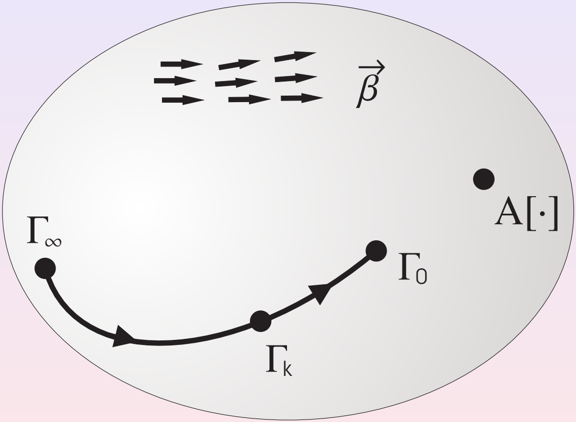
Quantum Einstein Gravity, Background Independence, and Asymptotic Safety

Holger Weyer

University of Mainz, Germany

"Renormalization Group & Related Topics"
Dubna, September 1-5, 2008

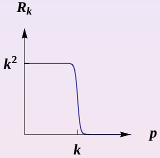
Theory space



Effective Average Action Approach

- The effective average action $\Gamma_k[g_{\mu\nu}, \cdots]$ is a scale dependent ("coarse grained") free energy functional for the metric.

- A Built-in Infrared Cutoff discriminates between high- and low-momentum modes.
    - Modes with $p > k$ are fully integrated out.
    - Modes with $p < k$ are suppressed by adding a "mass" term $\propto (\text{mass})^2 = \mathcal{R}_k(p^2)$ to the bare action.



- Γ_k interpolates between the bare action $S = \Gamma_{k\to\infty}$ and the standard effective action $\Gamma = \Gamma_{k\to 0}$.

- Γ_k satisfies an exact or functional Renormalization Group equation (FRGE), symbolically:

$$k\partial_k \Gamma_k = \tfrac{1}{2}\,\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right]$$

M. Reuter, Phys. Rev. D 57 (1998) 971

(a) Starting point: $\int \mathcal{D}\gamma_{\mu\nu}\ \exp\left(-S[\gamma_{\mu\nu}]\right)$
$S[\gamma_{\mu\nu}]$ is the classical action, assumed to be diffeomorphism invariant.

Construction of the Gravitational Average Action

M. Reuter, Phys. Rev. D 57 (1998) 971

(a) Starting point: $\int \mathcal{D}\gamma_{\mu\nu}\ \exp(-S[\gamma_{\mu\nu}])$
$S[\gamma_{\mu\nu}]$ is the classical action, assumed to be diffeomorphism invariant.

(b) Background field method:
decompose the quantum metric

$$\gamma_{\mu\nu}(x) = \bar{g}_{\mu\nu}(x) + h_{\mu\nu}(x)$$

- $\bar{g}_{\mu\nu}$: fixed background metric
  (arbitrary, but never concretely specified)
- $h_{\mu\nu}$: quantum fluctuations

(c) add background gauge fixing $S_{\rm gf}[h_{\mu\nu};\overline{g}_{\mu\nu}] +$ ghost terms

(d) add external sources

(c) add background gauge fixing $S_{\text{gf}}[h_{\mu\nu}; \overline{g}_{\mu\nu}]$ + ghost terms

(d) add external sources

(e) expand $h_{\mu\nu}$ in eigenmodes of $\overline{D}^2 \equiv \overline{g}^{\mu\nu} \overline{D}_\mu \overline{D}_\nu$ and introduce
IR-Cutoff in the spectrum of $\overline{D}^2$
$\Longrightarrow$ modes with $-\overline{D}^2$-eigenvalues $< k^2$ are suppressed

(f) define the generating functional for the connected Green's functions $W_k[\text{sources}; \bar{g}_{\mu\nu}]$

