Self-Similarity in Diffusion Processes on Networks

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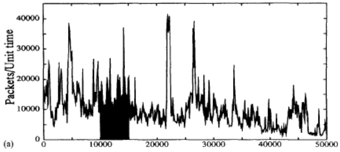
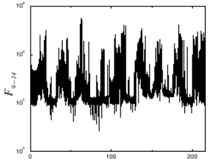
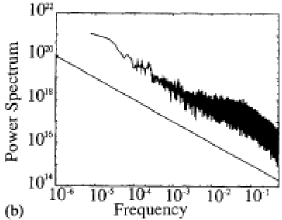
Outline

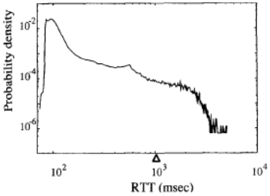
Empirical Evidence of complexity in traffic on networks

Internet traffic measurements: rtt, flow, jamming; time series









[Takayasu (1996), Moreno (2004), Barthelemy (2002), ...]

PHYSICS APPROACH: NUMERICAL

Purpose of physics research of transport and other nonlinear processes on networks:

- Looking for regularities/physics laws;
- Understanding the origin of *universal* behavior;
- Role of networks structure!
- Numerical experiment & Proper theoretical formulation;

Outline

INFORMATION TRAFFIC ON NETWORKS

MODEL*:

Traffic of information packets on networks of $N = 1000$ nodes and given topology (cSF,seG); Model with:

- Creation and assignment; (Rate: R)
- Navigation; (Local; Depth $r = 2$)
- Queuing; (LIFO queue; Buffer: $H = 1000$)
- delivery (at destination; traffic stationarity)

PARAMETERS:

- Creation rate R (alt. constant density ρ);
- Buffer size H; Searched depth $r = 2$;

[Tadić & Rodgers, Adv. Compl. Syst. (2002), Tadić, LNCS (2003)]

STATISTICAL MEASURES OF TRAFFIC

Statistical properties of traffic are monitored at *local* (individual Nodes and Edges) and *global* level of the entire network:

(a) Traffic Noise Signals and Power Spectra:

- Number pf packets processed by a node $\{h_i(t)\}$ w. T_{WIN};
- Number pf packets processed by an edge $\{f_{ij}(t)\}$ w. T_{WIN};
- Network load; Network delivery rate; ...

(b) various PDFs:

- Transit time of packets $P(T)$ (rtt);
- Waiting times in queues $P(t_w)$;
- Time intervals (return times) $P(\Delta t)$ to Nodes;Edges;
- Flow and Noise distributions; ...

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FCLT: Theoretical Background

In correlated dynamical systems, time-dependent distributions are nontrivial

- $P(T)$: 1st passage time statistics;
- $P(\Delta t)$: 1st return time statistics;

Computed numerically: related to **Limit Stochastic Processes** ;

Functional Central Limit Theorem: Set

$$S_n(t) = c^{-1}(S_{[nt]} - mnt) \Rightarrow S(t)$$

(*convergence in distribution*), $S(t)$ –stochastic process!
Other aspects: **Queuing processes on networks**;

Outline

NETWORK STRUCTURES & TRAFFIC DENSITY

▷ Packet interactions with queuing at nodes along the path:

▷ Work by BCN group [Ref.]—small network reconstruction to *minimize transport time*: two basic structures emerge:

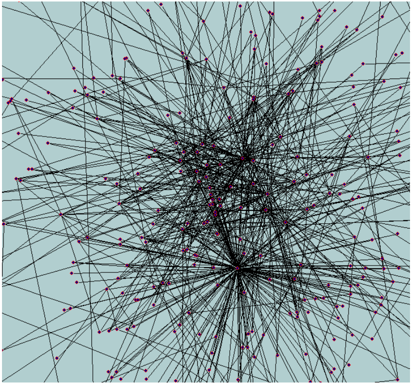
- Low traffic density: Highely Clustered Scale-Free Graph;
- High traffic density: Homogeneous (unclustered) Network;

▷ Here we simulate traffic on two prototype networks with such properties, $N = 1000$ nodes, and *traffic rules with local navigation*.

