Renormalization group for system with identical fields

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Simplest example: 2 scalar fields + fermions (see normalization)

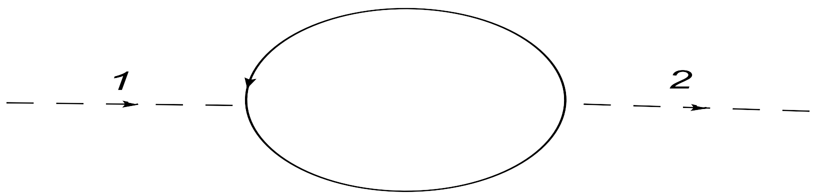
$$\begin{gathered}\mathcal{L}=\frac{1}{2}[(\partial_\mu\phi_1\,\partial_\mu\phi_1)+(\partial_\mu\phi_2\,\partial_\mu\phi_2)+2\varkappa(\partial_\mu\phi_1\,\partial_\mu\phi_2)]-\\ +i\bar{\psi}\overleftrightarrow{\hat{\partial}}\psi-m\bar{\psi}\psi-\sum m_{ij}^2\phi_i\phi_j/2-\\ -(2\pi)^2\left[\lambda_1\phi_1^4+\lambda_2\phi_2^4+\lambda_3\phi_1^2\phi_1^2+(\phi_1\phi_2)\left(\lambda_4\phi_1^2+\lambda_5\phi_2^2\right)\right]+\\ +4\pi(g_1\phi_1+g_2\phi_2)\bar{\psi}\psi\,.\end{gathered}$$

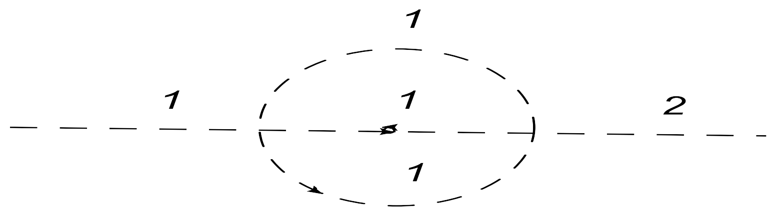
Similar systems were discussed in D.I. Kazakov, D.V. Shirkov, JINR-8974 (1975); G.M. Avdeeva et al. Sov. Yad. Fiz. **18** (1973) 1309; V.V. Belokurov et al. Phys. Lett. **47B** (1973) 359; M. Suzuki. Nucl. Phys. **B63** (1974) 269; T.L. Cutright, G.I. Chandour, I.F. Lyuksutov et al. Sov. JETP **68** (1973) 1817; I.F. Ginzburg Sov. Yad. Fiz. **25** (1977) 421; G. Gufan (197x-198x); Froggat et al., Wudka (199x-200x), and other papers devoted 2HDM – Two Higgs Doublet model.

Certainly, kinetic term of scalar fields can be diagonalized by the change of type

$$\phi_1 = a(\cos\alpha\,\Phi_1 + \sin\alpha\,\Phi_2)\,, \qquad \phi_2 = b(\cos\alpha\,\Phi_2 - \sin\alpha\,\Phi_1)$$

with suitable a, b, α. In this case form of potential is changed keeping the structure with $\lambda_i \to \Lambda_i$, $g_i \to G_i$. However, in loop corrections the terms, violating Z_2 symmetry ($\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$ or $\Phi_1 \to -\Phi_1$, $\Phi_2 \to \Phi_2$) generate mixed kinetic counter terms (term with $\varkappa$). That are terms with $\Lambda_{4,5}$ (in two and more loops) and Yukawa terms at $G_1 \neq 0$, $G_2 \neq 0$ (in one loop). Typical diagrams





$\Rightarrow$ mixed kinetic term must be included in the basic Lagrangian for renormalizability.

In spirit of general approach of renormalization group (RG) it become clear that in such models mixing among scalar fields varies with the change of scale μ, scalar fields cannot be separated from each other even at very small distances, renormalization group equations for corresponding models must be modified.
Unfortunately this effect was skipped in all papers, treated models with hardly broken Z_2 symmetry.

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We don't discuss *soft violation of Z_2 symmetry*, given by term m_{12}^2. This violation mixes fields at large distances and don't influence for situation at small distances.

There are two ways for RG analysis of this situation:

• To transform kinetic term to the diagonal form and construct RG scheme with mixing parameters transformed at each new iteration and each new scale.

