The pion-photon transition form factor in QCD: Facts and fancy

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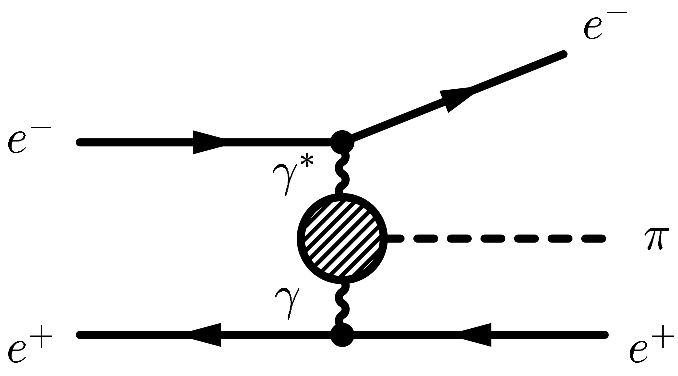
Outline:

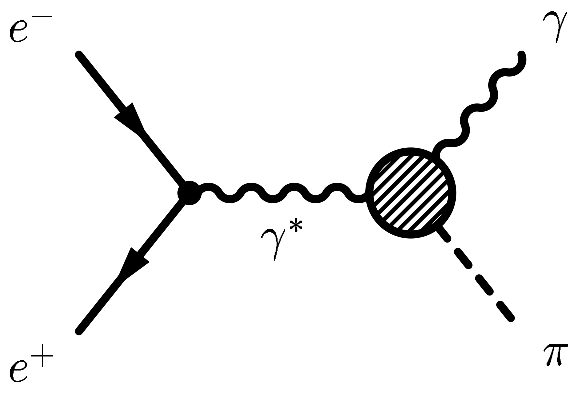
- **The $\pi\gamma$ trans. form factor in coll. factorization**
- **The new BaBar data**
- **Ways out**
- **The mod. pert. approach**
- **Generalization to η, η', η_c**
- **Summary**

based on ongoing work in collaboration with V. Braun and M. Diehl

Measuring the $\pi\gamma$ form factor



space-like



time-like

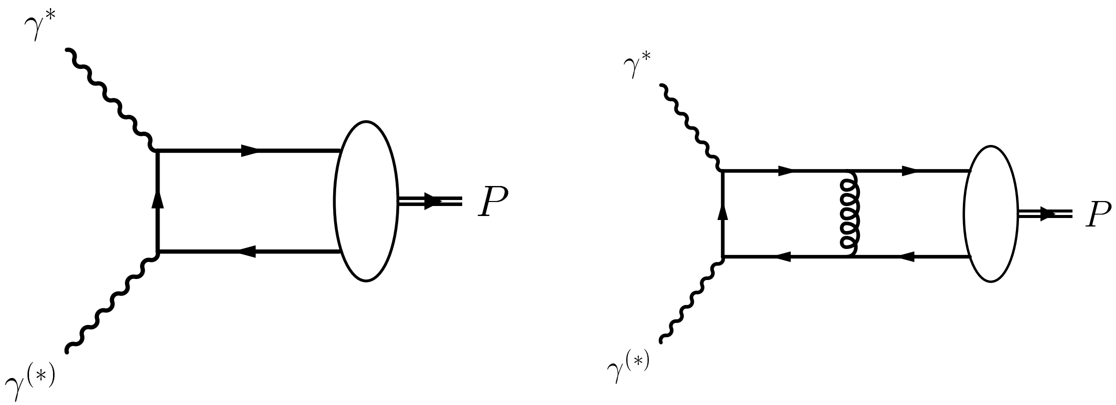
$\gamma^*\gamma\pi$ vertex: $\Gamma_{\mu\nu} = -ie^2 F_{\pi\gamma^*}(Q^2)\, \epsilon_{\mu\nu\alpha\beta} q^\alpha q'^\beta$

data from:
TPC/2γ(90), CELLO(91)
CLEO(95,98) $Q^2 \lesssim 8\,\mathrm{GeV}^2$
BaBar(09) $4 \lesssim Q^2 \lesssim 38\,\mathrm{GeV}^2$
also data on $\eta\gamma$, $\eta'\gamma$, $\eta_c\gamma$
L3(97), CLEO(95,98), BaBar(10)

