GLUONIC CONTRIBUTION TO THE PION TRANSITION FORM FACTOR

Nikolai Kochelev
Joint Institute for Nuclear Research,
Dubna, Russia

In collaboration with Vicente Vento
Valencia University, Spain

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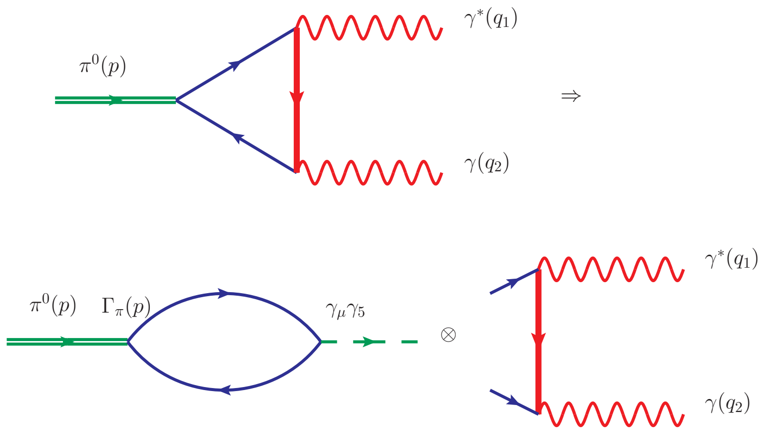
BaBar data on the $\pi^0 \to \gamma\gamma^*$ transition form factor

Transition form factor for the reaction $\pi^0 \to \gamma\gamma^*$ at large virtuality of one of photons Q^2 is the one of cornerstones for checking validity of perturbative QCD approach to the exclusive reactions. This approach is based on the assumption of factorization of short and large distance dynamics.



Asymptotically one can expect $F^{pQCD,As}_{\pi^0\gamma\gamma^*} = 2f_\pi/Q^2$ (Brodsky and Lepage (1980))

New result from BaBar Collaboration for pion transition form factor is in contradiction of such simple picture