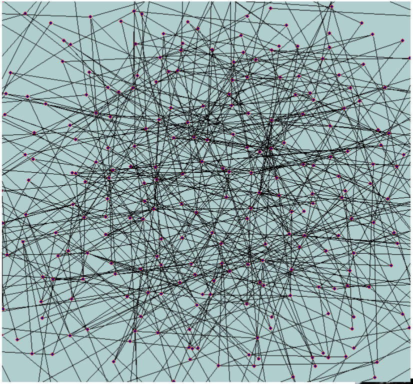
[Guimerà *et al.*, Phys. Rev. Lett. (2002).]

Two Network Structures

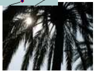
Low-traffic-density: cSF; *High-traffic-density: Homogeneous*



WebGraph

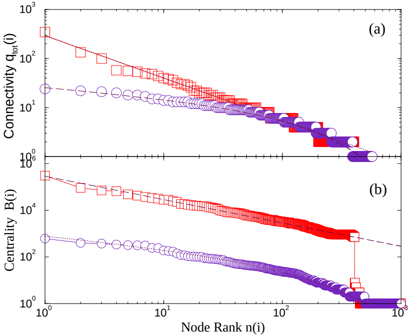


StatNet

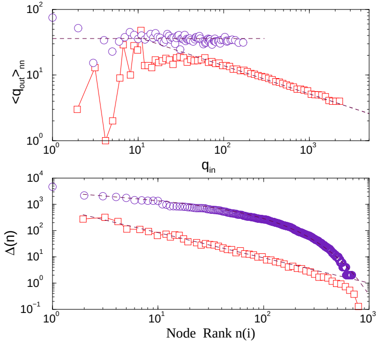


Correlations in Network Structures

▷Networks' topol.properties, relevant to traffic:



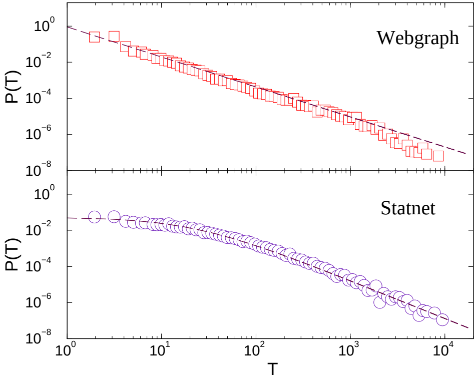
Connectivity; Centrality



Correlations; Clustering

[Tadić, in "Systems self-assembly", elsevier (2008)]

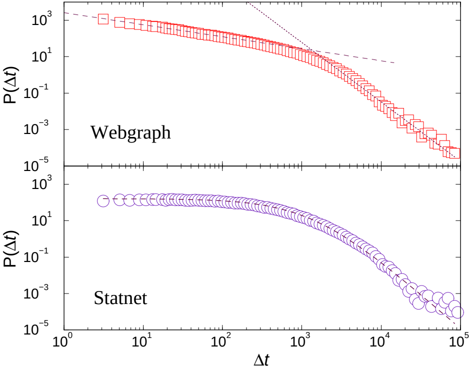
PDFs TIME STATISTICS: Travel times



Levy; q-exponential

[Tadić, Rodgers & Thurner, Int. J. Bifurcation & Chaos (2007)]

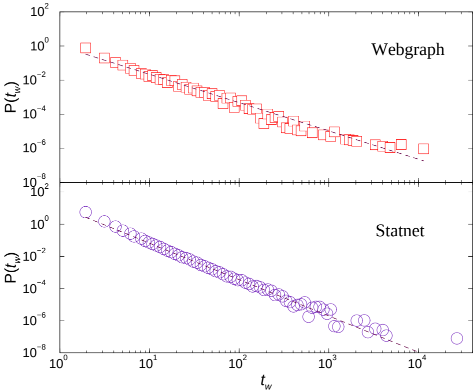
PDFs TIME STATISTICS: Return times



two slopes; q-exponential

[Tadić, Rodgers & Thurner, Int. J. Bifurcation & Chaos (2007)]