BaBar(06)
$\eta\gamma$ and $\eta'\gamma$ at $s = 112\,\mathrm{GeV}^2$

two-photon decay width of the mesons:
normalization of FF at $Q^2 = 0$

Theory: collinear factorization



$$F_{\pi\gamma}(Q^2) = \frac{\sqrt{2}f_\pi}{3}\int_0^1 dx\,\Phi_\pi(x,\mu_F)\,T_H(x,Q^2,\mu_R) \qquad \text{for large } Q^2$$

$$T_H^{\rm NLO} = \frac{1}{xQ^2}\left\{1 + C_F\frac{\alpha_s(\mu_R)}{2\pi}\left[\frac{1}{2}\ln^2 x - \frac{x\ln x}{2(1-x)} - \frac{9}{2} + \left(\frac{3}{2}+\ln x\right)\ln\frac{Q^2}{\mu_R^2}\right]\right\}$$

$$\Phi_\pi(x,\mu_F) = 6x(1-x)\left[1 + \sum_{n=2,4,\ldots} a_n(\mu_0)\left(\frac{\alpha_s(\mu_F)}{\alpha_s(\mu_0)}\right)^{\gamma_n/\beta_0} C_n^{3/2}(2x-1)\right]$$

f_π pion decay constant; μ_F, μ_R, μ_0 factorization, renormalization, initial scale

a_n embody soft physics convenient choice: $\mu_F = \mu_R = Q$ $\overline{MS}$ scheme

γ_n anomalous dimensions (pos. fractional numbers, growing with n)

LO: Brodsky-Lepage (80) NLO: del Aguila-Chase (81); Braaten (83)
Kadantseva et al(86)

LO: $Q^2 F_{\pi\gamma} = \frac{\sqrt{2} f_\pi}{3} \langle 1/x \rangle$ $\langle 1/x \rangle = 3\Big[1 + \sum a_n(\mu_F)\Big]$

due to evolution relative weights of the a_n vary with $\ln Q^2$

for $\ln Q^2 \to \infty$ $\Phi_\pi \to 6x(1-x) = \Phi_{\rm AS}$ $Q^2 F_{\pi\gamma} \to \sqrt{2} f_\pi$

Two virtual photons

$\overline{Q}^2 = \frac{1}{2}(Q^2 + Q'^2)$; $\qquad \omega = \frac{Q^2 - Q'^2}{Q^2 + Q'^2}$

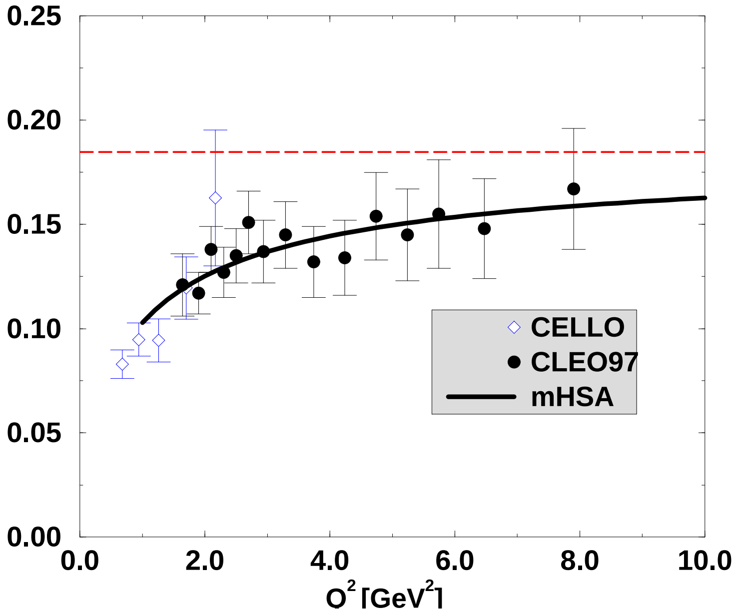
$$F_{\pi\gamma^*}(\overline{Q}^2, \omega) = \frac{\sqrt{2} f_\pi}{3\,\overline{Q}^2} \int_0^1 dx\, \frac{\Phi_\pi(x, \mu_F)}{1 - (2x-1)^2\omega^2} \left[1 + \frac{\alpha_s(\mu_R)}{\pi}\, \mathcal{K}(\omega, x)\right]$$

$$\text{for } \omega \to 0: \quad \overline{Q}^2 F_{\pi\gamma^*} = \frac{\sqrt{2} f_\pi}{3}\left[1 - \frac{\alpha_s}{\pi}\right] + \mathcal{O}(\omega^2)$$

$\omega \to 0$ limit	Cornwall(66)
a_n contribute to order ω^n	Diehl-K-Vogt(01)
α_s corrections	del Aguila-Chase (81)
α_s^2 corrections	Melic-Müller-Passek (03)

parameter-free QCD prediction (not end-point sens., power corr. small)
theor. status comparable with $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$,
Bjorken sum rule, Ellis-Jaffe sum rule, .. but no data

