Quantum Field Theory and Light Scalar Mesons

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OUTLINE

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2. Confinement, chiral dynamics and light scalar mesons.
3. Chiral shielding of $\sigma(600)$, chiral constraints, $\sigma(600)$, $f_0(980)$, and their mixing in $\pi\pi \to \pi\pi$ and $\phi \to \gamma\pi^0\pi^0$.
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Evidence for four-quark components of light scalars is given.

The priority of Quantum Field Theory in revealing the light scalar mystery is emphasized.

Introduction

The scalar channels in the region up to 1 GeV became a stumbling block of QCD. The point is that both perturbation theory and sum rules do not work in these channels because there are not solitary resonances in this region.

At the same time the question on the nature of the light scalar mesons is major for understanding the mechanism of the chiral symmetry realization, arising from the confinement, and hence for understanding the confinement itself.

Kategorischer Imperativ

Discussing the nature of the light scalar meson nonet ($f_0(600)$ (or $\sigma(600)$), $\kappa(700-900)$, $f_0(980)$, and $a_0(980)$) **requests** not only the explanation of the $f_0(980)$ and $a_0(980)$ **mass degeneracy but also the answer to the next real challenges**.

1. The copious $\phi\to\gamma f_0(980)$ decay and **especially** the copious $\phi\to\gamma a_0(980)$ decay, which looks as the decay **plainly** forbidden by the **Okubo-Zweig-Iizuka (OZI)** rule in the quark-antiquark model $a_0(980)=(u\bar u-d\bar d)/\sqrt{2}$.
2. **Absence** of $J/\psi\to a_0(980)\rho$ and $J/\psi\to f_0(980)\omega$ in contrast to **copious** $J/\psi\to a_2\rho$ and $J/\psi\to f_2\omega$.
3. **Absence** of $J/\psi\to\gamma f_0(980)$ in contrast to **copious** $J/\psi\to\gamma f_2(1270)$ and $J/\psi\to\gamma f_2'(1525)\phi$.
4. **Suppression** of $a_0(980)\to\gamma\gamma$ and $f_0(980)\to\gamma\gamma$ in contrast to **copious** $a_2(1320)\to\gamma\gamma$, $f_2(1270)\to\gamma\gamma$.

QCD, Chiral Limit, Confinement, σ-models

$$L = -(1/2)Tr\,(G_{\mu\nu}(x)G^{\mu\nu}(x)) + \bar{q}(x)(i\hat{D} - M)q(x).$$

M mixes Left and Right Spaces $q_L(x)$ and $q_R(x)$. But in chiral limit $M \to 0$ these spaces separate realizing $U_L(3) \times U_R(3)$ flavour symmetry, which, however, is broken by the gluonic anomaly up to $\mathrm{U_{vec}(1) \times SU_L(3) \times SU_R(3)}$.

As Experiment suggests, Confinement forms colourless observable hadronic fields and spontaneous breaking of chiral symmetry with massless pseudoscalar fields. There are two possible scenarios for QCD at low energy.

1. $U_L(3) \times U_R(3)$ non-linear σ-model.
2. $U_L(3) \times U_R(3)$ linear σ-model.

The experimental nonet of the light scalar mesons suggests

$U_L(3) \times U_R(3)$ linear σ-model.

History

Hunting the light σ and κ mesons had begun in the sixties already and a preliminary information on the light scalar mesons in Particle Data Group (PDG) Reviews had appeared at that time. But long-standing unsuccessful attempts to prove their existence in a conclusive way entailed general disappointment and an information on these states disappeared from PDG Reviews. One of principal reasons against the σ and κ mesons was the fact that both $\pi\pi$ and πK scattering phase shifts do not pass over 90^0 at putative resonance masses.

$SU_L(2) \times SU_R(2)$ linear σ model

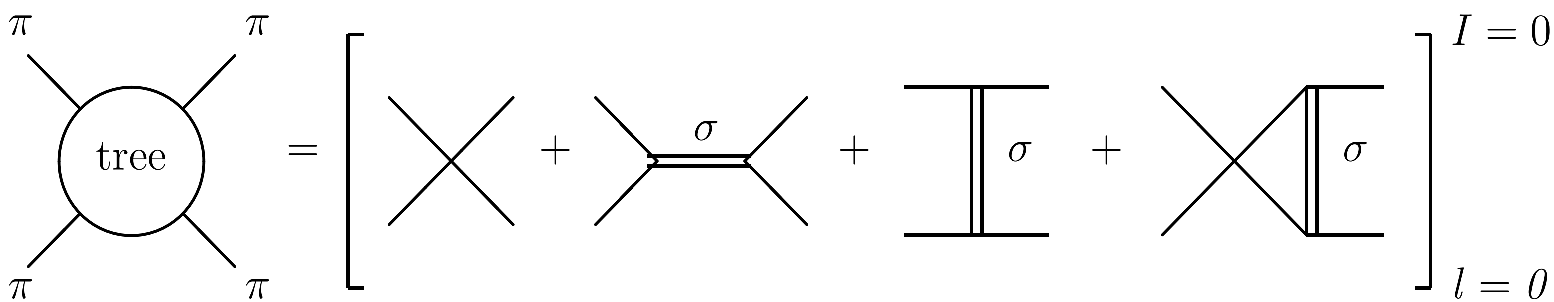
Situation changes when we showed that in the linear σ-model

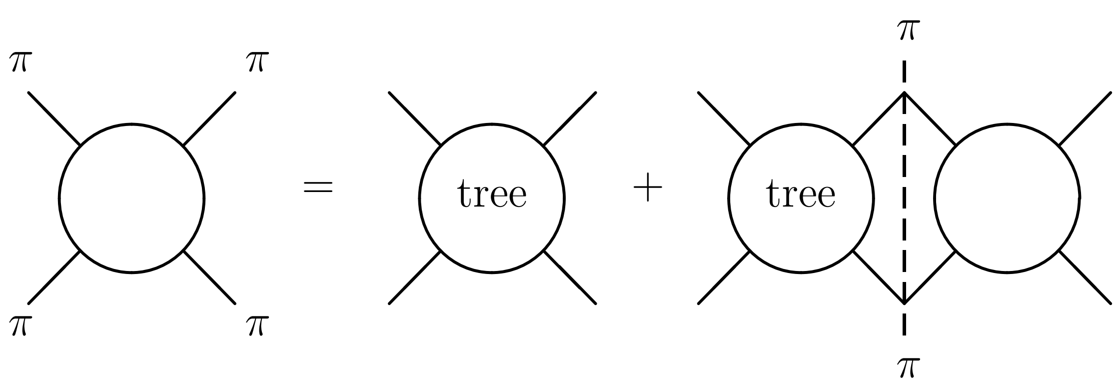
$$\mathbf{L} = \frac{1}{2}\left[(\partial_\mu \sigma)^2 + (\partial_\mu \vec{\pi})^2\right] - \frac{m_\sigma^2}{2}\sigma^2 - \frac{m_\pi^2}{2}\vec{\pi}^2$$

$$-\frac{m_\sigma^2 - m_\pi^2}{8f_\pi^2}\left[\left(\sigma^2 + \vec{\pi}^2\right)^2 + 4f_\pi \sigma\left(\sigma^2 + \vec{\pi}^2\right)\right]^2$$

there is a negative background phase which hides the σ meson (1993, 1994). It has been made clear that shielding wide lightest scalar mesons in chiral dynamics is very natural. This idea was picked up and triggered new wave of theoretical and experimental searches for the σ and κ mesons.

Our approximation





Our approximation

$$T_0^{0(tree)} = \frac{m_\pi^2 - m_\sigma^2}{32\pi f_\pi^2}\left[5 - 3\frac{m_\sigma^2 - m_\pi^2}{m_\sigma^2 - s}\right.$$

$$\left. -2\frac{m_\sigma^2 - m_\pi^2}{s - 4m_\pi^2}\ln\left(1 + \frac{s - 4m_\pi^2}{m_\sigma^2}\right)\right],$$

$$\mathrm{T}_0^0 = \frac{\mathrm{T}_0^{0(\mathrm{tree})}}{1 - \mathrm{i}\rho_{\pi\pi}\mathrm{T}_0^{0(\mathrm{tree})}} = \frac{\mathrm{e}^{2\mathrm{i}(\delta_{\mathrm{bg}}+\delta_{\mathrm{res}})} - 1}{2\mathrm{i}\rho_{\pi\pi}}$$

$$= \frac{1}{\rho_{\pi\pi}}\left(\frac{\mathrm{e}^{2\mathrm{i}\delta_{\mathrm{bg}}} - 1}{2\mathrm{i}}\right) + \mathrm{e}^{2\mathrm{i}\delta_{\mathrm{bg}}}\mathrm{T}_{\mathrm{res}},$$

$$\rho_\pi \equiv \rho_{\pi\pi} \equiv \rho_{\pi\pi}(m) = \sqrt{1 - 4m_\pi^2/m^2}.$$

Our approximation

$$T_{res} = \frac{1}{\rho_{\pi\pi}} \frac{\sqrt{s}\Gamma_{res}(s)}{M^2_{res} - s + \mathrm{Re}\Pi_{res}(M^2_{res}) - \Pi_{res}(s)} = \frac{e^{2i\delta_{res}} - 1}{2i\rho_{\pi\pi}}$$

$$T_{bg} = \frac{e^{2i\delta_{bg}} - 1}{2i\rho_{\pi\pi}} = \frac{\lambda(s)}{1 - i\rho_{\pi\pi}\lambda(s)}, \quad \lambda(s) = \frac{m^2_\pi - m^2_\sigma}{32\pi f^2_\pi} \Big[5 - 2\frac{m^2_\sigma - m^2_\pi}{s - 4m^2_\pi} \ln\left(1 + \frac{s - 4m^2_\pi}{m^2_\sigma}\right)\Big], \quad g_{\sigma\pi^+\pi^-} = -\frac{m^2_\sigma - m^2_\pi}{f_\pi}$$

$$\mathrm{Im}\Pi_{res}(s) = \frac{g^2_{res}(s)}{16\pi}\rho_{\pi\pi}, \quad \mathrm{Re}\Pi_{res}(s) = -\frac{g^2_{res}(s)}{16\pi}\lambda(s)\rho^2_{\pi\pi},$$

$$g_{res}(s) = \frac{g_{\sigma\pi\pi}}{|1 - i\rho_{\pi\pi}\lambda(s)|}, \quad M^2_{res} = m^2_\sigma - \mathrm{Re}\Pi_{res}(M^2_{res}).$$

Results in our approximation

$$T_0^2 = \frac{T_0^{2(tree)}}{1 - \imath\rho_{\pi\pi}T_2^{0(tree)}} = \frac{e^{2i\delta_0^2}-1}{2i\rho_{\pi\pi}}, \quad g_{\sigma\pi\pi} = \sqrt{\frac{3}{2}}\, g_{\sigma\pi^+\pi^-},$$

$$T_0^{2(tree)} = \frac{m_\pi^2 - m_\sigma^2}{32\pi f_\pi^2}\left[2 - 2\frac{m_\sigma^2 - m_\pi^2}{s-4m_\pi^2}\ln\left(1+\frac{s-4m_\pi^2}{m_\sigma^2}\right)\right].$$

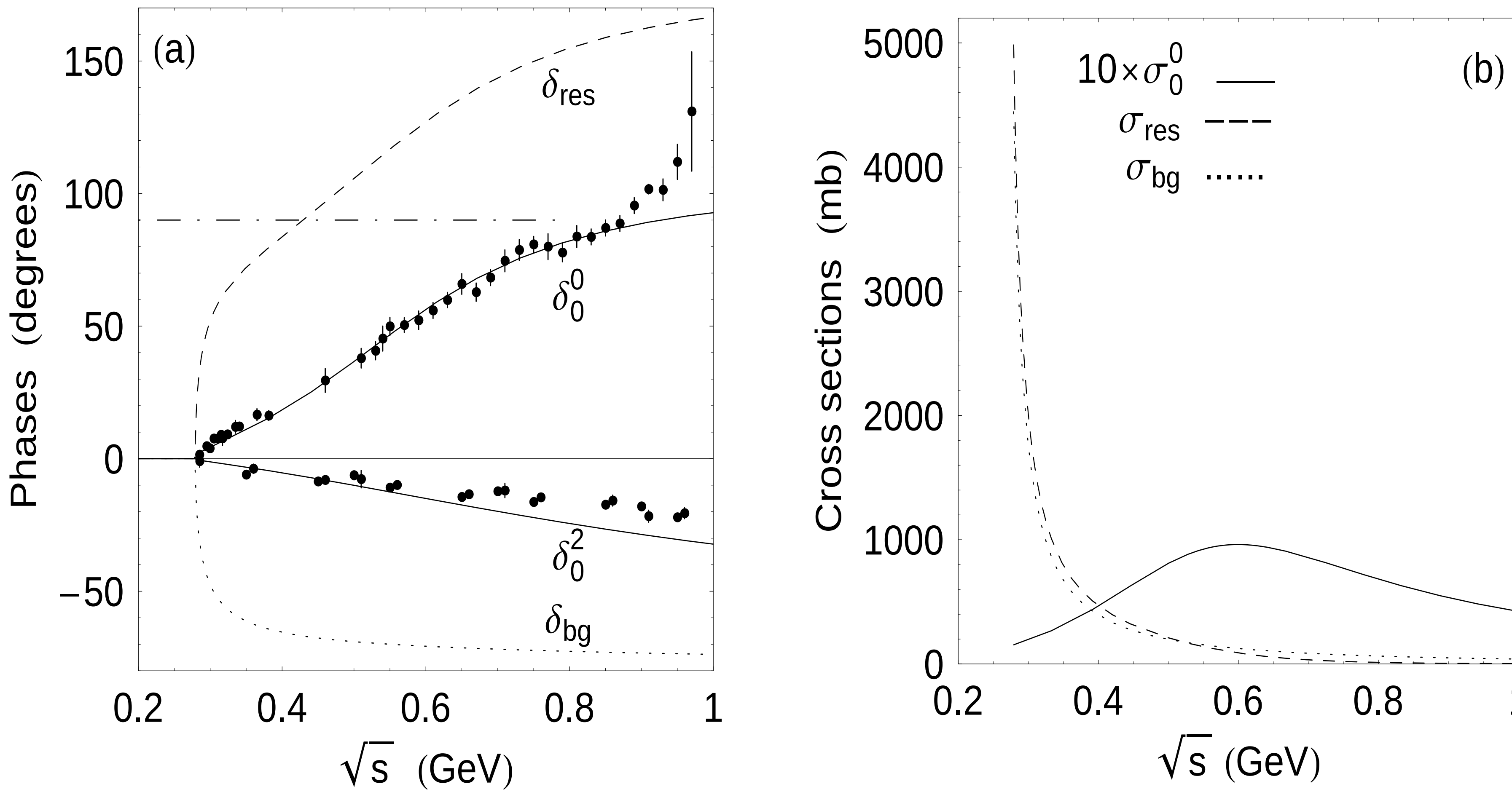
$$\mathrm{M_{res}} = 0.43\,\text{GeV},\ \Gamma_{\mathrm{res}}(\mathrm{M_{res}^2}) = 0.67\,\text{GeV},\ \mathrm{m}_\sigma = 0.93\,\text{GeV},$$

