PROPERTIES OF LIGHT MESONS IN THE RELATIVISTIC QUARK MODEL

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Ebert, Faustov, Galkin — Mod. Phys. Lett. A20, 1887-1894 (2005)
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OUTLINE

1. Relativistic quark model
2. Masses of pseudoscalar and vector light mesons
3. Decay constants of light mesons
4. Electromagnetic form factors of light mesons

RELATIVISTIC QUARK MODEL

Quasipotential equation of Schrödinger type:

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p},\mathbf{q};M)\Psi_M(\mathbf{q})$$

$\mathbf{p}$ - relative momentum of quarks
M - bound state mass ($M = E_1 + E_2$)
μ_R - relativistic reduced mass:

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}$$

$b(M)$ - on-mass-shell relative momentum in cms:

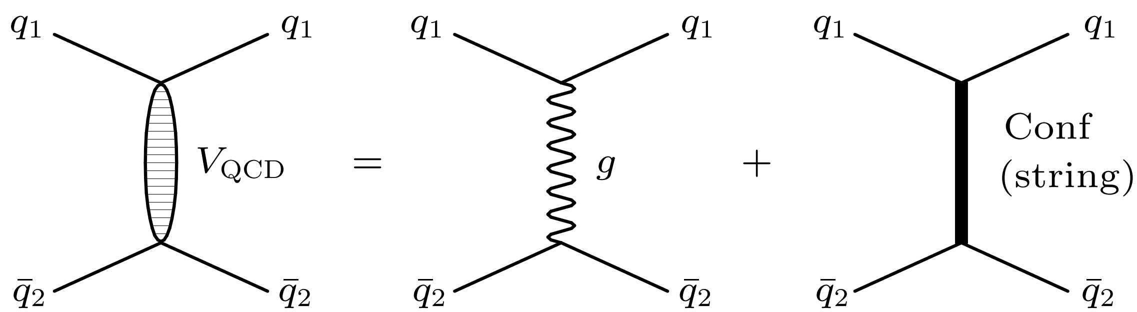
$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}$$

$E_{1,2}$ - center of mass energies:

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}$$

- Parameters of the model fixed from heavy meson sector

- $q\bar{q}$ **quasipotential**



$$V(\mathbf{p},\mathbf{q};M) = \bar{u}_1(p)\bar{u}_2(-p)\left\{\frac{4}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_1^{\mu}\gamma_2^{\nu} + V^V_{\rm conf}(\mathbf{k})\Gamma_1^{\mu}\Gamma_{2;\mu} + V^S_{\rm conf}(\mathbf{k})\right\}u_1(q)u_2(-q)$$

$\mathbf{k} = \mathbf{p} - \mathbf{q}$
$D_{\mu\nu}(\mathbf{k})$ - (perturbative) gluon propagator
$\Gamma_\mu(\mathbf{k})$ - effective long-range vertex with Pauli term:

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu,$$

κ - anomalous chromomagnetic moment of quark,

$$u^\lambda(p) = \sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}} \left(\begin{array}{c} 1 \\ \dfrac{\boldsymbol{\sigma}\mathbf{p}}{\epsilon(p)+m} \end{array} \right) \chi^\lambda,$$

with $\epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$.

- Lorentz structure of $V_{\rm conf} = V^V_{\rm conf} + V^S_{\rm conf}$

  In nonrelativistic limit

$$\left. \begin{array}{lcc} V^V_{\rm conf} & = & (1-\varepsilon)(Ar+B) \\ V^S_{\rm conf} & = & \varepsilon(Ar+B) \end{array} \right\} \quad {\rm Sum:} \quad (Ar+B)$$

ε - mixing parameter

$$V_{\rm Coul}(r) = -\,\frac{4}{3}\,\frac{\alpha_s}{r}$$

Parameters A, B, κ, ε and quark masses fixed from analysis of meson masses and radiative decays:

$\varepsilon = -1$ from heavy quarkonium radiative decays ($J/\psi \to \eta_c + \gamma$) and HQET

$\kappa = -1$ from fine splitting of heavy quarkonium ^3P_J states and HQET

$(1+\kappa) = 0 \Longrightarrow$ vanishing long-range chromomagnetic interaction (flux tube model)

Freezing of α_s for light quarks (Simonov, Badalyan)

