Round Table Italy-Russia@Dubna-2015

A fresh look at general isotropic scalaron cosmology: exact and asymptotic solution, inflationary perturbation theory

A.T.Filippov, JINR, Dubna

our aim is to spell out the mathematical structure

mostly using *differentiable maps* $\psi(\alpha)$ *and* $\alpha(\psi)$

which we call **PORTRAITS of COSMOLOGY** EU-Italy @ Dubna Round Table, 2014

Fundamental functions are: $\chi(\alpha) \equiv d\psi/d\alpha$ and $\bar{\chi}(\psi) \equiv d\alpha/d\psi$

Gauge invariant *equation for* $\chi^2(\alpha)$ *can be explicitly solved*

if we formally replace $v(\psi)$ *by* $\bar{v}(\alpha) \equiv v[\psi(\alpha)]$

We also discuss approaches in which the portraits are more fundamental than potentials

$\bar{\chi}(\psi)$ and $\chi(\alpha)$ satisfy

In more detail:

> the first-order differential equations depending only on the logarithmic derivative of the potential, $l(\psi) \equiv v'(\psi)/v(\psi)$. Once we know a general analytic solution for one of these χ-functions, we can explicitly derive all characteristics of the cosmological model.

In the α-version, the *whole dynamical system is integrable* for $k \neq 0$ and with any 'α-potential', $\bar{v}(\alpha) \equiv v[\psi(\alpha)]$, replacing $v(\psi)$. There is no *a priori* relation between the two potentials before deriving $\chi(\alpha)$ or $\bar{\chi}(\psi)$, which implicitly depend on the potential itself, but relations between the two pictures can be found by asymptotic expansions or by *inflationary perturbation theory*. We also consider alternative proposals – to specify a particular cosmology by guessing one of its portraits and then finding (reconstructing) the corresponding potential from the solutions of the dynamical equations.

The main subject of this paper is the mathematical structure of isotropic cosmologies, but some explicit applications of the results to a more rigorous treatment of the *chaotic inflation* models and to their comparison with the *ekpyrotic-bouncing* ones are outlined in the frame of our 'α-formulation' of isotropic scalaron cosmologies. In particular, we establish an *inflationary perturbation expansion* for $\chi(\alpha)$. When all the conditions for inflation are satisfied, which are: $v > 0$, $k = 0$, $\chi^2(\alpha) < 6$, and $\chi(\alpha)$ obeys a certain boundary (initial) condition at $\alpha \to -\infty$, the expansion is invariant under scaling of v and its first terms give the standard *inflationary parameters*, with higher-order corrections. When $v < 0$ and $6\bar{\chi}^2 < 1$ our general approach can be applied to studies of more complex *ekpyrotic* solutions alternative to inflationary ones.

The talk is mostly based on

ATF: arXiv:1506.01664 v.3

The general gauge invarinace and g.f. were introduced and discussed in the work on `discrete strings' (`86-`96)...
gauging (super)canon. symm

Generalization of Emden-Fowler eqn.: $\ddot{\psi} + 3\,\dot{\alpha}\dot{\psi} + v'(\psi)/2 = 0 \qquad v = a\psi^2 - b\psi^p$.

Exact and approxim. solutions `03-`10
N-Liouville and Toda systems (ATF, VdA)
BH-Cosm-Waves

quite NEW `14-`15

v.3 only !

A few remarks about non-isotropic and curved universes:
weak anisotropy (scalaron), and essential anisotropy (vecton). To be published , hopefully, soon.

SEPARATE and REDUCE !

spherically reduced metric $$e^{2\alpha}dr^2 + e^{2\beta}d\Omega^2(\theta,\phi) - e^{2\gamma}dt^2 + 2e^{2\delta}drdt$$