$$\Gamma_{\mathrm{res}}^{\mathrm{renorm}}(\mathrm{M_{res}^2}) = \frac{\Gamma_{\mathrm{res}}(\mathrm{M_{res}^2})}{(1+\mathrm{dRe}\Pi_{\mathrm{res}}(\mathrm{s})/\mathrm{ds}|_{\mathrm{s=M_{res}^2}})} = 0.53\,\text{GeV},$$

$$\Gamma_{\mathrm{res}}(\mathrm{s}) = \frac{\mathrm{g_{res}^2(s)}}{16\pi\sqrt{\mathrm{s}}}\rho_{\pi\pi}, \quad a_0^0 = 0.18\,m_\pi^{-1}, \quad a_0^2 = -0.04\,m_\pi^{-1},$$

$$g_{res}(M_{res}^2)/g_{\sigma\pi\pi} = 0.33,\ (\mathrm{s_A})_0^0 = 0.45\,\mathrm{m}_\pi^2,\ (\mathrm{s_A})_0^2 = 2.02\,\mathrm{m}_\pi^2.$$

Chiral Shielding in $\pi\pi \to \pi\pi$



The σ model. Our approximation. $\delta = \delta_{res} + \delta_{bg}$.

The σ pole in $\pi\pi \to \pi\pi$

$$T_0^0 \to \frac{g_\pi^2}{s - s_R},$$

$$\mathrm{g}_\pi^2 = (0.12 + \mathrm{i}0.21)\mathrm{GeV}^2,$$

$$s_R = (0.21 - i0.26)\mathrm{GeV}^2,$$

$$\sqrt{\mathrm{s_R}} = \mathrm{M_R} - \mathrm{i}\frac{\Gamma_\mathrm{R}}{2} = (0.52 - \mathrm{i}0.25)\mathrm{GeV}.$$

Considering the residue of the σ pole in T_0^0 as the square of its coupling constant to the $\pi\pi$ channel is not a clear guide to understand the σ meson nature for its great obscure imaginary part.

The σ propagator

$$\frac{1}{D_\sigma(s)} = \frac{1}{M^2_{res} - s + \text{Re}\Pi_{res}(M^2_{res}) - \Pi_{res}(s)} \, .$$

The σ meson self-energy $\Pi_{res}(s)$ is caused by the intermediate $\pi\pi$ states, that is, by the four-quark intermediate states if we keep in mind that the $SU_L(2) \times SU_R(2)$ linear σ model could be the low energy realization of the two-flavour QCD. This contribution shifts the Breit-Wigner (BW) mass greatly $m_\sigma - M_{res} = 0.50$ GeV. So, half the BW mass is determined by the four-quark contribution at least. The imaginary part dominates the propagator modulus in the region 300 MeV $< \sqrt{s} <$ 600 MeV. So, the σ field is described by its four-quark component at least in this energy (virtuality) region.

Chiral shielding in $\gamma\gamma \to \pi^+\pi^-$

$$T_S(\gamma\gamma \to \pi^+\pi^-) = T_S^{Born}(\gamma\gamma \to \pi^+\pi^-)$$

$$+8\alpha I_{\pi^+\pi^-}\, T_S(\pi^+\pi^- \to \pi^+\pi^-)$$

$$= T_S^{Born}(\gamma\gamma \to \pi^+\pi^-) + 8\alpha I_{\pi^+\pi^-}\left(\frac{2}{3}T_0^0 + \frac{1}{3}T_0^2\right)$$

in elastic region

$$= \frac{2}{3}e^{i\delta_0^0}\left\{T_S^{Born}(\gamma\gamma \to \pi^+\pi^-)\cos\delta_0^0 + 8\frac{\alpha}{\rho_{\pi\pi}}\mathrm{Re}I_{\pi^+\pi^-}\sin\delta_0^0\right\}$$

$$+\frac{1}{3}e^{i\delta_0^2}\left\{T_S^{Born}(\gamma\gamma \to \pi^+\pi^-)\cos\delta_0^2 + 8\frac{\alpha}{\rho_{\pi\pi}}\mathrm{Re}I_{\pi^+\pi^-}\sin\delta_0^2\right\}$$

Chiral shielding in $\gamma\gamma \to \pi^0\pi^0$

$$T_S(\gamma\gamma \to \pi^0\pi^0) = 8\alpha I_{\pi^+\pi^-} T_S(\pi^+\pi^- \to \pi^0\pi^0)$$

$$= 8\alpha I_{\pi^+\pi^-} \left(\frac{2}{3} T_0^0 - \frac{2}{3} T_0^2\right)$$

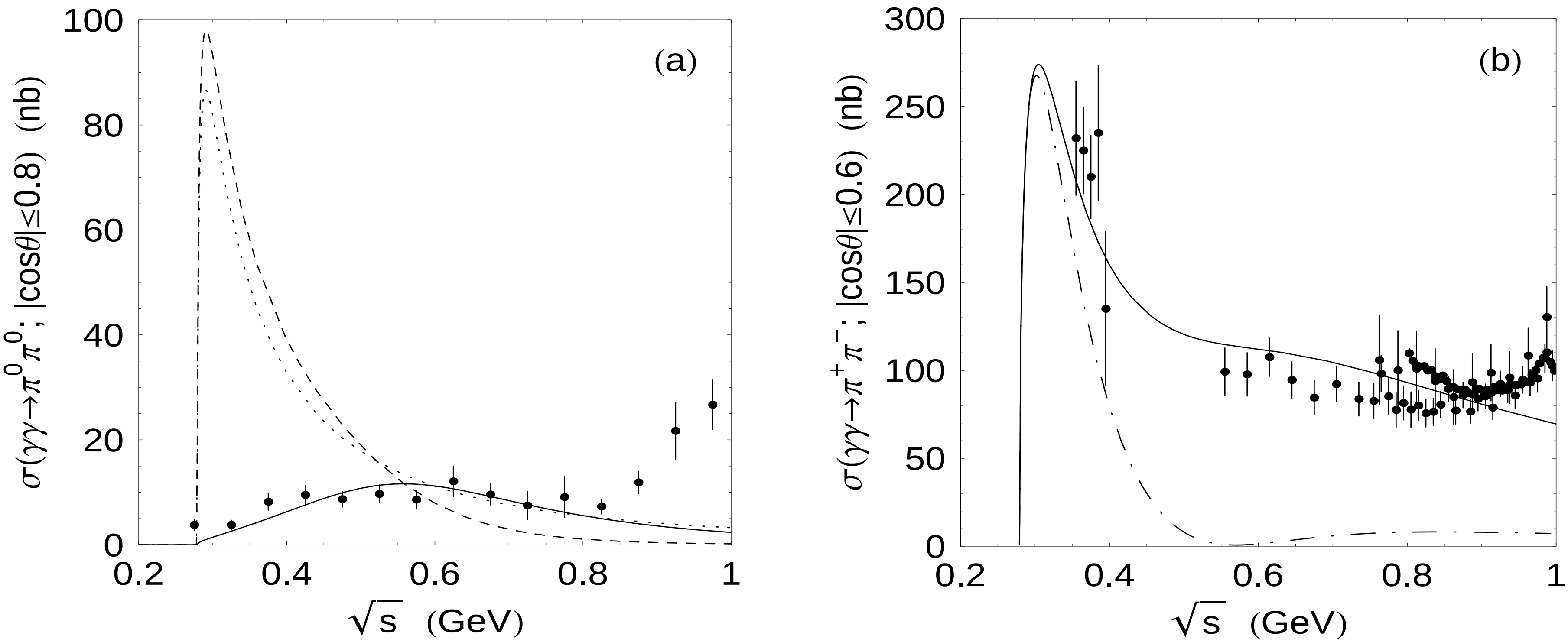
$$= \frac{2}{3} e^{i\delta_0^0} \left\{ T_S^{Born}(\gamma\gamma \to \pi^+\pi^-) \cos\delta_0^0 + 8\frac{\alpha}{\rho_{\pi\pi}} \mathrm{Re} I_{\pi^+\pi^-} \sin\delta_0^0 \right\}$$

$$-\frac{2}{3} e^{i\delta_0^2} \left\{ T_S^{Born}(\gamma\gamma \to \pi^+\pi^-) \cos\delta_0^2 + 8\frac{\alpha}{\rho_{\pi\pi}} \mathrm{Re} I_{\pi^+\pi^-} \sin\delta_0^2 \right\}$$

$$I_{\pi^+\pi^-} = \frac{m_\pi^2}{s}\left(\pi + i\ln\frac{1+\rho_{\pi\pi}}{1-\rho_{\pi\pi}}\right)^2 - 1\,, \quad s \geq 4m_\pi^2\,,$$

$$T_S^{Born}(\gamma\gamma \to \pi^+\pi^-) = \frac{8\alpha}{\rho_{\pi^+\pi^-}} \mathrm{Im} I_{\pi^+\pi^-}\,.$$

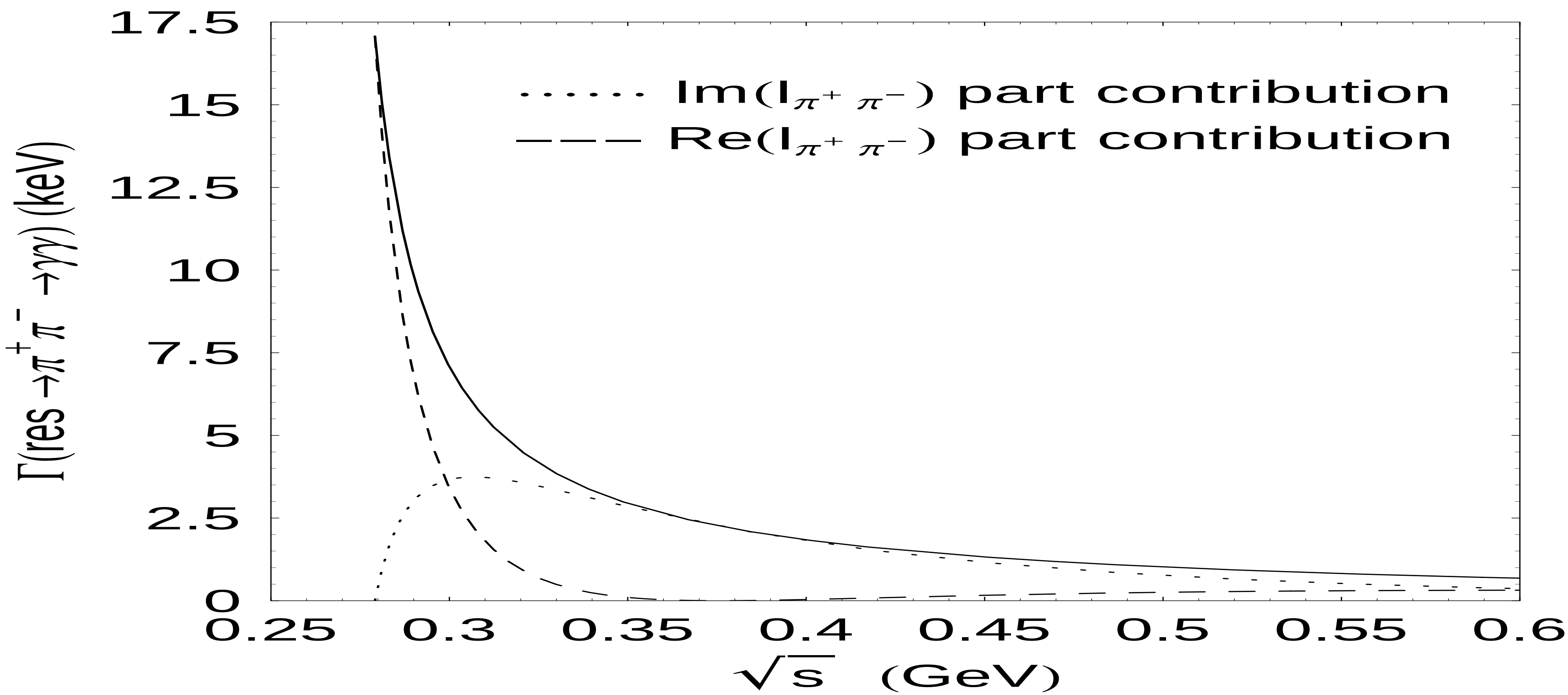
Chiral Shielding in $\gamma\gamma \to \pi\pi$



(a) The solid, dashed, and dotted lines are $\sigma_S(\gamma\gamma \to \pi^0\pi^0)$, $\sigma_{res}(\gamma\gamma \to \pi^0\pi^0)$, and $\sigma_{bg}(\gamma\gamma \to \pi^0\pi^0)$.