$\downarrow$

(f) define the generating functional for the connected Green's functions $W_k[\text{sources}; \bar{g}_{\mu\nu}]$

$\downarrow$

(g) Legendre transformation

$\downarrow$

effective average action $\Gamma_k[g_{\mu\nu}, \bar{g}_{\mu\nu}, \text{ghosts}]$

- $g_{\mu\nu}$ is the classical analogue to the quantum metric $\gamma_{\mu\nu}$

$$g_{\mu\nu} = \langle \gamma_{\mu\nu} \rangle = \bar{g}_{\mu\nu} + \langle h_{\mu\nu} \rangle$$

(f) define the generating functional for the connected Green's functions $W_k[\text{sources}; \bar{g}_{\mu\nu}]$

$\downarrow$

(g) Legendre transformation

$\downarrow$

effective average action $\Gamma_k[g_{\mu\nu}, \bar{g}_{\mu\nu}, \text{ghosts}]$

- $g_{\mu\nu}$ is the classical analogue to the quantum metric $\gamma_{\mu\nu}$

$$g_{\mu\nu} = \langle \gamma_{\mu\nu} \rangle = \bar{g}_{\mu\nu} + \langle h_{\mu\nu} \rangle$$

(h) derive functional RG equation

The UV-Limit of Quantum Einstein Gravity

If there exists a non-Gaussian fixed point Γ_*,

$$\beta_i(\Gamma_*) = 0, \qquad \forall i,$$

Quantum Einstein Gravity is nonperturbatively renormalizable ("asymptotically safe").

Weinberg, 1979

If there exists a non-Gaussian fixed point Γ_*,

$$\beta_i(\Gamma_*) = 0, \qquad \forall i,$$

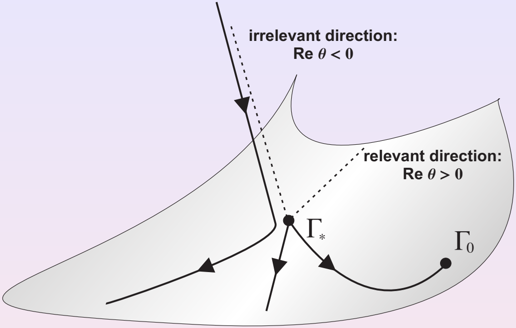
Quantum Einstein Gravity is nonperturbatively renormalizable ("asymptotically safe").

Weinberg, 1979

The quantum theory is described by a RG trajectory running inside the UV critical hypersurface of the FP, with

- initial point: $\Gamma_\infty =$ action infinitesimally close to Γ_*
- end point: $\Gamma_0 = \Gamma$

UV critical Hypersurface $\mathcal{S}_{\mathrm{UV}}$



- UV $\longrightarrow$ IR
- θ: critical exponent (neg. eigenvalue of lin. flow)

$$\Delta_{\mathrm{UV}} \equiv \dim \mathcal{S}_{\mathrm{UV}}$$
$$:= \# \text{ relevant directions}$$
$$:= \# \text{ free parameters in the asymptotically safe qft}$$

Quantum Einstein Gravity (QEG)

- "Background independent" quantization scheme:

  No special metric plays any role!

  The use of the background field technique provides "background independence".

- Fundamental action $S = \Gamma_*$ is a prediction:

  No special action plays any role!

  - input: field content and symmetries $\widehat{=}$ theory space
  - output: $\Gamma_* = S_{\text{Einstein-Hilbert}} +$ "more"

The Einstein-Hilbert action often is a reliable approximation, but not distinguished conceptually.

