Are there gravitational corrections to the running Yang-Mills coupling?

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Plan:

1. Introduction
2. Einstein-YM action
3. Diagrammatical approach and renormalization
4. Results and Discussions

Introduction

- Perturbatively quantized Einstein gravity is famously nonrenormalizable due to negative mass dimension of its coupling constant $\kappa \sim 1/M_P$.
- **But:** May be treated consistently as effective quantum field theory in low energy expansion (exp. in ∂) [Donoghue]

  $$S_{grav} = \int d^4x \sqrt{-g} \Big\{ \Lambda + \frac{2}{\kappa^2} R + c_1 \, R^2 + c_2 \, R_{\mu\nu} \, R^{\mu\nu} + \ldots \Big\}$$

  Higher order counterterms arise in loop expansions, Values of couplings $\Lambda, \kappa, c_1, c_2, \ldots$ from experiment. Higher parameters largely undetermined, e.g. $c_1, c_2 \leq 10^{74}$ [Stelle]
- **Important question** in this framework:
  Does inclusion of gravity alter the standard model running gauge couplings

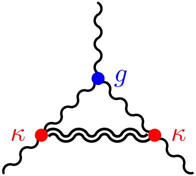
  $g_S(E), g_W(E), g_{el}(E)$ near $E \sim M_P$? [Robinson, Wilczek '06]

Gravitational contributions to YM β function

- Renormalized "running" coupling: $g_i(\mu) = \frac{Z_2^{3/2}(\mu)}{Z_1(\mu)}\, g_i \quad (\mu \to E)$

  Z_2, Z_1: wave fct/vertex renormalization constants

- Include gravitational effects, typical graph:

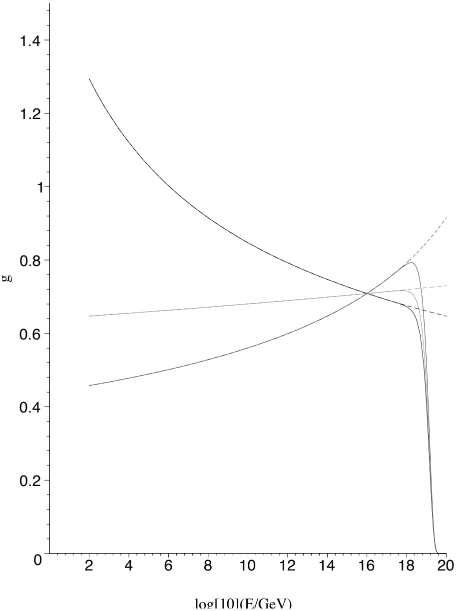


- Callan-Symanzik β-function ($g_i \to g$):

$$\boxed{\beta_g \equiv \frac{dg(E)}{d\ln E} = -\frac{b_0}{(4\pi)^2}\, g^3 + \frac{a_0}{(4\pi)^2}\, E^2\, \kappa^2\, g}$$

  b_0: pure YM contribution and claimed $\boxed{a_0 = -3/2}$ (Robinson/Wilczek '06)

Consequences for asymptotic freedom



- Result does not upset unification but shifts unification scale.
- Asymptotic freedom even for U(1) Maxwell theory ($b_0 = 0$)

$$\alpha_{el}(E) = \alpha_{el}(0)\ \exp[-\frac{3}{2}\ \frac{\kappa^2}{4\pi^2}\ E^2\,]\ ?!$$

- Consequences: Scenarios of BSM physics with large extra dimension and small gravitational scale (TeV) ⇒ effects observable at LHC ?!

Is this really correct?

However: Recent doubts/controversies concerning Robinson-Wilczek's work:

- Pietrykowski (PRL '07) & Toms ('07):
  (Method: U(1)-Maxwell theory , backgound field method with distinct gauge-fixing(s) to R-W, cut-off or dim. regularization)

  ⇒ Result of R-W is gauge-dependent, graviton correction to β-function vanishes.

- Our approach:
  Straightforward & involved diagrammatical calculation of the non-abelian Einstein-Yang-Mills system using simultaneously cut-off and dimensional regularisation.