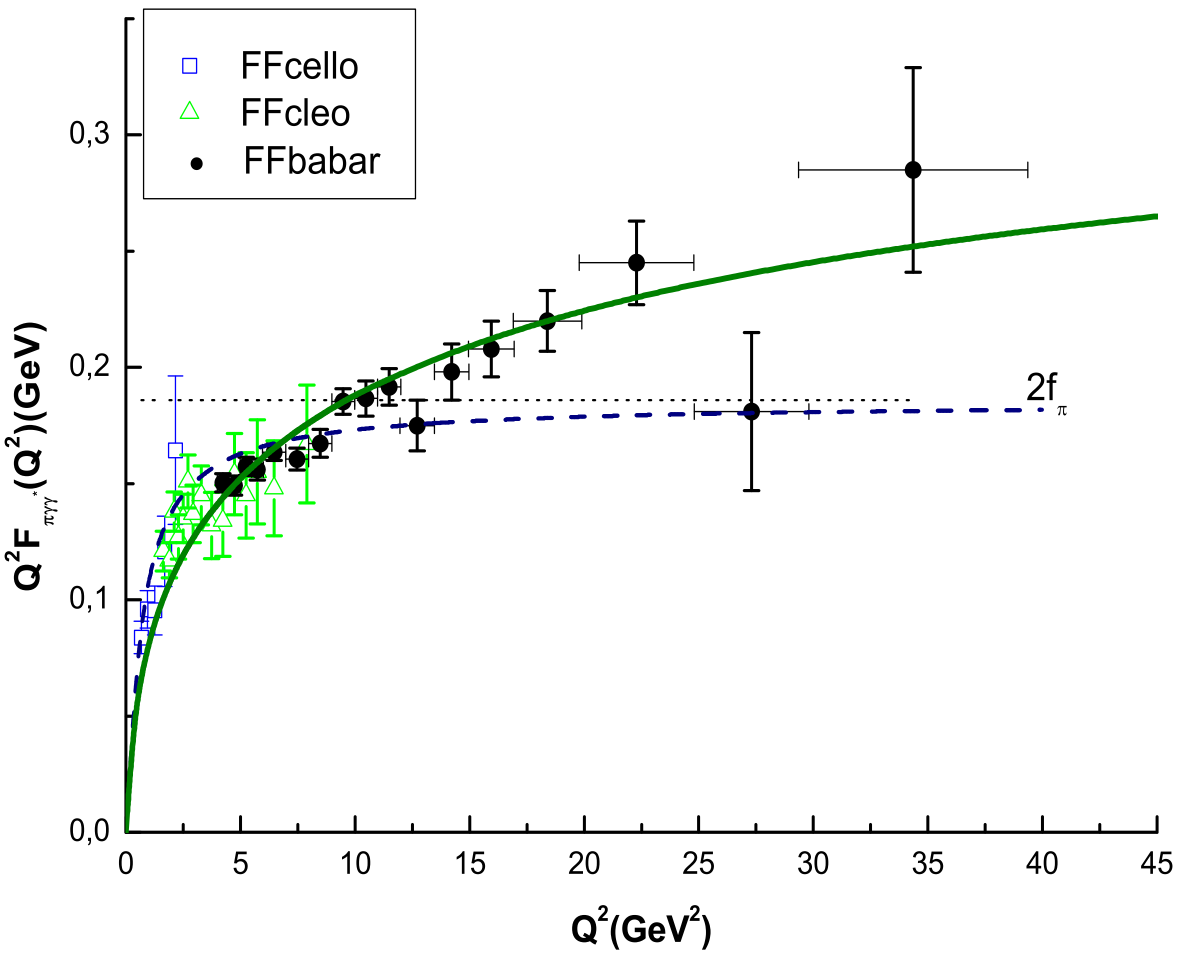


Figure 1: **Experimental results on the pion transition form factor**

Gluon coupling to the pion

N. K. and V. Vento, Phys. Rev. D 81, 034009 (2010)
The various ways for violation of naive perturbative factorization to explain BaBar data were discussed by Dorokhov, Radyushkin, Polyakov, Kroll, Kuraev, Noguera and Vento etc..

Here we present another possible explanation of the BaBar puzzle. It is based on the influence of gluonic component of the pion on the transition form factor.
How one can estimate gluon coupling to the pion?

Effective pion-gluon interaction

Let us propose a low-energy effective π^0 interaction with gluons of the following form

$$\mathcal{L}_{\pi gg}^{eff} = -\frac{1}{f_G^{\pi^0}}\pi^0 \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \widetilde{G}_{\mu\nu}^a .$$

Such type of Lagrangian density, describing the interaction of a pseudoscalar meson with gluons, was introduced many years ago by Cornwall and Soni (1984) to derive Witten's (1979) relation between the η' mass and the topological susceptibility, in a world without light quarks

$$\chi_t^{N_f=0} = -\frac{f_\pi^2}{6}(M_{\eta'}^2 + M_\eta^2 - 2M_K^2),$$

where the topological susceptibility is given by

$$\chi_t^{N_f=0} = i\int d^4x < 0|T\{Q_5(x)Q_5(0)\}|0 >_G,$$

and

$$Q_5(x) = \frac{\alpha_s}{8\pi} G^a_{\mu\nu}(x)\widetilde{G}^a_{\mu\nu}(x)$$

is the topological charge density.

The effective pion-gluon interaction is the analogue of the pion-quark effective interaction

$$\mathcal{L}_{\pi qq}^{eff} = -\frac{1}{f_\pi} M_q \bar{q} i\gamma_5 \vec{\tau} q \cdot \vec{\pi}$$

To derive the decay constant $f_G^{\pi^0}$, which sets the scale of the gluon nonperturbative interaction with the neutral pion, we will use a low energy theorem (LET) (Gross, Treiman and Wilczek (1979))

$$< 0|\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\widetilde{G}^a_{\mu\nu}|\pi^0 >= \frac{1}{2}\frac{m_d - m_u}{m_d + m_u}f_\pi M_\pi^2.$$

We stress that this matrix element is rather big due to the large light quark mass ratio

$$z = \frac{m_u}{m_d} = 0.35 - 0.6.$$

By using the effective pion-gluon interaction interaction and the LET we get

$$f_G^{\pi^0} = -\frac{2(1+z)}{(1-z)}\frac{\chi_t^{N_f=0}}{f_\pi^2 M_\pi^2}f_\pi.$$