Lagrangian $$e^{-\alpha+2\beta+\gamma}(2\beta'^2 + 4\beta'\gamma') - e^{\alpha+2\beta-\gamma}(2\dot\beta^2 + 4\dot\beta\dot\alpha) + 2ke^{\alpha+\gamma}$$

$$-\dot\beta' - \dot\beta\beta' + \dot\alpha\beta' + \dot\beta\gamma' \;=\; \frac{1}{2}\,\dot\psi\psi'$$ momentum constraint

$$\alpha = \alpha_0(t) + \alpha_1(r), \quad \beta = \beta_0(t) + \beta_1(r), \quad \gamma = \gamma_0(t) + \gamma_1(r)$$ separation of ***r*** and ***t***

isotropic solution isotropy condition **3**-dimensional curvature

$$\dot\alpha = \dot\beta\,, \quad \gamma' = 0\,, \quad \beta_1'' + \bar k\, e^{-2\beta_1} = 0\,, \quad 2\beta_1'' + 3{\beta_1'}^2 - \bar k\, e^{-2\beta_1} \;=\; 3k$$

$${\beta_1'}^2 - \bar k\, e^{-2\beta_1} \;=\; k$$ homogeneity + isotropy condition

homogeneous isotropic cosmology with ***scalaron***

$$\mathcal{L}^{(1)} = 6\bar k e^{\alpha+\gamma} - e^{2\beta}[e^{\alpha+\gamma}(V + 2\Lambda) - e^{\alpha-\gamma}(2\dot\beta^2 + 4\dot\beta\dot\alpha - \dot\psi^2)]$$

effective cosmological Lagrangian

$$\mathcal{L}^{(2)} = e^{3\alpha-\gamma}(\dot{\psi}^2 - 6\dot{\alpha}^2) - e^{3\alpha+\gamma}v(\psi) - 6ke^{\alpha+\gamma}$$

$\gamma + c\,\alpha = 0.$ $\quad \mathcal{L}_c = e^{(3+c)\alpha}(\dot{\psi}^2 - 6\dot{\alpha}^2) - e^{(3-c)\alpha}v(\psi) - 6ke^{(1-c)\alpha}$

$$\mathcal{H}_c \equiv \eta^2 - 6\,\xi^2 + e^{-2c\alpha}\,v(\psi) + 6k\,e^{-2(1+c)\alpha} = 0\,;$$

momentum - like variables $\eta\,,\xi$ $\qquad \dot{\psi} = \eta\,, \quad \dot{\alpha} = \xi$

$$2\,\dot{\eta} + 2(3+c)\,\eta\,\xi + e^{-2c\alpha}\,v'(\psi) = 0$$

$$6\,\dot{\xi} + (3+c)\,\eta^2 + c\,e^{-2c\alpha}\,v(\psi) + (1+c)\,6k\,e^{-2(1+c)\alpha} = 0$$

Equation for the **Hubble parameter** $H(\alpha) \equiv \bar{\xi}(\alpha)$

$$2\,\dot{\xi} + \eta^2 + 2c\,\xi^2 + 2ke^{-2(1+c)\,\alpha} = 0$$

Independence of *z* and potential

$$\frac{dz}{d\alpha} + \bar{\eta}^2 = 0\,; \qquad z \equiv \bar{\xi}^2 - ke^{-2}$$

***t*-reparametrizing**

$$d\tau \equiv e^{-c\alpha}\,dt\,, \qquad d/dt \equiv e^{-c\alpha}\,d/d\tau$$

Corresponding gauge transformations, invariance

$$\eta \equiv e^{-c\alpha}\,\bar{\eta}\,, \qquad \xi \equiv e^{-c\alpha}\,\bar{\xi}$$

$$d\psi/d\tau = \bar{\eta}\,, \qquad 2\,d\bar{\eta}/d\tau + 6\,\bar{\eta}\,\bar{\xi} + v'(\psi) = 0\,,$$

$$d\alpha/d\tau = \bar{\xi}\,, \qquad 2\,d\bar{\xi}/d\tau + \bar{\eta}^2 + 2k\,e^{-2\alpha} = 0\,.$$

$$\bar{\mathcal{H}} \equiv e^{2c\alpha}\mathcal{H}_c = \bar{\eta}^2 - 6\,\bar{\xi}^2 + v(\psi) + 6k\,e^{-2\alpha} = 0\,.$$