(b) The dashed-dotted line is $\sigma_S(\gamma\gamma \to \pi^+\pi^-)$. The solid line includes the higher waves from $T^{Born}(\gamma\gamma \to \pi^+\pi^-)$.

The $\sigma \to \gamma\gamma$ decay.



$$\mathbf{g}(\sigma \to \pi^+\pi^- \to \gamma\gamma,\, \mathrm{s}) = (\alpha/2\pi)\mathbf{I}_{\pi^+\pi^-} \times \mathbf{g}_{\mathrm{res}\,\pi^+\pi^-}(\mathrm{s}),$$

$$\Gamma(\sigma \to \pi^+\pi^- \to \gamma\gamma, \mathrm{s}) = \frac{1}{16\pi\sqrt{\mathrm{s}}}\left|\mathbf{g}(\sigma \to \pi^+\pi^- \to \gamma\gamma, \mathrm{s})\right|^2$$

Four-quark transition $\sigma \to \gamma\gamma$

So, the the $\sigma \to \gamma\gamma$ decay is described by the triangle $\pi^+\pi^-$-loop diagram $res \to \pi^+\pi^- \to \gamma\gamma$ ($I_{\pi^+\pi^-}$). Consequently, it is due to the four-quark transition because we imply a low energy realization of the two-flavour QCD by means of the the $SU_L(2) \times SU_R(2)$ linear σ model. As the previous Fig. suggests, the real intermediate $\pi^+\pi^-$ state dominates in $g(res \to \pi^+\pi^- \to \gamma\gamma)$ in the σ region $\sqrt{s} < 0.6$ GeV.

Thus the picture in the physical region is clear and informative. But, what about the pole in the complex s-plane? Does the pole residue reveal the σ indeed?

The σ pole in $\gamma\gamma \to \pi\pi$

$$\frac{1}{16\pi}\sqrt{\frac{3}{2}}\,T_S(\gamma\gamma \to \pi^0\pi^0) \to \frac{g_\gamma g_\pi}{s - s_R},$$

$$\mathrm{g}_\gamma \mathrm{g}_\pi = (-0.45 - \mathrm{i}0.19) \times 10^{-3}\,\mathrm{GeV}^2,$$

$$\mathrm{g}_\gamma / \mathrm{g}_\pi = (-1.61 + \mathrm{i}1.21) \times 10^{-3},$$

$$\Gamma(\sigma \to \gamma\gamma) = |\mathrm{g}_\gamma|^2 / \mathrm{M_R} \approx 2\,\mathrm{keV}.$$

It is hard to believe that anybody could learn the complex but physically clear dynamics of the $\sigma \to \gamma\gamma$ decay described above from the residues of the σ pole.

First Discussion and Conclusion

Heiri Leutwyler and collaborators obtained

$$\sqrt{s_R} = M_R - i\Gamma_R/2 = \left(441^{+16}_{-8} - i272^{+12.5}_{-9}\right) \times \text{MeV}$$

with the help of the Roy equation.

Our result agrees with the above only qualitatively.

$$\sqrt{s_R} = M_R - i\Gamma_R/2 = (518 - i250) \times \text{MeV}.$$

This is natural, because our approximation gives only a semiquantitative description of the data at $\sqrt{s} < 0.4$ GeV. We do not regard also for effects of the $K\bar{K}$ channel, the $f_0(980)$ meson, and so on, that is, do not consider the $SU_L(3) \times SU_R(3)$ linear σ model.

First Discussion and Conclusion

Could the above scenario incorporates the primary lightest scalar Bob Jaffe four-quark state? Certainly the direct coupling of this state to $\gamma\gamma$ via neutral vector pairs ($\rho^0\rho^0$ and $\omega\omega$), contained in its wave function, is negligible

$\Gamma(q^2\bar{q}^2 \to \rho^0\rho^0 + \omega\omega \to \gamma\gamma) \approx 10^{-3}$ keV

as we showed in 1982. But its coupling to $\pi\pi$ is strong and leads to $\Gamma(q^2\bar{q}^2 \to \pi^+\pi^- \to \gamma\gamma)$ similar to $\Gamma(res \to \pi^+\pi^- \to \gamma\gamma)$ in the above Fig..

Let us add to $T_S(\gamma\gamma \to \pi^0\pi^0)$ the amplitude for the the direct coupling of σ to $\gamma\gamma$ conserving unitarity

$$T_{direct}(\gamma\gamma \to \pi^0\pi^0) = s g^{(0)}_{\sigma\gamma\gamma} g_{res}(s) e^{i\delta_{bg}} \Big/ D_{res}(s) \,,$$

where $g^{(0)}_{\sigma\gamma\gamma}$ is the direct coupling constant of σ to $\gamma\gamma$, the factor s is caused by gauge invariance.

First Discussion and Conclusion

Fitting the $\gamma\gamma \to \pi^0\pi^0$ data gives a negligible value of $g^{(0)}_{\sigma\gamma\gamma}$,

$$\Gamma^{(0)}_{\sigma\gamma\gamma} = \left| M^2_{res} g^{(0)}_{\sigma\gamma\gamma} \right|^2 \Big/ (16\pi M_{res}) \approx 0.0034 \text{ keV},$$

in astonishing agreement with our prediction (1982).

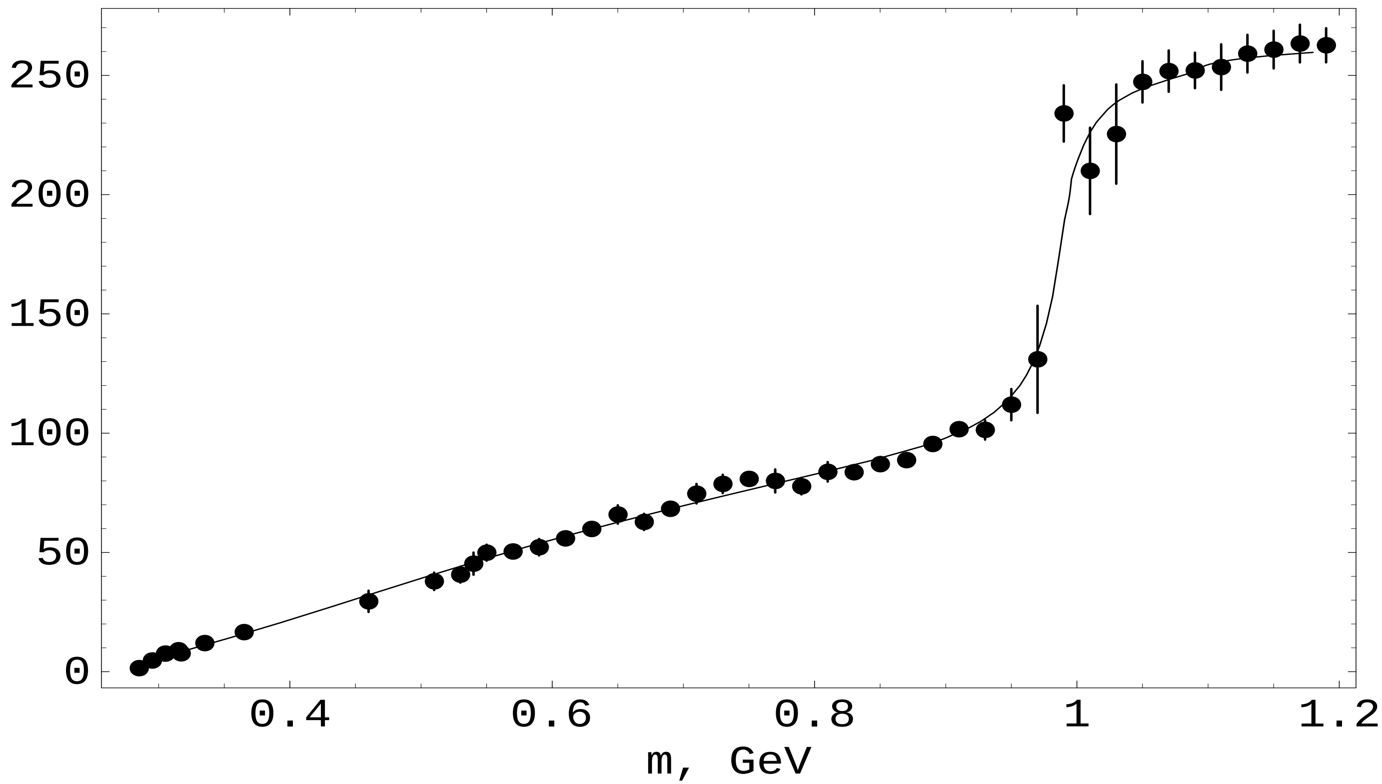
The majority of current investigations of the mass spectra in scalar channels does not study particle production mechanisms. That is why such investigations are only preprocessing experiments, and the derivable information is very relative.

The only progress in understanding the particle production mechanisms could essentially advance us in revealing the light scalar meson nature, as is evident from the foregoing.

Troubles and Expectancies

In theory the principal problem is impossibility to use the linear σ-model in the tree level approximation inserting widths into σ meson propagators because such an approach breaks the both unitarity and Adler self-consistency conditions. The comparison with the experiment requires the non-perturbative calculation of the process amplitudes. Nevertheless, now there are the possibilities to estimate odds of the $U_L(3)\times U_R(3)$ linear σ-model to underlie physics of light scalar mesons in phenomenology, taking into account the idea of chiral shielding, our treatment of $\sigma(600)$-$f_0(980)$ mixing based on quantum field theory ideas, and Adler's conditions.

Phenomenological Shielding, $\delta_0^0 = \delta_B^{\pi\pi} + \delta_{res}$



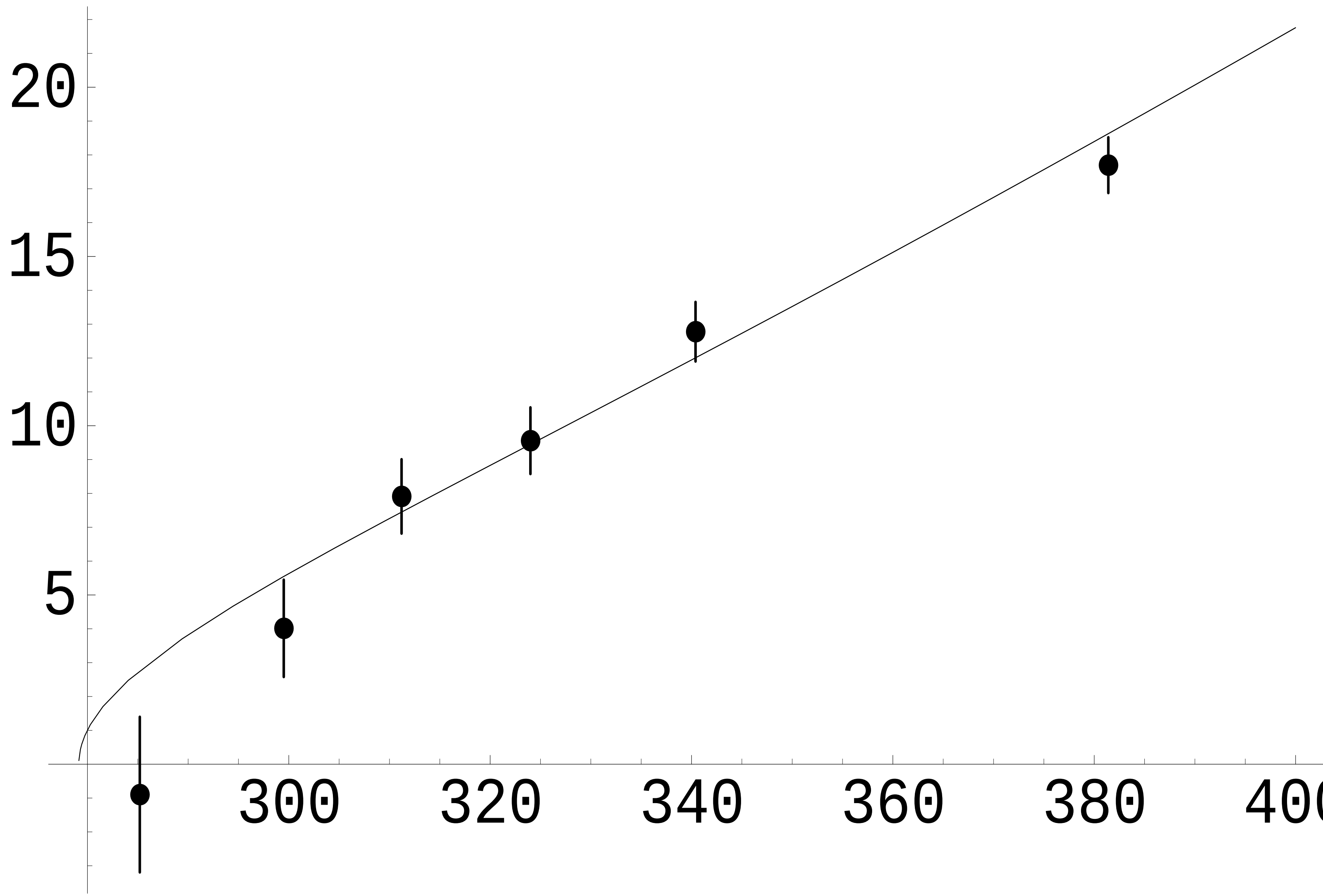
$$g^2_{\sigma\pi^+\pi^-}/4\pi = 0.99\,\text{GeV}^2, \quad g^2_{\sigma K^+K^-}/4\pi = 2\cdot 10^{-4}\,\text{GeV}^2$$

$$g^2_{f_0\pi^+\pi^-}/4\pi = 0.12\,\text{GeV}^2, \quad g^2_{f_0 K^+K^-}/4\pi = 1.04\,\text{GeV}^2$$