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln \frac{\mu^2 + M_0^2}{\Lambda^2}}, \quad \beta_0 = 11 - \frac{2}{3} n_f, \quad \mu = \frac{2m_1 m_2}{m_1 + m_2},$$

$$M_0 = 2.24\sqrt{A} = 0.95 \text{ GeV}$$

Quasipotential parameters:

$A = 0.18$ GeV^2, $B = -0.30$ GeV,

$\Lambda = 0.413$ GeV (from M_ρ)

Quark masses:

$m_b = 4.88$ GeV $m_s = 0.50$ GeV

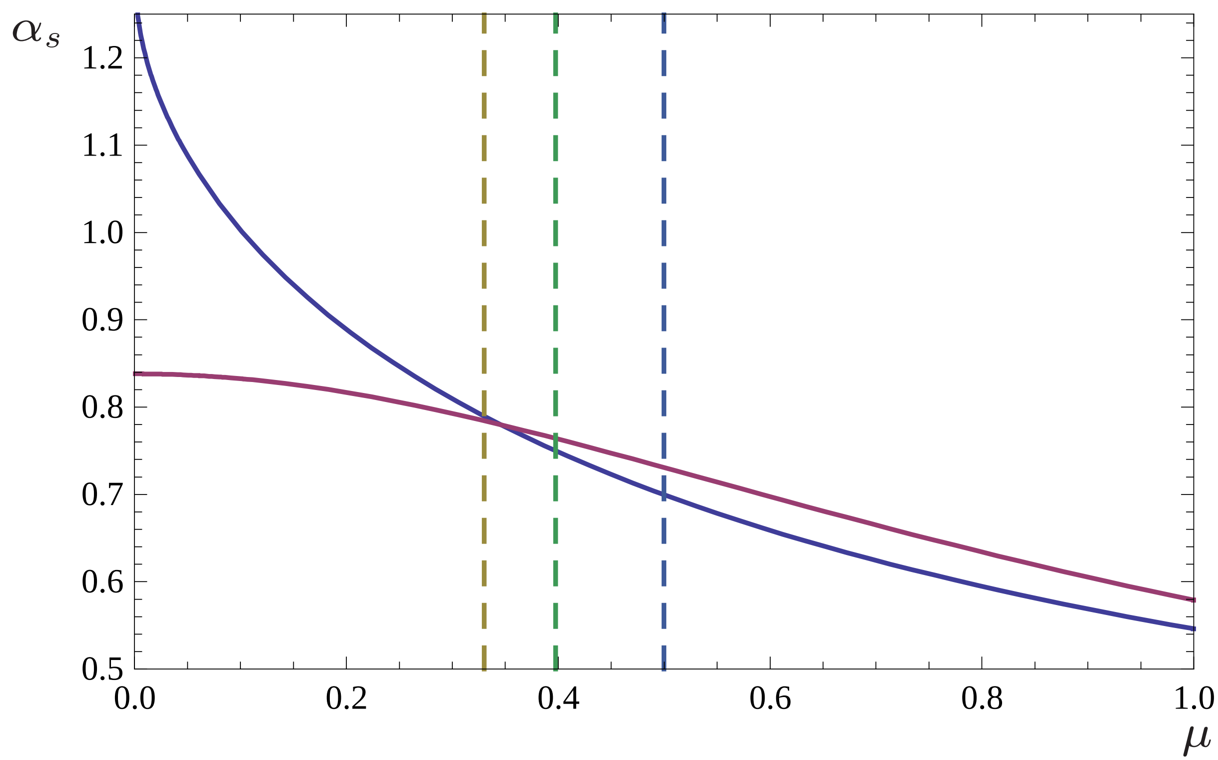
$m_c = 1.55$ GeV $m_{u,d} = 0.33$ GeV

Alternative representation for $\alpha_s(\mu)$ (Shirkov, Zayakin)

$$\alpha_E^{mod}(\mu) = \frac{4\pi}{\beta_0}\left\{\frac{1}{l_2} + \frac{1}{1-\exp(l_2)}\right\} \longrightarrow \frac{4\pi}{\beta_0},\ \mu \to 0$$

$$l_2 = l + b\ln\sqrt{l^2 + 2\pi^2} \qquad b = 4\pi\beta_1/\beta_0^2, \qquad l = \ln(\mu^2/\Lambda^2), \qquad \beta_1 = \frac{153 - 19n_f}{6\pi}$$

$$\mu_{12} = 2m_1 m_2/(m_1 + m_2); \qquad \Lambda = 920 \text{ MeV}$$



MASSES OF LIGHT MESONS

The quasipotential of qq interaction is extremely nonlocal in configuration space for arbitrary quark masses. To make it local