Situation before the advent of the BaBar data



$Q^2 F_{\pi\gamma}$

$Q'^{\,2} = 0$

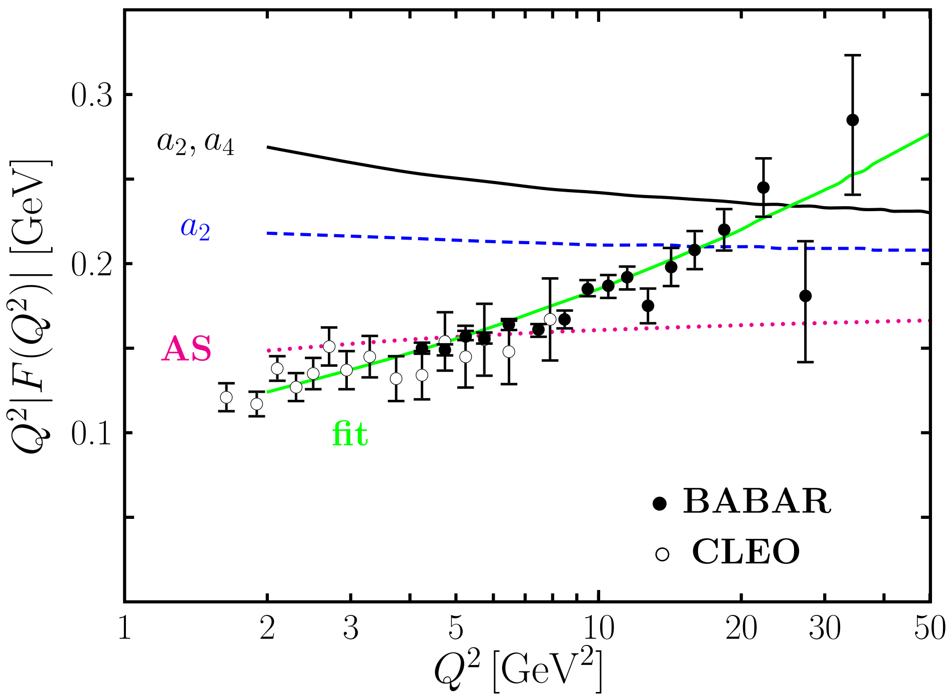
close to NLO result evaluated from asymptotic distribution amplitude

remaining $\simeq 10\%$ can be explained easily but differently:

non-asymptotic DA, low renormalization scale, twist-4 effects, quark transverse momenta, ...

K-Raulfs (95), Ong (96), Musatov-Radyushkin (97), Brodsky-Pang-Robertson (98), Yakovlev-Schmedding (00), Diehl-K-Vogt (01), Bakulev et al (03), ...

The new situation



BaBar (09)

strong increase with Q^2

green: $\sqrt{2} f_\pi \left(Q^2/10\,\mathrm{GeV}\right)^{0.25}$

(to guide the eyes)

AS: NLO corr. < 0

$a_2(1\,\mathrm{GeV}) = 0.39$

$a_2(1\,\mathrm{GeV}) = 0.39, \;\; a_4 = 0.24$

we have to worry:

a substantial increase of FF is difficult to accomodate in fixed order pQCD

corr. due to a_n (> 0) only shift NLO pred. upwards, don't change shape

(except unplausible solutions like $a_2 \simeq 4, \quad a_4 \simeq -3.5$

in conflict with lattice QCD: $a_2(1\,\mathrm{GeV}) = 0.252 \pm 0.143$ Braun et al (06))

Ways out

flat DA $\Phi \equiv 1$:

Polyakov(09) $Q^2 F_{\pi\gamma} \sim \int_0^1 dx \big[x + M^2/Q^2\big]^{-1} = \ln\,[Q^2/M^2 + 1]$

Radyushkin(09) with Gaussian w.f. $\sim k_\perp^2/x(1-x)$

$$Q^2 F_{\pi\gamma} \sim \int_0^1 \frac{dx}{x}\Big\{1 - \exp\big[-\frac{xQ^2}{2(1-x)\sigma}\big]\Big\} \rightarrow \ln\,[Q^2/2\sigma]$$

broad DA also found from AdS/QCD $\sim \sqrt{x(1-x)}$ Brodsky-de Teramond(06)

but not by Mikhailov et al (10) QCD sum rules

Dorokhov(10) non-pert. effects (chiral quarks, instantons) $\rightarrow \ln\,[Q^2/M_q^2]$

dispersion (LCSR) approach: Khodjamirian(09)

corrections due to long-distance, hadron-like component of photon

quark-transverse momenta and resummation of Sudakov-like effects:

Braun-Diehl-K, Li-Mishima(09)

