Possible explanation of BaBar anomaly with the use of Sudakov vertex

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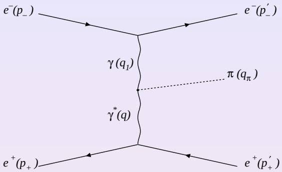
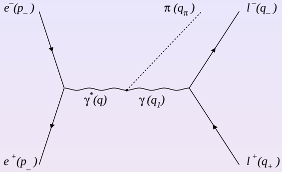
Process $e^+e^- \to \pi_0 e^+e^-$

The process $e^+e^- \to \pi_0 e^+e^-$:





is described my two diagrams:

$$M_{sc} = \frac{2s(4\pi\alpha)^2}{q^2 q_1^2}\left[\mathbf{q}\times\mathbf{q_1}\right]_z V(Q^2) N_+ N_-, \qquad M_{ann} = \frac{(4\pi\alpha)^2}{q_1^2 q^2} J^\mu J^{\nu(l)} V\left(s\right)\epsilon_{\mu\nu\alpha\beta} q^\alpha q_1^\beta, \tag{1}$$

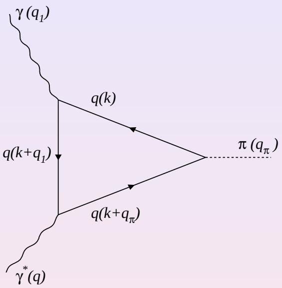
where

$$N_+ = \frac{p_-^\mu}{s}\left[\bar v\left(p'_+\right)\gamma_\mu v\left(p_+\right)\right], \qquad J^\mu = \left[\bar v\left(p_+\right)\gamma^\mu u\left(p_-\right)\right],$$
$$N_- = \frac{p_+^\mu}{s}\left[\bar u\left(p'_-\right)\gamma_\mu u\left(p_-\right)\right], \qquad J^{\nu(l)} = \left[\bar u_l\left(q_-\right)\gamma^\nu v_l\left(q_+\right)\right], \tag{2}$$

and $V(Q^2)$ is the vertex, which related with the neutral pion form factor $F(Q^2)$ as:

$$V(Q^2) = \frac{M_q^2}{2\pi^2 F_\pi Q^2} F\left(\frac{Q^2}{M_q^2}\right). \tag{3}$$

Pion form factor for vertex $\pi^0 \to 2\gamma$ can be parameterized in different manners.



In the approach based on QCD collinear factorization theorem (G. P. Lepage and S. J. Brodsky, Phys. Lett. **B87**, 359 (1979))

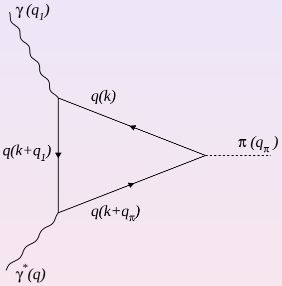
$$V^{BL}(Q^2) = \frac{2F_\pi}{3} \int_0^1 \frac{dx}{xQ^2} \phi_\pi(x,s), \tag{4}$$

and in the papers S. V. Mikhailov and N. G. Stefanis, Nucl. Phys. **B821**, 291 (2009); M. V. Polyakov, JETP Lett. **90**, 228 (2009) different forms of pion wave function $\phi_\pi(x,s)$ was used. Also in the paper L. Ametller,L. Bergstrom,A. Bramon, and E. Masso, Nucl. Phys. **B228**, 301 (1983); A. E. Dorokhov, (2009), arXiv:0905.4577 was pointed that pion form factor in the frames of the constituent quark model has the double logarithmic asymptotic at large momentum transfer.

$$V(Q^2) = \frac{m_\pi^2}{m_\pi^2 + t} \frac{1}{2\arcsin^2(\frac{m_\pi}{2M_Q})} \left\{ 2\arcsin^2(\frac{m_\pi}{2M_Q}) + \frac{1}{2}\ln^2\frac{\beta_Q+1}{\beta_Q-1} \right\}, \tag{5}$$

where $\beta_Q = \sqrt{1 + \frac{4M_Q^2}{Q^2}}$.