Einstein-Hilbert Truncation

M. Reuter, Phys. Rev. D 57 (1998) 971

- ansatz:

$$\Gamma_k[g_{\mu\nu}, \bar{g}_{\mu\nu}, \cdots] = -\frac{1}{16\pi\, G_k} \int d^4x \sqrt{g}\, \{R(g) - 2\,\Lambda_k\}$$
$$+ \text{classical gauge fixing} + \text{ghost terms}$$

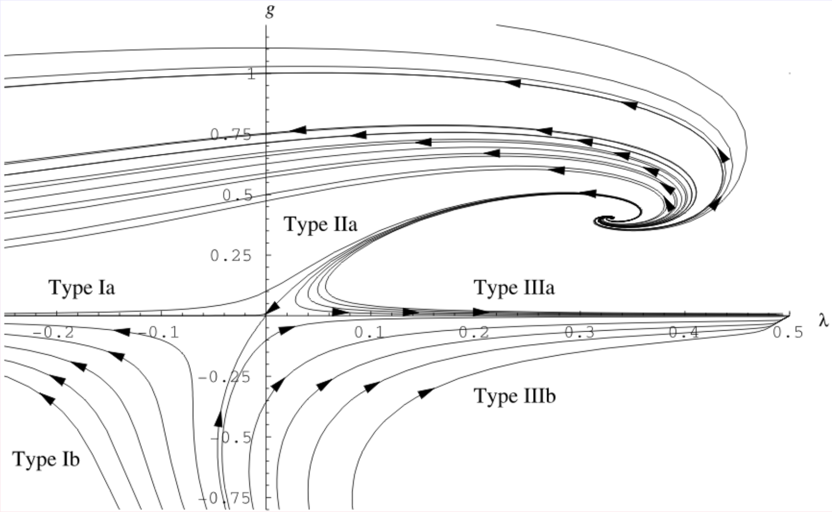
- 2 running parameters:

Newton's constant	G_k,	dim.less	$g_k \equiv k^2\, G_k$
cosmological constant	Λ_k,	dim.less	$\lambda_k \equiv \Lambda_k / k^2$

- RG flow described by sytem of ODEs

$$k\partial_k\, g_k = \beta_g(g_k, \lambda_k)$$
$$k\partial_k\, \lambda_k = \beta_\lambda(g_k, \lambda_k)$$

EH Flow



M. Reuter and F. Saueressig, Phys. Rev. D 65 (2002) 065016

Conformally Reduced Quantum Einstein Gravity

M. Reuter and H.W., arXiv:0801.3287 [hep-th]
M. Reuter and H.W., arXiv:0804.1475 [hep-th]

- Is a simplified version of QEG.
- disentangles conceptual from computational problems
- gauge fixing issues play no role

Conformally Reduced Quantum Einstein Gravity

M. Reuter and H.W., arXiv:0801.3287 [hep-th]
M. Reuter and H.W., arXiv:0804.1475 [hep-th]

- Is a simplified version of QEG.
- disentangles conceptual from computational problems
- gauge fixing issues play no role

- Only the conformal factor of the metric is quantized: all metrics are taken to be conformal to $\widehat{g}_{\mu\nu}$:

$$\gamma_{\mu\nu} = \chi^2 \, \widehat{g}_{\mu\nu}$$
$$\overline{g}_{\mu\nu} = \chi_{\mathrm{B}}^2 \, \widehat{g}_{\mu\nu}$$
$$g_{\mu\nu} = \langle\chi^2\rangle \, \widehat{g}_{\mu\nu}$$

Reference metric $\widehat{g}_{\mu\nu}$ is non-dynamical
and concretely chosen once and for all !

- uses the same methods as full QEG:

  effective average action approach with starting point

$$\int \mathcal{D}\chi \, \exp(-S[\chi])$$

  background field method, decomposing

$$\chi(x) = \chi_{\rm B}(x) + f(x)$$

Conformally Reduced Quantum Einstein Gravity

- uses the same methods as full QEG:

  effective average action approach with starting point

  $$\int \mathcal{D}\chi \ \exp(-S[\chi])$$

  background field method, decomposing

  $$\chi(x) = \chi_{\mathrm{B}}(x) + f(x)$$

- has the same qualitative features as full QEG

- shows the importance of "background independence" for the RG flow:
  It is a scalar-like theory, but its RG behavior is very different from that of a standard scalar matter field in a rigid background spacetime.