Simple examples from upside down standpoint

$$v = v_0 \equiv 2\Lambda \qquad \eta = \eta_0 \exp[-(3+c)\alpha] \qquad t_c = \int d\alpha/\dot{\alpha}(\alpha)$$

$$6\dot{\alpha}^2 e^{2c\alpha} = v_0 + 6k\, e^{-2\alpha} + \eta_0^2\, e^{-6\alpha}$$

$$\text{when } c = -3 \qquad \psi = \eta_0\,(t - t_0)$$

$$\eta_0^2\, e^{-6\alpha} = v_0\, \sinh^2[\sqrt{3/2}\,\eta_0\,(t - t_0)]$$

Portrait: $= v_0\, \sinh^2(\sqrt{3/2}\,\psi)$

***K=0* exponential case, *c=-3*:** $v = v_0 e^{g\psi} \qquad \psi + g\alpha = C_0(t - t_0)$

$$2\,\ddot{\psi} + e^{6\alpha}\, v'(\psi) = 0\,, \qquad 2\,\ddot{\alpha} - e^{6\alpha}\, v(\psi) = 0$$

$$e^{-(g\psi + 6\,\alpha)} = 2\bar{g}C_1^{-2}\cosh^2[\psi + g\alpha + C_0(t_0 - t_1)/2]$$

Integrable bi--Liouville

$$v \;=\; v_1\, e^{g_1 \psi} \;+\; v_2\, e^{g_2 \psi}$$

$$6\alpha + g_i \psi = \mu_i\, \psi_i \qquad\qquad g_2 = 6/g_1$$

$$\mathcal{L}_c = -\dot{\psi}_1^2 + \dot{\psi}_2^2 + v_1\, e^{\mu_1 \psi_1} + v_2\, e^{\mu_2 \psi_2}$$

How to derive the potential in simple cases?

$$2\,\dot{\xi} + \eta^2 + 2ke^{-2\alpha} = 0\,, \quad 2\,\dot{\eta} + 6\,\eta\,\xi + v'(\psi) = 0\,,$$

$$v(\psi) = 6\,\xi^2 - \eta^2 - 6\,k\,e^{-2\alpha}\,.$$

The first eqn. can be solved if $\quad 1.\ \dot{\xi} = C_0 \text{ or } 2.\ \dot{\eta} = C_0.$

When $C_0 = 0$ $\qquad \eta = k_0\, e^{-\alpha(\tau)}\,, \quad (k_0^2 \equiv -2k)$

Hubble function $\qquad \dot{\alpha}(\tau) \equiv \xi = \xi_0\,, \quad \alpha(\tau) = \xi_0\,(\tau - \tau_0)\,,$

$$\chi(\alpha) = \frac{d\psi}{d\alpha} = \frac{\eta}{\xi} = \frac{k_0}{\xi_0}\, e^{-\alpha}\,; \qquad \chi:\ \alpha \mapsto \psi,$$

portrait $\qquad \tilde{\psi} \equiv (\psi - \psi_0) = \int \chi(\alpha) = -\frac{k_0}{\xi_0}\, e^{-\alpha}$

$$\bar{v}(\alpha) \ = \ 6\,\xi_0^2 \ + \ 2\,k_0^2\, e^{-2\alpha} \ = \ \bar{v}\,[\alpha(\psi)]$$

potential $\qquad = \ 6\,\xi_0^2 \ + \ 2\,\xi_0^2\,(\psi - \psi_0)^2 = v(\psi)\,.$

Gauge-invariant equations for differentials of the map

Definiitions

$$\chi(\alpha) \equiv d\psi/d\alpha \equiv \dot{\psi}/\dot{\alpha} \equiv \eta/\xi\,, \quad \bar{\chi}(\psi) \equiv d\alpha/d\psi \equiv \dot{\alpha}/\dot{\psi} \equiv \xi/\eta$$

$$\frac{d}{dt} = \dot{\alpha}\frac{d}{d\alpha} = \xi\frac{d}{d\alpha} = \dot{\psi}\frac{d}{d\psi} = \eta\frac{d}{d\psi} \qquad \frac{d}{d\psi} = \frac{d\alpha}{d\psi}\frac{d}{d\alpha} = \bar{\chi}(\psi)\frac{d}{d\alpha}$$

$$2\frac{d\chi}{d\alpha} \equiv \xi^{-3}(\xi\dot{\eta} - \dot{\xi}\eta) = (\chi^2 - 6)\left(\chi + [\,\ln v(\psi)]'\right) +$$

$$+\, 2k\,\xi^{-2}e^{-2(1+c)\alpha}\left(\chi + 3\,[\,\ln v(\psi)]'\right)$$

equation for $\chi(\alpha)$ $\qquad \bar{v}(\alpha) \equiv v[\psi(\alpha)] \qquad \bar{l}'(\alpha) \equiv [\,\ln \bar{v}(\alpha)]'$

$$\frac{d\chi^2}{d\alpha} = (\chi^2 - 6)\left(\chi^2 + \bar{l}'(\alpha)\right) + \frac{2k}{\xi^2(\alpha)}\,e^{-2(1+c)\alpha}\left(\chi^2 + 3\,\bar{l}'(\alpha)\right)$$