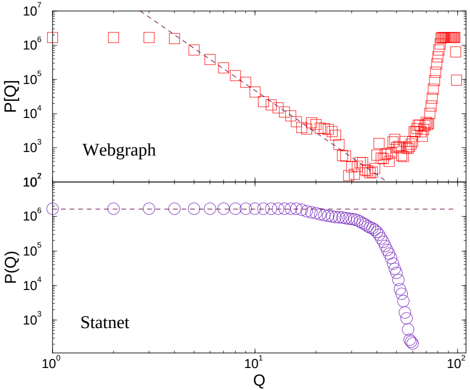
PDFs QUEUING STATISTICS: Waiting times



Power-laws: ρ-dependent!

[Tadić, Rodgers & Thurner, Int. J. Bifurcation & Chaos (2007)]

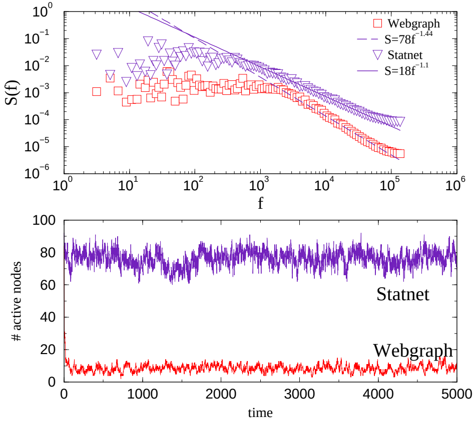
PDFs QUEUING STATISTICS: Queue lengths



[Tadić, Rodgers & Thurner, Int. J. Bifurcation & Chaos (2007)]

Outline

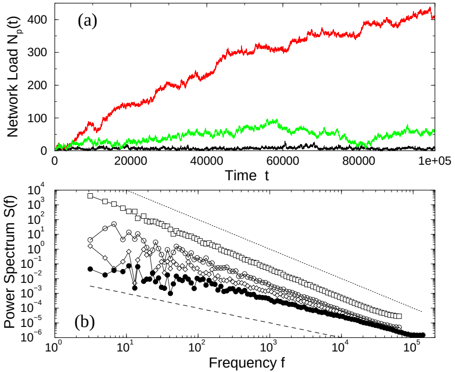
TIME SERIES ANALYSIS: cont.DENSITY



$\rho = const$: 2 Nets

[Tadić, Rodgers & Thurner, Int. J. Bifurcation & Chaos (2007)]

TIME SERIES ANALYSIS: const.RATE

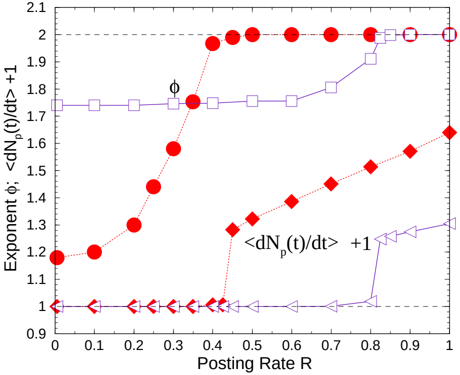


R varied: WG

[Tadić, Rodgers & Thurner, Int. J. Bifurcation & Chaos (2007)]

Jamming on Networks

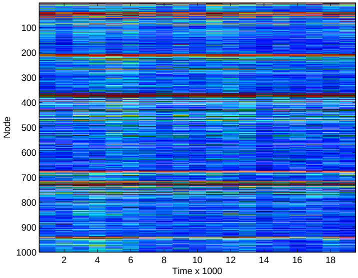
Def. jamming rate: $J \equiv < dN_p(t)/dt > = R - \lambda$