Adjusting the IR Cutoff $\mathcal{R}_k[\chi_{\rm B}]$

- The coarse graining scale k^{-1} of $\Gamma_k[g, \overline{g}\,]$ should be a proper rather than a coordinate length.
- "background independence" of Γ_k and its RG flow: $\Longrightarrow$ k^{-1} can be proper only w. r. t. $g_{\mu\nu}$ or $\overline{g}_{\mu\nu}$.

Adjusting the IR Cutoff $\mathcal{R}_k[\chi_B]$

- The coarse graining scale k^{-1} of $\Gamma_k[g, \bar{g}]$ should be a proper rather than a coordinate length.
- "background independence" of Γ_k and its RG flow: $\Longrightarrow$ k^{-1} can be proper only w. r. t. $g_{\mu\nu}$ or $\bar{g}_{\mu\nu}$.

our choice in QEG:

k^2 is a cutoff in the spectrum of $-\bar{\Box} \equiv -(D^\mu D_\mu)(\bar{g})$.

- Typical structures (periods, $\cdots$) of the $-\bar{\Box}$-eigenfunction with eigenvalue k^2 have a $\bar{g}$-proper size of order k^{-1}.

Adjusting the IR Cutoff $\mathcal{R}_k[\chi_{\rm B}]$

- The IR cutoff $\mathcal{R}_k$ suppresses $(-\overline{\Box})$-eigenfunctions with eigenvalues $< k^2$ by giving them a "mass" of order k. Those with larger eigenvalues must remain "massless".

- replacement:

$$(-\overline{\Box}) \longrightarrow (-\overline{\Box}) + k^2\, R^{(0)}\big(-\frac{\overline{\Box}}{k^2}\big)$$

replacement for $\chi_{\rm B} = const$ and $\overline{g}_{\mu\nu} = \chi_{\rm B}^2\ \widehat{g}_{\mu\nu}$:

$$(-\widehat{\Box}) \longrightarrow (-\widehat{\Box}) + \chi_{\rm B}^2\, k^2\, R^{(0)}\big(-\frac{\widehat{\Box}}{\chi_{\rm B}^2\, k^2}\big)$$

Adjusting the IR Cutoff $\mathcal{R}_k[\chi_{\rm B}]$

- The IR cutoff $\mathcal{R}_k$ suppresses $(-\overline{\Box})$-eigenfunctions with eigenvalues $< k^2$ by giving them a "mass" of order k. Those with larger eigenvalues must remain "massless".

- replacement:

$$(-\overline{\Box}) \longrightarrow (-\overline{\Box}) + k^2\, R^{(0)}\big(-\tfrac{\overline{\Box}}{k^2}\big)$$

replacement for $\chi_{\rm B} = const$ and $\overline{g}_{\mu\nu} = \chi_{\rm B}^2\ \widehat{g}_{\mu\nu}$:

$$(-\widehat{\Box}) \longrightarrow (-\widehat{\Box}) + \chi_{\rm B}^2\, k^2\, R^{(0)}\big(-\tfrac{\widehat{\Box}}{\chi_{\rm B}^2\, k^2}\big)$$

Cf. standard QFT on a rigid background spacetime:

k^2 is in the spectrum of $-\widehat{\Box}$!