Einstein-YM action

- Einstein-YM action:

$$S_{\text{EYM}} = \int d^4x \sqrt{-\mathbf{g}} \left(\frac{2}{\kappa^2} \mathbf{R} - \tfrac{1}{2} \mathbf{g}^{\mu\rho} \mathbf{g}^{\nu\sigma} \operatorname{Tr} [F_{\mu\nu} F_{\rho\sigma}] \right)$$

$$\mathbf{R} = \text{Ricci scalar}$$

$$F_{\mu\nu} = \nabla_\mu \mathcal{A}_\nu - \nabla_\nu \mathcal{A}_\mu - ig [\mathcal{A}_\mu, \mathcal{A}_\nu]$$

- Field decomposition:

$$\mathbf{g}_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}$$

Background field: $\eta_{\mu\nu}$

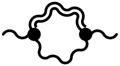
Quantum field: $h_{\mu\nu}$

- Add gauge fixing terms & ghost contributions
- Expand action around background

Our diagrammatical approach

- Calculate UV-divergent graviton contributions to Z-factors from loop corrections to gluon propagator and vertex.
- Propagator renormalization ($\rightarrow Z_2$)

(a)



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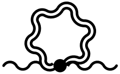
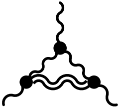


$\sim \kappa^2$

- Vertex renormalization ($\rightarrow Z_1$)

(c)



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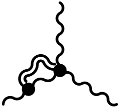

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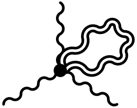
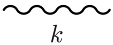


$\sim g\kappa^2$

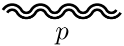
The Gluon and Graviton Propagators

- Expand S_{EYM} via $\mathbf{g}_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}$ and work out propagators and vertices.
- Gluon propagator in Feynman gauge ($\partial_\mu A^\mu = 0$)



$$\mu \quad \nu = -\frac{\eta_{\mu\nu}}{k^2 + i0}$$

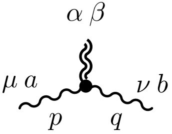
- Graviton propagator in harmonic gauge ($\partial_\nu h^{\mu\nu} - \frac{1}{2}\partial^\mu h^\nu_\nu = 0$)



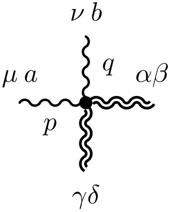
$$\alpha\beta \quad \gamma\delta = \frac{i\left(I^{\alpha\beta,\gamma\delta} - \frac{1}{d-2}\eta^{\alpha\beta}\eta^{\gamma\delta}\right)}{p^2 + i0}$$

with $I^{\mu\nu,\alpha\beta} \equiv \frac{1}{2}(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha})$.

The Gluon–Graviton Vertices with Two Gluon Lines



$$\begin{aligned}= -i\kappa\delta^{ab}\Big[&P^{\mu\nu,\alpha\beta}p\cdot q + \eta^{\mu\nu}p^{(\alpha}q^{\beta)} + \tfrac{1}{2}\eta^{\alpha\beta}p^{\nu}q^{\mu}\\ &-p^{\nu}\eta^{\mu(\alpha}q^{\beta)} - q^{\mu}\eta^{\nu(\alpha}p^{\beta)}\Big]\end{aligned}$$

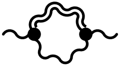


$$\begin{aligned}= \frac{i}{2}\kappa^{2}\delta^{ab}\Big[&(p^{\nu}q^{\mu} - p\cdot q\,\eta^{\mu\nu})P^{\alpha\beta,\gamma\delta}\\ &+p\cdot q(2I^{\mu\nu,\alpha(\gamma}\eta^{\delta)\beta} + 2I^{\mu\nu,\beta(\gamma}\eta^{\delta)\alpha}\\ &\quad -I^{\mu\nu,\alpha\beta}\eta^{\gamma\delta} - I^{\mu\nu,\gamma\delta}\eta^{\alpha\beta})\\ &+2p^{(\alpha}q^{\beta)}P^{\mu\nu,\gamma\delta} + 2p^{(\gamma}q^{\delta)}P^{\mu\nu,\alpha\beta}\\ &+\big\{2p^{\alpha}\eta^{\nu[\mu}\eta^{\beta](\gamma}q^{\delta)} + 2p^{\gamma}\eta^{\nu[\mu}\eta^{\delta](\alpha}q^{\beta)}\\ &\quad -p^{\nu}(q^{\alpha}P^{\mu\beta,\gamma\delta} + q^{\beta}P^{\alpha\mu,\gamma\delta}\\ &\qquad +q^{\gamma}P^{\alpha\beta,\mu\delta} + q^{\delta}P^{\alpha\beta,\gamma\mu})\big\}\\ &+\{(p,\mu)\leftrightarrow(q,\nu)\}\Big]\end{aligned}$$

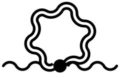
with $P^{\mu\nu,\alpha\beta} \equiv \frac{1}{2}(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \eta^{\mu\nu}\eta^{\alpha\beta})$.