The modified perturbative approach

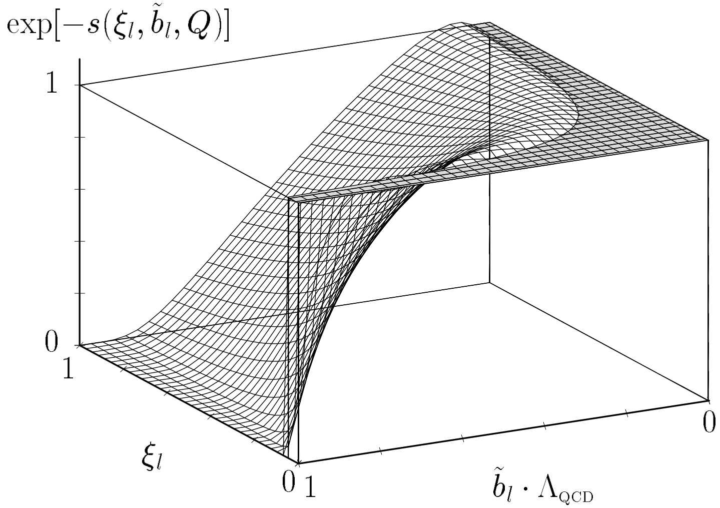
LO pQCD + quark transv. momenta + Sudakov suppr. Sterman et al (89,92)

$\Longrightarrow$ coll. fact. a. for $Q^2 \to \infty$ ($k_\perp$ fact. based on work by Collins-Soper)

Sudakov factor: higher order pQCD in NLL, resummed to all orders

$$S \propto \ln \frac{\ln (xQ/\sqrt{2}\Lambda_{QCD})}{\ln (1/b\Lambda_{QCD})} + \mathrm{NLL} + \mathrm{RG}(\mu_F, \mu_R) \Longrightarrow e^{-S}$$

exponentiation in b space ($q - \bar{q}$ separation)



with $e^{-S} = 0$ for $b > 1/\Lambda_{QCD}$

$$\hat{\Psi}_\pi(x, b, \mu_F) = 2\pi \frac{f_\pi}{\sqrt{6}} \Phi_\pi(x, \mu_F) \exp\left[-\frac{x(1-x)b^2}{4\sigma_\pi^2}\right]$$

factorization scale $\mu_F = 1/b$

b plays role of IR cut-off:

interface between soft gluons (in wave fct)

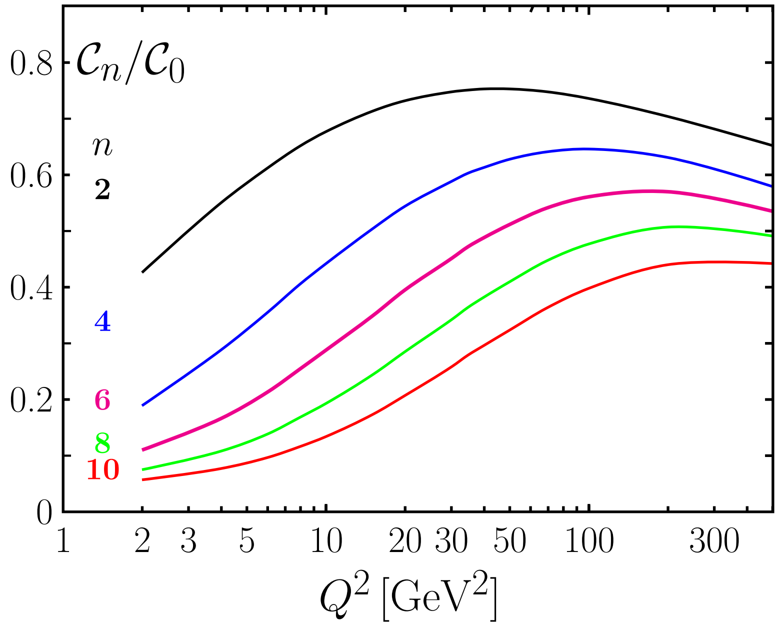
and (semi-)hard gluons in Sudakov f. and T_H

$$F_{\pi\gamma} = \int_0^1 dx \int_0^{1/\Lambda_{QCD}} db^2 \; \hat{\Psi}_\pi \left[\frac{2}{\sqrt{3}\pi} K_0(\sqrt{x}Qb)\right] e^{-S}$$

A remarkable property

with the Gegenbauer expansion

$$Q^2 F_{\pi\gamma} = \sqrt{2} f_\pi \mathcal{C}_0(Q^2, \mu_0, \sigma_\pi) \Big[1 + \sum_{n=2,4,\cdots} a_n(\mu_0)\, \mathcal{C}_n/\mathcal{C}_0 \Big]$$



$Q^2 \to \infty$: $\mathcal{C}_0 \to 1$ and $\mathcal{C}_n \to 0$
due to evolution

low Q^2: strong suppr. of higher terms

increasing Q^2: higher n terms become
gradually more important

only lowest few Gegenbauer terms influence results on $F_{\pi\gamma}$

Φ_{AS} suffices for low Q^2 (see fit to CLEO data)