Truncations employed

The conformally reduced Einstein-Hilbert ("CREH") truncation

$$\begin{aligned}\Gamma_k[\bar{f};\chi_{\mathrm{B}}] \equiv \Gamma_k[\phi,\chi_{\mathrm{B}}] &= -\frac{1}{16\pi\, G_k}\int \mathrm{d}^4x\,\sqrt{g}\,\big(R(g)-2\Lambda_k\big)\Big|_{g_{\mu\nu}\to\phi^2\,\widehat{g}_{\mu\nu}} \\ &= \frac{3}{4\pi\, G_k}\int \mathrm{d}^4x\,\sqrt{\widehat{g}}\,\left\{\tfrac{1}{2}\,\phi\,\widehat{\Box}\,\phi - \tfrac{1}{12}\,R(\widehat{g})\,\phi^2 + \tfrac{1}{6}\,\Lambda_k\,\phi^4\right\}\end{aligned}$$

- Γ_k depends only on the combination $\phi = \chi_{\mathrm{B}} + \bar{f}$.
- Theory space $\{G,\Lambda\} \sim \{g,\lambda\}$ is 2-dimensional.

Truncations employed

The local potential approximation (LPA)

$$\Gamma_k[\phi, \chi_{\rm B}] = \frac{3}{4\pi\, G_k} \int d^4x\, \sqrt{\widehat{g}}\; \left\{ \tfrac{1}{2}\, \phi\, \widehat{\Box}\, \phi - F_k(\phi) \right\}$$

- Theory space $\{G, F(\,\cdot\,)\} \sim \{g, Y(\,\cdot\,)\}$ is ∞-dimensional.

dimensionless variables

$$Y_k(\varphi) = k^2\, F_k(\varphi/k), \qquad \varphi = k\,\phi,$$
$$g_k \equiv k^2\, G_k, \qquad \lambda_k \equiv \Lambda_k/k^2$$

Flow Equations

R^4 topology and optimized cutoff function

$$k\partial_k\, G_k = \eta_{\mathrm{N}}\big(G_k, [F_k]\big)\, G_k$$

$$k\partial_k\, F_k(\phi) = \eta_{\mathrm{N}}\, F_k(\phi) - \frac{G_k}{24\pi}\,\left(1 - \tfrac{1}{6}\,\eta_{\mathrm{N}}\right)\,\frac{\phi^6\, k^6}{\phi^2\, k^2 + F_k''(\phi)}$$

$$\eta_{\mathrm{N}} = \cdots$$

R^4 topology and optimized cutoff function

$$k\partial_k\, G_k = \eta_{\mathrm{N}}\big(G_k, [F_k]\big)\, G_k$$

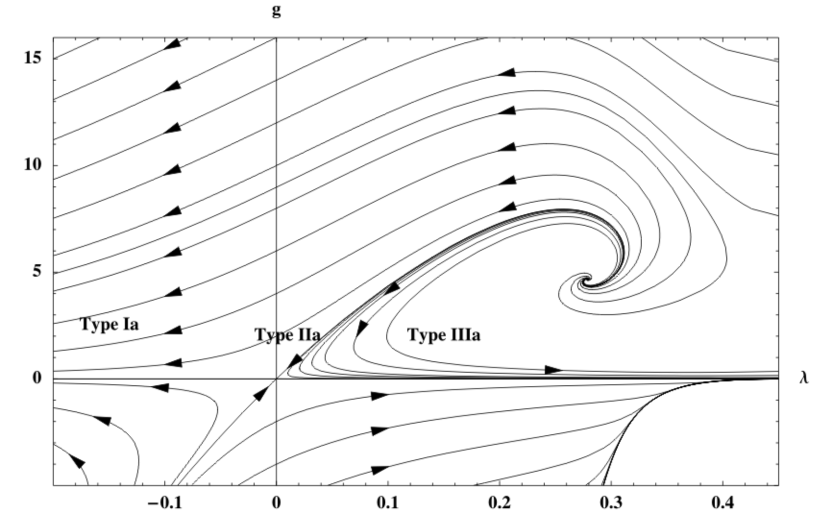
$$k\partial_k\, F_k(\phi) = \eta_{\mathrm{N}}\, F_k(\phi) - \frac{G_k}{24\pi}\,\left(1 - \tfrac{1}{6}\,\eta_{\mathrm{N}}\right)\,\frac{\phi^6\, k^6}{\phi^2\, k^2 + F_k''(\phi)}$$

$$\eta_{\mathrm{N}} = \cdots$$

Compare to QFT on rigid background:

$$\frac{\phi^6\, k^6}{\phi^2\, k^2 + F_k''(\phi)} \longrightarrow \frac{k^6}{k^2 + F_k''(\phi)}$$