The Divergences of the Two Gluon Graphs

- Calculate divergent pieces of gluon propagator in cut-off ($|k^2| < \Lambda$) and dimensional regularization ($\int d^4x \to \int d^dx$, $d = 4 - \epsilon$):



$$= \frac{i}{16\pi^2}\kappa^2(q^2\eta^{\mu\nu} - q^\mu q^\nu)\,\delta^{ab}\Big[-\frac{3}{2}\begin{Bmatrix}\Lambda^2\\ 0\end{Bmatrix} - \frac{q^2}{6}\begin{Bmatrix}\log\Lambda^2\\ \frac{2}{\epsilon}\end{Bmatrix} + \text{finite}\Big]$$



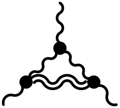
$$= \frac{i}{16\pi^2}\kappa^2(q^2\eta^{\mu\nu} - q^\mu q^\nu)\,\delta^{ab}\,\frac{3}{2}\begin{Bmatrix}\Lambda^2\\ 0\end{Bmatrix}$$

Quadratic divergencies cancel!

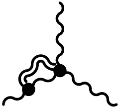
- Already implies absence of gravitational corrections to the β-function of g in abelian theory!
- Log divergences related to dim 6-operator.

The Divergences of the Three Gluon Graphs

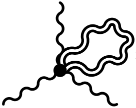
Computed in cut-off and dim-reg, divergent parts:



$$= \frac{1}{16\pi^2} g\kappa^2 f^{abc} \left\{ \left(\eta^{\mu\nu}\left(p^\rho\left(\tfrac{5}{6} p\cdot q + \tfrac{1}{4} q\cdot k\right) - q^\rho\left(\tfrac{5}{6} q\cdot p + \tfrac{1}{4} p\cdot k\right)\right) + \dots\right) - \tfrac{5}{6}\left(k^\mu k^\nu (p-q)^\rho + \dots\right) - \tfrac{1}{4}\left(p^\rho q^\mu k^\nu - p^\nu q^\rho k^\mu\right) \right\} \left\{ \begin{matrix} \log \Lambda^2 \\ \frac{2}{\epsilon} \end{matrix} \right\}$$



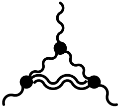
$$= \frac{1}{16\pi^2} g\kappa^2 f^{abc} \left\{ \left[\left(\eta^{\mu\nu}\left(p^\rho\left(-\tfrac{7}{6} p\cdot q - \tfrac{1}{6} p\cdot k - \tfrac{3}{4} q\cdot k\right) - q^\rho(\dots)\right) + \dots\right) + \left(k^\mu k^\nu (p-q)^\rho + \dots\right) + \tfrac{3}{4}\left(p^\rho q^\mu k^\nu - p^\nu q^\rho k^\mu\right) \right] \left\{ \begin{matrix} \log \Lambda^2 \\ \frac{2}{\epsilon} \end{matrix} \right\} + \tfrac{3}{2}\left(\eta^{\mu\nu}(p-q)^\rho + \dots\right) \left\{ \begin{matrix} \Lambda^2 \\ 0 \end{matrix} \right\} \right\}$$



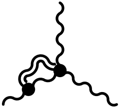
$$= -\frac{1}{16\pi^2} g\kappa^2 f^{abc} \frac{3}{2}\left(\eta^{\mu\nu}(p-q)^\rho + \dots\right) \left\{ \begin{matrix} \Lambda^2 \\ 0 \end{matrix} \right\}$$

The Quadratic Divergences of the Three Gluon Graphs

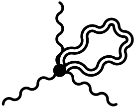
- Again cancellation of the quadratic divergencies (trivial in dim. reg.)



$$\Big|_{\mathcal{O}(\Lambda^2)} = 0$$



$$\Big|_{\mathcal{O}(\Lambda^2)} = + \frac{1}{16\pi^2} g\kappa^2 f^{abc} \frac{3}{2} (\eta^{\mu\nu}(p-q)^\rho + \dots) \begin{Bmatrix} \Lambda^2 \\ 0 \end{Bmatrix}$$

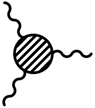


$$\Big|_{\mathcal{O}(\Lambda^2)} = - \frac{1}{16\pi^2} g\kappa^2 f^{abc} \frac{3}{2} (\eta^{\mu\nu}(p-q)^\rho + \dots) \begin{Bmatrix} \Lambda^2 \\ 0 \end{Bmatrix}$$

- ⇒ Main result: $\boxed{\beta_g\big|_{\mathcal{O}(\kappa^2)} = 0}$ No alteration of assymptotic freedom

