Threshold resummation beyond two loop in QCD.

V. Ravindran

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- Factorisation of soft and collinear contributions
- Resummation
- Soft-plus-Virtual at N^3LO_{pSV} for total cross section, rapidity distribution
- Soft-plus-Virtual at N^2LO_{pSV} for large p_T distributions
- Conclusions

In collaboration with

J. Blümlein, W.L. van Neerven, J. Smith

Factorisation Theorem in Quantum Chromodynamics(QCD)

Hadronic cross section in terms of partonic cross sections convoluted with appropriate PDF:

$$2S\, d\sigma^{P_1 P_2}\left(\tau, m_h^2\right) = \sum_{ab} \int_\tau^1 \frac{dx}{x} \Phi_{ab}\left(x, \mu_F\right) 2\hat{s}\, d\hat{\sigma}^{ab}\left(\frac{\tau}{x}, m_h^2, \mu_F\right)$$

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- Large number of Feynman diagrams
- Complexity of the integrals (Ultraviolet, Soft and Collinear divergences)
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- To identify and resum the important contributions to all orders

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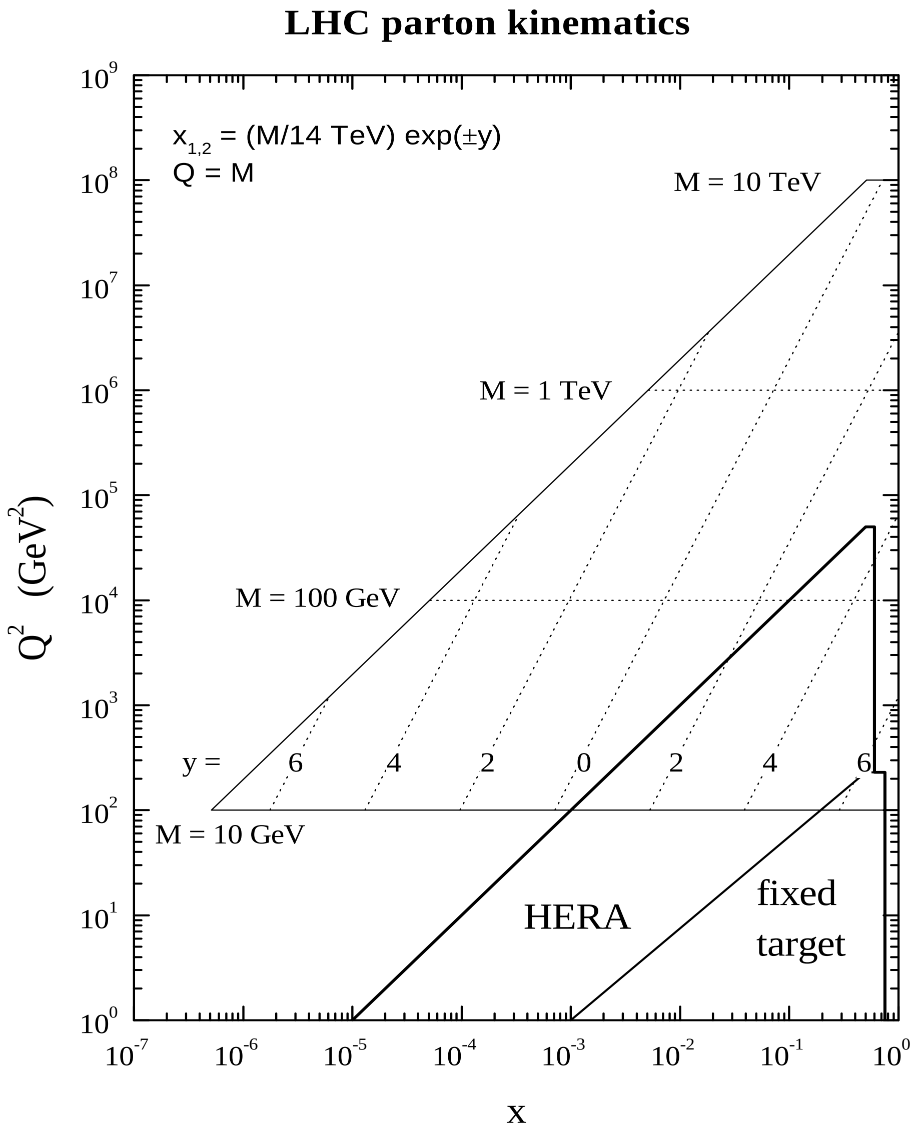
Partron distribution functions have been extracted very precisely at DESY

PDF from LHC

[*Martin, Roberts, Stirling, Thorne*]



$g + g \longrightarrow H$ at LHC

- Hinchcliff,Dawson, Djuoadi, Spira, Zerwas, Laenen, Moch, Catani, Grazzini, Harlander, Kilgore, Anastasiou, Melnikov, van Neerven, Smith, Ravindran, many more

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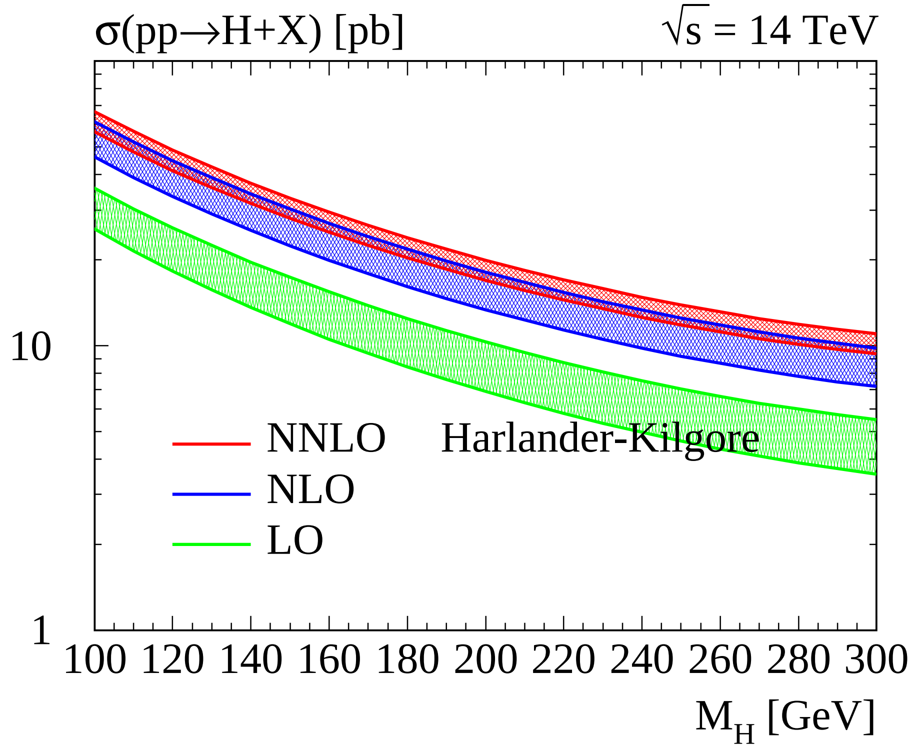

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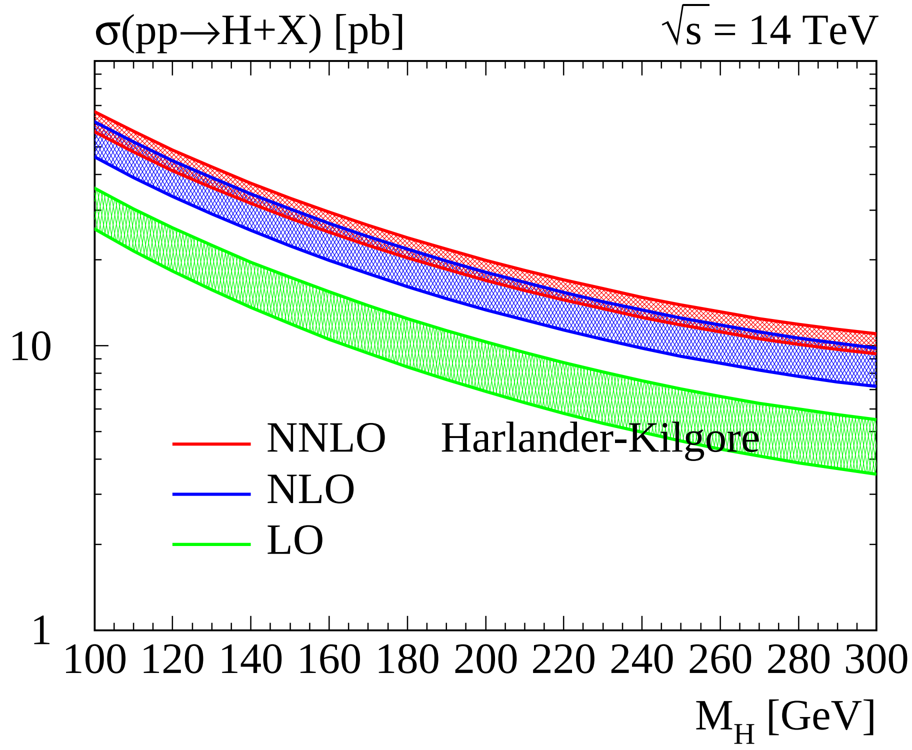