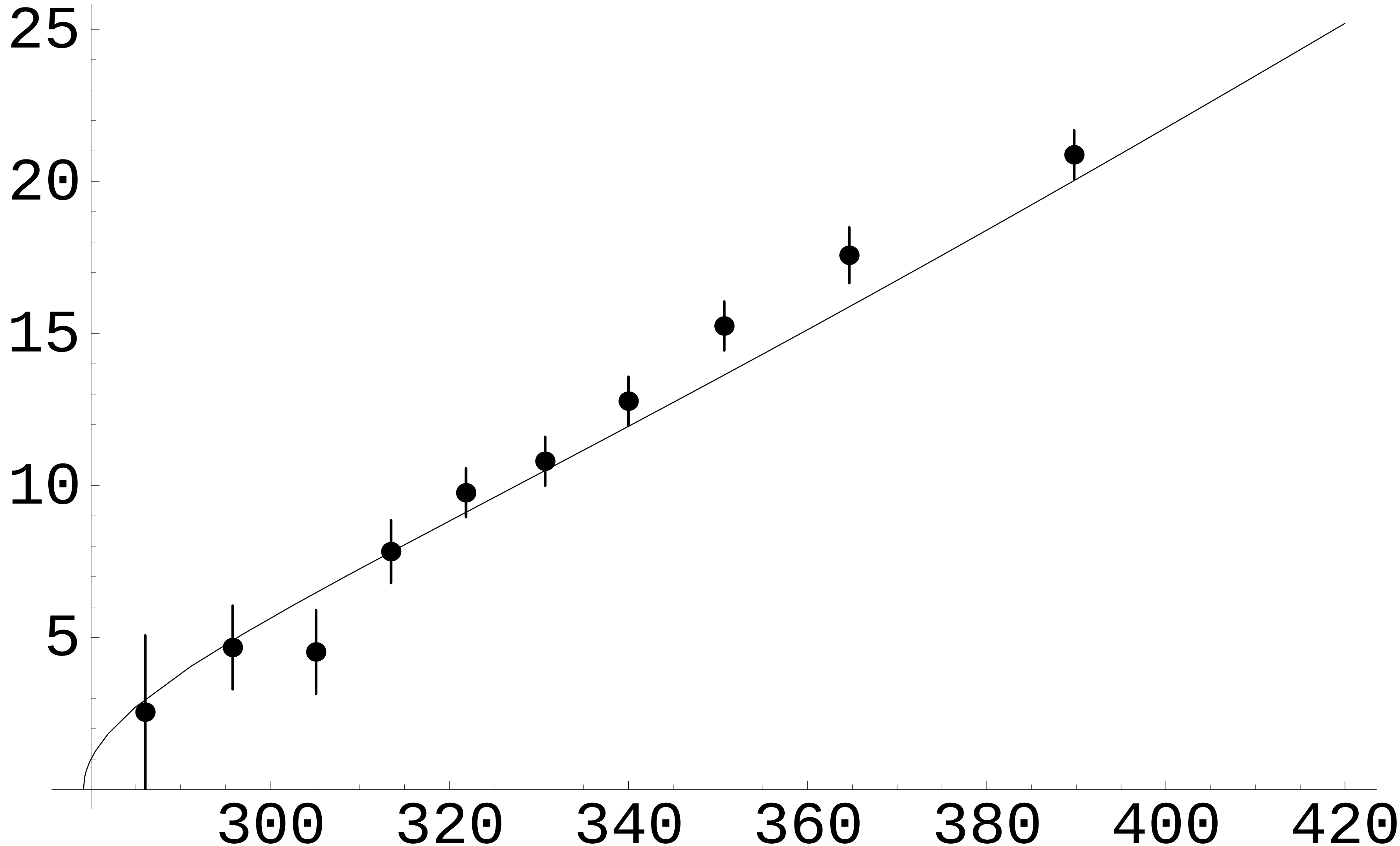
$$m_\sigma = 679\,\text{MeV}\,,\ \Gamma_\sigma = 498\,\text{MeV}\,,\ m_{f_0} = 989\,\text{MeV}\,,$$