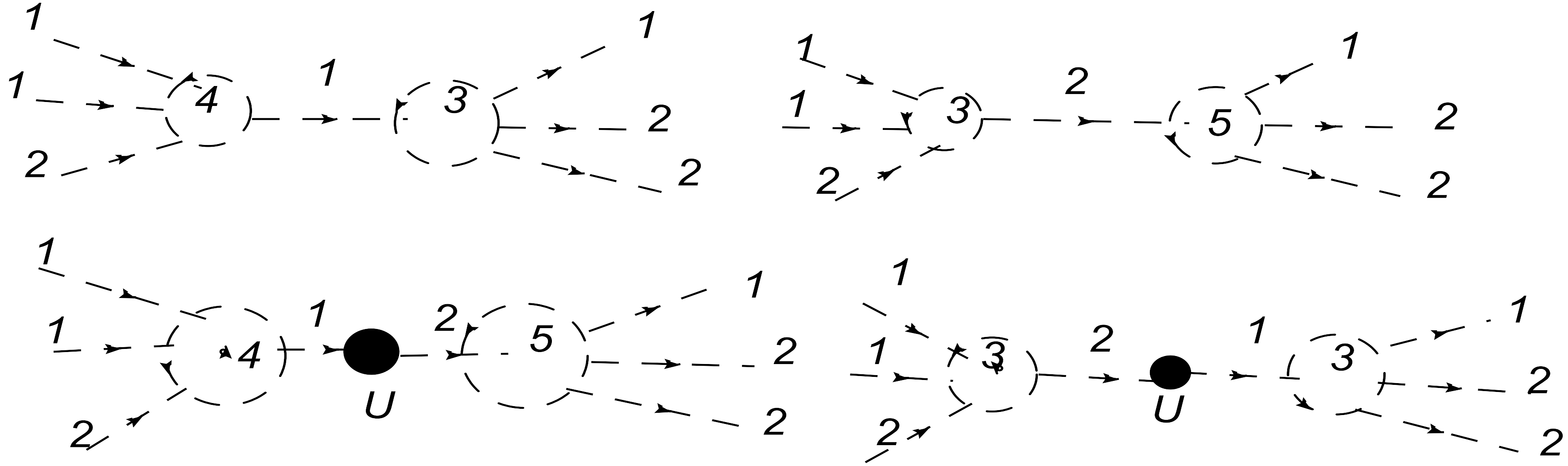
• To develop standard RG scheme considering mixed kinetic and terms as perturbation at the same level as standard terms in $\mathcal{L}_{int}$. The diagonalization of kinetic term can be performed at final stage if necessary.

We develop second way, assuming

$$1 \gtrsim \varkappa \gg \lambda_i \sim g_i^2 \, .$$

In this approach parameter $\varkappa$ generates new invariant charge $\mathcal{U}$.

For example, typical tree digrams for the process $\phi_1\phi_1\phi_2 \to \phi_1\phi_2\phi_2$ have form



Here open blob i corresponds full vertex $\rho_i \leftarrow \lambda_i$, dark blob corresponds full kinetic mixing $\mathcal{U} \leftarrow \varkappa$.

Modified RG equations for ultraviolet region (at $k^2 \gg m_{ij}^2$)

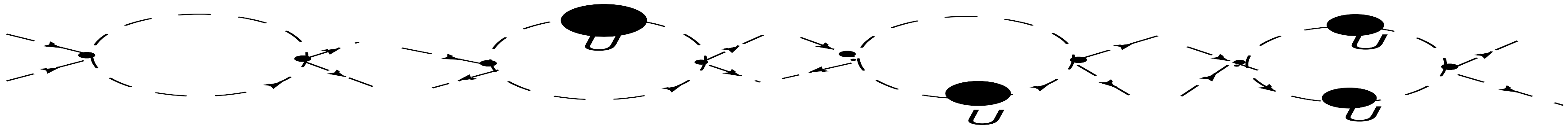
As usual (see Bogolyubov, Shirkov book) we consider 4 scalar-vertexes Δ_{abcd}, 3-vertexes fermion-scalar Γ_a, nominators of fermion and matrix boson propagators s and d_{ab}, defined in the considered region via complete propagators as $S = \dfrac{\hat{k} s(k^2)}{k^2}$ and $D = \dfrac{1}{k^2}\begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix}$ (Here $a,\, b,\, c,\, d = 1,\, 2$).