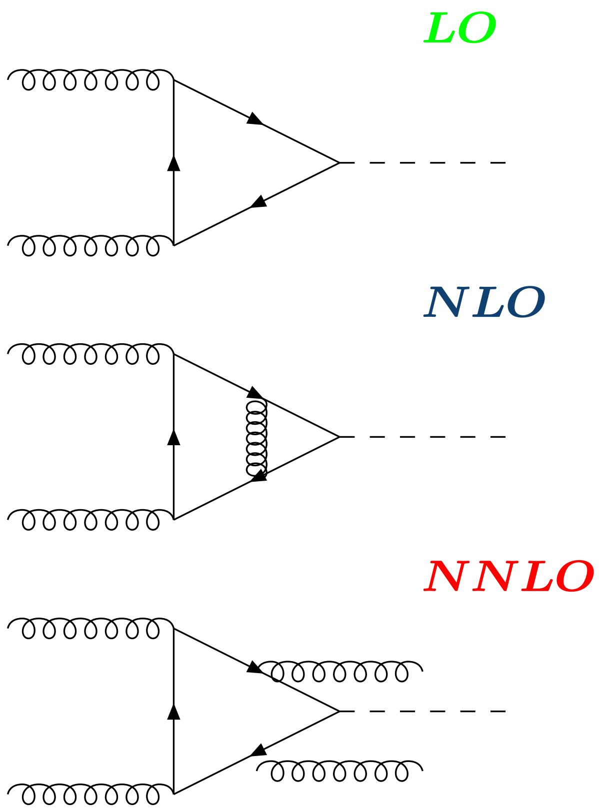

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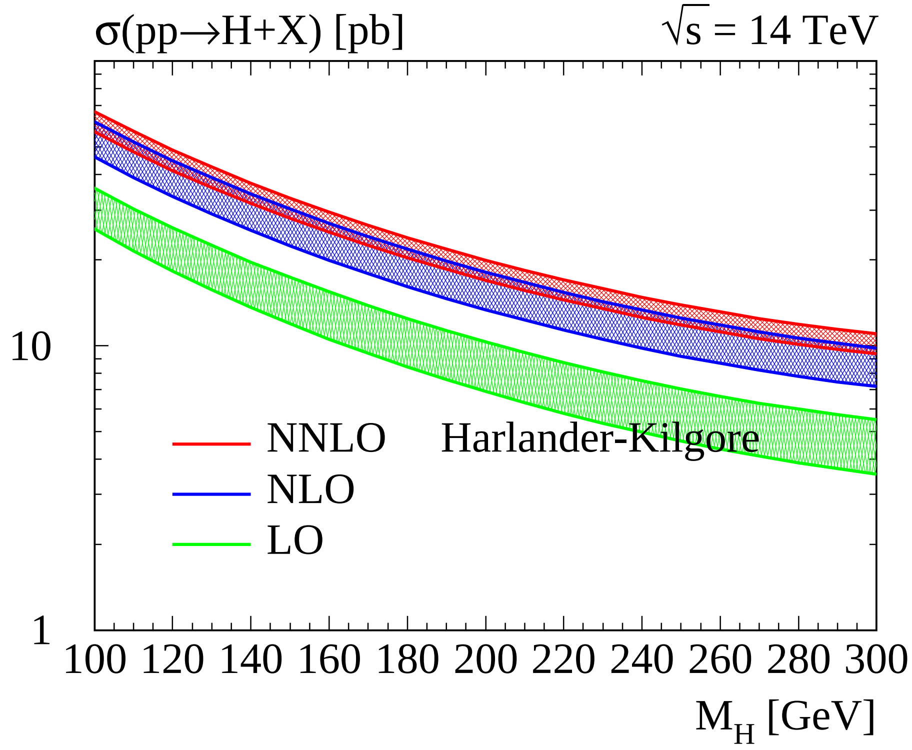

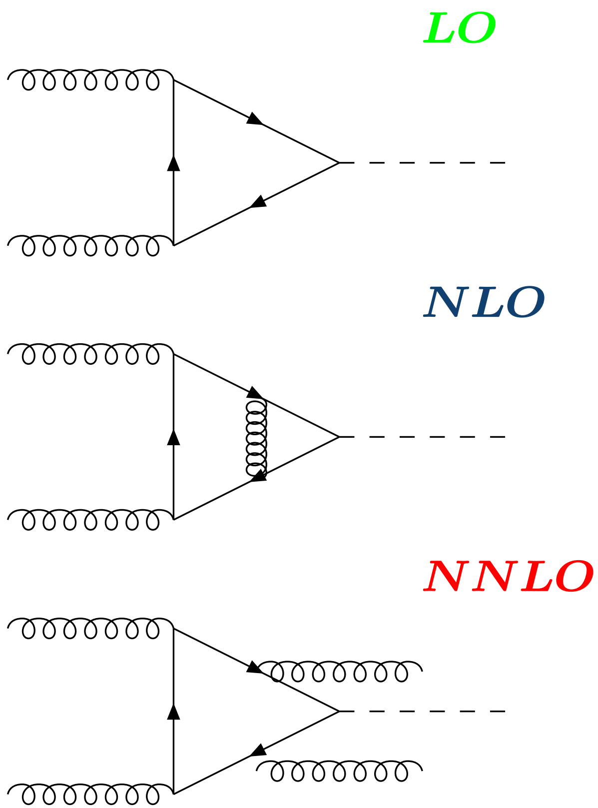

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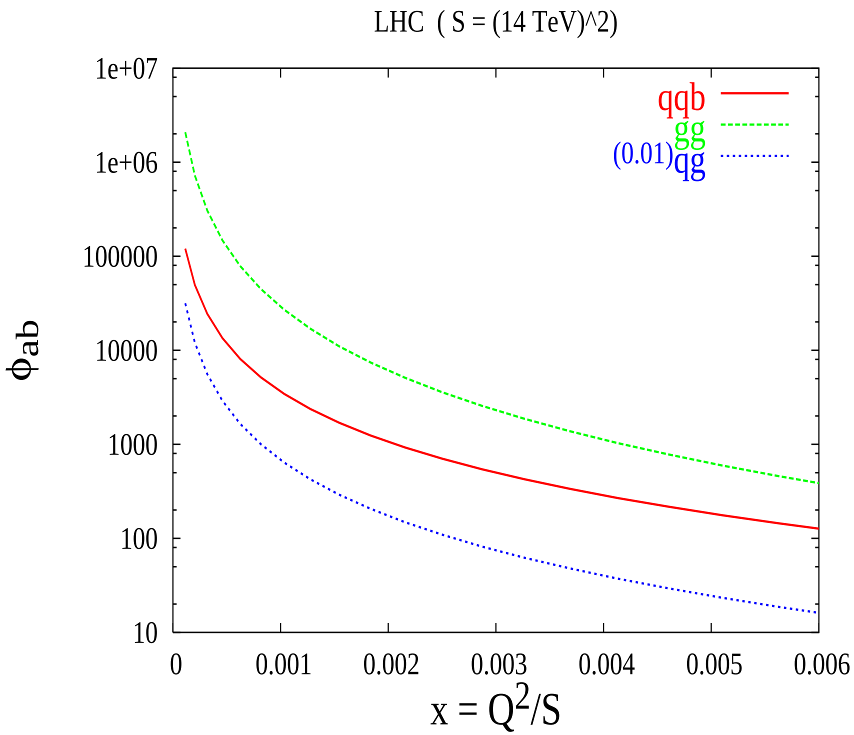

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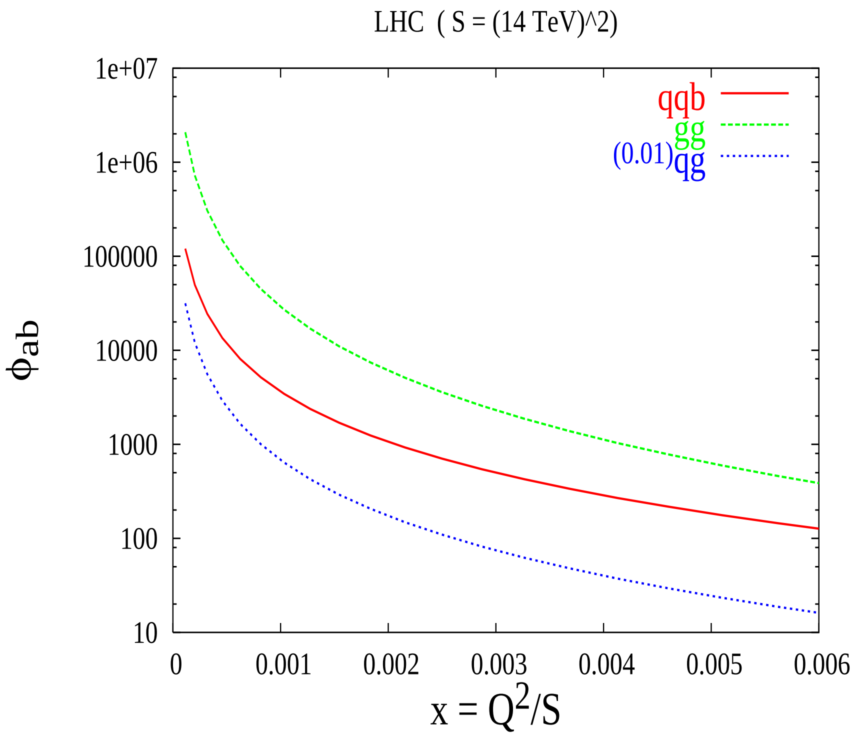

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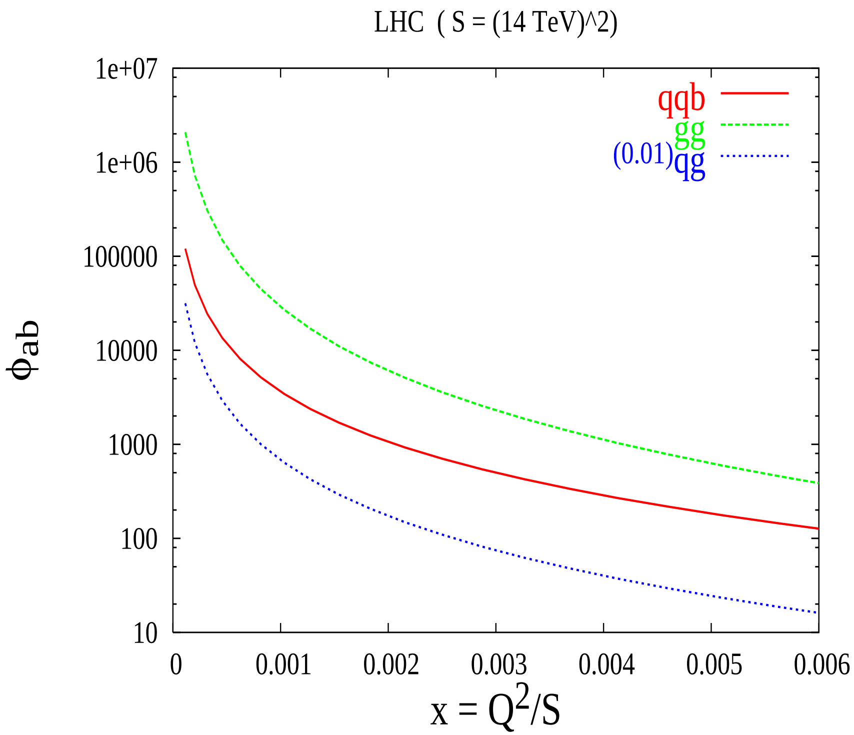

