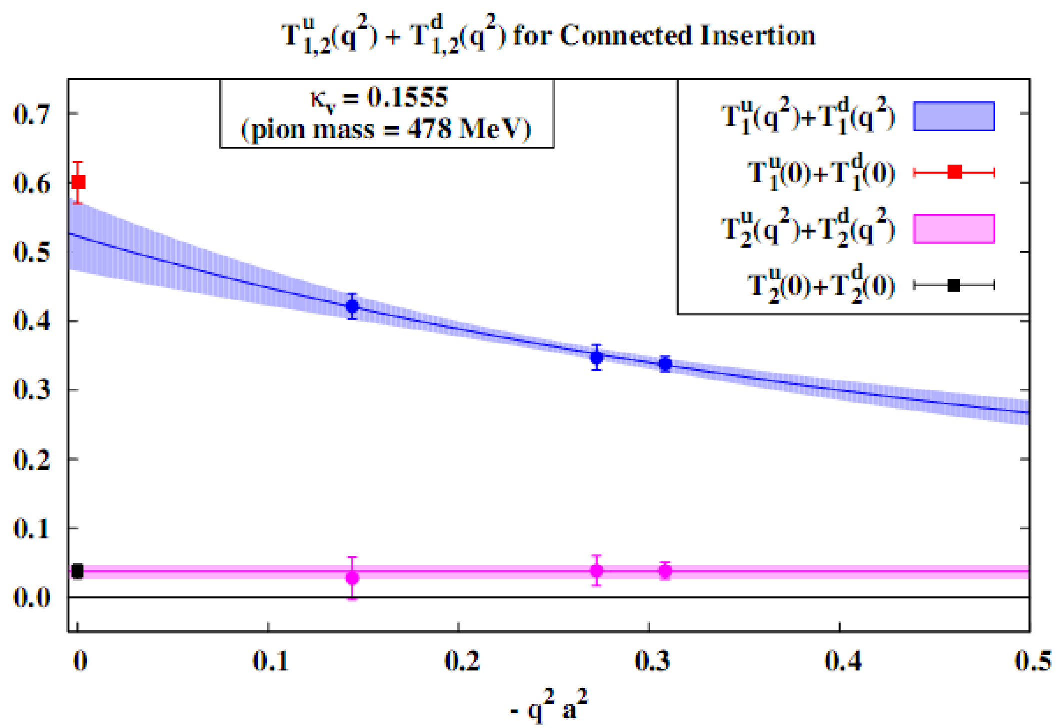
Generalization of Equivalence principle

- Various arguments: AGM ≈ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)



Recent lattice study (M. Deka et al. arXiv:1312.4816; cf plenary talk of K.F. Liu)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs



Extended Equivalence Principle=Exact EquiPartition

- In pQCD – violated
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Supported by generic smallness of E (isoscalar AMM)

Sum rules for EMT (and OAM)

- First (seminal) example: X. Ji’s sum rule (’96). Gravity counterpart – OT’99
- Burkardt sum rule – looks similar: can it be derived from EMT?
- Yes, if provide correct prescription to gluonic pole (OT’14)

Pole prescription and Burkardt SR

- Pole prescription (dynamics!) provides ("T-odd") symmetric part!
  $$\sum \int \int dx_1 dx_2 \frac{T(x_1,x_2)}{x_1 - x_2 + i\varepsilon} = 0$$
- SR: $\sum \int dx T(x,x) = 0$ (but relation of gluon Sivers to twist 3 still not founs – prediction!)
- Can it be valid separately for each quark flavour: nodes (related to "sign problem")?
- Valid if structures forbidden for TOTAL EMT do not appear for each flavour
- Structure contains besides S gauge vector n: If GI separation of EMT – forbidden: SR valid separately!

Another manifestation of post-Newtonian (E)EP for spin 1 hadrons

- Tensor polarization - coupling of gravity to spin in forward matrix elements - inclusive processes

- Second moments of tensor distributions should sum to zero

$$\langle P,S|\bar{\psi}(0)\gamma^{\nu}D^{\nu_1}...D^{\nu_n}\psi(0)|P,S\rangle_{\mu^2} = i^{-n}M^2S^{\nu\nu_1}P^{\nu_2}...P\nu_n\int_0^1 C_q^T(x)x^n dx$$

$$\sum_q \langle P,S|T_i^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^{\mu}P^{\nu}(1-\delta(\mu^2)) + 2M^2S^{\mu\nu}\delta_1(\mu^2)$$

$$\langle P,S|T_g^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^{\mu}P^{\nu}\delta(\mu^2) - 2M^2S^{\mu\nu}\delta_1(\mu^2)$$

$$\sum_q \int_0^1 C_i^T(x)x dx = \delta_1(\mu^2)$$ =0 for ExEP

HERMES – data on tensor spin structure function

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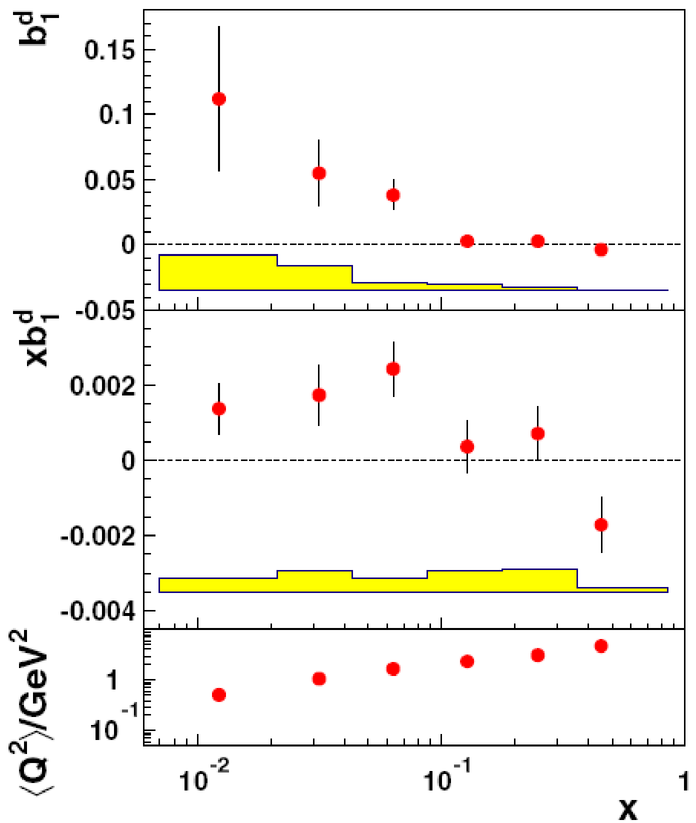
- Isoscalar target – proportional to the sum of u and d quarks – combination required by EEP
- Second moments – compatible to zero better than the first one (collective glue << sea) – for valence:

$$\int_0^1 C_i^T(x)dx = 0$$



Are more accurate data possible?

- HERMES – unlikely
- JLab may provide information about collective sea and glue in deuteron and indirect new test of Equivalence Principle

CONCLUSIONS

- Spin-gravity interactions may be probed directly in gravitational (inertial) experiments and indirectly – studing EMT matrix element
- Torsion and EP are tested in EDM experiments
- SR's for deuteron tensor polarization- indirectly probe EP and its extension separately for quarks and gluons

- BACKUP SLIDES

EEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides g=2 identically!
- Experimental test at time –like region possible