Generally not integrable

$$2\,\frac{d\bar{\chi}}{d\psi} = (6\,\bar{\chi}^2 - 1)\,(1 + \,\bar{\chi}\,l'(\psi))\,;$$

$$x = \sqrt{2/3}\,\psi,\;\; z(x) = \sqrt{6}\,\bar{\chi}.$$

$$dz/dx = (z^2 - 1)(z\,u(x) + 1) \qquad u(x) \equiv d\ln\sqrt{v}/dx$$

Asymptotic at large *x*

$$\bar{\chi}(\psi)/\sqrt{6} \;\equiv\; z(x) = -[\,1 - e^{-2\,(x+c_0)}\,v(x)\,]\,[\,1 + e^{-2\,(x+c_0)}\,v(x)\,]^{-1} \;\; +$$

$$u(x) = v_0(\,\cosh^{\varepsilon} a\bar{x}\,)^{2a\,(1-a)}\,, \quad \varepsilon = \pm 1$$

Integrable if $a = 1/3$

$$z = y + \,z_a = y + (a-1)/u\,.$$

New variable *y(x)*

if $a = 1/3$ y^2 term vanishes: $y' = u\,y^3 - (u + 1/3u)\,y$

$$w \equiv y^{-2}: \qquad w'(x) = 2(u + 1/3u)\,w - 2u\,.$$

$$z(x) = w^{-1/2} + \,\coth(2x/3)$$

General solutions

important **Ansatz**

$$z(x) = -\frac{1-\varepsilon e^{-2y}}{1+\varepsilon e^{-2y}} = -\tanh^{\varepsilon} y(x)$$

$$dy/dx = 1 - u(x)\,\tanh^{\varepsilon}(y)$$

main equation in ***psi*** **version**

Logarithmic potential

$$u(x) = \tilde{v}'(x)/2\,\tilde{v}(x)$$

'inflation potential' $v(\psi) = \psi^2$

Asymptotic portrait **(example)**

$$\sqrt{6}\,\alpha(\psi) = \psi_0 - \psi + \bar{C}_0\,[1 - (3\,\psi^2 + \sqrt{6}\,\psi + 1)\exp(-\sqrt{6}\,\psi) + \ldots]$$

The Solution for *k=0*

$$\chi^2(\alpha) = 6 - e^{6\alpha}\,\bar{v}(\alpha)\left[\,C_0 + \int e^{6\alpha}\bar{v}(\alpha)\,\right]^{-1}$$

$$\bar{v}(\alpha) = v_0\exp(g\alpha):$$

$$\chi^2\,(\alpha) = 6 - (g+6)\,[\,1 + C_1 e^{-(6+g)\alpha}\,]^{-1}$$

Now it is possible to derive $\psi(\alpha)$ by integrating $\psi'(\alpha) \equiv \chi(\alpha)$ along the 'physical' paths

complex α-plane and thus to find the 'portrait'

$$\frac{d\eta^2}{d\alpha} + 2(3+c)\,\eta^2 + e^{-2c\alpha}\,\bar{v}'(\alpha) = 0$$

$$\frac{d\xi^2}{d\alpha} + 2c\,\xi^2 + \eta^2 + 2k\,e^{-2(1+c)\alpha} = 0$$

$$y(\alpha) = e^{2c\alpha}\,\eta^2 \text{ and } x(\alpha) = e^{2c\alpha}\,\xi^2$$

x gives y > gives potential !

constraint

$$y(\alpha) - 6\,x(\alpha) + \bar{v}(\alpha) + 6k\,e^{-2\alpha} = 0$$

Exact solution $$y(\alpha) = 6\,e^{-6\alpha}\,I(\alpha) - \bar{v}(\alpha)$$

$$I(\alpha) \equiv [\, C_0 + \int e^{6\alpha} \bar{v}(\alpha) \,]$$

$$x(\alpha) = e^{-6\alpha} I(\alpha) \, + \, k \, e^{-2\alpha} \, .$$

$$\dot{\alpha} = e^{-c\alpha} \sqrt{x(\alpha)} =$$

Exact result!