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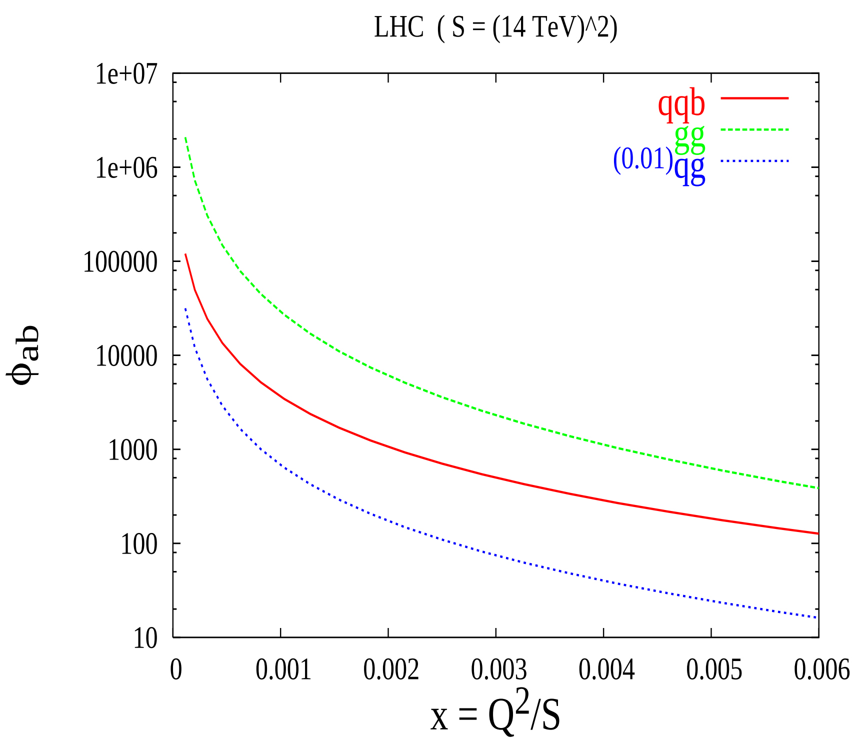

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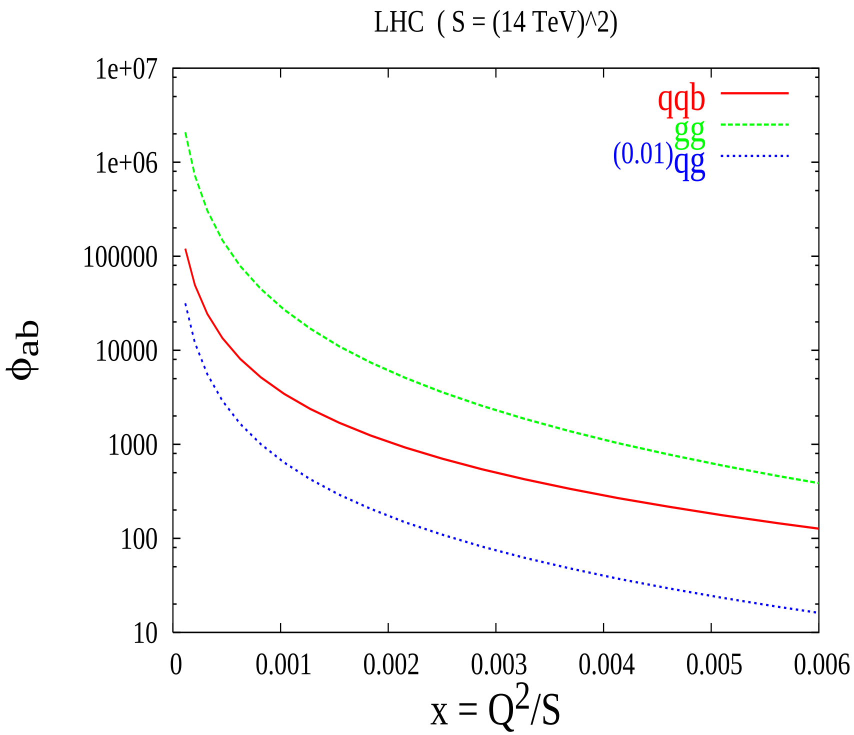

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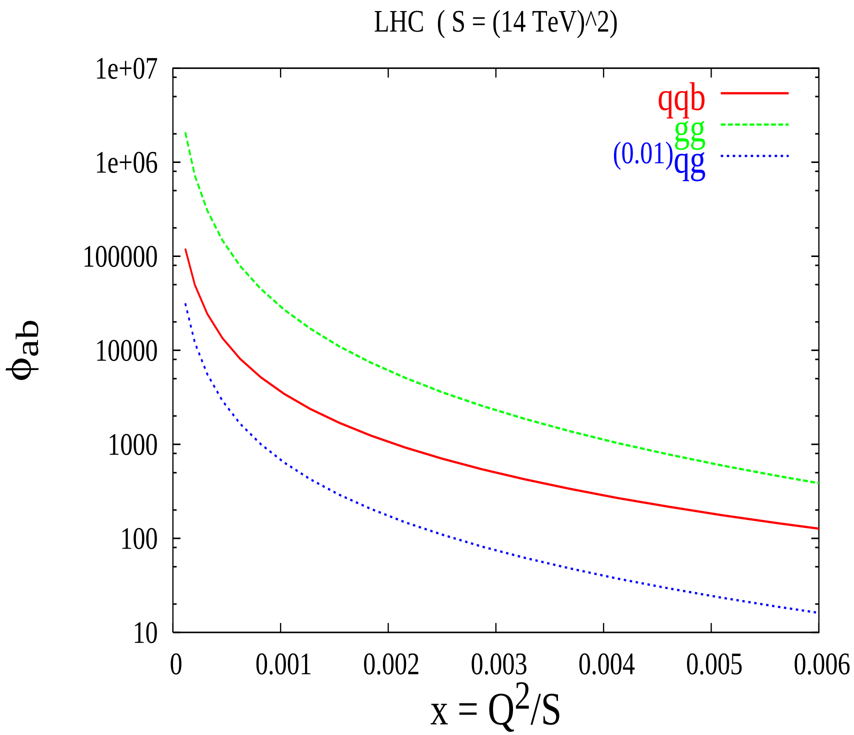

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Catani et al, Harlander and Kilgore

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- $\mathcal{C}^{(0)}$:

$$\mathcal{C}^{(0)} = \mathcal{C}_0^{\delta}\, \delta(1-z) + \sum_{k=0}^{\infty} \mathcal{C}_0^{(k)} \left(\frac{\ln^k(1-z)}{(1-z)} \right)_+$$

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  OR

  Extract from "Form factors and DGLAP kernels" using

  1) Factorisation theorem 2) Renormalisation Group Invariance

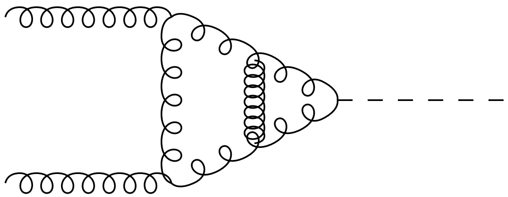
  3) Sudakov Resummation

Soft gluon cancellation

Soft gluon cancellation

$V.\ Ravindran$

$\boldsymbol{\sigma^V_{ab}(z, Q^2, \varepsilon_s, \varepsilon_c)}$ - Virtual soft gluon
$(|\boldsymbol{F^I}|^2)$



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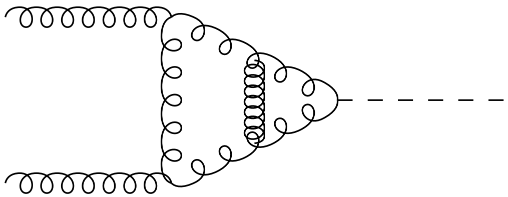

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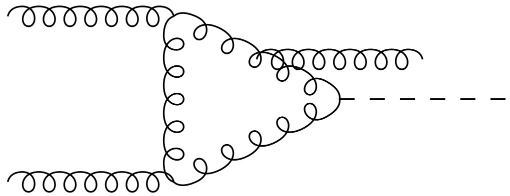

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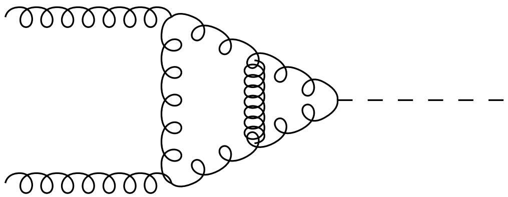

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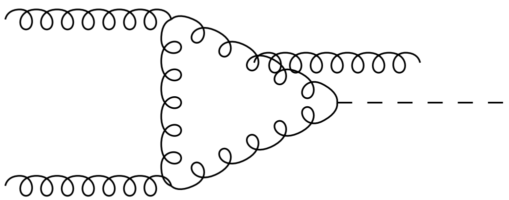

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- ε_c - collinear parton regulator

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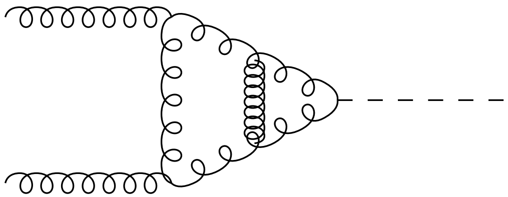

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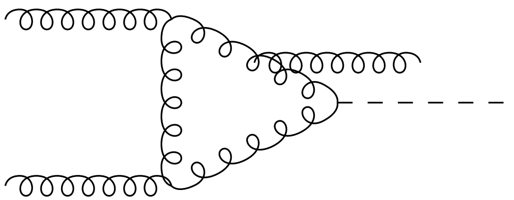

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Only collinear partons remain

Factorisation of Collinear partons

Due to the massless partons, collinear singularities appear in

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$$\Delta(z,Q^2) = \delta(1-z) + \alpha_s(Q^2)\left(a_{11}\delta(1-z) + \frac{a_{12}}{(1-z)_+} + a_{13}\left(\frac{\ln(1-z)}{1-z}\right)_+ \right.$$
$$\left. + R_1(z)\right) + \alpha_s^2(Q^2)\left(\cdots + \cdots + \cdots + R_2(z)\right) + \cdots$$

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Soft plus Virtual at N^3LO and beyond

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Using "factorisation" of UV, Soft and Collinear:

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$$\hat{a}_s = \frac{\hat{g}_s^2}{16\pi^2} \qquad m=\frac{1}{2}\quad \text{for}\quad \text{DIS}/e^+e^-, \quad m=1 \qquad \text{for}\quad \text{DY}, \text{Higgs}$$

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Most general solution:

$$\begin{aligned}\Phi^I(\hat{a}_s, q^2, \mu^2, z, \varepsilon) \quad &= \quad \Phi^I(\hat{a}_s, q^2(1-z)^{2m}, \mu^2, \varepsilon) \\ &= \quad \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2(1-z)^{2m}}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \left(\frac{i\, m\, \varepsilon}{2(1-z)}\right) \hat{\phi}^{\,I,(i)}(\varepsilon)\end{aligned}$$