the $l = I = 0\ \pi\pi$ scattering length $a_0^0 = 0.223\,m_{\pi^+}^{-1}$

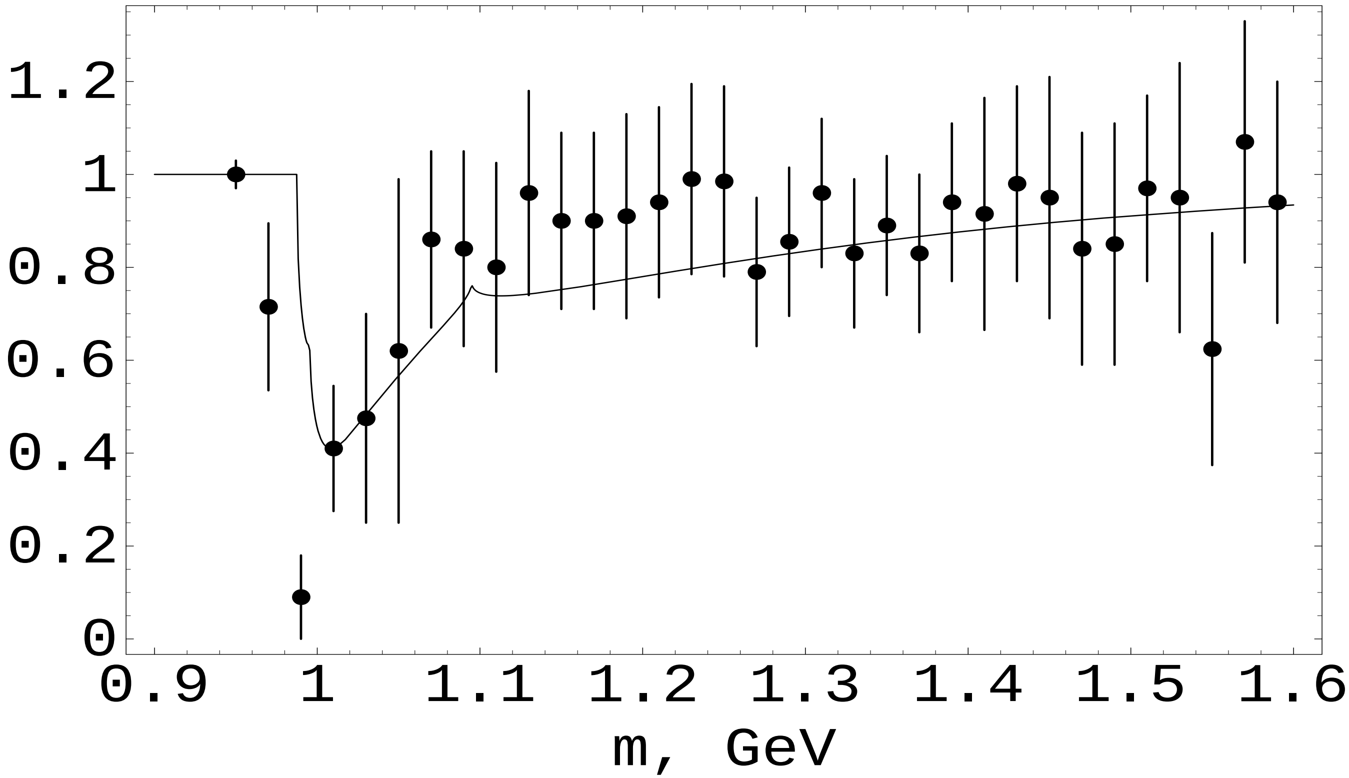
δ_0^0, comparison with BNL data



δ_0^0, comparison with NA48 data



Inelasticity, η_0^0

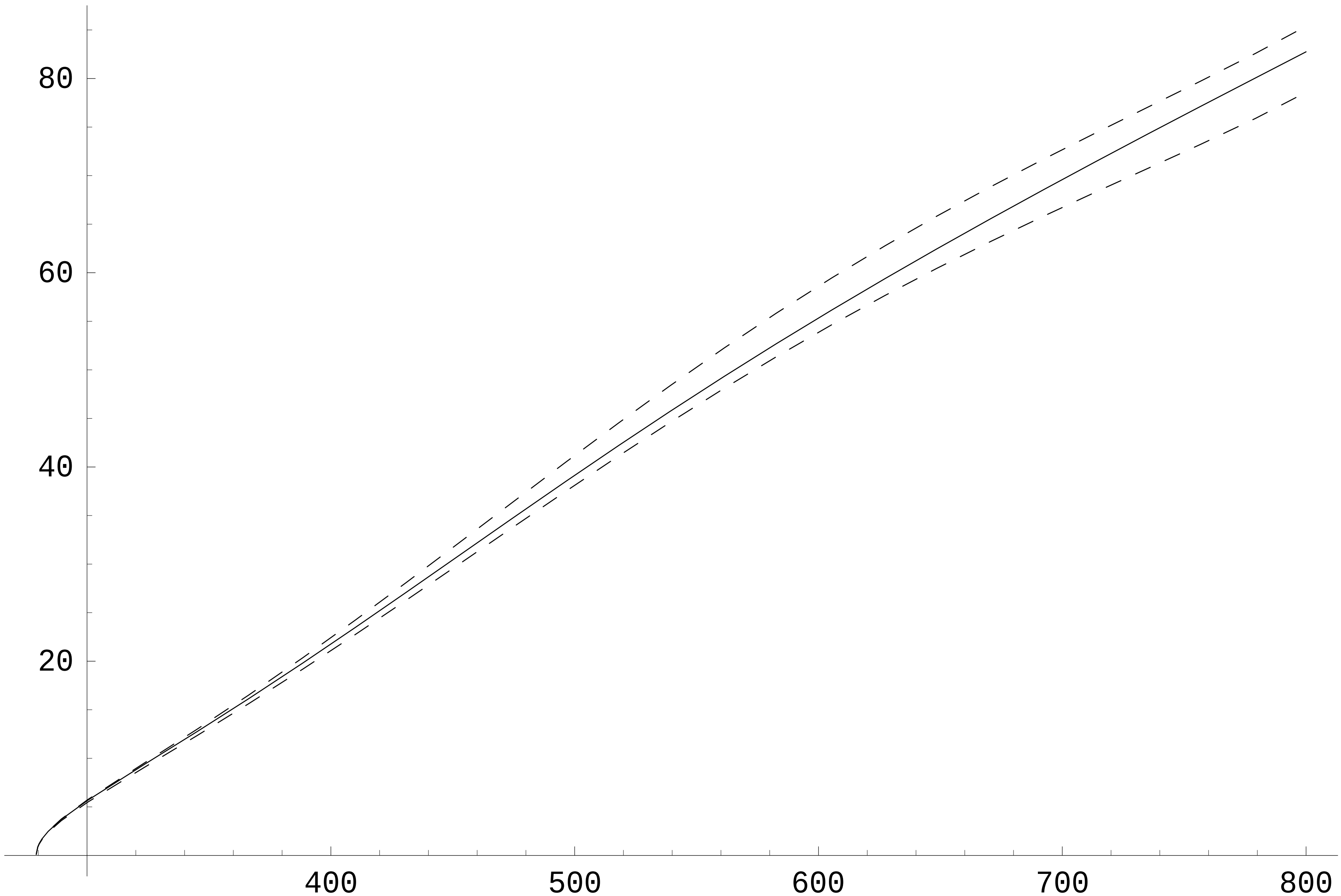


$$\eta_0^0 =$$

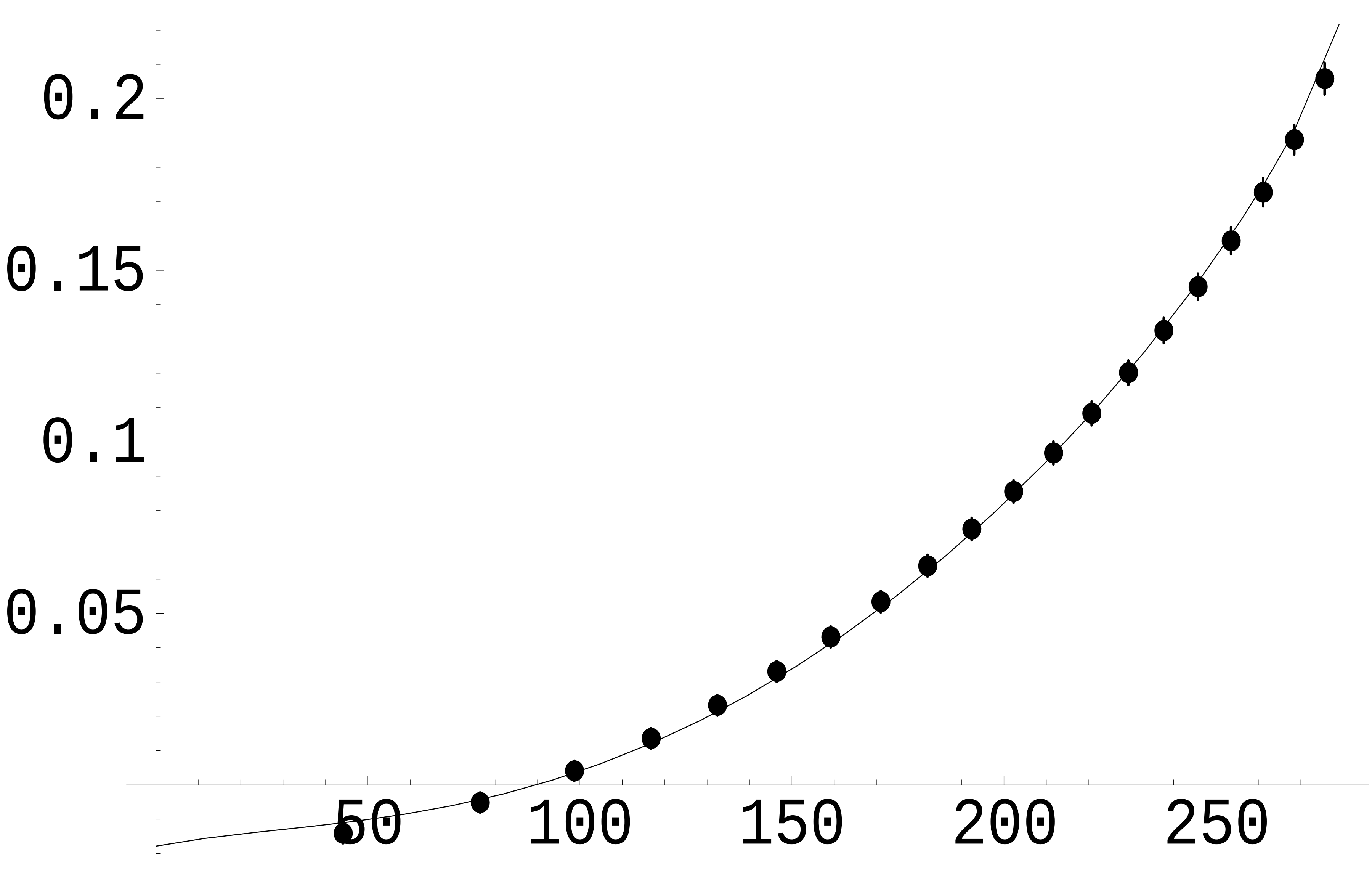
$$\sqrt{1 - 4\rho_{K^+}|T_0^0(\pi\pi \to K^+K^-)|^2 - 4\rho_{K^0}|T_0^0(\pi\pi \to K^0\bar{K}^0)|^2}$$

$$\rho_{K^+} \equiv \rho_{K^+K^-} \equiv \rho_{K^+K^-}(m) = \sqrt{1 - 4m_{K^+}^2/m^2}\,, \ldots$$

δ_0^0, comparison with CGL band



T_0^0, ● is Heiri Leutwyler's calculation



Four-quark Model

The nontrivial nature of the well-established light scalar resonances $f_0(980)$ and $a_0(980)$ is no longer denied practically anybody. As for the nonet as a whole, even a dope's look at PDG Review gives an idea of the four-quark structure of the light scalar meson nonet, $\sigma(600)$, $\kappa(700-900)$, $f_0(980)$, and $a_0(980)$, inverted in comparison with the classical P-wave $q\bar{q}$ tensor meson nonet, $f_2(1270)$, $a_2(1320)$, $K_2^*(1420)$, $\phi_2'(1525)$. Really, while the scalar nonet cannot be treated as the P-wave $q\bar{q}$ nonet in the naive quark model, it can be easy understood as the $q^2\bar{q}^2$ nonet, where σ has no strange quarks, κ has the s quark, f_0 and a_0 have the $s\bar{s}$-pair. Similar states were found by Jaffe in 1977 in the MIT bag.

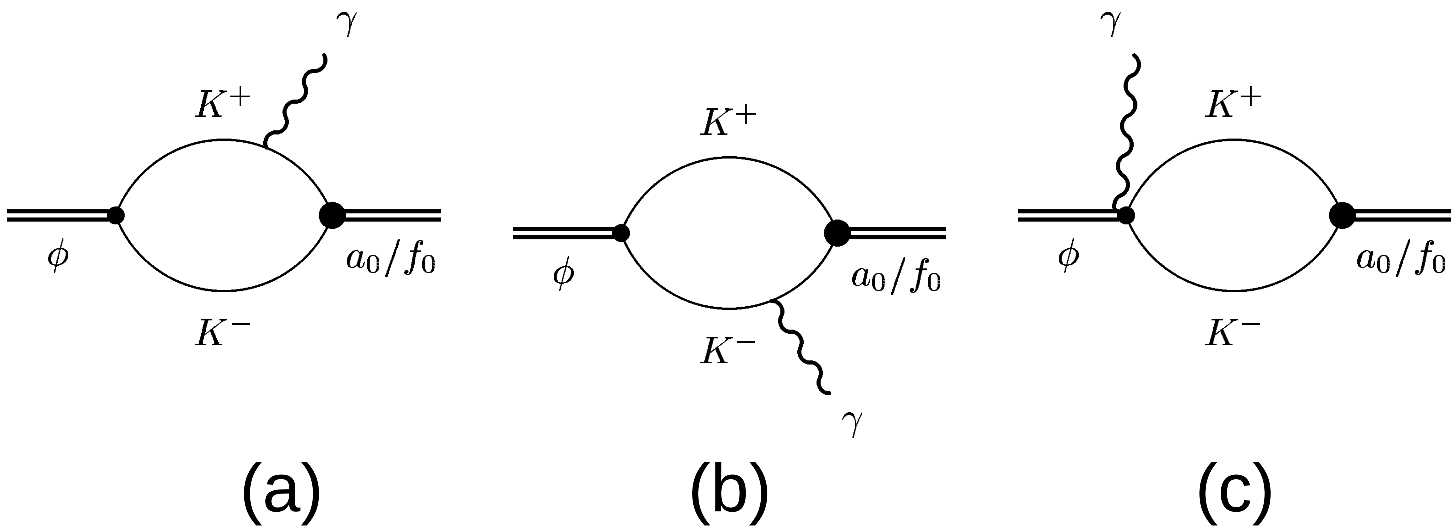
Radiative Decays of ϕ-Meson

Ten years later we showed that $\phi \to \gamma a_0 \to \gamma\pi\eta$ and $\phi \to \gamma f_0 \to \gamma\pi\pi$ can shed light on the problem of $a_0(980)$ and $f_0(980)$ mesons.

Now these decays are studied not only theoretically but also experimentally. The first measurements (1998, 2000) were reported by SND and CMD-2. After (2002) they were studied by KLOE in agreement with the Novosibirsk data but with a considerably smaller error.

Note that $a_0(980)$ is produced in the radiative ϕ meson decay as intensively as $\eta'(958)$ containing $\approx\ 66\%$ of $s\bar{s}$, responsible for $\phi \approx s\bar{s} \to \gamma s\bar{s} \to \gamma\eta'(958)$. It is a clear qualitative argument for the presence of the $s\bar{s}$ pair in the isovector $a_0(980)$ state, i.e., for its four-quark nature.

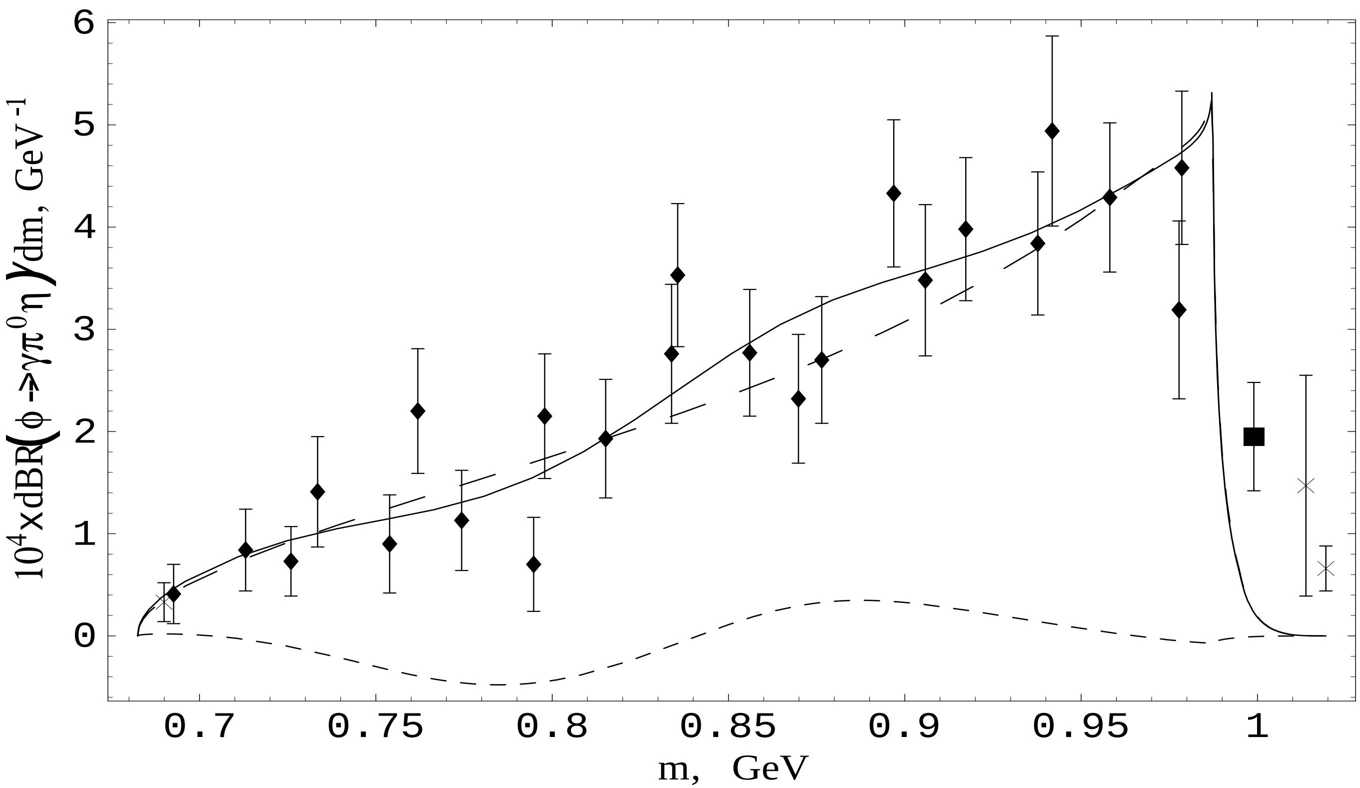
K^+K^--Loop Model



When basing the experimental investigations, we suggested one-loop model $\phi \to K^+K^- \to \gamma a_0/f_0$. This model is used in the data treatment and is ratified by experiment.

Gauge invariance gives the conclusive arguments in favor of the K^+K^- - loop transition as the principal mechanism of $a_0(980)$ and $f_0(980)$ meson production in the ϕ radiative decays.