$$= e^{-(1+c)\alpha} \, [\, e^{-4\alpha} I(\alpha) \, + \, k \,]^{1/2}$$

$$\chi^2(\alpha) \equiv y(\alpha)/x(\alpha) =$$

EXACT FORMULA for any potential and curvature

$$[\, 6 \, e^{-6\alpha} I(\alpha) \, - \, \bar{v}(\alpha) \,] \, [\, e^{-6\alpha} I(\alpha) \, + \, k \, e^{-2\alpha} \,]^{-1}$$

Another way to derive the potential in the new version of cosmology

(from matter to gravity)

$$\bar{v}(\alpha) = -y(\alpha) + 6J(\alpha)\,, \quad x(\alpha) = ke^{-2\alpha} + J(\alpha)$$

WHERE

$$J(\alpha) \equiv C_1 + \int_{\alpha}^{\alpha_+} y(\alpha)$$

α-version,

is it a universal key?

My answer – YES !

A fresh look at inflation

We begin with the standard conditions for inflation and the parameters accessible to measurements. In our language the obvious necessary condition for inflation is $\bar{\chi}(\psi) \gg 1$ on a small interval of ψ, or, equivalently, $\chi^2(\alpha) \ll 1$ on the corresponding large interval $(\alpha_i < \alpha < \alpha_f)$, where $\alpha_f - \alpha_i = N \sim 50$ is the so-called number of e-foldings (see [3], [15]). Using equations (11) and (63) with $k = 0$ [16] we derive the exact relations

$$\underline{-2\,\dot{\xi}/\xi^2 \equiv -x'(\alpha)/x(\alpha)} = y(\alpha)/x(\alpha) \equiv \chi^2(\alpha) \equiv 2\,\hat{\epsilon}\,.$$

$$\hat{r} \equiv \dot{\psi}^2/v(\psi) \equiv y(\alpha)/\bar{v}(\alpha) = \frac{\hat{\epsilon}}{3}\,(1 - \hat{\epsilon}/3)^{-1} \simeq \frac{\hat{\epsilon}}{3}$$

It the standard approach to inflation cosmologists use one more parameter $\hat{\eta}$ defined by

$$-2\,\dot{\eta}/\eta\xi \equiv -y'(\alpha)/y(\alpha) \equiv 2(\hat{\eta} - \hat{\epsilon})\,, \qquad \hat{\eta} = \chi^2(\alpha) - \chi'(\alpha)/\chi(\alpha)$$

$$6x(\alpha) \;=\; 6e^{-6\alpha}I(\alpha) = C_0 e^{-6\alpha} + \sum_{0}^{N}(-6)^{-n}\bar{v}^{(n)}(\alpha) + (-6)^{-(N+1)}e^{-6\alpha}\int_{-\infty}^{\alpha} e^{6\alpha}\bar{v}^{(n+1)}(\alpha)$$

$$\hat{r}(\alpha) \equiv \frac{y(\alpha)}{\bar{v}(\alpha)} = 6\,\frac{x(\alpha)}{\bar{v}(\alpha)} - 1 =$$

for the **inflationary** solution with

$$C_0 = 0$$

we find the **perturbative expansion** (by iterations)

$$= \frac{\chi^2}{6}\left(1 - \frac{\chi^2}{6}\right)^{-1} = \sum_1^{\infty}(-1)^n \frac{\bar{v}^{(n)}(\alpha)}{6^n\,\bar{v}(\alpha)}$$

$$\chi = -l'(\psi)\left(1 - \frac{\chi^2}{6}\right)\left[1 + \sum_2^{\infty} \frac{D_n * v(\psi)}{(-6)^{n-1} v\, l'}\right] \qquad d_\alpha^n = \chi D_n$$

$$d_\alpha \equiv \chi d_\psi$$

$$d_\alpha^2 = \chi[\,\chi d_\psi^2 + (d_\psi \chi) d_\psi]$$

<<definitioins>>

$$\frac{D_2 * v}{v\, l'(\psi)} = \frac{\chi}{l'}[\, l'' + (l')^2] + (d_\psi \chi)$$

The **corrections** to the standard inflationary expression

$$\chi_2 = -l'(\psi)\left[1 - \frac{1}{6}(l')^2\right]\left\{1 + \frac{1}{6}[2l'' + (l')^2]\right\} = -l'(\psi)\left[1 + \frac{1}{3}l'' + O(\chi_1^4)\right]$$

$$\hat{\eta}_2 = \chi_2^2 - d_\psi\, \chi_2 = \hat{\eta}_1 + (1/3)\,[\,2\, l^{(2)}\, \hat{\eta}_1 + l^{(1)} l^{(3)} - (l^{(2)})^2]$$