The Sum of the Divergences of the Three Gluon Graphs

- Remaining logarithmic divergencies:



$$\begin{aligned} = \frac{1}{16\pi^2} g\kappa^2 f^{abc}\Big[&(\eta^{\mu\nu}(p^\rho(-\tfrac{1}{3}p\cdot q - \tfrac{1}{6}p\cdot k - \tfrac{1}{2}q\cdot k) \\ &\qquad - q^\rho(-\tfrac{1}{3}q\cdot p - \tfrac{1}{6}q\cdot k - \tfrac{1}{2}p\cdot k) + \dots) \\ &+ \tfrac{1}{6}(k^\mu k^\nu (p-q)^\rho + \dots) \\ &+ \tfrac{1}{2}(p^\rho q^\mu k^\nu - p^\nu q^\rho k^\mu)\Big] \left\{\begin{matrix} \log\Lambda^2 \\ \frac{2}{\epsilon} \end{matrix}\right\} \end{aligned}$$

- Lead to counterterms of dimension 6 operators

The Dimension Six Terms

- Using Bianchi identities naively 3 possible structures:

$$\begin{aligned} \mathcal{O}_1 &\equiv \mathrm{Tr}[\,(D_\mu F_{\nu\rho})^2\,] \\ \mathcal{O}_2 &\equiv \mathrm{Tr}[\,(D_\mu F^\mu{}_\nu)^2\,] \\ \mathcal{O}_3 &\equiv i\ \mathrm{Tr}[\,F_\mu{}^\nu\,F_\nu{}^\rho\,F_\rho{}^\mu\,] \end{aligned}$$

- However, are linearly related

$$\mathcal{O}_2 = \frac{1}{2}\,\mathcal{O}_1 - 2\,g\,\mathcal{O}_3 + \text{total derivatives}$$

- Forces us to extend original EYM action by

$$\mathcal{L}_{\text{dim6}} = d_1\mathcal{O}_1 + d_2\mathcal{O}_2$$

- Turns out that log divergent terms are cancelled by $\mathcal{O}_2$ alone.

The Effective Lagrangian

- In summary we thus have the renormalized extended lagrangian

$$
\begin{aligned}
\mathcal{L} =& \mathcal{L}_{\text{ext}} + \mathcal{L}_{\text{CT}} \\
\mathcal{L}_{\text{ext}} =& \mathcal{L}_{\text{EYM, ren}} + d_2 \ \mathrm{Tr}[(D_\mu F^{\mu\nu})^2] \\
\mathcal{L}_{\text{CT}} =& \delta_2 \, (\partial A)^2 + g\, \delta_1^{3g} \, A^2 \partial A + \delta_1^{2d_2} \, (\partial^2 A)^2 + g\, \delta_1^{3d_2} \, \partial^2 A \, \partial A A + \mathcal{O}(A^4)
\end{aligned}
$$

- We find the κ dependent countertems:

$$
\delta_2 \Big|_{\mathcal{O}(\kappa^2)} = \delta_1^{3g} \Big|_{\mathcal{O}(\kappa^2)} = 0
$$

$$
\delta_1^{2d_2} \Big|_{\mathcal{O}(\kappa^2)} = \delta_1^{3d_2} \Big|_{\mathcal{O}(\kappa^2)} = \frac{1}{(4\pi)^2} \frac{1}{6} \kappa^2 \left\{ \begin{matrix} \log(\frac{\Lambda^2}{\mu^2}) \\ \frac{2\,\mu^{-\epsilon}}{\epsilon} \end{matrix} \right\}
$$

- Which yields the β-function contributions:

$$
\boxed{\beta_g \Big|_{\mathcal{O}(\kappa^2)} = 0, \qquad \beta_{d_2} \Big|_{\mathcal{O}(\kappa^2)} = \frac{1}{(4\pi)^2} \frac{1}{3} \kappa^2}
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Summary

- Considered Einstein-Yang-Mills system in framework of effective field theories.
- Controversies in background field approach for gravitational contributions to Yang-Mills β-function.
- Potential effect not visible in dimensional regularization.
- Performed conceptually clean diagramatic computation of one-loop renormalization of gluon 2 and 3 point functions.
- $\beta_g\Big|_{\mathcal{O}(\kappa^2)} = 0$
- Gravity enforces dimension 6 counterterm $\mathcal{L}_{\mathsf{CT}} = d_2 \; \mathrm{Tr}[(D_\mu F^{\mu\nu})^2]$ whose β_{d_2} receives gravitational contributions.