Solution to (Soft)Sudakov Equation

$$q^2 \frac{d}{dq^2} \Phi^I \left(\hat{a}_s, q^2, \mu^2, z, \varepsilon\right) \quad = \quad \frac{1}{2}\left[\overline{K}^I \left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, z, \varepsilon\right) + \overline{G}^I \left(\hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, z, \varepsilon\right)\right]$$

Solution to (soft) Sudakov equation:

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where

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\end{aligned}$$

- All the poles are known upto three loop
- All the poles and finite terms are known upto two loop level

Threshold Resummation

- Alternate derivation for the threshold resummation formula in z space for both DY and DIS:

$$\begin{aligned}
\Phi^I_P(\hat{a}_s, q^2, \mu^2, z, \varepsilon) &= \left(\frac{m}{1-z}\left\{\int_{\mu_R^2}^{q^2(1-z)^{2m}\delta_P} \frac{d\lambda^2}{\lambda^2} A_I\left(a_s(\lambda^2)\right)\right.\right.\\
&\left.\left. + \overline{G}^I_P\left(a_s\left(q^2(1-z)^{2m}\delta_P\right), \varepsilon\right)\right\}\right)_+\\
&+ \delta(1-z) \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2\delta_P}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i\, \hat{\phi}_P^{I,(i)}(\varepsilon)\\
&+ \left(\frac{m}{1-z}\right)_+ \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{\mu_R^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i\, \overline{K}^{\,I,(i)}(\varepsilon)
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- The threshold exponents D_i^I for DY and B_i^I for DIS are related to $\overline{G}_P^I(\varepsilon = 0)$.
- $\overline{G}_P^I(\varepsilon = 0)$ upto three loop gives D_i^I and B_i^I for $i = 1, 2, 3$ (Moch,Vermaseren,Vogt)

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- Expansion of $\mathcal{C}e^{\left(2\Phi_P^I\right)}$ leads to soft part of the cross section.
- Soft part of Wilson Coeffient of $F_2(x, Q^2)$ structure functions upto "four loops" can be reproduced (Moch,Vogt,Vermaseren)

Hadro production in e^+e^- annihilation at N^3LO

Blümlein and VR

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$$\Phi_{DIS}(\hat{a}_s, Q^2, \mu^2, x_{Bj}, \varepsilon) = \Phi_{ee}(\hat{a}_s, q^2, \mu^2, x_{ee}, \varepsilon)$$

$$P_{II}(x_{Bj}) = \tilde{P}_{II}(x_{ee}) \qquad \textbf{Distributions}$$

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- From DIS results, we can predict soft plus virtual part of the coefficient functions for hadro production in e^+e^- annihilation upto three loop level.

$$N^3LO \quad coefficient \quad function \quad C_{ee}^{(3),sv}(\alpha_s, z) \qquad \text{New} \quad \text{result}$$

Scale variation at N^3LO_{pSV} for Higgs production

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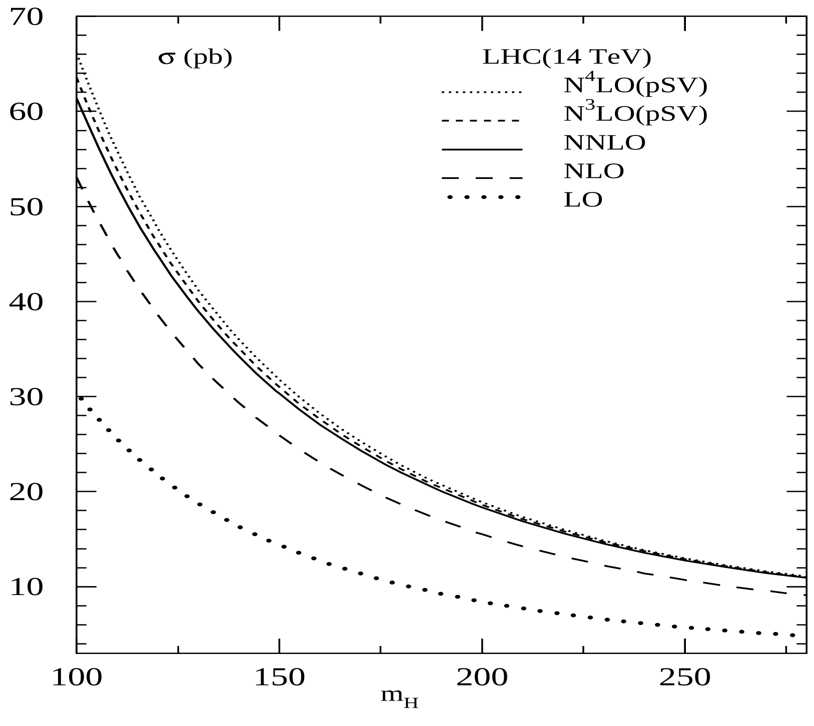

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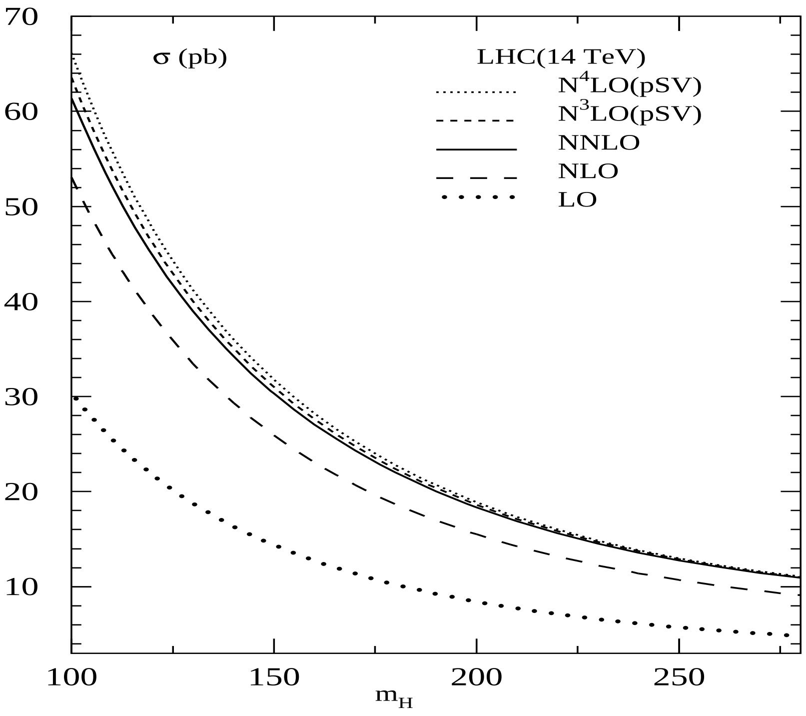

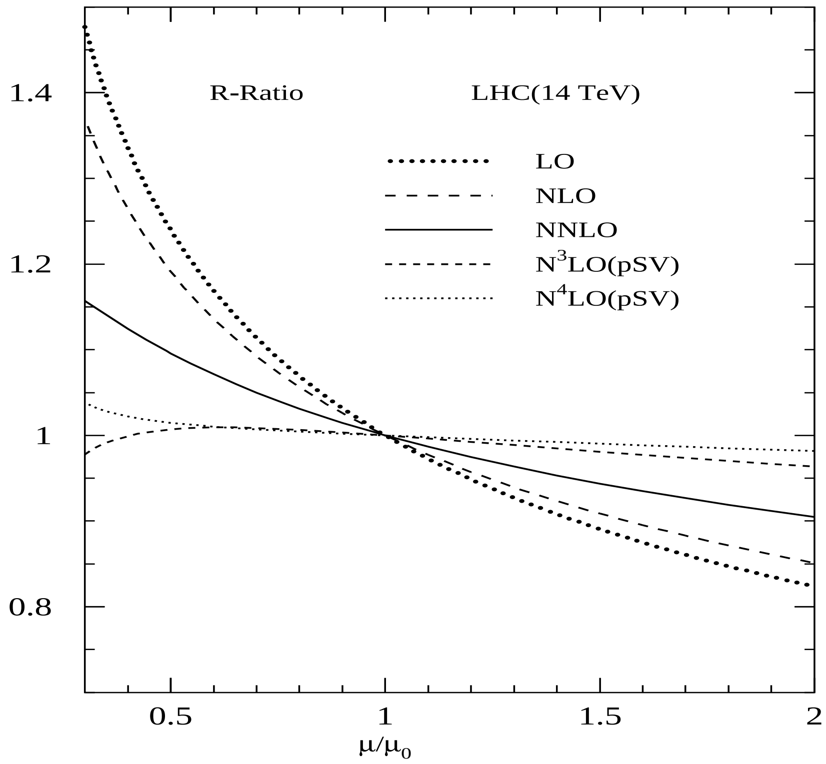

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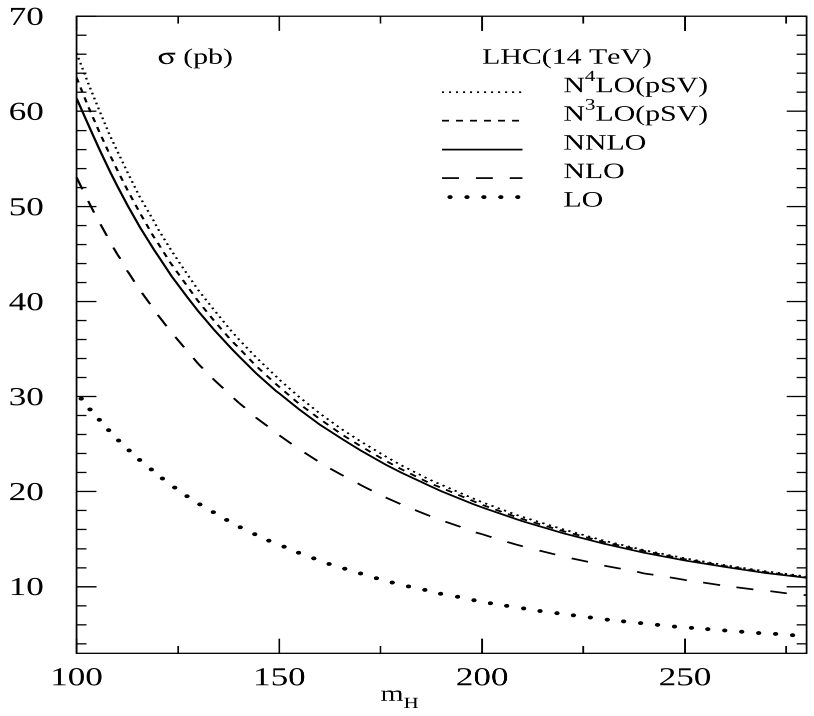