Nature of corrections in m.p.a.

can be understood by replacing e^{-S} by $\Theta(1/\Lambda_{\rm QCD} - b)$ and wave fct $\propto \delta(k_\perp^2)$:

$$\int_0^{\Lambda_{\rm QCD}^{-1}} b db K_0(\sqrt{x}Qb) = \frac{1}{xQ^2}\Big[1 - \frac{\sqrt{x}Q}{\Lambda_{\rm QCD}}\, K_1\big(\frac{\sqrt{x}Q}{\Lambda_{\rm QCD}}\big)\Big]$$

(coll. fact: suppression of large b only by pert. prop.)

$\sqrt{x}Q \gg \Lambda_{\rm QCD}$: K_1 term exponentially suppressed

$\sqrt{x}Q \sim \Lambda_{\rm QCD}$: K_1 term of $\mathcal{O}(1)$

multiplication with distr. ampl. and integration over x

$$F_{\pi\gamma} \sim 1 + a_2 + a_4 + \ldots - 8\frac{\Lambda_{\rm QCD}^2}{Q^2}(1 + 6a_2 + 15a_4 + \ldots) + \mathcal{O}\Big(\frac{\Lambda_{\rm QCD}^4}{Q^4}\Big)$$

Sudakov factor provides series of power suppressed terms which come from region of soft quark momenta $(x, 1-x \to 0)$ and grow with Gegenbauer index n

intrinsic transverse momentum: power suppressed terms from all x
which do not grow with n

Fit to BaBar data

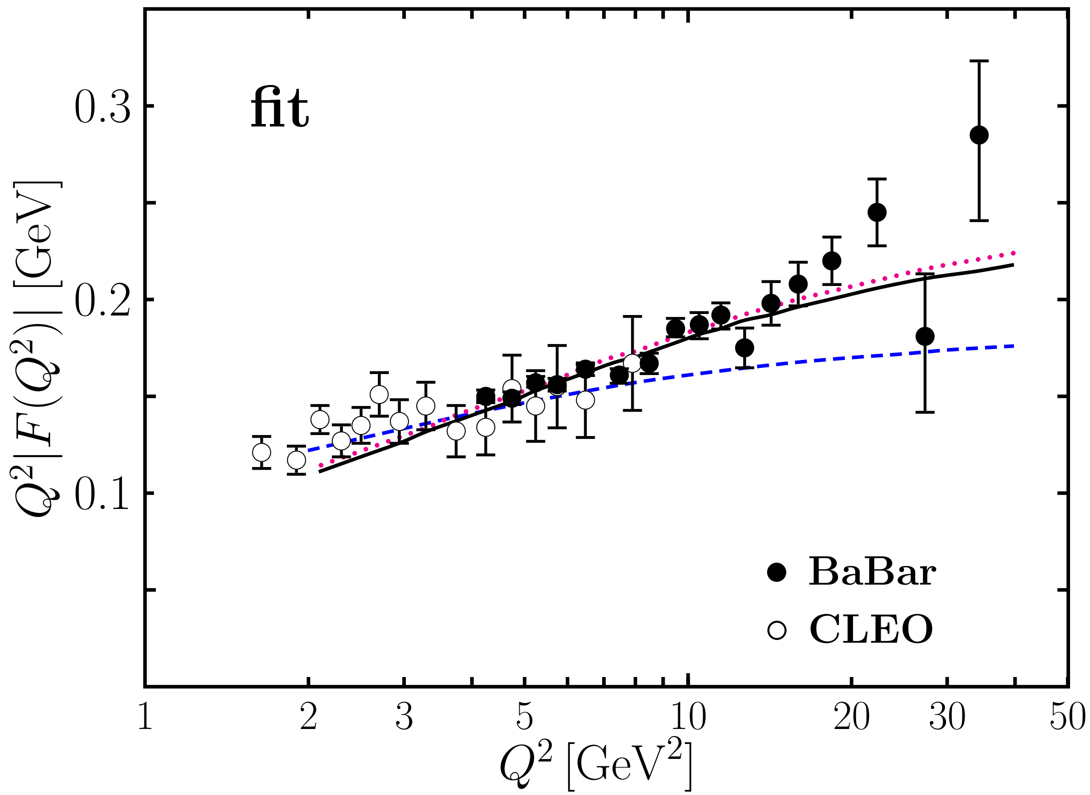
present data allow to fix only one Gegenbauer coefficient Braun-Diehl-K.

fit to CLEO and Babar data (initial scale $1\,\mathrm{GeV}$):

$a_2 = 0.25$ (fixed from lattice Braun(06))