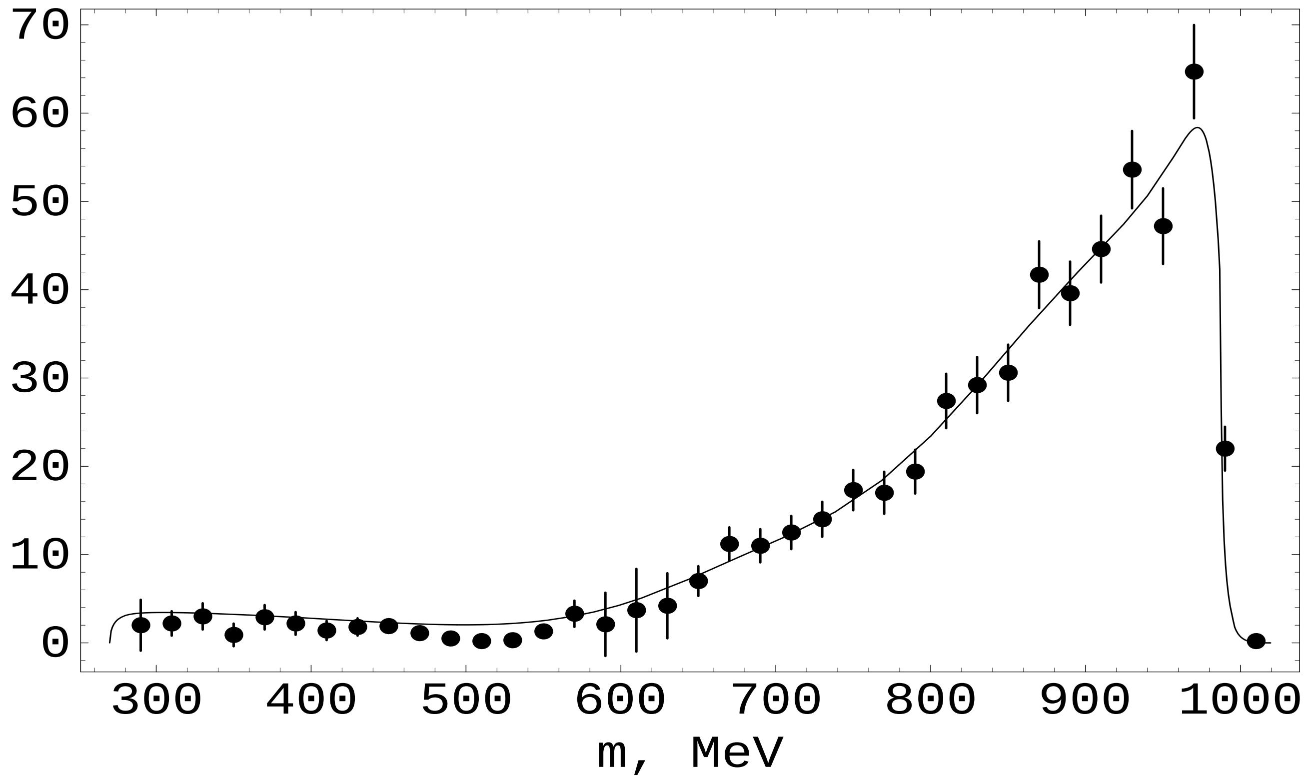
$\phi \to \gamma\pi^0\eta$, KLOE



$$\frac{dBR(\phi \to K^+K^- \to \gamma a_0 \to \gamma\pi^0\eta\,,\,m)}{dm} =$$

$$= \frac{4|g(m)|^2\omega(m)p_{\pi\eta}(m)}{\Gamma_\phi\,3(4\pi)^3 m_\phi^2}\left|\frac{g_{a_0K^+K^-}g_{a_0\pi\eta}}{D_{a_0}(m)}\right|^2$$

$\phi \to \gamma\pi^0\pi^0$, KLOE



$$\frac{dBR(\phi \to K^+K^- \to \gamma(\sigma + f_0) \to \gamma\pi^0\pi^0\,,\ m)}{dm} =$$

$$= \frac{16|g(m)|^2\omega(m)p_{\pi\eta}(m)}{\Gamma_\phi\, 3\pi m_\phi^2}\left|T_0^0(K^+K^- \to \pi^0\pi^0)\right|^2$$

Spectra and Gauge Invariance

To describe the experimental spectra

$$\mathrm{S_R(m)} \equiv \frac{\mathrm{dBR}(\phi \to \gamma \mathrm{R} \to \gamma \mathrm{ab}\,,\ \mathrm{m})}{\mathrm{dm}} =$$

$$\frac{2}{\pi}\frac{\mathrm{m}^2\Gamma(\phi \to \gamma \mathrm{R}\,,\ \mathrm{m})\Gamma(\mathrm{R} \to \mathrm{ab}\,,\ \mathrm{m})}{\Gamma_\phi |\mathrm{D_R(m)}|^2}$$

$$= \frac{4|\mathrm{g_R(m)}|^2\omega(\mathrm{m})\mathrm{p_{ab}(m)}}{\Gamma_\phi\, 3(4\pi)^3\mathrm{m}_\phi^2}\left|\frac{\mathrm{g_{Rab}}}{\mathrm{D_R(m)}}\right|^2,$$

$$\mathrm{R} = \mathrm{a_0},\ \mathrm{f_0},\ \mathrm{ab} = \pi^0\eta,\ \pi^0\pi^0,$$

$|g_R(m)|^2$ should be smooth at $m \le 0.99$ GeV (the photon energy $\omega(m) = (m_\phi^2 - m^2)/2m_\phi \le 29$ MeV). But gauge invariance requires $A\left[\phi(p) \to \gamma(k)R(q)\right] =$

Spectra and Gauge Invariance

$$= G_R(m)\left[p_\mu e_\nu(\phi) - p_\nu e_\mu(\phi)\right]\left[k_\mu e_\nu(\gamma) - k_\nu e_\mu(\gamma)\right],$$

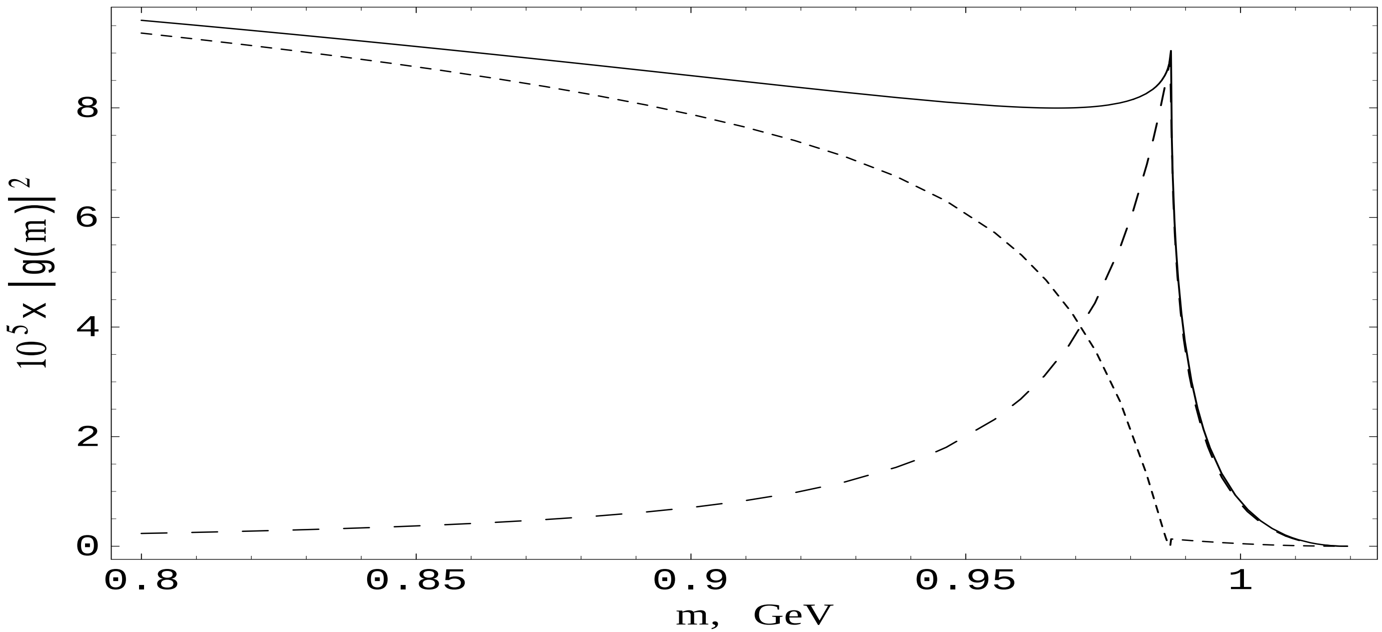
$$g_R(m) = -2(pk)G_R(m) = -2\omega(m)m_\phi G_R(m)$$

Stopping the function $(\omega(m))^2$ at $\omega(990\,\text{MeV}) = 29$ MeV with the help of the form-factor $1/\left[1 + (R\omega(m))^2\right]$ requires a prohibitive $R \approx 100$ GeV^{-1}. $R \approx 10$ GeV^{-1} allows us to obtain an effective maximum of the mass spectrum only near 900 MeV.

So stopping the $g_R(m)$ function is the crucial point in understanding the mechanism of the production of $a_0(980)$ and $f_0(980)$ resonances in the ϕ radiative decays.

The K^+K^--loop model $\phi \to K^+K^- \to \gamma R$ solves this problem in the elegant way: fine threshold phenomenon is discovered.

New Threshold Phenomenon



The universal in K^+K^--loop model function $|g(m)|^2 = |g_R(m)/g_{RK^+K^-}|^2$ is drawn with the solid line. The contribution of the imaginary part is drawn with the dashed line. The contribution of the real part is drawn with the dotted line.

K^+K^--Loop Mechanism is established

In truth this means that $a_0(980)$ and $f_0(980)$ are seen in the radiative decays of ϕ meson owing to K^+K^- intermediate state.

So, the mechanism of production of $a_0(980)$ and $f_0(980)$ mesons in the ϕ radiative decays is established at a physical level of proof.

WE ARE DEALING WITH THE FOUR-QUARK TRANSITION.

A radiative four-quark transition between two $q\bar{q}$ states requires creation and annihilation of an additional $q\bar{q}$ pair, i.e., such a transition is forbidden according to the OZI rule, while a radiative four-quark transition between $q\bar{q}$ and $q^2\bar{q}^2$ states requires only creation of an additional $q\bar{q}$ pair, i.e., such a transition is allowed according to the OZI rule. The large N_C expansion supports this conclusion.

$a_0(980)/f_0(980) \to \gamma\gamma$ & $q^2\bar{q}^2$-Model

Recall that twenty six years ago the suppression of $a_0(980) \to \gamma\gamma$ and $f_0(980) \to \gamma\gamma$ was predicted in our work based on the $q^2\bar{q}^2$ model,

$\Gamma(a_0(980) \to \gamma\gamma) \sim \Gamma(f_0(980) \to \gamma\gamma) \sim 0.27$ keV.

Experiment supported this predicton

$\Gamma(a_0 \to \gamma\gamma) = (0.19 \pm 0.07^{+0.1}_{-0.07})/B(a_0 \to \pi\eta)$ keV, Crystal Ball

$\Gamma(a_0 \to \gamma\gamma) = (0.28 \pm 0.04 \pm 0.1)/B(a_0 \to \pi\eta)$ keV, JADE.

When in the $q\bar{q}$ model it was anticipated

$$\Gamma(a_0 \to \gamma\gamma) = (1.5 - 5.9)\Gamma(a_2 \to \gamma\gamma)$$
$$= (1.5 - 5.9)(1.04 \pm 0.09) \text{ keV.}$$

The wide scatter of the predictions is connected with different reasonable guesses of the potential form.

$f_0(980)/a_0(980) \to \gamma\gamma$

The $a_0 \to K^+K^- \to \gamma\gamma$ model describes adequately data and correspond the four-quark transition $a_0 \to q^2\bar{q}^2 \to \gamma\gamma$, $\langle\Gamma(a_0 \to K^+K^- \to \gamma\gamma\rangle \approx 0.3$ keV.

$\Gamma(f_0 \to \gamma\gamma) = (0.31 \pm 0.14 \pm 0.09)$ keV, Crystal Ball,
$\Gamma(f_0 \to \gamma\gamma) = (0.24 \pm 0.06 \pm 0.15)$ keV, MARK II.

When in the $q\bar{q}$ model it was anticipated

$$\Gamma(f_0 \to \gamma\gamma) = (1.7 - 5.5)\Gamma(f_2 \to \gamma\gamma)$$
$$= (1.7 - 5.5)(2.8 \pm 0.4) \text{ keV.}$$

The wide scatter of the predictions is connected with different reasonable guesses of the potential form.

Dynamics of $\gamma\gamma \to \pi\pi$

$$T_S(\gamma\gamma \to \pi^+\pi^-,\, s) = T_S^{Born}(\gamma\gamma \to \pi^+\pi^-,\, s)$$

$$+ 8\alpha \mathrm{I}_{\pi^+\pi^-}(\mathrm{s})\, \mathrm{T_S}(\pi^+\pi^- \to \pi^+\pi^-,\, \mathrm{s})$$

$$+ 8\alpha \mathrm{I}_{\mathrm{K^+K^-}}(\mathrm{s})\, \mathrm{T_S}(\mathrm{K^+K^-} \to \pi^+\pi^-,\, \mathrm{s})$$