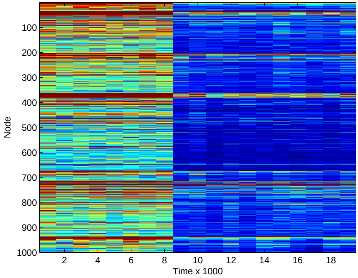
OP: 2 Nets

[Tadić, Rodgers & Thurner, Int. J. Bifurcation & Chaos (2007)]

RW: Number of VISITS per NODE



Advanced.Search: Free-flow

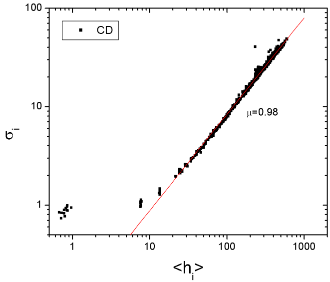


Random.Diff: Jamming

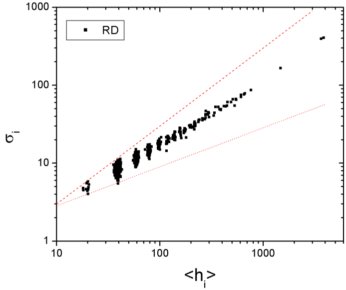
[Tadić (2008)]

Scaling in Time-Series Fluctuations: NODES

Scaling relation: $\sigma \sim < X >^{\mu}$; Exponent μ increses with T_{WIN}



Preferred Navigation

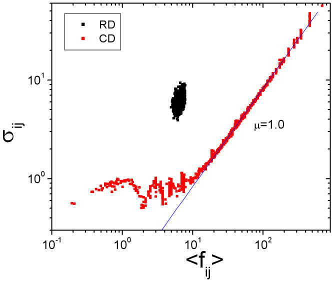


Random Diffusion

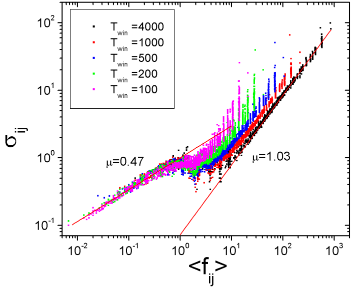
[Kujawski,Tadić & Rodgers, New J. Phys. **9**, 154 (2007)]

Scaling in Time-Series Fluctuations: EDGES

Dynamic **preferential behavior** *(navigation rule) or static* **topological preference** *(SF structure)* is necessary for the scaling to occur:



RD & Preferred Navigation



Preferred Navigation: T_{WIN}

[Kujawski Tadić & Rodgers, New J. Phys. 9, 154 (2007)]

TREES ARE SPECIAL?

- Spectra of the Laplacian **L**
  related to the network structure **A**;
- Random Walks
  on Trees & Modular Networks;

Outline

1. Motivation: & Goals
2. Dense Traffic on Networks
   - Model & Statistical Measures
3. Two Dynamical Classes of Behavior
   - Associated with prototype Network Structures
   - Jamming Transition on Networks
4. Diffusion on Trees
   - Laplacian Spectra: between Structure and Dynamics
   - Random Walks on Trees & Modular Graphs

THEORY

$L_{ij}^{(2)} = \delta_{ij} - \frac{1}{q_i}A_{ij}$; (similarity: $S_{ij} = \frac{1}{\sqrt{q_i}}\delta_{ij}$) $\Rightarrow L_{ij}^{(3)} = \delta_{ij} - \frac{1}{\sqrt{q_i q_j}}A_{ij}$;

RETURN-TIME DISTRIBUTION (autocorrelator):

$$P(\Delta t) = \int_0^\infty d\lambda^L e^{-\lambda^L \Delta t}\rho(\lambda^L) \; ; \qquad (1)$$