- ⋆ heavy quarks: nonrelativistic v/c or heavy quark $1/m_Q$ expansion

- ⋆ light quarks: highly relativistic, substitution

$$\epsilon_q(p) \equiv \sqrt{m_q^2 + \mathbf{p}^2} \to E_q = \frac{M^2 - m_{q'}^2 + m_q^2}{2M}$$

$q\bar{q}$ potential

$$V_{q\bar{q}}(r) = V_{\rm SI}(r) + V_{\rm SD}(r)$$

spin-independent potential for S-states ($\langle \mathbf{L}^2 \rangle = 0$)

$$V_{\rm SI}(r) = V_{\rm Coul}(r) + V_{\rm conf}(r) + \frac{(E_1^2 - m_1^2 + E_2^2 - m_2^2)^2}{4(E_1+m_1)(E_2+m_2)} \Bigg\{ \frac{1}{E_1E_2} V_{\rm Coul}(r)$$
$$+ \frac{1}{m_1m_2}\Bigg(1 + (1+\kappa)\Bigg[(1+\kappa)\frac{(E_1+m_1)(E_2+m_2)}{E_1E_2}$$
$$- \left(\frac{E_1+m_1}{E_1} + \frac{E_1+m_2}{E_2}\right)\Bigg]\Bigg) V_{\rm conf}^V(r) + \frac{1}{m_1m_2} V_{\rm conf}^S(r) \Bigg\}$$
$$+ \frac{1}{4}\left(\frac{1}{E_1(E_1+m_1)}\Delta\tilde{V}_{\rm Coul}^{(1)}(r) + \frac{1}{E_2(E_2+m_2)}\Delta\tilde{V}_{\rm Coul}^{(2)}(r)\right)$$
$$- \frac{1}{4}\left[\frac{1}{m_1(E_1+m_1)} + \frac{1}{m_2(E_2+m_2)} - (1+\kappa)\left(\frac{1}{E_1m_1} + \frac{1}{E_2m_2}\right)\right]$$
$$\times \Delta V_{\rm conf}^V(r) + \frac{(E_1^2 - m_1^2 + E_2^2 - m_2^2)}{8m_1m_2(E_1+m_1)(E_2+m_2)} \Delta V_{\rm conf}^S(r)$$

spin-dependent potential

$$V_{\rm SD}(r) = \frac{2}{3E_1E_2}\Bigg[\Delta\bar{V}_{\rm Coul}(r) + \left(\frac{E_1-m_1}{2m_1} - (1+\kappa)\frac{E_1+m_1}{2m_1}\right)$$
$$\times \left(\frac{E_2-m_2}{2m_2} - (1+\kappa)\frac{E_2+m_2}{2m_2}\right)\Delta V_{\rm conf}^V(r)\Bigg]\mathbf{S}_1\mathbf{S}_2$$

Table 1: Masses of light S-wave mesons (in MeV)

Meson	State	Theory				Experiment
	$n^{2S+1}L_J$	our	Godfrey Isgur	Maris Roberts	Koll et al.	PDG
π	1^1S_0	154	150	138	140	139.57
ρ	1^3S_1	776	770	742	785	775.8(5)
π'	2^1S_0	1292	1300		1331	1300(100)
ρ'	2^3S_1	1486	1450		1420	1465(25)
K	1^1S_0	482	470	497	506	493.677(16)
K^*	1^3S_1	897	900	936	890	891.66(26)
K'	2^1S_0	1538	1450		1470	
$K^{*\prime}$	2^3S_1	1675	1580		1550	1717(27)
ϕ	1^3S_1	1038	1020	1072	990	1019.46(2)
ϕ'	2^3S_1	1698	1690		1472	1680(20)

Table 2: Masses of light ground state mesons (in MeV)

Meson	State	Theory		Experiment
	$n^{2S+1}L_J$	α_s	α_E^{mod}	PDG
π	1^1S_0	154	154	139.57
ρ	1^3S_1	776	776	775.8(5)
K	1^1S_0	482	483	493.677(16)
K^*	1^3S_1	897	905	891.66(26)
ϕ	1^3S_1	1038	1051	1019.46(2)

