Conformal invariance and the expressions for $C_F^4 \alpha_s^4$ terms in the Bjorken and the Gross-Llewellyn Smith sum rules

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Recently the result of analytical calculation of α_s^4-correction in $SU(3)$ was obtained for $D^{NS}(Q^2) = Q^2 \int_0^\infty \frac{R(s)}{(s+Q^2)^2} ds = 3 \sum_F Q_F^2 C_D^{NS}$ $(\mathbf{Baikov}, \mathbf{Chetyrkin}, \mathbf{Kuhn}\ (\mathbf{08}))$. It is desirable to check or understand the analytical structure of the α_s^4 result for C_D^{NS}. Why?!

The part of QED variant of this result reads:

$$C_D^{NS}(a) = \left[1 + \frac{3}{4}a - \frac{3}{32}a^2 - \frac{69}{128}a^3 + \left(\frac{4157}{2048} + \frac{\mathbf{96}}{\mathbf{256}}\zeta_{\mathbf{3}}\right)a^4\right]$$

$(\mathbf{Baikov}, \mathbf{Chetyrkin}, \mathbf{Kuhn}\ (\mathbf{07}))$ where $a = \alpha/\pi$.

What is interesting?? The appearence of ζ_3 at $O(a^4)$.

Indeed, at a^3 calculations by **S.G.Gorishny, A.L.Kataev, S.A.Larin, L.R.Surguladze(91)** the numbers were **rational**. At a^2 level **C. Bender at al (77)** claimed without proof that the origin of rationality is realted to the property of conformal symmetry of $C_D^{NS}(a)$. However, in $SU(4)$ SYM theories ζ_3 and other transcendalities appear in analogs RG-functions though in other combinations (**without rational numbers!)A.V.Kotikov,L.N.Lipatov(07) J.M.Drummond,G.P.Korchemsky,E.Sokatchev unproved property of "maximal transcedentality".**

It is desirable to understand the status of BChK result.

Proposal: use Crewther relation in the p-space :

Gabadadze,Kataev(95); Kataev (96) Derivation:

Consider the AVV 3-point function

$T^{abc}_{\mu\alpha\beta}(p,q) = i\int <0|TA^a_\mu(y)V^b_\alpha(x)V^c_\beta(0)|0> e^{ipx+iqy}dxdy$

$= d^{abc}T_{\mu\alpha\beta}(p,q)$ where $A^a_\mu(x) = \overline{\psi}\gamma_\mu\gamma_5(\lambda^a/2)\psi$,

$V^a_\mu(x) = \overline{\psi}\gamma_\mu(\lambda^a/2)\psi$ are the A and V NS currents.

The r.h.s. can be expanded in a basis of

3 tensor structures under the condition $(pq) = 0$ as

$T_{\mu\alpha\beta}(p,q) = \xi_1(q^2,p^2)\epsilon_{\mu\alpha\beta\tau}p^\tau$

$+\xi_2(p^2,q^2)(q_\alpha\epsilon_{\mu\beta\rho\tau}p^\rho q^\tau - q_\beta\epsilon_{\mu\alpha\rho\tau}p^\rho q^\tau)$

$+\,\xi_3(p^2,q^2)(p_\alpha\epsilon_{\mu\beta\rho\tau}p^\rho q^\tau + p_\beta\epsilon_{\mu\alpha\rho\tau}p^\rho q^\tau)$

Taking the divergency of A current one can get $\xi_1(q^2,p^2)$:

$q_\beta T_{\mu\alpha\beta}(p,q) = \epsilon_{\mu\alpha\rho\tau}q^\rho p^\tau\xi_1(q^2,p^2)$

The conservation of the V currents implies

$lim|_{p^2\to\infty}p^2\xi_3(q^2,p^2) = -\xi_1(q^2,p^2)$

The definition of polarized Bjorken SR

$Bjp(Q^2) = \int_0^1 [g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)]dx$

$= \frac{1}{6}|\frac{g_A}{g_V}|C_{Bjp}(a_s)$ and the Gross-Llewellyn Smith SR

$GLS(Q^2) = \frac{1}{2}\int_0^1 \left[F_3^{\nu p}(x,Q^2) + F_3^{\overline{\nu}p}(x,Q^2)\right]dx$

$= 3C_{GLS}(a_s)$ where $a_s = \alpha_s/\pi$.