As it was to be expected, the strength of the coupling of the neutral pion to gluons is related to the violation of isospin symmetry and

proportional to the difference of the d- and u-quark masses

$$\frac{1}{f_G^{\pi^0}} \propto m_d - m_u.$$

For the value of the mass ratio m_u/m_d which is the one allowed by the Particle Data Group, one obtains

$$R = \frac{f_G^{\pi^0}}{f_\pi} \simeq 28.1 - 51.1.$$

Instanton induced gluonic contribution to the pion transition form factor

The contribution arising from the pion-gluon interaction to the transition form factor of two photons to the pion $\gamma^*(q_1)\gamma^*(q_2) \to \pi^0(p)$, where q_1 and q_2 are the photon momenta and $q_1 + q_2 = p$ can be estimated by using instanton model for QCD vacuum. We consider the case where all virtualities of the incoming and the outcoming particles are in the Euclidean domain $Q_1^2 = -q_1^2 \geq 0$, $Q_2^2 = -q_2^2 \geq 0$, $P^2 = -p^2 \geq 0$. The single instanton contribution to the pion form factor coming from the pion-gluon interaction is associated to the diagrams in Figure below.

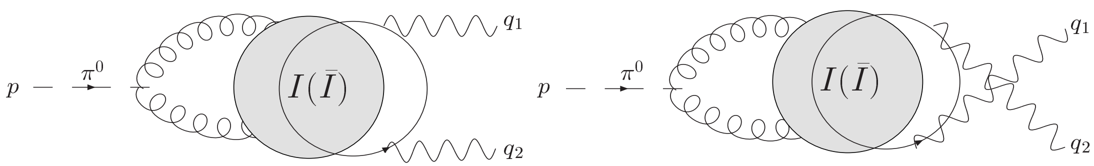


Figure 2: Gluonic contribution to the pion transition form factor. Symbol I ($\bar{I}$) denotes instanton(antiinstanton).

The amplitude for the $\pi^0 \to \gamma^*\gamma^*$ interaction via an instanton with center z_0 has the following form

$$
\begin{aligned}
T_{\mu\nu}(p,q_1,q_2) \quad &= \quad \epsilon_{\mu\nu\alpha\beta}\sum_i e_i^2 \frac{4}{f_G^{\pi^0}\pi^4}\int n_I(\rho)\rho^2 d\rho \times \\
&\int d^4z_0 \int d^4z_1 \int d^4x \int d^4y e^{-ipz_1} e^{iq_1x} e^{iq_2y} Q_5^I(\bar{z}_1)) \times \\
&\frac{h_{\bar{x}}h_{\bar{y}}}{\Delta^2}\left\{\frac{1}{\Delta^2}(h_{\bar{y}}\Delta^\alpha\bar{y}^\beta - h_{\bar{x}}\Delta^\beta\bar{x}^\alpha) + h_{\bar{x}}h_{\bar{y}}\bar{x}^\alpha\bar{y}^\beta\right\},
\end{aligned}
$$

where

$$
Q_5^I(\bar{z}_1)) = \frac{6\rho^4}{\pi^2(\bar{z}_1^2+\rho^2)^4}
$$

is the topological charge density of the instanton in the space-time point z_1, $n_I(\rho)$ is the instanton density, ρ is the instanton size, $\Delta = \bar{x} - \bar{y}$, $h_{\bar{x}} = 1/(\bar{x}^2+\rho^2)$, $h_{\bar{y}} = 1/(\bar{y}^2+\rho^2)$ and the notation $\bar{w} \equiv w - z_0$ for any variable w has been introduced. The sum runs over the light quark flavors, i.e. $i = u, d, s$.

The final result for the gluon contribution to the pion transition form factor induced by instantons is

$$F(P^2,Q_1^2,Q_2^2)_g^I = \frac{4 < e^2 >}{f_\pi R}\int d\rho n_I(\rho)\rho^4 S(\rho,P^2,Q_1^2,Q_2^2)$$

where

$$S(\rho,P^2,Q_1^2,Q_2^2) = \Phi_1(\sqrt{z_3})\int_0^1 dt\left\{I(t,z_1,z_2,z_3)+(1-t)I(t,z_2,z_1,z_3)\right\},$$

$$I(t,z_1,z_2,z_3) \quad = \quad \int_0^\infty d\alpha\frac{\alpha(\alpha+1)\Phi_2(Z(\alpha,t,z_1,z_2,z_3))}{(\alpha+1-t)^3 Z^2(\alpha,t,z_1,z_2,z_3)},$$

and

$$Z(\alpha,t,z_1,z_2,z_3) = \sqrt{(\alpha+1)(t\alpha z_1+tz_2+(1-t)z_3)/(\alpha+1-t)}.$$

The functions

$$\Phi_1(z) = \frac{z^2K_2(z)}{2},\ \Phi_2(z) = zK_1(z)$$

behave as $\Phi_{1,2}(z) \to 1$ in the limit $z \to 0$.