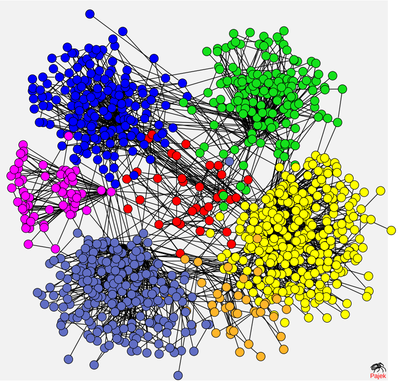
Computed in Ref[*], min.connected nodes important!:

$$P(\Delta t) = B(\Delta t)^\eta \times \exp\left[-3\left(\frac{a}{2}\right)^{2/3}(\Delta t)^{1/3}\right] \; ; \qquad (2)$$

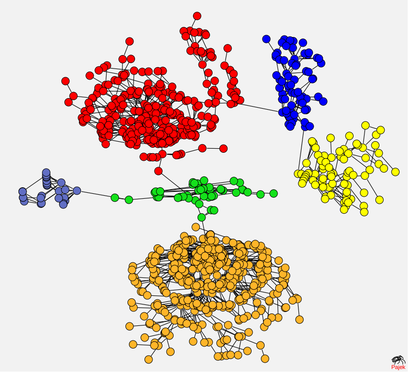
p_0^{eq}, $\eta = -7/30$ for trees; $\eta = -1/18$ for tree-like graphs

[*Samukhin & Dorogovtsev (2007)]

SPECTRA of TWO NETWORKS



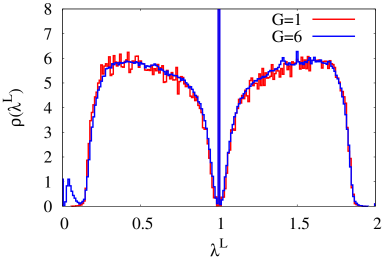
Clustered mdular network



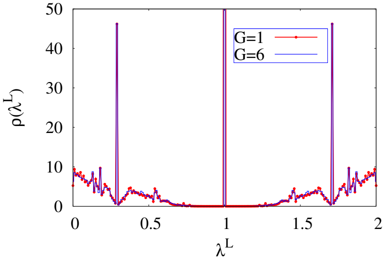
Tree-of-trees

[Mitrović & Tadić (2008)]

SPECTRAL DENSITY



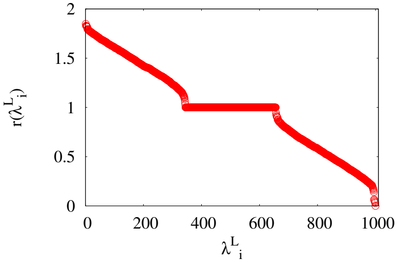
Clustered modular: M=2



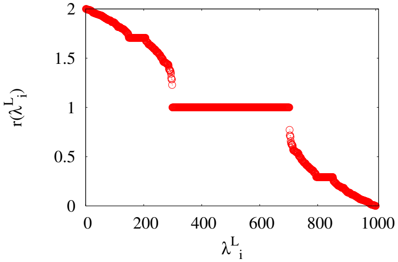
Trees

[Mitrović & Tadić (2008)]

EIGENVALUES RAKNING



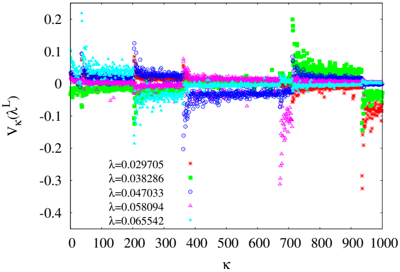
Clustered modular network



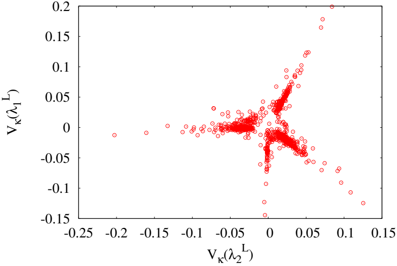
Tree-of-trees

[Mitrović & Tadić (2008)]