Typical definitions for invariant charges are similar to well known, for example

$$\rho_4 = d_{11}^{3/2} d_{22}^{1/2} \Delta_{1112}\,; \quad \sigma_1 = s d_{11}^{1/2} \Gamma_1\,; \quad U_{12} = d_{11}^{1/2} d_{22}^{1/2} d_{12}\,.$$

In this simplified case $U_{12} = U_{21}$ so that we will skip label at U often.

Now, for example, typical 4 scalar vertex diagrams in one loop approximation have form



Similar correction must be included in fermion polarization operator. Corresponding β-functions are calculated easily via known loop integrals with new simple combinatorics. In the one-loop approximation

$$\frac{d\rho_1}{dL} \equiv \beta_{\lambda 1} = 9\rho_1^2 + \rho_3^2 - 4\sigma_1^4 + 4\sigma_1^2\rho_1 + \frac{9}{4}\rho_4^2 + \\ +18\varkappa\rho_1\rho_4 + 9\varkappa^2\rho_1\rho_3 + \frac{9}{2}\varkappa^2\rho_4^2\,;$$

$$\frac{d\sigma_1}{dL} \equiv \beta_{g1} = \frac{5}{2}\sigma_1^3 + \frac{1}{2}\sigma_1\sigma_2^2 + \varkappa\sigma_1^2\sigma_2\,;$$

$$\frac{dU}{dL} \equiv \beta_{\varkappa} = 2\sigma_1\sigma_2 + \varkappa(\sigma_1^2 + \sigma_2^2)\,.$$

Remind: The phenomenon takes place even in pure scalar theory but in one loop it appears due to Yukawa coupling.

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This system is much more complicated than that obtained in the case of Z_2 symmetry.

Application to Two Higgs Doublet Model (2HDM), etc.

2HDM is the simplest extension of the minimal SM for description of EWSB – contains two scalar weak isodoublets ϕ_1 and ϕ_2 with identical hypercharge, interacting with fermion fields via Yukawa interaction. Isoscalar combinations of the field operators

$$x_1 = \phi_1^\dagger \phi_1, \quad x_2 = \phi_2^\dagger \phi_2, \quad x_3 = \phi_1^\dagger \phi_2\,, \quad x_{3^*} \equiv x_3^\dagger = \phi_2^\dagger \phi_1\,.$$

The most general renormalizable Higgs potential is

$$V = -\frac{1}{2}\left[m_{11}^2 x_1 + m_{22}^2 x_2 + \left(m_{12}^2 x_3 + h.c.\right)"\right] +$$
$$+\frac{\lambda_1 x_1^2 + \lambda_2 x_2^2}{2} + \lambda_3 x_1 x_2 + \lambda_4 x_3 x_3^\dagger + \left[\frac{\lambda_5 x_3^2}{2} + \lambda_6 x_1 x_3 + \lambda_7 x_2 x_3 + h.c.\right].$$

Here "blue" terms violate Z_2 symmetry hardly and generate field mixing at small distances, as it was discussed for simple example above.

In accordance with presented analysis large series of papers considered RG equations in this model, neglecting mixing terms, is not completely correct
(M.Carena, J. Ellis, A. Pilaftsis, M. Wagner, hep-ph 0003160; c.D. Froggart, R. Nevzorov, H.B. Nielsen et al. hep-ph0708.2901 and earlier, R. Santos, A. Barroso, hep-ph 9701257, J. Gunion + F. Haber ,,,, J. Wudka... , many others.)

In my opinion, the discussed field mixing at small distances, varied with the change of distances is strongly unnatural phenomenon.
I hope that the principe, formulated Ukrainian philosopher Gregory Skovoroda works:

Thanks to God, created all useless to be complex and all complex to be useless.

The mentioned difficulty is absent in models where Z_2 symmetry either pure or broken softly (by mass terms) and in Yukawa interaction each right fermion is coupled to only one Higgs boson (Models I and II). It forbids, for example Model III for Yukawa sector.

Certainly, some generalized rotation in ϕ_1, ϕ_2 plane can transform Lagrangian with softly broken Z_2 symmetry to the "most general" form. However, in this "hidden soft Z_2 symmetry form" (I.F. G., M. Krawczyk) field mixing in kinetic term cannot appear, the cancellation of different contributions is due to relations among couplings, appeared at above transformation from explicitly soft Z_2 violated form.