In equations the notations are $z_1 = Q_1^2\rho^2$, $z_2 = Q_2^2\rho^2$, $z_3 = P^2\rho^2$ and $< e^2 >= \sum_i e_i^2$.

For an estimate we use Shuryak's instanton liquid model, where the density is given by

$$n_I(\rho) = n_0\delta(\rho - \rho_c)$$

and

$$n_0 \approx 1/2fm^{-4}, \quad \rho_c \approx 1/3fm.$$

Within this simple model for the instanton distribution the result for the form factor is

$$F(P^2, Q_1^2, Q_2^2)_g^I = \frac{4 < e^2 > f_I}{\pi^2 f_\pi R} S(\rho_c, P^2, Q_1^2, Q_2^2),$$

where $f_I = \pi^2 n_0 \rho_c^4$ is so-called instanton packing fraction in the QCD vacuum. The calculation above was done for the case when all external momenta are Euclidean. In order to compare with BaBar data we have to perform an analytic continuation of the pion virtuality to the physical point of the pion on-shell $P^2 \to -m_\pi^2$.

In the limit $Q_1^2 \gg Q_2^2$ which is valid for BaBar kinematics, the formulas for the real and the imaginary parts of form factor are,

$$Re(F(m_\pi^2, Q_1^2, Q_2^2))_g^I \approx \frac{4 < e^2 > f_I}{\pi^2 f_\pi R} \frac{[log(Q_1^2/m_\pi^2) log(Q_1^2/Q_2^2) + \pi^2/6]}{\rho_c^2 Q_1^2},$$

$$Im(F(m_\pi^2, Q_1^2, Q_2^2))_g^I \approx \frac{4 < e^2 > f_I}{\pi f_\pi R} \frac{log(Q_1^2/Q_2^2)}{\rho_c^2 Q_1^2}.$$

The imaginary part of form factor arises because the pion may decay in this calculation into a quark-antiquark pair since confinement, which forbids this decay, is not explicitly implemented.

There are two features of singlet gluon induced contribution:

- **Dependence on the large photon virtuality Q_1^2 proportional to $log^2(Q_1^2)/Q_1^2$, which is much stronger than that of the flavor nonsinglet part, which in most of the models is of the form $1/Q_1^2$;**

- **It has strong chiral enhancement since the massless logs appear governed by the pion mass as $log(Q_1^2/m_\pi^2)$.**

Comparison with BaBar data

It should be pointed out that in spite of the smallness of instanton packing fraction $f_I \approx 0.06$, using the single instanton approximation as above is only valid for values of the momentum transfers $Q_1, Q_2 \gg 1/R_I$, where $R_I \approx 3\rho_c$ is the distance between the instantons in instanton liquid model. For smaller photon virtualities it is necessary to include the contributions arising from multiinstanton configurations. With an average size of the instanton in the QCD vacuum $\rho_c \approx 1/3 fm$ for the region $Q_1^2, Q_2^2 \geq 1/\rho_c^2 \geq \mu^2 = 0.35 GeV^2$ the validity of a single instanton approximation is assured.

In order to compare our results with the BaBar data we perform an extrapolation of their results from $Q_2^2 = 0$ to $Q_2^2 = 0.35 GeV^2$. We also add to our model for full form factor its flavor nonsinglet part according vector meson dominance (VMD) model

$$F(Q_1^2, Q_2^2)_q^{VMD} = \frac{1}{4\pi^2 f_\pi} \frac{1}{(1 + Q_1^2/M_\rho^2)(1 + Q_2^2/M_\rho^2)}.$$