EIGENVECTOR LOCALIZATION: Modules



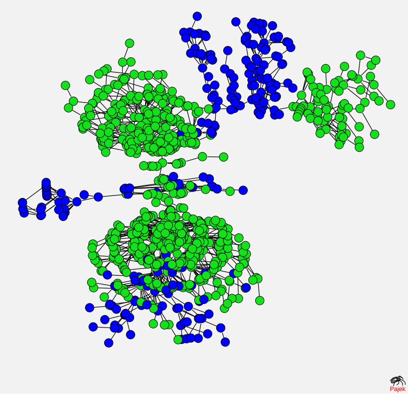
Clustered modular network



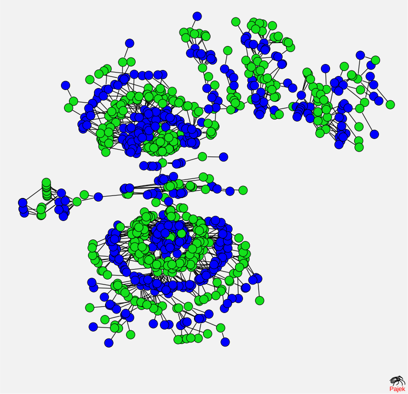
scatter-plot

[Mitrović & Tadić (2008)]

EIGENVECTOR LOCALIZATION: Trees



$\lambda \sim 0$

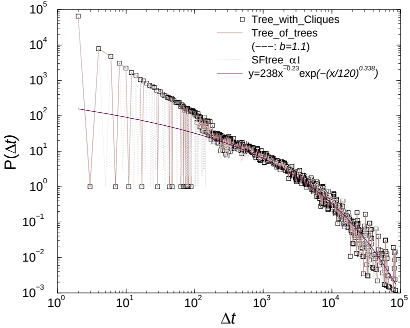


$\lambda = 2$

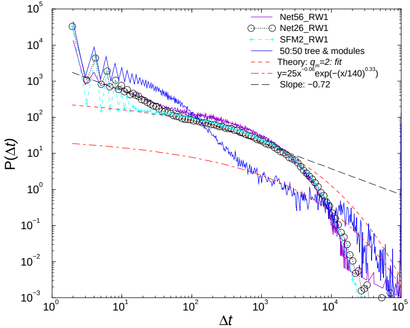
[Mitrović & Tadić (2008)]

Outline

Return-Times of RW



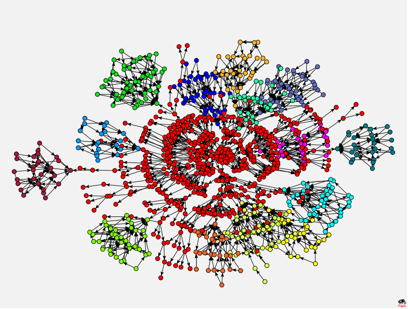
$M = 1$



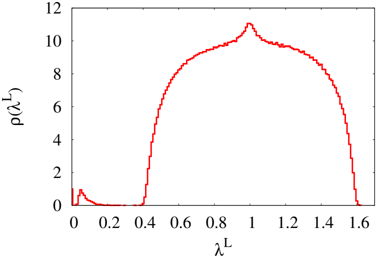
$M \geq 2$; ++

[Mitrović & Tadić (2008)]

Return-Times of RW



net: half-half



modular, $M = 5$

[Mitrović & Tadić (2008)]

CONCLUSIONS

- **Collective dynamic phenomena:**
  can be *related to networks structure*
- **Laplacian spectra:**
  tools *between structure and dynamics*
- **Trees** are different: *trees & tree-like graphs; clustered, correlated, modular graphs; tree-representation of a graph—limited;*

ACKNOWLEDGMENTS

- **Collaboration:**
  - Marija Mitrović (SCLab Institute of Physics, Belgrade)
  - Bernard Kujawski (Brunel University, London)
  - Geoff Rodgers (Brunel University, London)
  - Stefan Thurner (Medical University, Vienna)
- **Projects:** P1-0044 (Slovenia); MRTN-CT-2004-005728 (FP6); COST-P10;
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