$a_4 = 0.07 \pm 0.10$ $\sigma = 0.42 \pm 0.07\,\mathrm{GeV}^{-1}$ (trans. size parameter)

dashed line: Φ_{AS} K.-Raulfs(95)



Extension to $\eta\gamma$ and $\eta'\gamma$

$P = \eta, \eta'$: $\qquad F_{P\gamma} = F^8_{P\gamma} + F^1_{P\gamma}$

$F^i_{P\gamma}$ as $F_{\pi\gamma}$ except of diff. wave fct. and charge factors

octet-singlet basis favored because of evolution behavior:

flavor-octet part as for pion

flavor-singlet part: due to mixing with the two-gluon Fock component

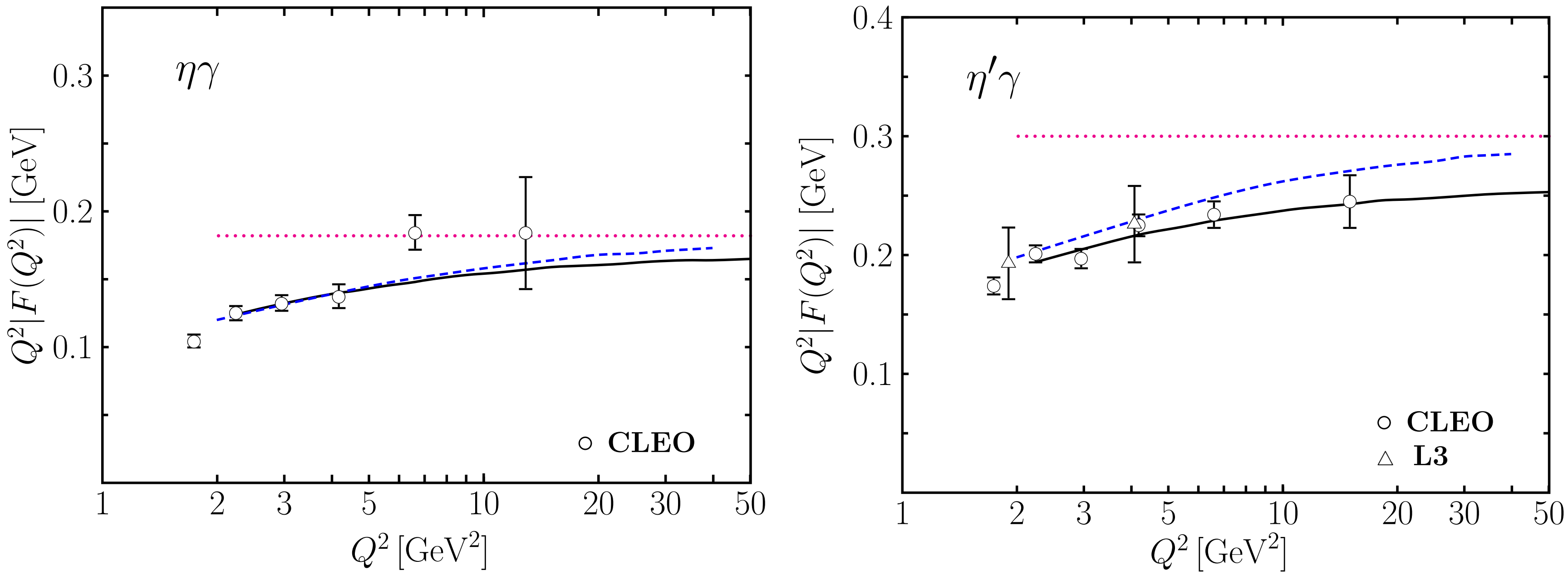
if intrinsic glue is small $a^g_n(\mu_0) \simeq 0$: evolution with $\gamma_n^{(+)} \simeq \gamma_n$

to NLO: also direct contribution from gg Fock state (K-Passek(03))

quark-flavor mixing scheme (Feldmann-K-Stech (98))

$$
\begin{aligned}
Q^2 F_{\eta\gamma} &= \cos\theta_8 F^8 - \sin\theta_1 F^1 \qquad \Longrightarrow \frac{2}{3} f_8 \\
Q^2 F_{\eta'\gamma} &= \sin\theta_8 F^8 + \cos\theta_1 F^1 \qquad \Longrightarrow \frac{4}{\sqrt{3}} f_1 \\
f_8 &= 1.26 f_\pi \quad f_1 = 1.17 f_\pi \quad \theta_8 = -21.2^\circ \quad \theta_1 = -9.2^\circ
\end{aligned}
$$