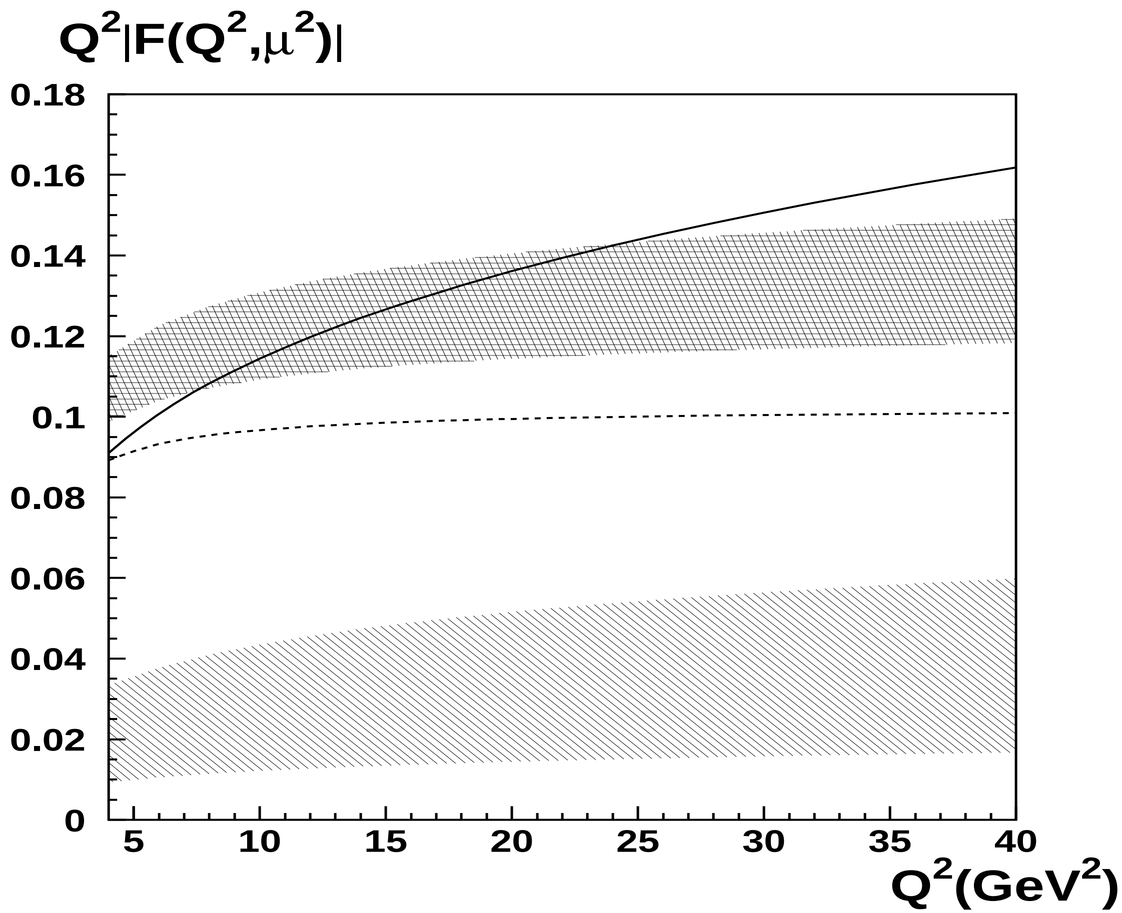


Figure 3: Contributions to the pion transition form factor compared with BaBar extrapolation (solid line): gluonic contribution (lower band), VMD contribution (dashed line), and their sum (upper band).

Discussion

- Behavior of the flavor singlet gluonic part of the form factor as function of Q^2 is determined by shape of the decay of the non-zero modes in the instanton field.

- At the same time VMD-like behavior of the flavor nonsinglet part of pion form factor can be easily reproduced within a non local chiral quark model based on the quark zero-modes dominance in the instanton vacuum (Dorokhov).

- Due to the weaker decay of the quark nonzero modes with respect to zero modes there is a harder Q^2 dependence of the flavor singlet part of the form factor in comparison with the flavor nonsinglet part.

- Taking into account some uncertainties in our estimates related to the poorly known ratio of the u- and d- quark masses, as well as uncertainties in the parameters of the instanton model, we conclude, that the new contribution related to the gluonic component of pion might explain the anomalous behavior of the pion transition form factor found by the BaBar Collaboration.

CONCLUSION

- **There is rather large coupling of neutral pion to gluons due to isospin violation.**

- **Estimation of gluon contribution to pion transition form factor within instanton model shows some possibility to explain BaBar data.**