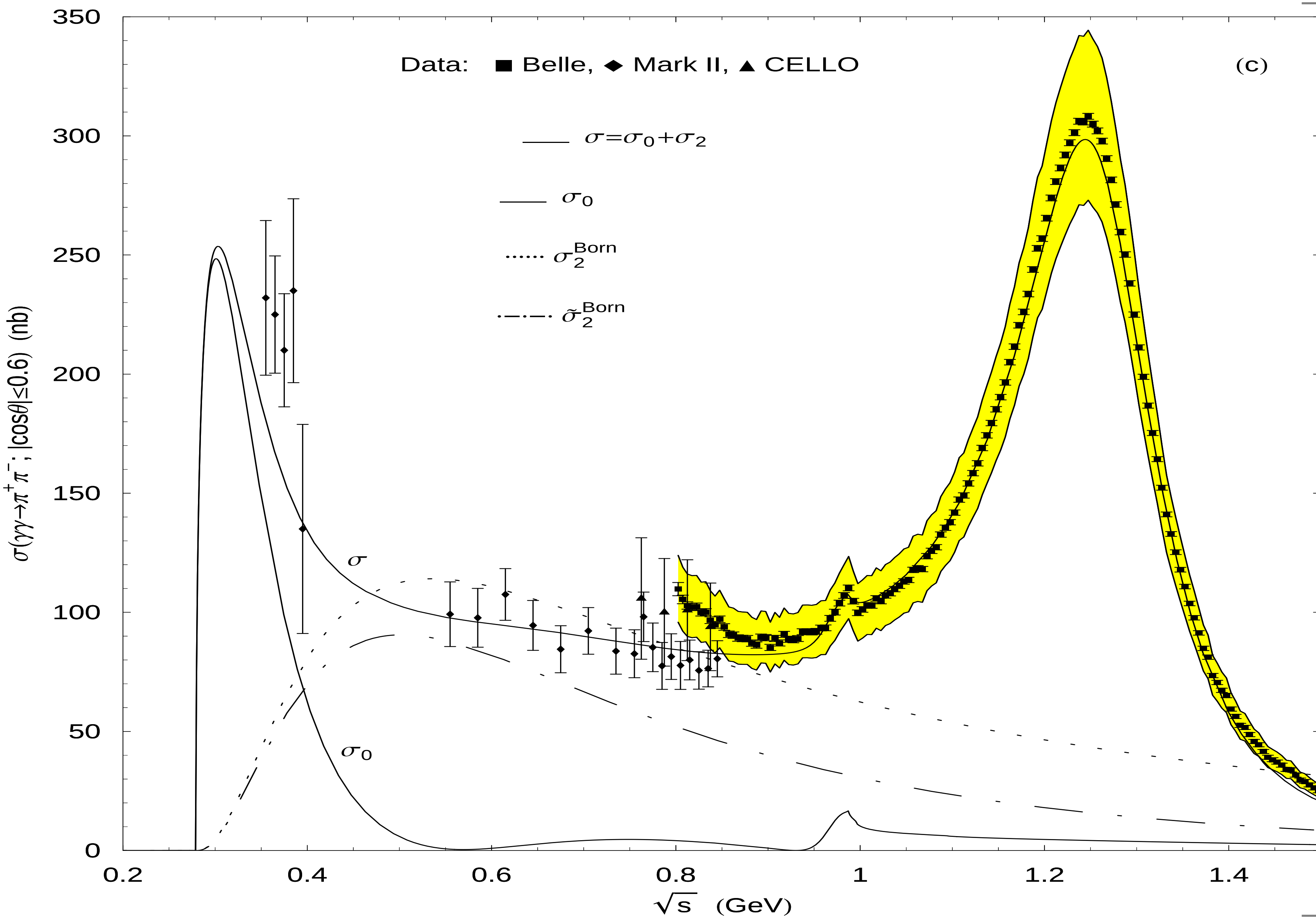
$$+ \mathrm{T_S^{direct}}(\gamma\gamma \to \mathrm{res} \to \pi^+\pi^-),$$

$$T_S(\gamma\gamma \to \pi^0\pi^0,\, s) = 8\alpha \mathrm{I}_{\pi^+\pi^-}(\mathrm{s})\, \mathrm{T_S}(\pi^+\pi^- \to \pi^0\pi^0,\, \mathrm{s})$$

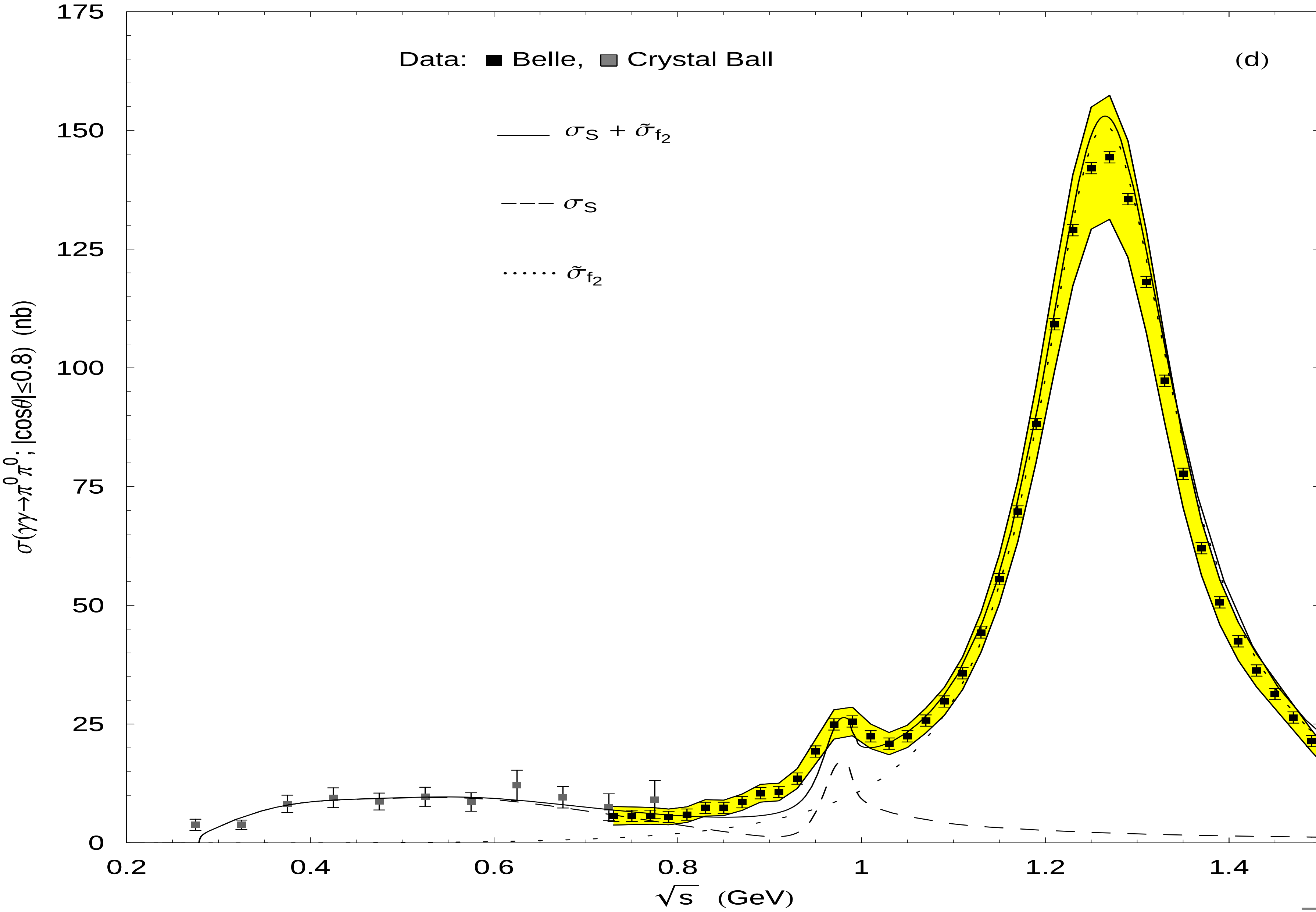
$$+ 8\alpha \mathrm{I}_{\mathrm{K^+K^-}}(\mathrm{s})\, \mathrm{T_S}(\mathrm{K^+K^-} \to \pi^0\pi^0,\, \mathrm{s})$$

$$+ \mathrm{T_S^{direct}}(\gamma\gamma \to \mathrm{res} \to \pi^0\pi^0).$$

The Belle data on $\gamma\gamma \to \pi^+\pi^-$



The Belle data on $\gamma\gamma \to \pi^0\pi^0$



The $f_0 \to K^+K^- \to \gamma\gamma$ approximation

$$\langle \Gamma_{f_0 \to K^+K^- \to \gamma\gamma} \rangle \approx 0.2 \text{ keV}$$

The $\sigma \to \pi^+\pi^- \to \gamma\gamma$ approximation

$$\langle \Gamma_{\sigma \to \pi^+\pi^- \to \gamma\gamma} \rangle \approx 0.45 \text{ keV}$$

The direct $f_0 \to \gamma\gamma$ & $\sigma \to \gamma\gamma$ decays

$$\Gamma^{direct}_{\sigma \to \gamma\gamma} \ll 0.1\text{keV}, \quad \Gamma^{direct}_{f_0 \to \gamma\gamma} \ll 0.1\text{keV}$$

$a_0^0(980) - f_0(980)$ mixing

Mixing $a_0^0(980)$ and $f_0(980)$ was discovered theoretically as a threshold phenomenon in our work of 1979. Now it is timely to study this phenomenon experimentally.

At HADRON-2007 VES (Protvino, A.M. Zaitsev) have presented preliminary data on $f_1(1420) \to \pi^0 a_0^0(980) \to \pi^0 f_0(980) \to 3\pi$ in agreement with our calculation of 1981.

The main contribution $a_0^0(980) \to K^+K^- + K^0\bar{K}^0 \to f_0(980)$

$$\Pi_{a_0 f_0}(m) = \frac{g_{a_0 K^+K^-} g_{f_0 K^+K^-}}{16\pi} \left[i \left(\rho_{K^+K^-}(m) - \rho_{K^0\bar{K}^0}(m) \right) - \frac{\rho_{K^+K^-}(m)}{\pi} \ln \frac{1+\rho_{K^+K^-}(m)}{1-\rho_{K^+K^-}(m)} + \frac{\rho_{K^0\bar{K}^0}(m)}{\pi} \ln \frac{1+\rho_{K^0\bar{K}^0}(m)}{1-\rho_{K^0\bar{K}^0}(m)} \right] \approx$$

$a_0^0(980) - f_0(980)$ mixing

$$\approx \frac{g_{a_0K^+K^-}g_{f_0K^+K^-}}{16\pi}\left[i\left(\rho_{K^+K^-}(m) - \rho_{K^0\bar{K}^0}(m)\right)\right],$$

where $m \geq 2m_{K^0}$, in the region $0 \leq m \leq 2m_K$,
$\rho_{K\bar{K}}(m) = \sqrt{1 - 4m_K^2/m^2}$ should be replaced by
$i|\rho_{K\bar{K}}(m)|$. In the region between the K^+K^- and $K^0\bar{K}^0$ thresholds, which is 8 MeV wide,

$$|\Pi_{a_0f_0}(m)| \approx \frac{|g_{a_0K^+K^-}g_{f_0K^+K^-}|}{16\pi}\sqrt{\frac{2(m_{K^0} - m_{K^+})}{m_{K^0}}}$$

$$\approx 0.127|g_{a_0K^+K^-}g_{f_0K^+K^-}|/16\pi \gtrsim 0.032\,\text{GeV}^2.$$

This contribution dominates for two reason.

$a_0^0(980) - f_0(980)$ mixing

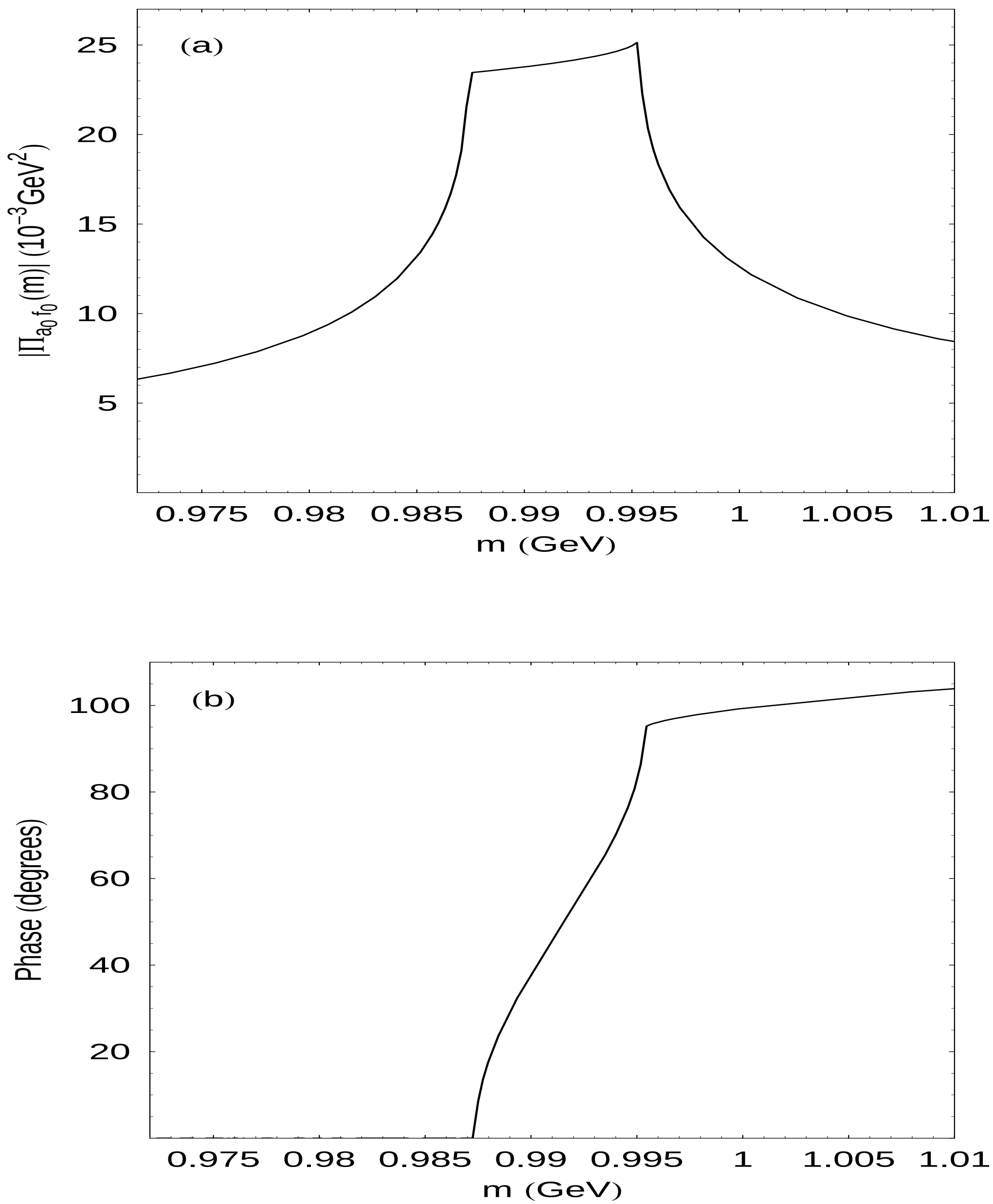
i) It has the $\sqrt{m_d - m_u} \sim \sqrt{\alpha}$ order. As for effects of the $m_d - m_u \sim \alpha$ order, they are small. Such effects were considered partly due to $a_0^0(908) \to \eta\pi^0 \to \pi^0\pi^0 \to f_0(980)$. A clear idea of the magnitude of effects of the $m_d - m_u \sim \alpha$ order gives $|\Pi_{a_0 f_0}(m)|$ at $m < 0.95$ and $m > 1.05$ in the next Fig. (a).[a]

ii) The strong coupling of $a_0^0(980)$ and $f_0(980)$ to the $K\bar{K}$ channels $|g_{a_0K^+K^-} g_{f_0K^+K^-}|/4\pi \gtrsim 1$ GeV^2.