Results for $\eta\gamma$ and $\eta'\gamma$



preliminary BaBar data; ICHEP 2010, Paris

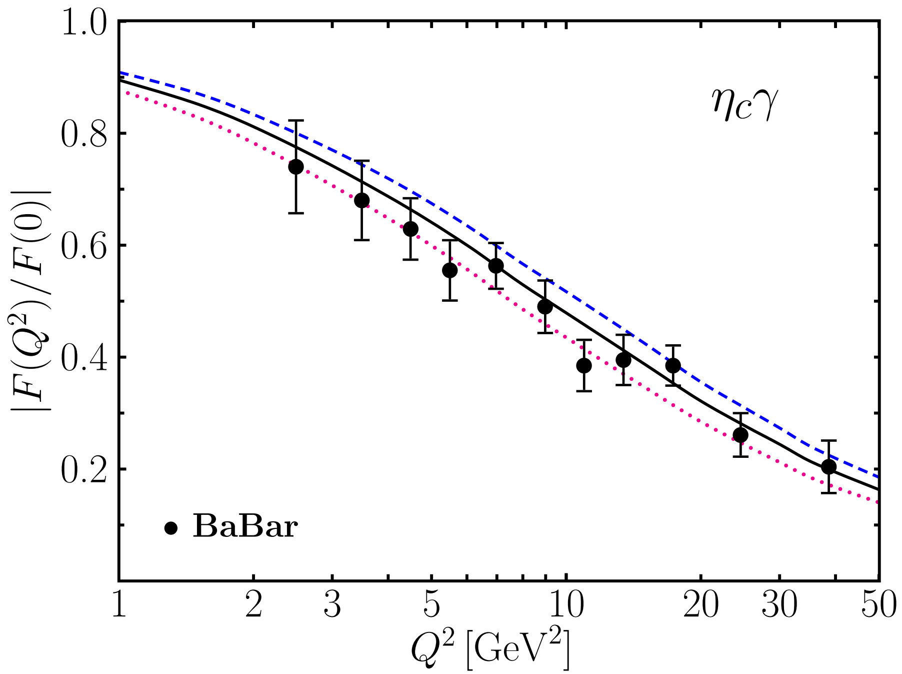
dashed: $\Phi_{\rm AS}$ Feldmann-K.(97)

dotted: asymptotic behavior

solid: $\sigma_8 = \sigma_1 = 0.76 \pm 0.06\,\mathrm{GeV}^{-1}$

$a_2^8(\mu_0) = -0.10 \pm 0.09 \quad a_2^1(\mu_0) = -0.20 \pm 0.07$

Extension to $\eta_c\gamma$



data BaBar(10)

$$T_H = \frac{2\sqrt{6}\, e_c^2}{xQ^2 + (1+4x(1-x))m_c^2 + k_\perp^2}$$

2nd scale, Sudakov unimportant

$$\Phi_{\eta_c} = Nx(1-x)\exp\left[-\sigma_{\eta_c}^2 M_{\eta_c}^2 \frac{(x-1/2)^2}{x(1-x)}\right]$$

Wirbel-Stech-Bauer(85)

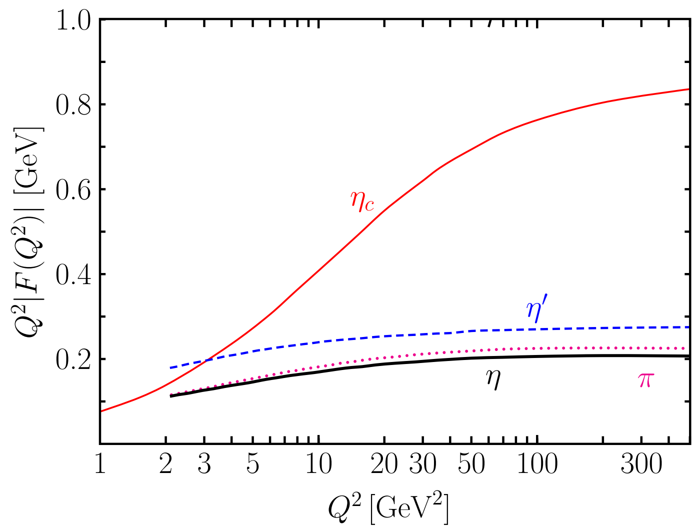
solid (dashed, dotted) line:

$m_c = 1.35(1.49, 1.21)\,\text{GeV}$

PDG: $m_c = 1.25 \pm 0.09\,\text{GeV}$

in contrast to $\pi\gamma$ form factor:

behavior predicted Feldmann-K(97)



The time-like region

collinear factorization to LO accuracy: time-like = space-like (at $s = Q^2$)