$C_{Bjp}(a_s)$ can be found from OPE of 2 NS V currents

$i\int TV_\alpha^a(x)V_\beta^b(0)e^{ipx}dx|_{p^2\to\infty} \approx C_{\alpha\beta\rho}^{P,abc}A_\rho^c(0) + ...$

$C_{\alpha\beta\rho}^{P,abc} \sim id^{abc}\epsilon_{\alpha\beta\rho\sigma}\frac{p^\sigma}{P^2}C_{Bjp}(a_s)$ and $P^2 = -p^2$.

For GLS SR one should consider the OPE of the

A and V NS currents

$i\int TA_\mu^a(x)V_\nu^b(0)e^{iqx}dx|_{q^2\to\infty} \approx C_{\mu\nu\alpha}^{V,ab}V_\alpha(0) + ...$

where $C_{\mu\nu\alpha}^{V,ab} \sim i\delta^{ab}\epsilon_{\mu\nu\alpha\beta}\frac{q^\beta}{Q^2}C_{GLS}(a_s)$ and $Q^2 = -q^2$.

$C_D^{NS}(a_s)$ is the Adler D-function of the NS A currents

$D^{NS}(a_s) = -12\pi^2 q^2 \frac{d}{dq^2}\Pi_{NS}(q^2) = 3\sum_F Q_F^2 C_D^{NS}(a_s)$

$i\int < 0|TA_\mu^a(x)A_\nu^b(0)|0 > e^{iqx}dx = \delta^{ab}(g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi_{NS}$

$\xi_2(q^2,p^2)|_{|p^2|\to\infty} \to \frac{1}{p^2}C_{Bjp}(a_s)\Pi_{NS}(a_s)$

$q^2\frac{d}{dq^2}\xi_2(q^2,p^2)|_{|p^2|\to\infty} \to \frac{1}{p^2}C_{Bjp}(a_s)C_D^{NS}(a_s)$

On the other hand, it was shown in that in a conformal invariant (c-i) limit

$T_{\mu\alpha\beta}^{abc}(p,q)|_{c-i} = d^{abc}K(a_s)\Delta_{\mu\alpha\beta}^{1-loop}(p,q)$

In other words, in a conformal invariant limit one has

$$
\begin{aligned}
\xi_1^{c-i}(q^2,p^2) &= K(a_s)\xi_1^{1-loop}(q^2,p^2) \\
\xi_2^{c-i}(q^2,p^2) &= K(a_s)\xi_2^{1-loop}(q^2,p^2) \\
\xi_3^{c-i}(q^2,p^2) &= K(a_s)\xi_3^{1-loop}(q^2,p^2)
\end{aligned}
$$

In view of the Adler-Bardeen theorem the amplitude $\xi_1(q^2,p^2)$ has no radiative corrections, $K(a_s) = 1$ and $C_{Bjp}(a_s(Q^2))C_D^{NS}(a_s(Q^2))|_{c-i} = 1$

The results of **Baikov, Chetyrkin and Kuhn** are equivalent to the QCD result

$$C_D^{NS}(a_s) = \Big[1 + \tfrac{3}{4}\mathrm{C_F}a_s - \tfrac{3}{32}\mathrm{C_F^2}a_s^2 - \tfrac{69}{128}\mathrm{C_F^3}a_s^3 + \left(\tfrac{4157}{2048} + \tfrac{96}{256}\zeta_{\mathbf{3}}\right)\mathrm{C_F^4}a_s^4\Big]$$

where $\mathrm{C_F} = (N^2-1)/(2N)$.

The scheme-independent contributions to Bjorken polarized and GLS sum rule are

$$C_{Bjp}(a_s) = C_{GLS}(a_s)$$

$$= 1 - \tfrac{3}{4}\mathrm{C_F}a_s + \tfrac{21}{32}\mathrm{C_F^2}a_s^2 - \tfrac{3}{128}\mathrm{C_F^3}a_s^3 - \left(\tfrac{4823}{2048} + \tfrac{3}{8}\zeta_{\mathbf{3}}\right)\mathrm{C_F^4}a_s^4$$

Conclusion

- Up to now we have no theoretical arguments pro or contra existing ζ_3 in the conformal-invariant limit of the QCD result
- It will be highly desirable to find solution of this question, keeping in mind the existence of "maximal transcedentality"principle
- It is desirable to check by direct calculation the result for Bjp and GLS - the most convincing solution of this question