Our corrections to the number of e-foldings

$$N_e = \int_{\psi_i}^{\psi_f} d\psi\, \bar{\chi}(\psi) = \int_{\psi_f}^{\psi_i} \frac{d\psi}{l'(\psi)} \left[1 - \frac{1}{3} l'' + O(\chi_1^4) \right] = N_e^{(0)}(\psi) + \frac{1}{3} \ln[\, l'(\psi)]_i^f$$

$$N_e = [\, \psi^2/4N + \ln\psi/3\,]_f^i \text{ when } v = v_0 \psi^{2N}$$

Possible relation to the ψ **version:** $z(x)$ = $-\tanh^{\varepsilon} y(x)$

$$dy/dx = 1 - u(x)\, \coth(y) \qquad u(x) = \tilde{v}'(x)/2\,\tilde{v}(x) \qquad z = \sqrt{6}\,\bar{\chi}\,, \quad x = \sqrt{3/2}\,\psi$$

asymptotic expansion of $y(x)$ in powers of $1/x \sim u(x) \sim l'$ for $v = v_0 x^{2N}$:

$$y(x) = \sum_0^{\infty} y_{2n+1}\, x^{-(2n+1)} \qquad y_1 = N\,, \quad y_3 = \frac{1}{3}N^3 - N^2, \dots,$$

if $u(x) = \sum_1^{\infty} u_n\, e^{-ngx}$ $\qquad y = \sum_1^{\infty} y_n\, e^{-ngx}$

SHORT SUMMARY

1. The *cosmological dynamical equations* are formulated *in different gauges and versions*. We illustrate relations between them on simple solutions and by integrable models.
2. The *general properties of gauge independent* χ*-equations* (26)-(27), describing the main (α, ψ) portraits of isotropic cosmologies, are established in α and ψ versions.
3. Equations (29)-(30) allow us to derive the complete solution if $\chi(\alpha)$ or $\bar{\chi}(\psi)$ are known. Taking into account equations (31) we can in addition derive $\bar{v}(\alpha)$ or $v(\psi)$.
4. We discussed *different ways to determine cosmologies not using potentials*. A most natural one seems to first derive $\chi^2(\alpha)$ using (32), with the Hubble function $\xi^2(\alpha)$ as an input.
5. Although the χ-equations depend only on $v'(\psi)/v(\psi)$ and are thus insensitive to the sign of $v(\psi) \equiv \bar{v}(\alpha)$, *this sign is critically important for global properties of the solutions*. From (28), (44), (75) it follows that the solutions in the intervals with $v(\psi) > 0$ are isolated from those in the intervals with $v(\psi) < 0$ and must be studied separately.
6. We mostly considered potentials not changing the sign and studied in detail models with *positive potentials for which inflationary scenarios are natural*. We also can use and actually used our solutions and their expansions near the points ψ_0 where $v(\psi_0) = 0$ and thus $v'(\psi)/v(\psi)$ behaves as $(\psi - \psi_0)^{-1} \to \pm\infty$. This is a problem in the ψ-version because $(\chi^2 - 6)$ may change the sign with the potential, as follows from (44), (75). But in the α-version it is no problem at all, as can be seen from from expression (62) for $\chi^2(\alpha)$.

7. Probably, the *most important results* are presented in Section 4, where we have found the *exact solution of all equations for arbitrary* $\bar{v}(\alpha)$ *and* k. The necessary condition for inflation is $\chi^2(\alpha) < 6$ ($6\bar{\chi}^2(\psi > 1)$. To derive from $\chi(\alpha)$ standard inflationary scenarios we first suppose that the spatial curvature vanishes, $k = 0$. Then, by fixing the arbitrary integration constant, $C_0 = 0$, we preserve the v-scale invariance of inflationary solution χ and derive its expansion from Eq.(82) as a sum, the n-th term of which for $n \geq 1$ has the form:

$$-l'(\psi) \sum_k c_n(k_1, ..., k_{2n}) \prod_{i=1}^{2n} [\, l^{(i)}(\psi)\,]^{k_i}\,, \quad \text{where} \; \sum_i i k_i = 2n\,, \;\; k_i \geq 0\,.$$

This *inflationary perturbation expansion* can be obtained by the well-defined recursive algebraic iterations and gives higher-order corrections to the inflationary parameters $\hat{\epsilon}$, $\hat{\eta}$, N_e .