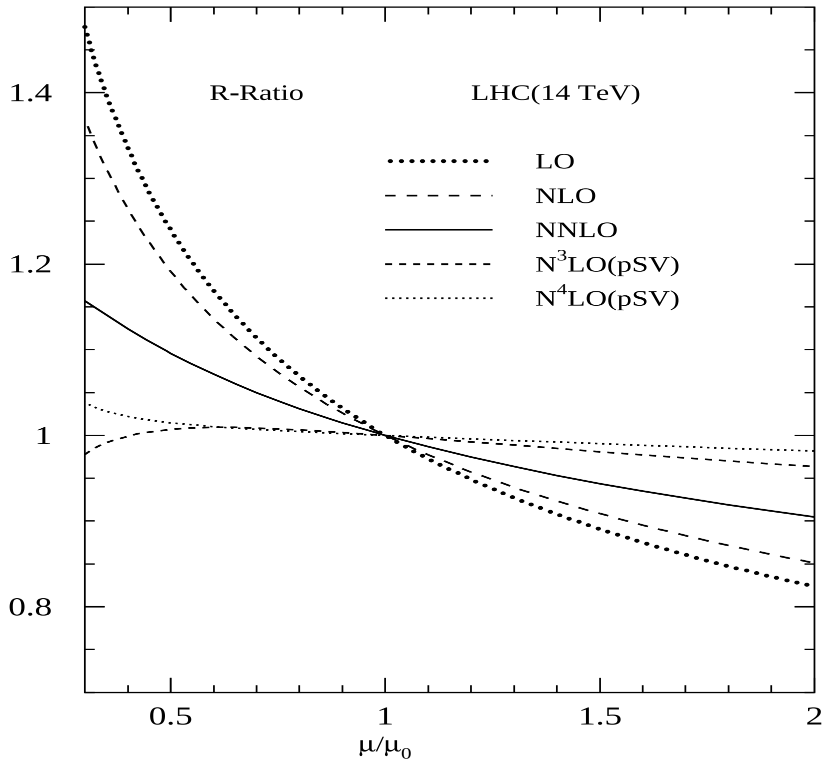

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Less-Inclusive Drell-Yan and Higgs Productions

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$$\frac{d\sigma^I}{dx} = \sigma^I_{\rm Born}(x_1^0, x_2^0, q^2) W^I(x_1^0, x_2^0, q^2)\ , \qquad I = q, b, g \quad ,$$

The x_i^0 $(i = 1, 2)$ are related to the kinematical variables q^2 and x.

$$x = x_F = \frac{2(p_1 - p_2)\cdot q}{S}, \qquad \text{and} \qquad x = Y = \frac{1}{2}\ln\left(\frac{p_2 \cdot q}{p_1 \cdot q}\right) .$$

$$W^I(x_1^0, x_2^0, q^2) \quad = \quad \sum_{ab=q,\overline{q},g} \int_0^1 dx_1 \int_0^1 dx_2\, \mathcal{H}^I_{ab}(x_1, x_2, \mu_F^2)$$

$$\times \int_0^1 dz_1 \int_0^1 dz_2\, \delta(x_1^0 - x_1 z_1)\, \delta(x_2^0 - x_2 z_2)$$

$$\times \Delta^I_{d,ab}(z_1, z_2, q^2, \mu_F^2, \mu_R^2)\ .$$

Here, μ_R is the renormalisation scale and μ_F the factorisation scale.

Rapidity of Drell-Yan and its Scale dependence at NNLO

Anastasiou, Dixon, Melnikov, Petriello

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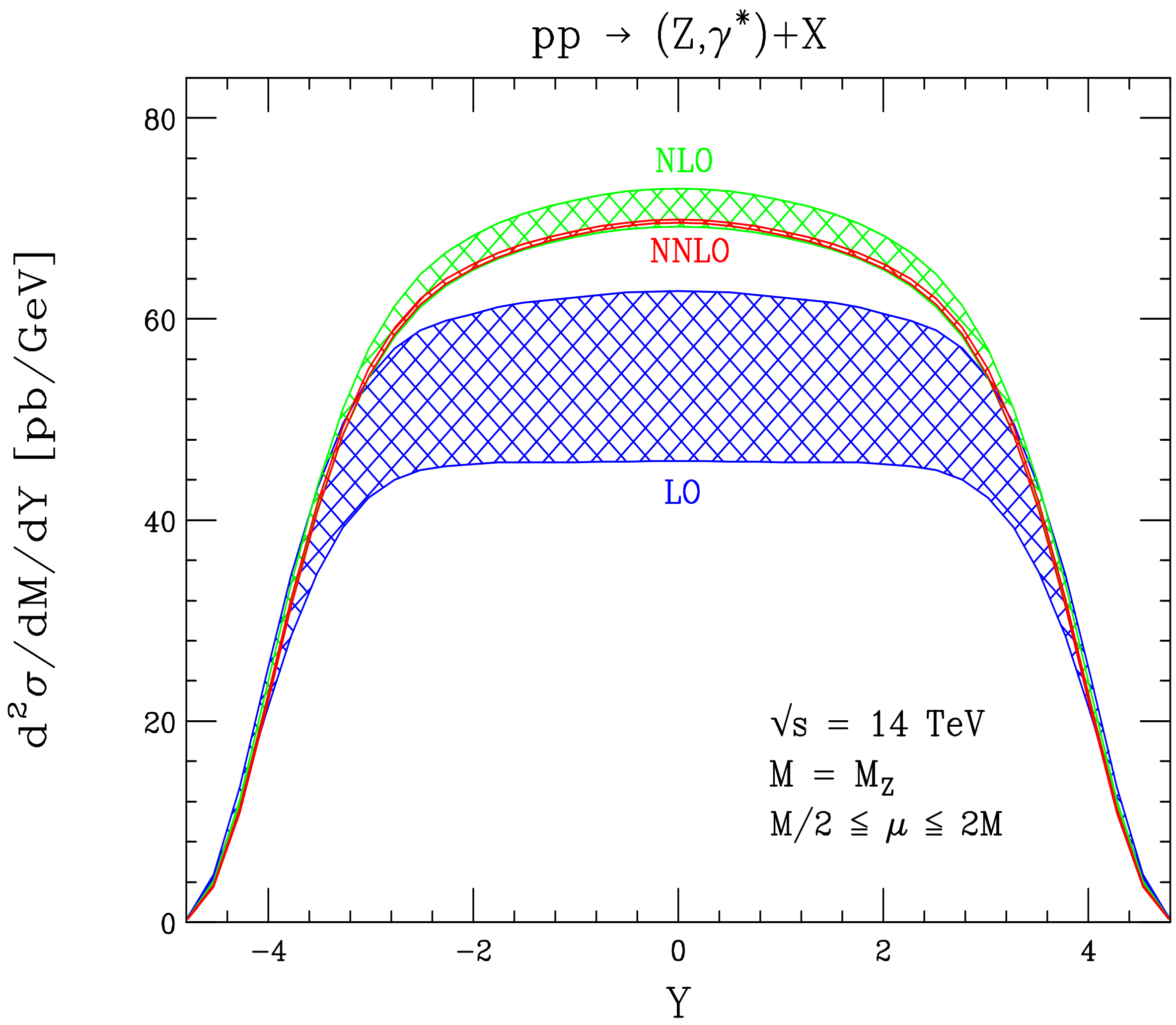

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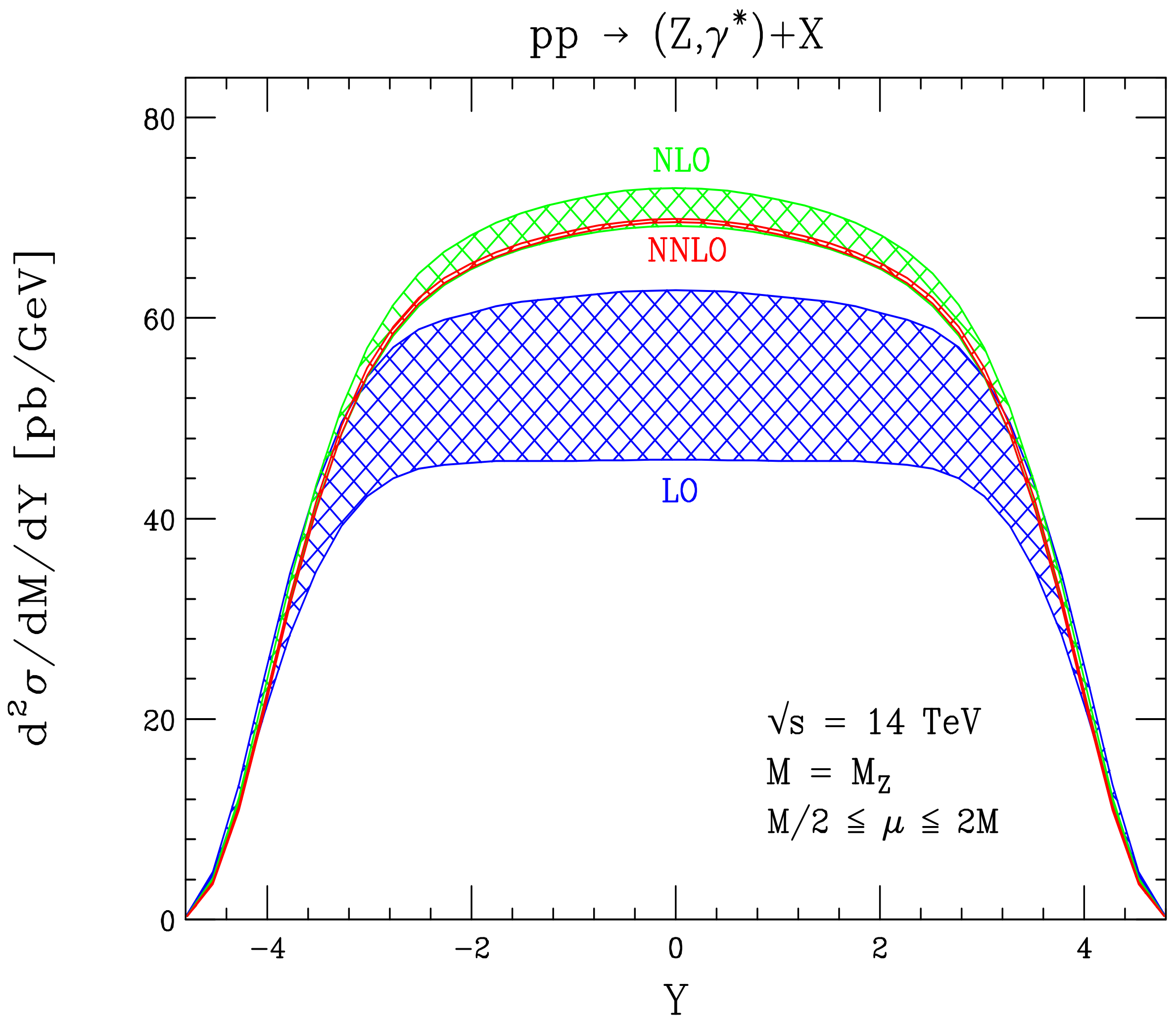