CREH flow: $Y_k(\varphi) = -\lambda_k \varphi^4/6$



Results

Two inequivalent quantization schemes:

(a) rigid background quantization

(b) "background independent" quantization

RG flows are very different:

(a) standard $(-\phi^4)$-theory: no NGFP, asymptotically free
(Symanzik, 1973)

(b) NGFP exists: asymptotically safe

The beta functions depend on the topology:

for example R^4, S^4, $\cdots$

Results

Gaussian Fixed Point (GFP)

fixed point potential $Y_*^{\text{GFP}}(\varphi) = c\,\varphi^2$

Scaling dimensions at the GFP are shifted by 2 units:

for scaling field φ^n, $n \in \mathbb{R}$, say:

(a) rigid background quantization: $\theta = n - 4$

(b) "background independent" quantization: $\theta = n - 2$

Results

Gaussian Fixed Point (GFP)

fixed point potential $Y_*^{\text{GFP}}(\varphi) = c\,\varphi^2$

Scaling dimensions at the GFP are shifted by 2 units:

for scaling field φ^n, $n \in \mathbb{R}$, say:

(a) rigid background quantization: $\theta = n - 4$

(b) "background independent" quantization: $\theta = n - 2$

Running of the cosmological constant near the GFP:

(a) rigid background quantization: $\Lambda_k \propto \ln(k)$

(b) "background independent" quantization: $\Lambda_k \propto G_0\, k^4$

consistent with full QEG and approaches for summing up zero-point energies

Non-Gaussian Fixed Point (NGFP)

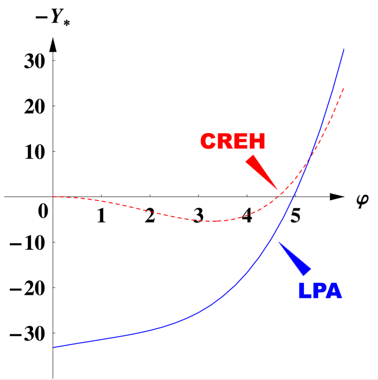
R^4 topology:

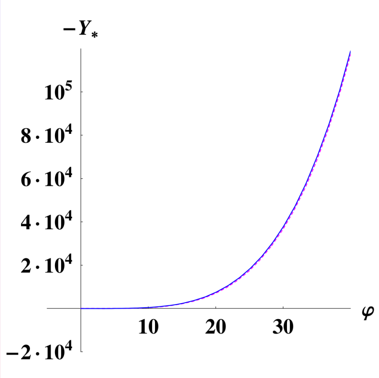
$$g_*^{\mathrm{NGFP}} = g_*^{\mathrm{CREH}}, \qquad Y_*^{\mathrm{NGFP}}(\varphi) = y_* + \frac{1}{6}\,\lambda_*^{\mathrm{CREH}}\,\varphi^4$$

Scaling dimensions and scaling fields at the NGFP depend on the theory space chosen:

for example $\{\varphi^m\}$, with $m \in \mathbb{N}$, $m \in \mathbb{Z}$, $m \in \mathbb{R}$, $m \in \mathbb{C}$, $\cdots$

S^4 topology (numerical solution)





Corresponds to infinitely many couplings approaching a non-trivial fixed point !

Summary

- The gravitational average action is a "background independent" approach to quantum gravity.
- "Background independence" seems to be a (the ?) crucial prerequisite for asymptotic safety.
- RG flow of the conformal factor is typical of the full set of metric degrees of freedom.