8. When $v(\psi) < 0$ and thus $\chi^2(\alpha) > 6$, $6\bar{\chi}^2(\psi) < 1$, it is also convenient to use expansions of $\bar{\chi}(\psi)$ when it is small or close to $1/6$. In the last case we have derived *asymptotic approximation* (54) for $\psi \to \infty$ valid *for a broad class of potentials* $v(\psi)$. We should mention an interesting one-parameter class of 'bouncing' solutions (see (59) and (60)), which exist when $v'(\psi)/v(\psi) \sim 1/\psi$, and a special solution (56) that probably is a separatrix. The *global picture of solutions* with such properties are of great interest for ekpyrotic-bouncing scenarios and *must be studied in future.*

I believe that *visualization of these structures*, drawing the (α, ψ) portraits, and using perturbative expansions for concrete inflationary, ekpyrotic, bouncing and other, more strange isotropic cosmologies may stimulate their better theoretical understanding.

THE

END

A generic example of a portrait of static states and cosmologies

See ATF, arXiv:hep-th/9605008v2, gr-qc/9612058

Lagrangian: $L \equiv W\dot{\phi}^2 + \dot{\phi}F + hV + Z\dot{\psi}^2 = 0, \quad F \equiv \dot{h}/h$

second integral:

$$(F/W)^2 + 4\bar{g}_1 h + 2\bar{g}_2 C_0^2 \log h = \bar{C}_1$$

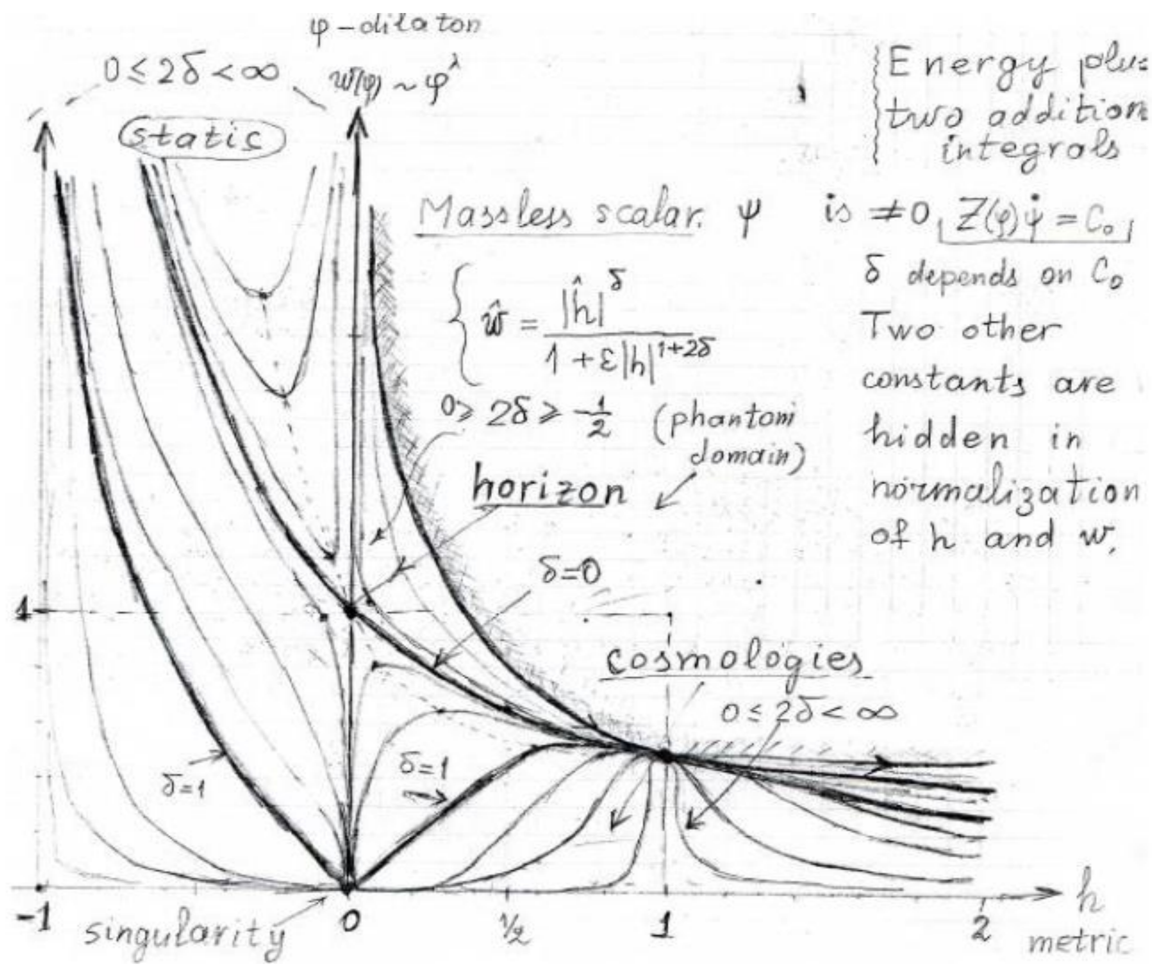
$$V = W(\bar{g}_4 w^2 - \bar{g}_1), \quad Z^{-1} = W(\bar{g}_3 + \bar{g}_2 \log w),$$