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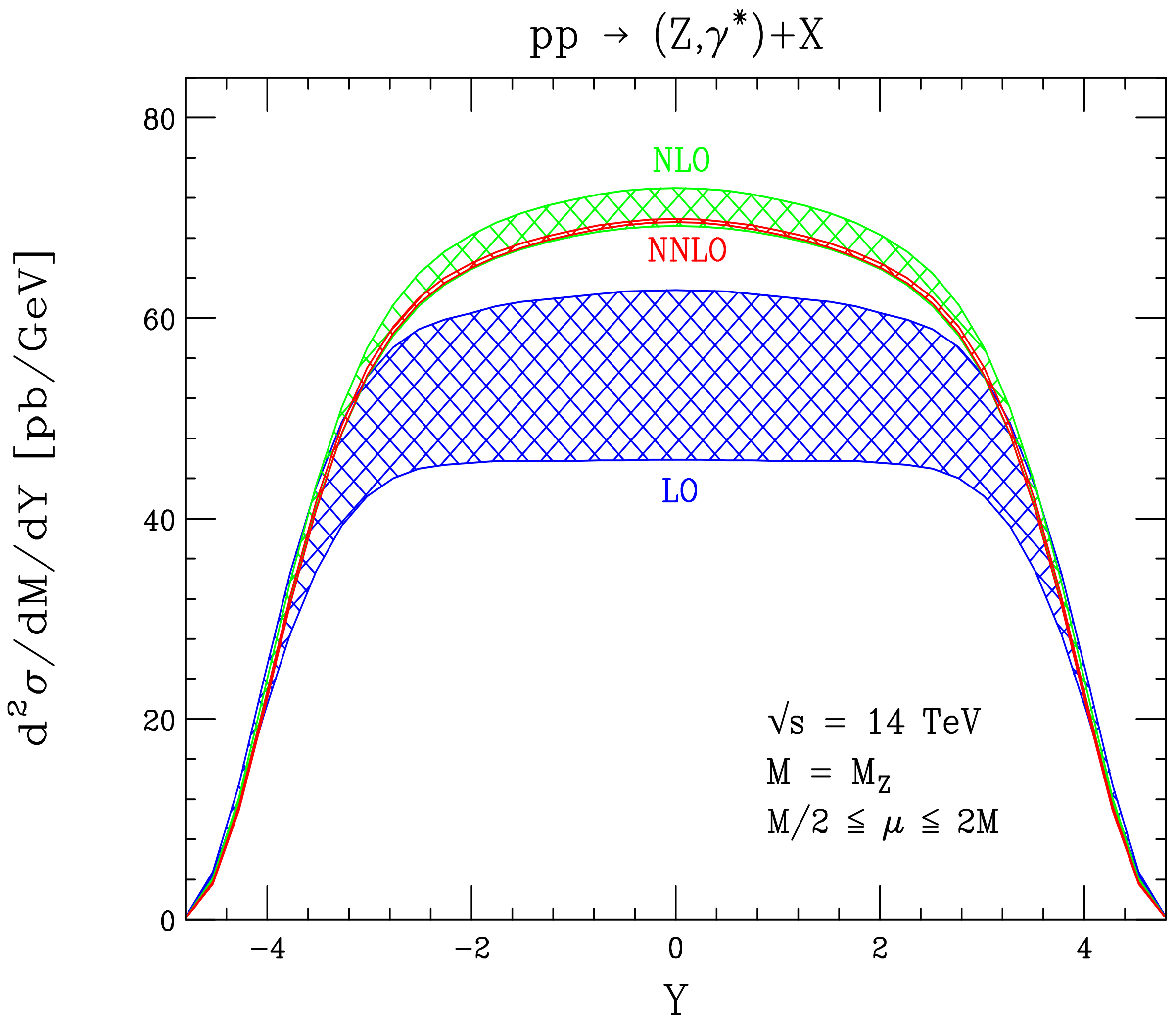

- NNLO exact reduces the scale uncertainity significantly
- Also "most difficult" computation in QCD
  What is next?

Soft Gluons

Soft Gluons

We first study the contributions coming from the soft gluons.

$$\Delta^{I}_{d,ab}(z_1, z_2, q^2, \mu_F^2, \mu_R^2) = \Delta^{\rm hard}_{I,ab}(z_1, z_2, q^2, \mu_F^2, \mu_R^2) + \delta_{a\bar{b}}\Delta^{\rm sv}_{d,I}(z_1, z_2, q^2, \mu_F^2, \mu_R^2),$$

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The soft-plus-virtual parts of the differential cross sections ($\Delta^{\text{sv}}_{d,I}(z_1, z_2, q^2, \mu_R^2, \mu_F^2)$) are found to be

$$\Delta^{\text{sv}}_{d,I}(z_1, z_2, q^2, \mu_R^2, \mu_F^2) = \mathcal{C}\exp\left(\Psi^{I}_{d}(q^2, \mu_R^2, \mu_F^2, z_1, z_2, \varepsilon)\right)\Big|_{\varepsilon=0},$$

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The symbol "$\mathcal{C}$" means convolution.

$$\begin{aligned}\mathcal{C}e^{f(z_1,z_2)} &= \delta(1-z_1)\delta(1-z_2) + \frac{1}{1!}f(z_1,z_2) + \frac{1}{2!}f(z_1,z_2)\otimes f(z_1,z_2)\\ &\quad + \frac{1}{3!}f(z_1,z_2)\otimes f(z_1,z_2)\otimes f(z_1,z_2) + \cdots.\end{aligned}$$

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The function $f(z_1, z_2)$ is a distribution of the kind $\delta(1-z_j)$,

$$\mathcal{D}_i(z_j) = \left[\frac{\ln^i(1-z_j)}{(1-z_j)}\right]_+ \qquad i = 0, 1, \cdots, \quad \text{and} \quad j = 1, 2\,,$$

Rapidity distribution $d\sigma/dy$ of Higgs at N^3LO_{pSV}

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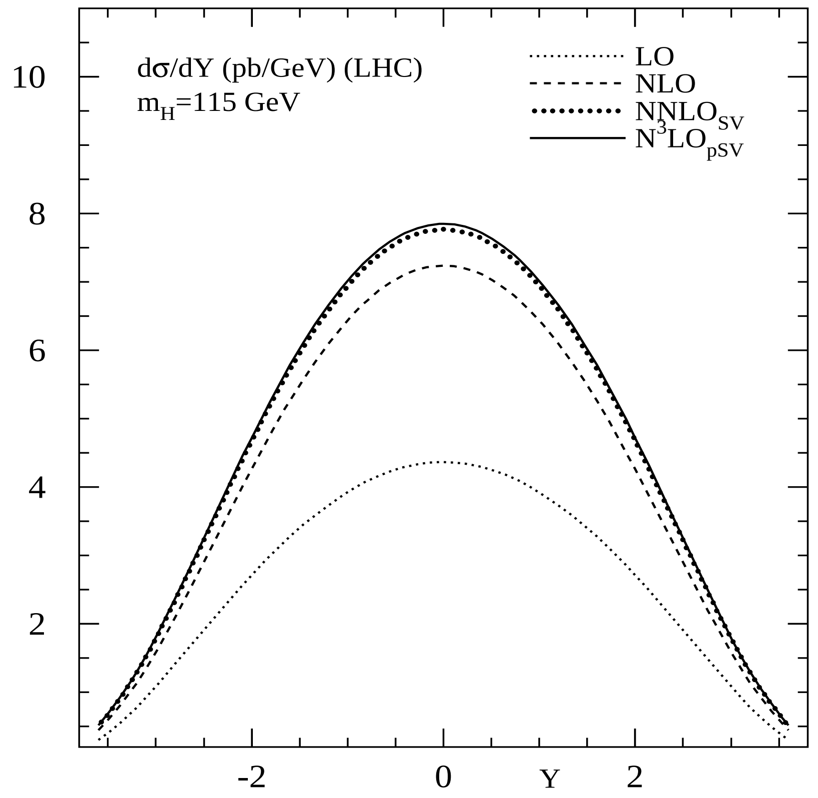

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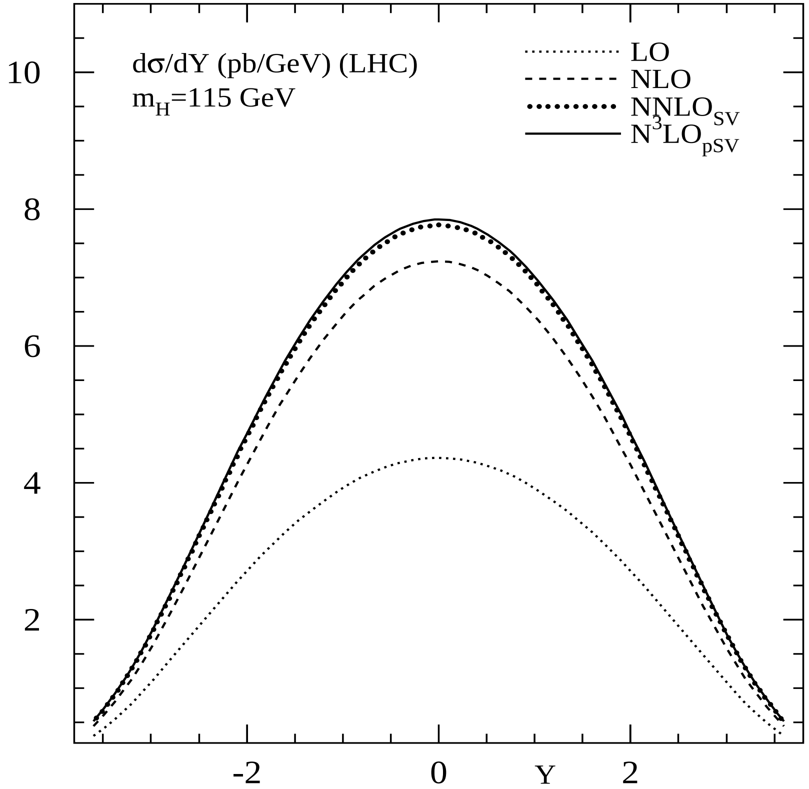