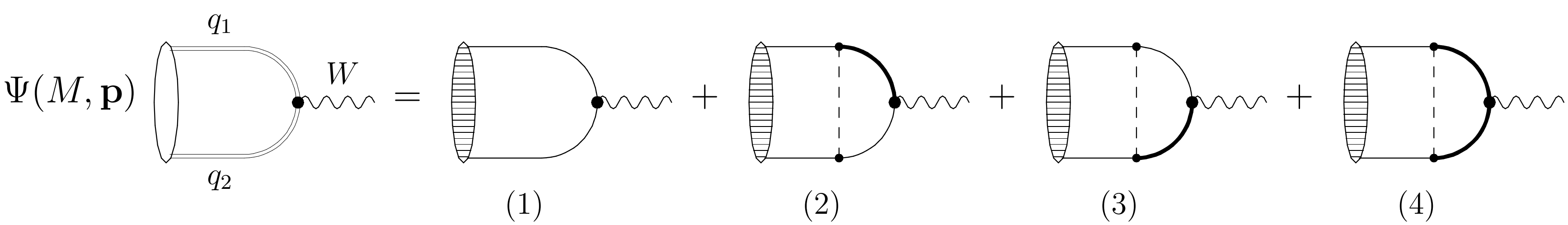
DECAY CONSTANTS

$$\langle 0|\bar{q}_1\gamma^\mu\gamma_5 q_2|P(\mathbf{K})\rangle = i f_P K^\mu,$$
$$\langle 0|\bar{q}_1\gamma^\mu q_2|V(\mathbf{K},\varepsilon)\rangle = f_V M_V \varepsilon^\mu,$$

$\mathbf{K}$ – meson momentum, ε^μ – polarisation vector, M_V – vector meson mass.

$$\left\langle 0|J^W_\mu|M(\mathbf{K})\right\rangle = \int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left\{\gamma_\mu(1-\gamma_5)\Psi(M,p)\right\},$$

$\Psi(M,p)$ – two-particle Bethe-Salpeter wave function



- completely relativistic treatment of light quark
- relativistic contributions of the negative-energy states are taken into account (bold lines)

$$f_{P,V} = f^{(1)}_{P,V} + f^{(2+3)}_{P,V} + f^{(4)}_{P,V},$$

contributions $f^{(2+3)}_{P,V}$ and $f^{(4)}_{P,V}$ are new

$$\Psi(M,\mathbf{p}) = \int \frac{dp^0}{2\pi}\Psi(M,p) = \Phi_M(p)\chi_{ss'}\phi_{q_1q_2}.$$

$$f_{P,V}^{(1)} = \sqrt{\frac{12}{M}} \int \frac{d^3p}{(2\pi)^3} \left(\frac{\epsilon_1(p)+m_1}{2\epsilon_1(p)}\right)^{1/2} \left(\frac{\epsilon_2(p)+m_2}{2\epsilon_2(p)}\right)^{1/2}$$
$$\times \left\{1 + \lambda_{P,V} \frac{\mathbf{p}^2}{[\epsilon_1(p)+m_1][\epsilon_2(p)+m_2]}\right\} \Phi_{P,V}(p), \qquad \lambda_P = -1, \quad \lambda_V = 1/3$$

$$f_{P,V}^{(2+3)} = \sqrt{\frac{12}{M}} \int \frac{d^3p}{(2\pi)^3} \left(\frac{\epsilon_1(p)+m_1}{2\epsilon_1(p)}\right)^{1/2} \left(\frac{\epsilon_2(p)+m_2}{2\epsilon_2(p)}\right)^{1/2} \left[\frac{M-\epsilon_1(p)-\epsilon_2(p)}{M+\epsilon_1(p)-\epsilon_2(p)}\right.$$
$$\left.\times \frac{\mathbf{p}^2}{\epsilon_1(p)[\epsilon_1(p)+m_1]} \left\{1 + \lambda_{P,V} \frac{\epsilon_1(p)+m_1}{\epsilon_2(p)+m_2}\right\} + (1 \leftrightarrow 2)\right] \Phi_{P,V}(p),$$