The "resonancelike" behavior of the $a_0^0(980) - f_0(980)$ mixing modulus and phase of the amplitude $\Pi_{a_0 f_0}(m)$ is clearly illustrated in the next Figs. (a) and (b).

[a] Note that $|\Pi_{\rho^0\omega}| \approx |\Pi_{\pi^0\eta}| \approx 0.0036$ GeV^2 $\sim m_d - m_u$.

$a_0^0(980) - f_0(980)$ mixing



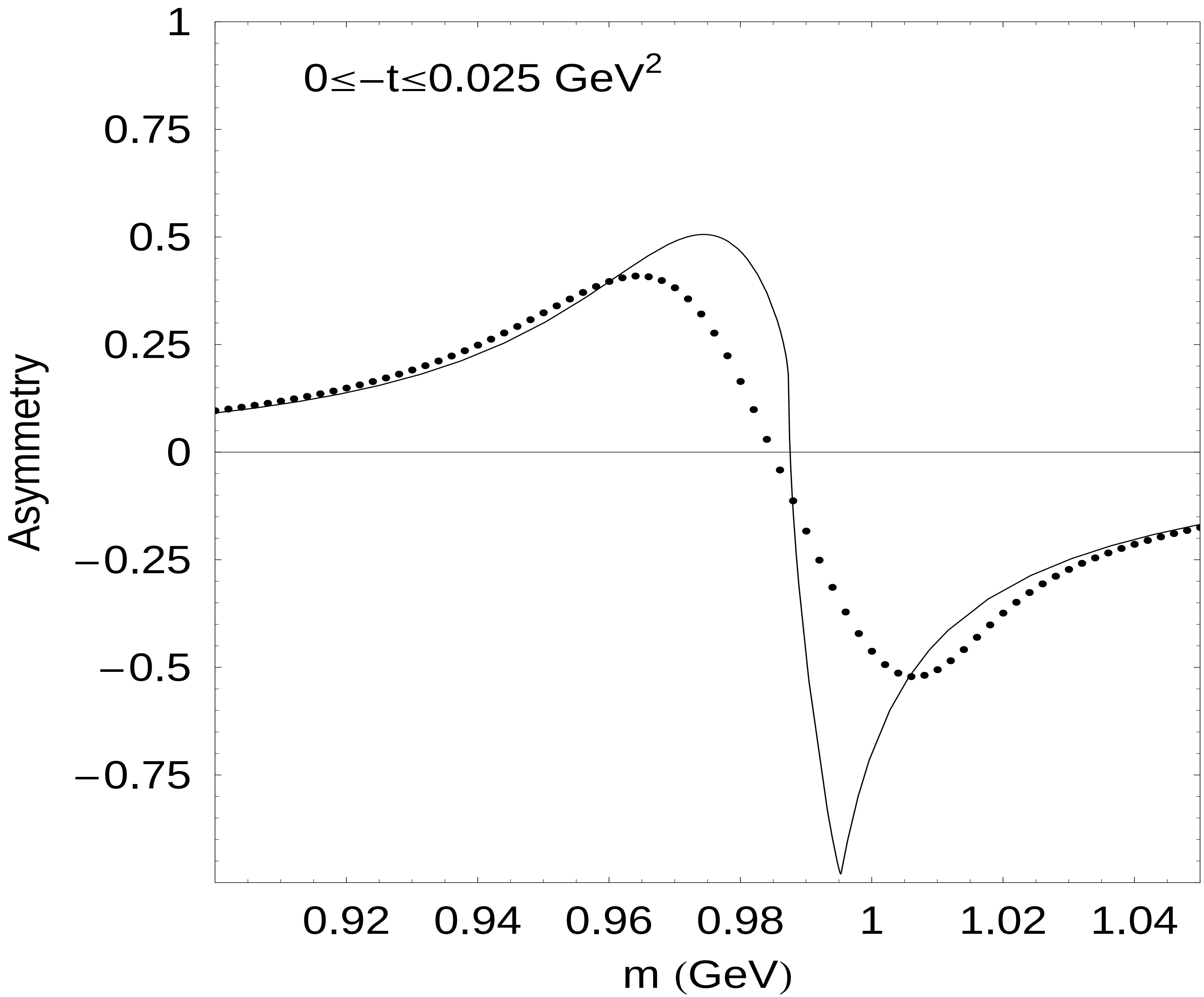
Polarization Phenomena

The phase jump suggest the idea to study the $a_0^0(980) - f_0(980)$ mixing in polarization phenomena. If a process amplitude with a spin configuration is dominated by the $a_0^0(980) - f_0(980)$ mixing then a spin asymmetry of a cross section jumps near the $K\bar{K}$ thresholds. An example is $\pi^- p \to \big(a_0^0(980) + f_0(980)\big)\, n \to \eta\pi^0\, n$.

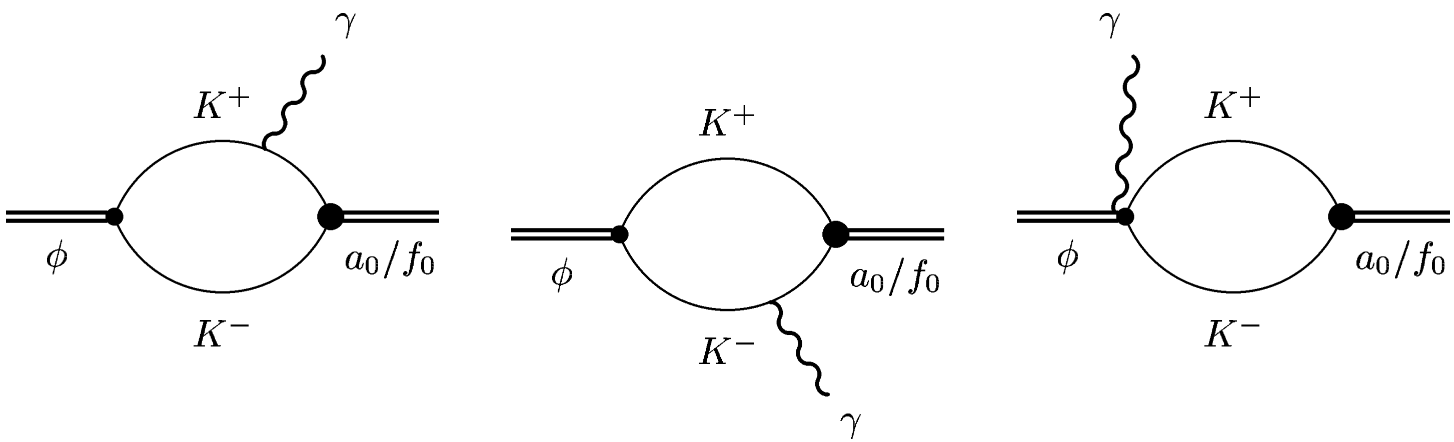
$$\frac{d^3\sigma}{dtdmd\psi} = \frac{1}{2\pi}[\,|M_{++}|^2+|M_{+-}|^2+2\,\Im(M_{++}M^*_{+-})\,P\cos\psi\,]$$

The dimensionless normalized spin asymmetry $A(t,m) = 2\,\Im(M_{++}M^*_{+-})/[\,|M_{++}|^2+|M_{+-}|^2\,]$, $-1 \le A(t,m) \le 1$.

Spin Assymetry



Why a_0 and f_0 are not $K\bar{K}$ molecules



(a) **(b)** **(c)**

$$T\left\{\phi(p)\to\gamma\left[a_0(q)/f_0(q)\right]\right\} = (a) + (b) + (c)$$

Every diagram is divergent hence should be regularized in a gauge invariant manner, for example, the Pauli-Wilars one.

$$\overline{T}\left\{\phi(p)\to\gamma\left[a_0(q)/f_0(q)\right], M\right\} = \overline{(a)} + \overline{(b)} + \overline{(c)}$$

$$\overline{T}\left\{\phi(p)\to\gamma\left[a_0(q)/f_0(q)\right], M\right\} = \epsilon^{\nu}(\phi)\epsilon^{\mu}(\gamma)\overline{T}_{\nu\mu}(p,q) =$$
$$\epsilon^{\nu}(\phi)\epsilon^{\nu}(\gamma)\left[\overline{a}_{\nu\mu}(p,q) + \overline{b}_{\nu\mu}(p,q) + \overline{d}_{\nu\mu}(p,q)\right]$$

Why a_0 and f_0 are not $K\bar{K}$ molecules

$$\overline{a}_{\nu\mu}(p,q) =$$

$$-\frac{i}{\pi^2}\int\left\{\frac{(p-2r)_\nu(p+q-2r)_\mu}{(m_K^2-r^2)[m_K^2-(p-r)^2][m_K^2-(q-r)^2]} - \frac{(p-2r)_\nu(p+q-2r)_\mu}{(M^2-r^2)[M^2-(p-r)^2][M^2-(q-r)^2]}\right\}dr$$

$$\overline{b}_{\nu\mu}(p,q) =$$

$$-\frac{i}{\pi^2}\int\left\{\frac{(p-2r)_\nu(p-q-2r)_\mu}{(m_K^2-r^2)[m_K^2-(p-r)^2][m_K^2-(p-q-r)^2]} - \frac{(p-2r)_\nu(p-q-2r)_\mu}{(M^2-r^2)[M^2-(p-r)^2][M^2-(p-q-r)^2]}\right\}dr$$

Why a_0 and f_0 are not $K\bar{K}$ molecules

$$\overline{d}_{\nu\mu}(p,q) = -\frac{i}{\pi^2}\,2g_{\nu\mu}\int dr\,\times$$

$$\left\{\frac{1}{(m_K^2-r^2)[m_K^2-(q-r)^2]} - \frac{1}{(M^2-r^2)[M^2-(q-r)^2]}\right\}$$

where M is the regulator field mass, $M\to\infty$ in the end

$$\overline{T}[\phi\to\gamma(a_0/f_0), M\to\infty] \to T^{Phys}[\phi\to\gamma(a_0/f_0)]$$

We can shift the integration variables in the regularized amplitudes and easy check the gauge invariance condition

$$\epsilon^\nu(\phi)k^\mu\overline{T}_{\nu\mu}(p,q) = \epsilon^\nu(\phi)(p-q)^\mu\overline{T}_{\nu\mu}(p,q) = 0.$$

It is instructive to consider how the gauge invariance condition

$\epsilon^\nu(\phi)\epsilon^\mu(\gamma)\overline{T}_{\nu\mu}(p,p) = 0$ holds true.

Why a_0 and f_0 are not $K\bar{K}$ molecules

$$\epsilon^\nu(\phi)\epsilon^\mu(\gamma)\overline{T}_{\nu\mu}(p,p) =$$
$$\epsilon^\nu(\phi)\epsilon^\mu(\gamma)T^{m_K}_{\nu\mu}(p,p) - \epsilon^\nu(\phi)\epsilon^\mu(\gamma)T^{M}_{\nu\mu}(p,p) =$$
$$(\epsilon(\phi)\epsilon(\gamma))(1-1) = 0$$

The superscript m_K refers to the non-regularized amplitude and the superscript M refers to the the regulator field amplitude.
So, the contribution of the (a), (b), and (d) diagrams does not depend on a particle mass in the loops (m_K or M) at $p = q$. [a]
But, the meaning of these contributions is radically different.

$e^\nu(\phi)e^\mu(\gamma)T^{m_K}_{\nu\mu}(p,p)$ is caused by intermediate momenta (a few GeV) in the loop, whereas the regulator field contribution is caused fully by high momenta ($M \to \infty$) and teaches us how to allow for

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[a] A typical example of such integrals is $2\int_0^\infty \frac{m^2 x}{(x+m^2)^3}dx = 1$.

Why a_0 and f_0 are not $K\bar{K}$ molecules

high K virtualities in gauge invariant way. It is clear that

$$\epsilon^{\nu}(\phi)\epsilon^{\mu}(\gamma)\mathrm{T}_{\nu\mu}^{\mathrm{M}\to\infty}(\mathrm{p},\mathrm{q}) \to \epsilon^{\nu}(\phi)\epsilon^{\mu}(\gamma)\mathrm{T}_{\nu\mu}^{\mathrm{M}\to\infty}(\mathrm{p},\mathrm{p}) \equiv$$

$$\epsilon^{\nu}(\phi)\epsilon^{\mu}(\gamma)T_{\nu\mu}^{M}(p,p) \equiv (\epsilon(\phi)\epsilon(\gamma)).$$

So, the regulator field contribution tends to the subtraction constant when $M \to \infty$. The finiteness of this constant hides its high momentum origin and gives rise to an illusion of a nonrelativistic physics in the decays under discussion for some theorists.