portrait

$$w = \frac{|h|^{\delta}}{|1 + \epsilon |h|^{1+2\delta}|}$$

with normalized *h, w* .

first integral: $C_0 = Z\dot{\psi}$



INTRODUZIONE

A root to inflation and all that…

**Italia and 1965-ICTP workshop*

**Weak interaction* of baryons and leptons through *11-dimensional space-time* with 4-dimensional brane of our world in it. `Unifying' W-EM, CP-viol. (`66-`67 with BA).*

**Disrete relativistic strings* and bound states by *gauging (super) canonical groups* (`86-`94)*

**Quantizations of BH and Cosmol. >>* regarded as discrete gauge theories (`94-`96 with VdA and MC, Torino)*

**Gravity + scalaron*, integrable models, *unifying BH+Cosm+Waves.* Search for 2-dim and 1-dim solutions (`02-`06)*

**Multiexponential models* (`06-`09)*

**WEE-inspired affine extension* of gravity: *GR+vecton*. Intrinsically non-isotropic cosmology. *Reduct. to scalaron*, fresh view of comsology, `08..*

***3 workshops Constr.dyn. &QG* (Ru.-It.)*

Acc. Sc. Torino - Memorie Sc. Fis.
19-20 (1995-1996), 129-158.

FISICA TEORICA

Discrete Gauge Theories

Memoria di A.T. FILIPPOV*
presentata dal Socio corrispondente Vittorio de ALFARO
nell'adunanza del 15 Novembre 1995

RELAZIONE
letta ed approvata nell'adunanza del 14 Febbraio 1996
sulla Memoria di A.T. FILIPPOV dal titolo

DISCRETE GAUGE THEORIES

L'autore ha dedicato molte ricerche allo studio di diversi tipi di teorie a numero discreto di gradi di libertà che possiedano invarianza per trasformazioni di gauge che danno luogo, nella formulazione canonica, a vincoli di prima classe le cui parentesi di Poisson formano un'algebra di Lie. Il lavoro in questione espone i caratteri generali di questi sistemi e discute gli aspetti fondamentali del problema della loro quantizzazione.

Tali sistemi sono definiti a partire da una azione che nella formulazione hamiltoniana mostra un'invarianza rispetto a gruppi di trasformazioni delle coordinate canoniche che dipendono da una o più classi di funzioni arbitrarie del tempo. I gruppi di gauge associati sono sottogruppi del gruppo canonico (simplettico) e gli invarianti di gauge formati con le variabili canoniche e con i potenziali di gauge sono le quantità fondamentali per la quantizzazione.

Tra i sistemi di questo tipo si annoverano i sistemi di particelle relativistiche e le teorie di campo topologiche. Sono anche di questo tipo le approssimazioni discrete della teoria di stringa. Inoltre hanno notevole interesse i sistemi, definiti nell'ambito della Relatività Generale, che si ottengono limitandosi a considerare un numero finito di gradi di libertà. In tal caso si ha un sottoinsieme di trasformazioni di gauge dovute alla invarianza per riparametrizzazioni del tempo coordinato; la funzione Hamiltoniana propria è nulla ed è sostituita da una combinazione lineare dei vincoli, e i vincoli sono almeno bilineari nei momenti.