$$f_{P,V}^{(4)} = \sqrt{\frac{12}{M}} \int \frac{d^3p}{(2\pi)^3} \left(\frac{\epsilon_1(p)+m_1}{2\epsilon_1(p)}\right)^{1/2} \left(\frac{\epsilon_2(p)+m_2}{2\epsilon_2(p)}\right)^{1/2} \frac{M-\epsilon_1(p)-\epsilon_2(p)}{M+\epsilon_1(p)+\epsilon_2(p)}$$
$$\times \left\{-\lambda_{P,V} - \frac{\mathbf{p}^2}{[\epsilon_1(p)+m_1][\epsilon_2(p)+m_2]}\right\} \left[\frac{(1-\varepsilon)m_1^2 m_2^2}{\epsilon_1^2(p)\epsilon_2^2(p)} + \frac{\mathbf{p}^2}{[\epsilon_1(p)+m_1][\epsilon_2(p)+m_2]}\right] \Phi_{P,V}(p)$$

$$f_{P,V}^{\mathrm{NR}} = \sqrt{\frac{12}{M_{P,V}}}\, |\Psi_{P,V}(0)|$$

Table 3: Different contributions to the pseudoscalar and vector decay constants of light and heavy mesons (in MeV).

Decay constant	f_M^{NR}	$f_M^{(1)}$	$f_M^{(2+3)} + f_M^{(4)}$	$(f_M^{(2+3)} + f_M^{(4)})/f_M^{(1)}$	f_M
f_π	1290	515	-391	-76%	124
f_ρ	490	402	-183	-46%	219
f_K	783	353	-198	-56%	155
f_{K^*}	508	410	-174	-42%	236
f_ϕ	511	415	-170	-41%	245
f_D	376	275	-41	-15%	234
f_{D^*}	391	334	-24	-7%	310
f_{D_s}	436	306	-38	-12%	268
$f_{D_s^*}$	447	367	-52	-14%	315
f_B	259	210	-21	-10%	189
f_{B^*}	280	235	-16	-7%	219
f_{B_s}	300	238	-20	-8%	218
$f_{B_s^*}$	316	264	-13	-5%	251
f_{B_c}	538	433	-8	-1.8%	425
$f_{B_c^*}$	545	503	-4	-0.8%	499

Table 4: Pseudoscalar and vector decay constants of light mesons (in MeV).

Constant	our	Godfrey	Maris	Koll	He	Ali Khan	Milc	Experiment
f_π	124	180	131	219	138	$126.6(6.4)$	$129.5(3.6)$	$130.70(10)(36)$
f_K	155	232	155	238	160	$152.0(6.1)$	$156.6(3.7)$	$155.5(1.0)(2)$
f_ρ	219	220	207		238	$239.4(7.3)$		$\left\{ \begin{array}{c} 220(2)^{*} \\ 209(4)^{**} \end{array} \right.$
f_{K^*}	236	267	241		241	$255.5(6.5)$		$217(5)^{\dagger}$
f_ϕ	245	336	259			$270.8(6.5)$		$229(3)^{\ddagger}$

* obtained using experimental value for $\Gamma_{\rho^0 \to e^+ e^-}$.

** obtained using experimental value for $\Gamma_{\tau \to \rho \nu_\tau}$.

† obtained using experimental value for $\Gamma_{\tau \to K^* \nu_\tau}$.

‡ obtained using experimental value for $\Gamma_{\phi \to e^+ e^-}$.

Table 5: Pseudoscalar decay constants of heavy-light mesons (in MeV).

Constant	Quark Model		Lattice		QCD sum rules			Experiment
	our	Cvetic	Ali Khan	Milc	Narison	Penin	Jamin	PDG
f_D	234	230(25)	225(14)(40)	201(3)(17)	203(20)	195(20)		$205.8(8.9)$
f_{D_s}	268	248(27)	267(13)(48)	249(3)(16)	235(24)			273(10)
f_{D_s}/f_D	1.15	1.08(1)		1.24(1)(7)	1.15(4)			
f_B	189	196(29)	208(10)(29)	216(9)(19)(6)	203(23)	206(20)	210(19)	227(52)
f_{B_s}	218	216(32)	250(10)(35)	259(32)	236(30)		244(21)	
f_{B_s}/f_B	1.15	1.10(1)		1.20(3)(1)	1.16(4)		1.16	

ELECTROMAGNETIC FORM FACTORS OF LIGHT MESONS

$$\langle M(P_F)|J_\mu|M(P_I)\rangle = F_P(Q^2)(P_I+P_F)_\mu, \qquad Q^2 = -(P_F-P_I)^2$$

Conservation of electric charge $\longrightarrow$ normalisation condition: $F_P(0)=1$

- completely relativistic calculation in the wide range of space-like momenta $Q^2 \geq 0$
- contributions of negative-energy states are taken into account
- calculated pion form factor for large Q^2 has the asymptotic behaviour $F_\pi(Q^2) \sim \alpha_s(Q^2)/Q^2$ predicted by quark counting rules and perturbative QCD

Mean charge radius squared of the pseudoscalar meson $(P=\pi, K)$:

$$\langle r^2\rangle_P = -6\left[\frac{\mathrm{d}F_P(Q^2)}{\mathrm{d}Q^2}\right]_{Q^2=0}.$$

Table 6: Charge radii of pseudoscalar mesons.