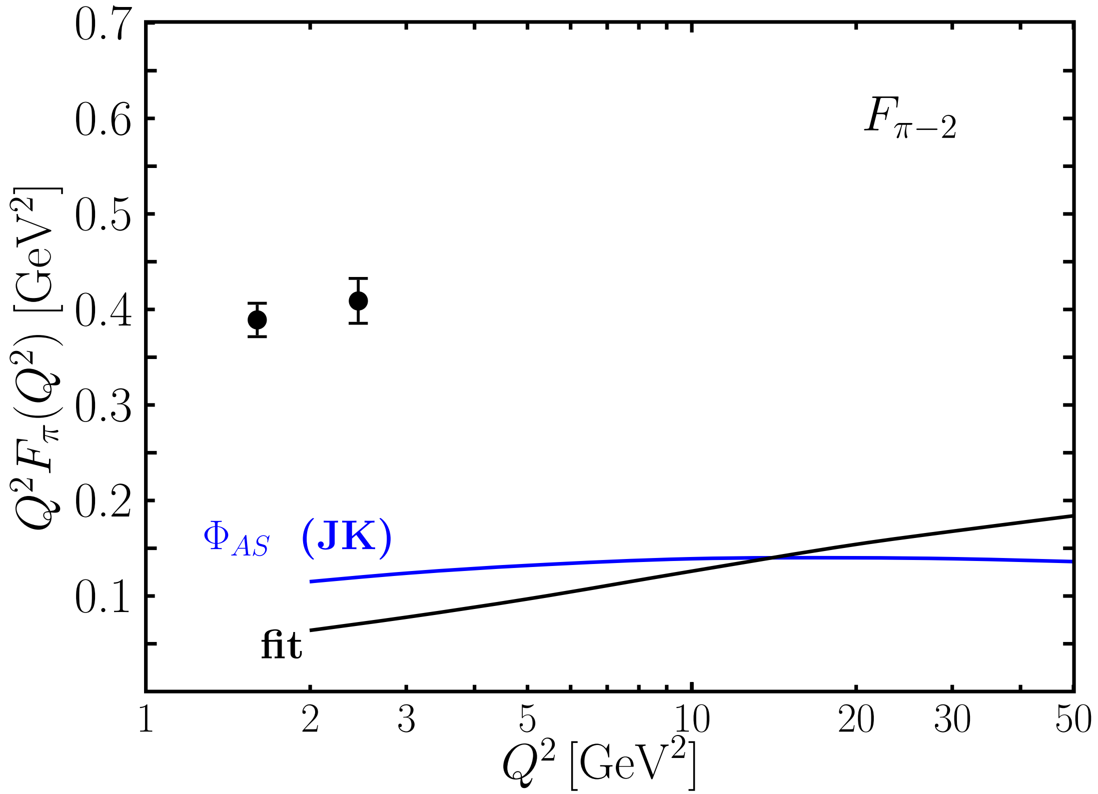
within m.p.a. (as proposed by Gousset-Pire(94) for pion elm. FF)

$$1/(xQ^2 + k_\perp^2) \longrightarrow 1/(-xs + k_\perp^2 - \imath\epsilon) \qquad \text{or} \qquad K_0(\sqrt{x}Qb) \longrightarrow \frac{\imath\pi}{2} H_0^{(1)}(\sqrt{x}sb)$$

analytic continuation of Sudakov f. not well understood (Magnea-Sterman(90))
(probably leads to an oscillating phase)
Gousset-Pire: take space-like Sudakov factor

estimate:
$s = 112\,\mathrm{GeV}^2$: $\quad s|F_{\eta\gamma}| = 0.23\,\mathrm{GeV} \quad s|F_{\eta'\gamma}| = 0.23\,\mathrm{GeV}$
BaBar(06): $s|F_{\eta\gamma}| = 0.229 \pm 0.031\,\mathrm{GeV} \quad s|F_{\eta'\gamma}| = 0.251 \pm 0.021\,\mathrm{GeV}$
ratio time-like/space-like about 1.14

Consequences for the pion elm. form factor



perturbative contribution:

with distr. amplitude from best fit to $\pi\gamma$ form fator

with Φ_{AS} Jakob-K.(93)

Summary

even this simple excl. observable, believed to be understood very well, is subject to strong power suppressed corrections visible even at Q^2 as large as $40\,\mathrm{GeV}^2$

casts severe doubts on every attempt to explain other excl. observables within coll. factorization frame work (e.g. pion or proton FF)

quark-transverse momenta and Sudakov suppressions is one way to estimate power corrections; existing data on $P\gamma$ trans. form factor $(P = \pi, \eta, \eta', \eta_c)$ can well be described within that approach. One Gegenbauer coeff. of DA can be determined from data
preserves standard asymptotics $Q^2 F_{\pi\gamma} \to \sqrt{2} f_\pi$

$\pi\gamma$ form factor should be remeasured by BELLE