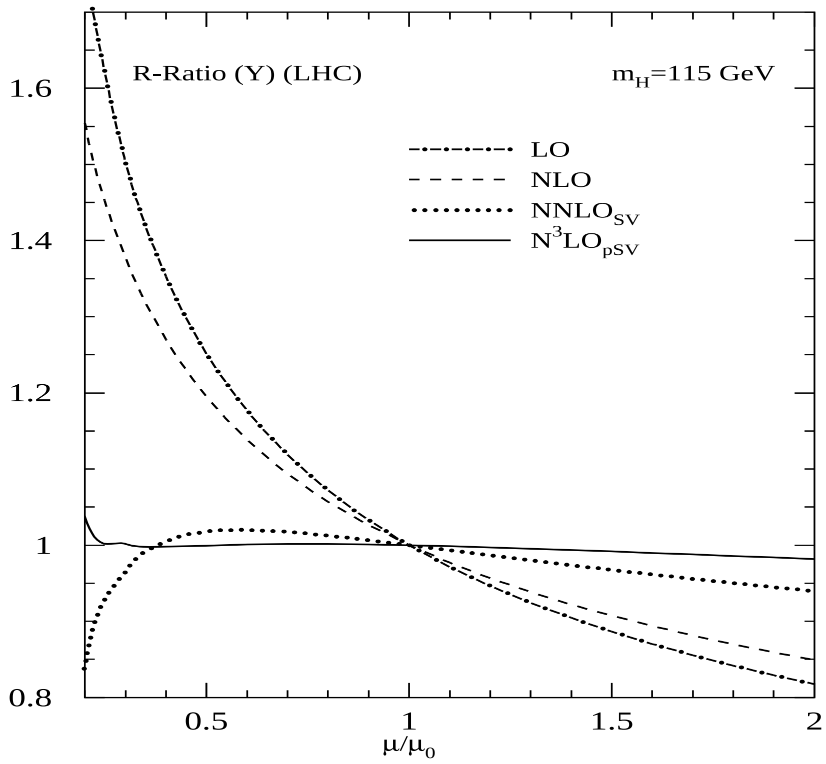

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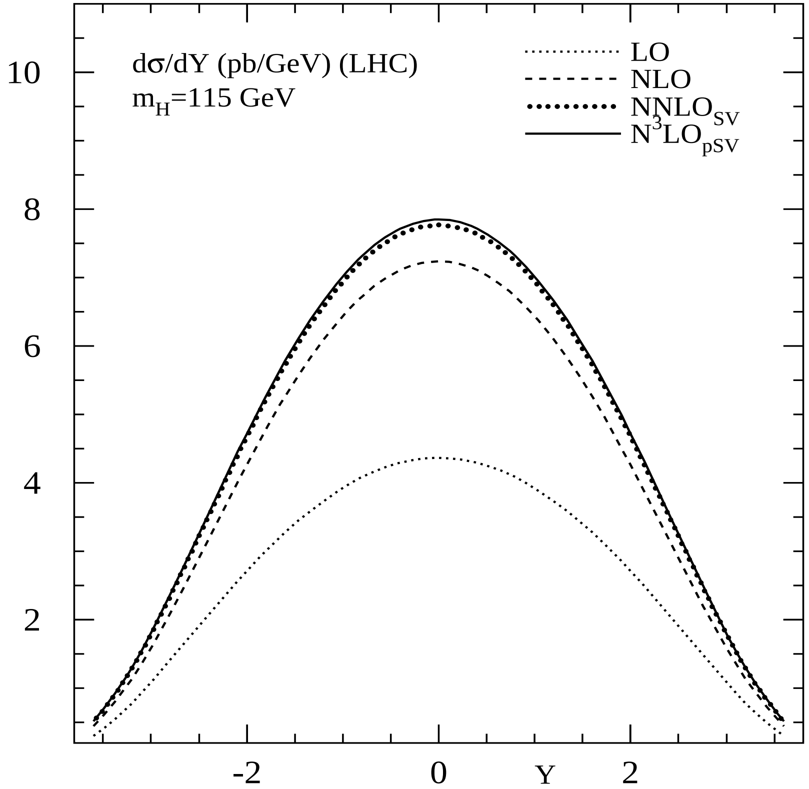

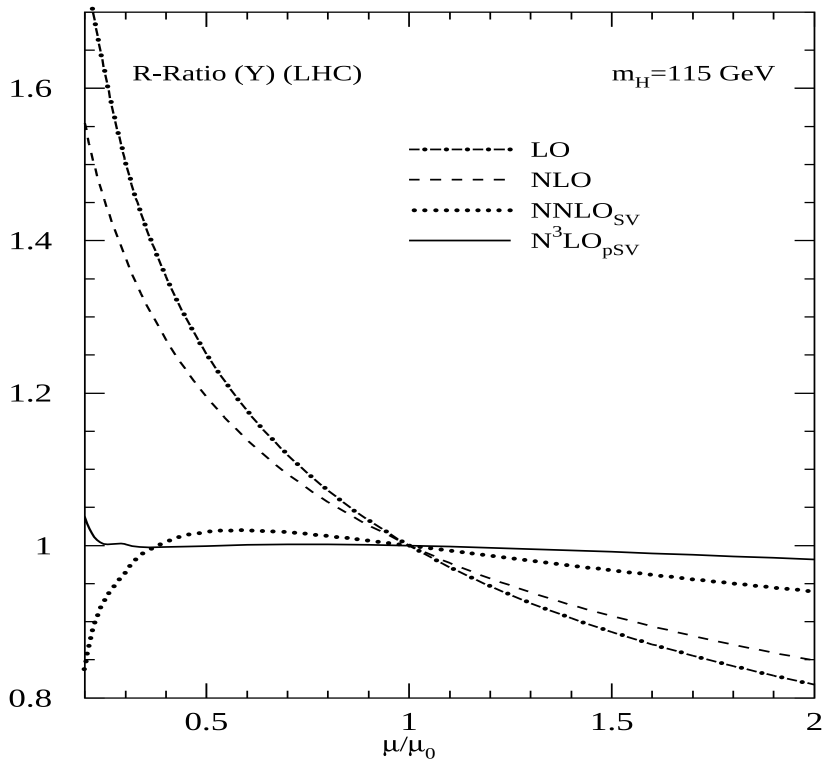

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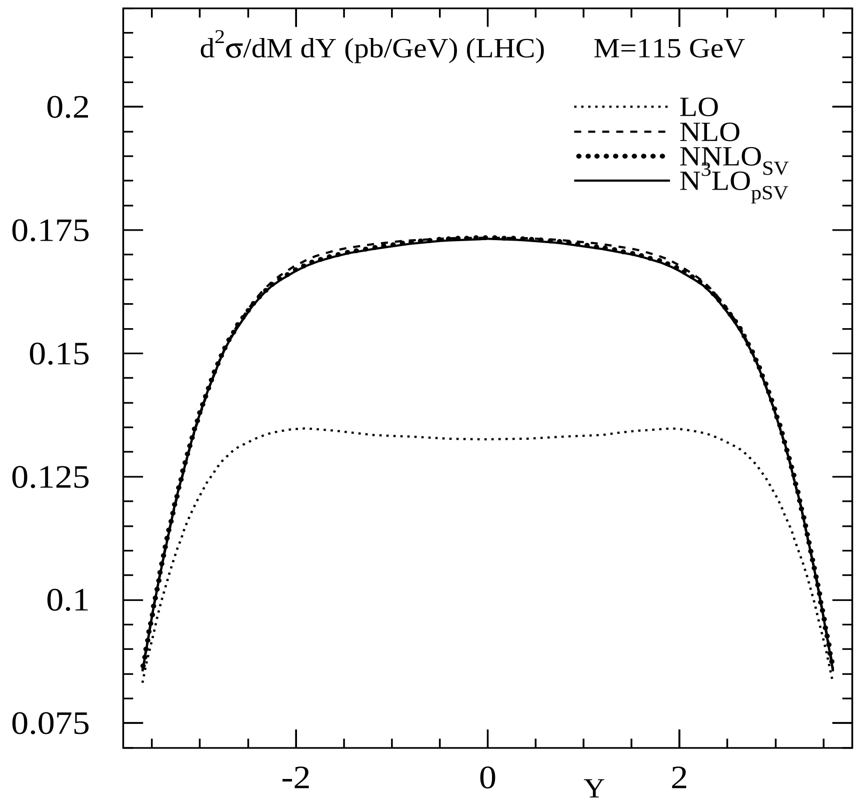

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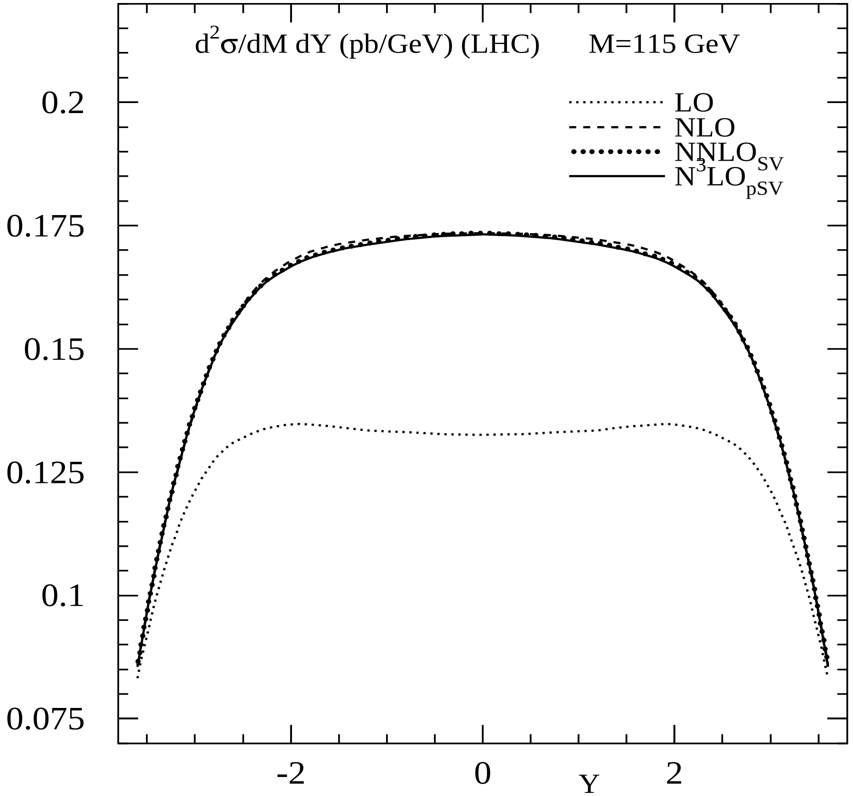