Charge radius	our	Godfrey	Maris	He	Lattice	Experiment
$\sqrt{\langle r^2\rangle_\pi}$ (fm)	0.66	0.66	0.67	0.63	0.63 ± 0.1	0.672±0.08
$\sqrt{\langle r^2\rangle_{K^\pm}}$ (fm)	0.57	0.59	0.62	0.60		0.560±0.031
$\langle r^2\rangle_{K^0}$ (fm^2)	-0.072	-0.09	-0.086	-0.062		-0.076±0.018

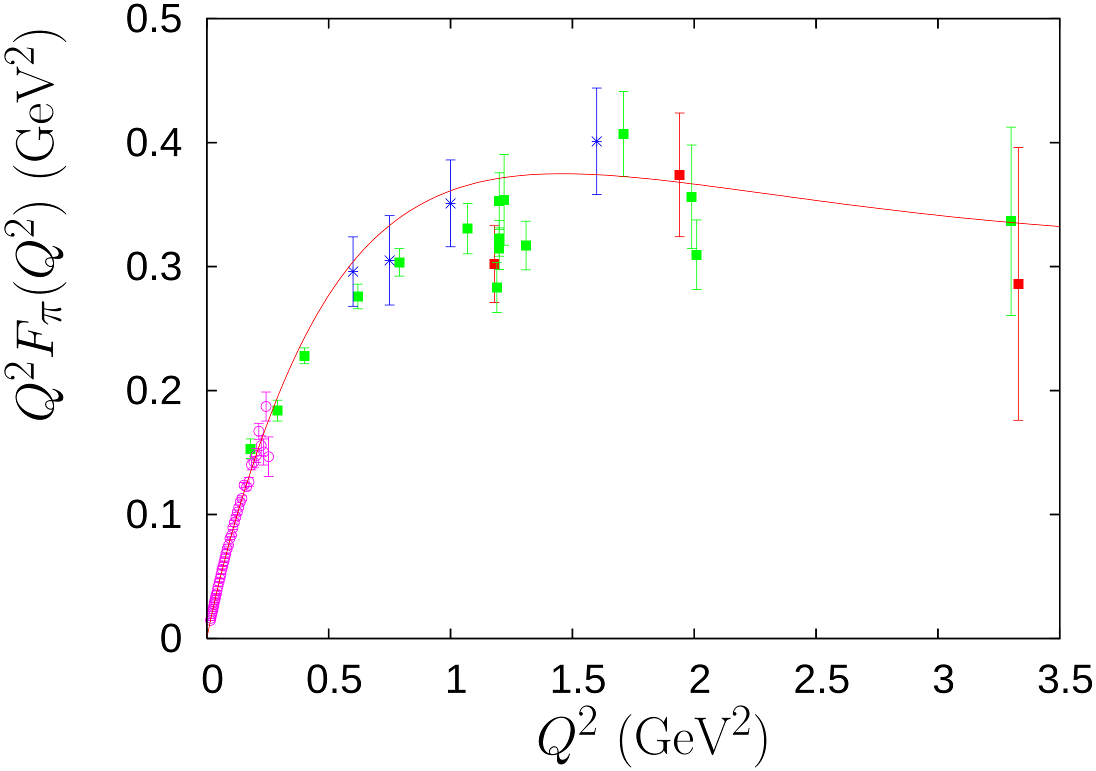


Figure 1: The product of Q^2 and the form factor of the charged pion in comparison with experimental data.

CONCLUSIONS

- Light mesons (pseudoscalar and vector) can be described as composite bound systems of highly relativistic constituent light quarks
- Account of contribution of light-quark negative-energy states to decay constants and electromagnetic form factors is crucial
- Infrared behaviour of QCD coupling $\alpha_s(\mu)$ essentially influences obtained results