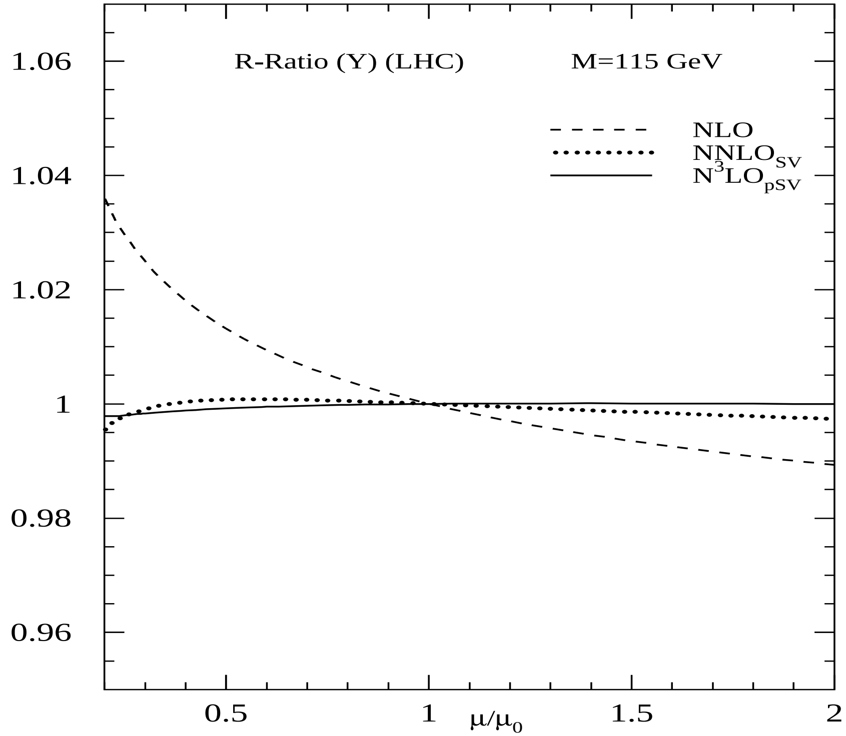

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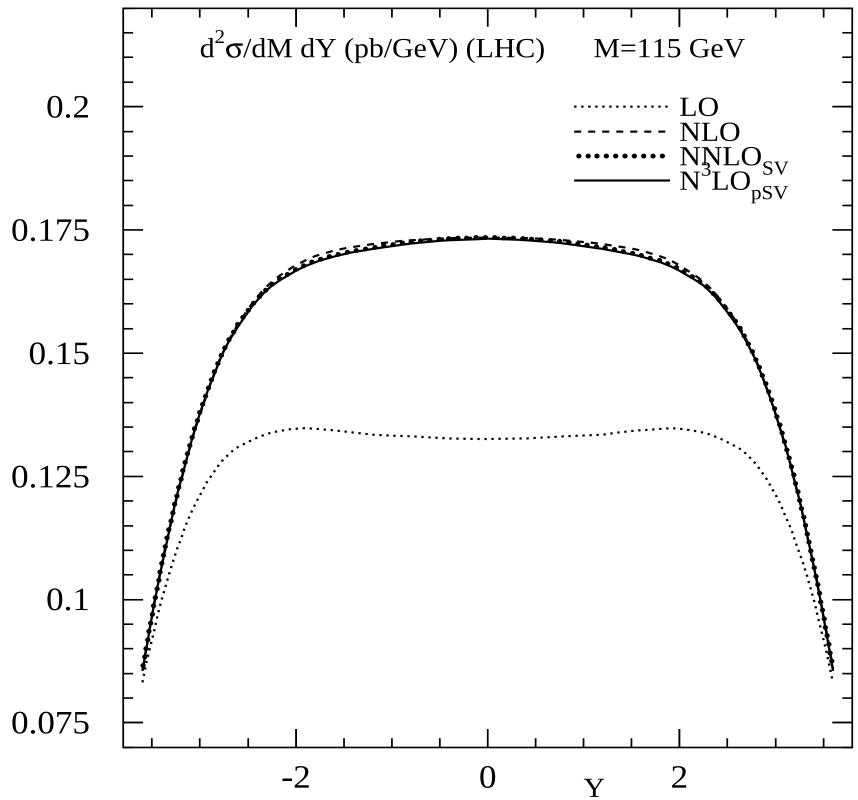

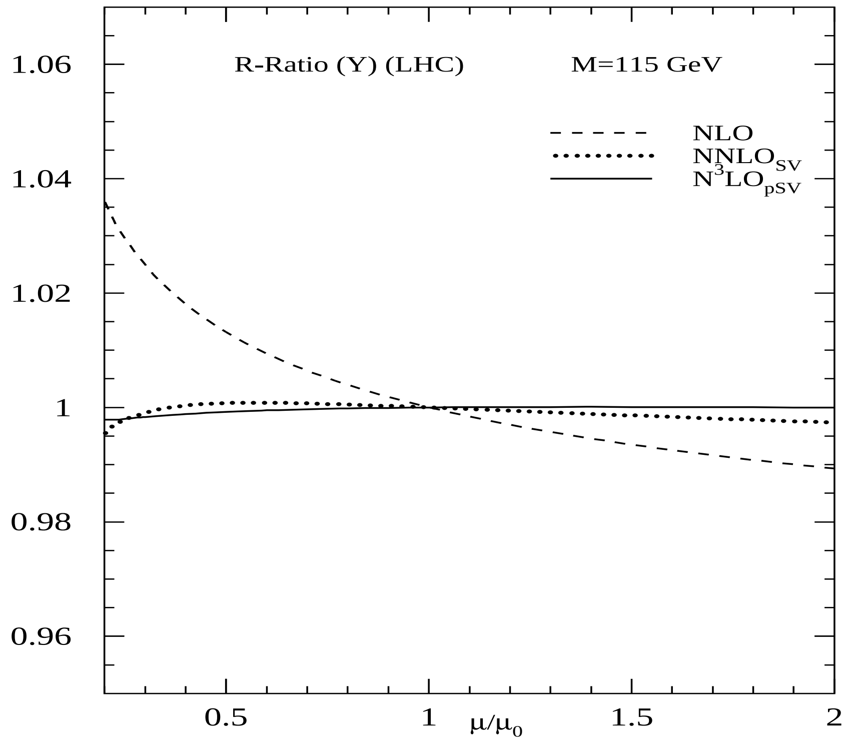

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Distributions:

$$\delta(s_4), \qquad \left(\frac{\ln^i(s_4/p_T^2)}{s_4}\right)_+ \qquad\qquad i = 0, 1, 2, \cdots$$

where

$$\int_0^{p_T^2} \frac{ds_4}{s_4} f(s_4) \left(\frac{\ln^i s_4/p_T^2}{s_4}\right)_+ = \int_0^{p_T^2} \frac{ds_4}{s_4} \Big(f(s_4) - f(0)\Big) \left(\frac{\ln^i s_4/p_T^2}{s_4}\right)$$

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$$\Delta_{ab}(p_T) = \left(\frac{d^2\sigma^{(0)}}{dtdu}\right)^{-1} \frac{d^2\sigma}{dtdu} \qquad t, u \quad - \quad Mandelstam \quad variables$$

The threshold region:

$$s_4 = s + t + u \quad \longrightarrow \quad 0$$

Distributions:

$$\delta(s_4), \qquad \left(\frac{\ln^i(s_4/p_T^2)}{s_4}\right)_+ \qquad\qquad i = 0, 1, 2, \cdots$$

where

$$\int_0^{p_T^2} \frac{ds_4}{s_4} f(s_4) \left(\frac{\ln^i s_4/p_T^2}{s_4}\right)_+ = \int_0^{p_T^2} \frac{ds_4}{s_4} \Big(f(s_4) - f(0)\Big) \left(\frac{\ln^i s_4/p_T^2}{s_4}\right)$$

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At leading order: $a + b \longrightarrow \gamma + c$ is proportional to $\delta(s_4)$

NLO p_T distributions

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Contributions:

1) Real emission: $\boldsymbol{a + b \longrightarrow \gamma(p_T) + c + d}$

2) Virtual corrections: $\boldsymbol{a + b \longrightarrow \gamma(p_T) + c}$

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$$\Delta^{NLO}(p_T) \quad = \quad \delta(s_4) + a_s(\mu_R^2)\left(c_0\delta(s_4) + \sum_{i=1,2} c_i(\mu_F^2, t_i)\left(\frac{\ln^{i-1}(s_4/p_T^2)}{s_4}\right)_+ + Reg(t_i)\right)$$

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arise due to

1) Outgoing soft gluons

2) Collinear partons(incoming as well as outgoing massless quarks and gluons)

Computation of NNLO is highly non-trivial:

- two loop boxes, one loop corrections to $2 \longrightarrow 3$
- $2 \longrightarrow 4$ phase space

Multi-parton amplitudes in QCD

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Consider a process:

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The amplitude :

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Cross section $< \mathcal{M}|\mathcal{M} >$ from "no-bremstraulung" processes factorises

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Collinear Singularities are removed by mass factorisation:

$$\ln\left(\Delta^{fact}\right) = -\sum_{I=in} \mathcal{C}\, \ln\Gamma_I\left(\hat{a}_s, \mu^2, \mu_F^2, s_4, t_I\right)$$

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"$+t_I$"-distribution:

$$\left(\frac{\ln^i(s_4/t_I)}{s_4}\right)_{+t_I} = \left(\frac{\left(\ln(s_4/Q^2) - \ln(t_I/Q^2)\right)}{s_4}\right)_+ + \frac{\left(-\ln(t_I/Q^2)\right)^{i+1}}{i+1}\delta(s_4)$$

Soft distribution functions

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Bremstraulung diagrams contribute to soft divergences:

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Solution to (soft) Sudakov equation:

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Most general solution:

$$\begin{aligned}\Phi^I(\hat{a}_s, \mu^2, s_4, t_I, \varepsilon) &= \Phi^I(\hat{a}_s, s_4^2/t_I^2, \mu^2, \varepsilon) \\ &= \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{s_4^{2m}\, s}{s^{2m}\, \mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \left(\frac{i\, m\, \varepsilon}{2 s_4}\right) \hat{\phi}^{\,I,(i)}(\varepsilon)\end{aligned}$$

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\ln \Delta_d \quad &= \quad \delta(s_4)\Big[\sum_{J=I,a_s} \ln Z_J \left(\hat{a}_s, \mu^2, \mu_R^2, \varepsilon\right) \\
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- $F^I \left(\hat{a}_s, \mu^2, s, \varepsilon\right)$ and $\Phi^I \left(\hat{a}_s, \mu^2, s_4, t_I, \varepsilon\right)$ are known to two loop level, only poles are known to three loop level.
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- Large p_T distributions for prompt photon, Drell-Yan and Higgs production can also be obtained using resummed approach.