We suppose Sudakov type of radiative corrections in one of the vertexes (V. V. Sudakov, Sov. Phys. JETP **3**, 65 (1956); E. A. Kuraev and V. S. Fadin, Yad. Fiz. **27**, 1107 (1978)).



$$F\left(Q^2/M_q^2\right) = -\int \frac{d^4k}{i\pi^2} \times$$
$$\times \frac{Q^2 R_S(Q^2, p_1^2, p_2^2)}{(k^2 - M_q^2 + i0)(p_1^2 - M_q^2 + i0)(p_2^2 - M_q^2 + i0)}, \quad (6)$$

where $p_1 = k + q_1$, and $p_2 = k + q_\pi$ and Sudakov vertex function R_S (J. J. Carazzone, E. C. Poggio, and H. R. Quinn, Phys. Rev. **D11**, 2286 (1975); J. M. Cornwall and G. Tiktopoulos, Phys. Rev. **D13**, 3370 (1976)) is:

$$R_S(Q^2, p_1^2, p_2^2) = \exp\left(-\frac{\alpha_s C_F}{2\pi} \ln\frac{Q^2}{|p_1^2|} \ln\frac{Q^2}{|p_2^2|}\right), \quad (7)$$

where $Q^2 \gg |p_{1,2}^2| \gg M_q^2$ and $C_F = \left(N^2 - 1\right)/(2N) = 4/3$. We use here the the Goldberger-Treiman relation on the quark level $F_\pi = M_q / g_{q\bar{q}\pi} = 93$ MeV.

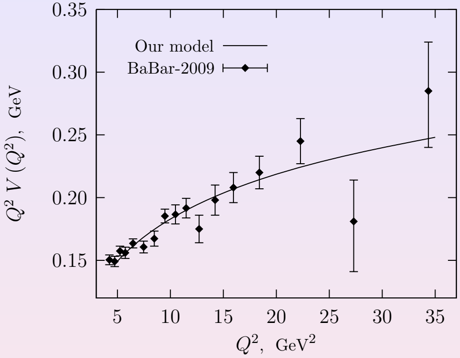
The
cross section of process $e^+e^- \to e^+e^-\pi^0$ is

$$\frac{d\sigma}{dQ^2} = \frac{\alpha^4}{4Q^2}V^2(Q^2)J(Q^2), \quad (8)$$

$$J(Q^2) = \frac{1}{2}L_s^2 + L_s(L_e-1) - (L_e+1),$$

where $L_s = \ln\frac{s}{Q^2+M^2}$,
$L_e = \ln\frac{Q^2}{m_e^2}$ and $V(Q^2)$ is the Sudakov vertex:

$$V(Q^2) = A\frac{M_q^2}{2\pi F_\pi \alpha_s C_F}\Phi(z_B), \quad (9)$$



where $\Phi(z_B) = \int_0^1 \frac{dx}{x}\left(1 - e^{-z_B x(1-x)}\right), \qquad z_B = \frac{C_F\alpha_s}{2\pi}\ln^2\frac{Q^2}{BM_q^2}.$

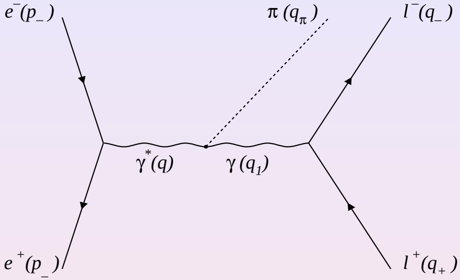
Quantities A and B can be considered as a positive fitting parameters of order of unity. We fixed their values as $A = 0.49$ and $B = 0.23$ (which corresponds to effective quark mass $m_q \approx 135$ MeV) by fitting the BaBar data (The BABAR, B. Aubert *et al.*, Phys. Rev. **D80**, 052002 (2009)).

The annihilation channel of $e^+e^- \to \ell^+\ell^-\pi^0$, $\ell = \mu, \tau$ process:

$$e^+(p_+) + e^-(p_-) \to \gamma^*(q) \to \to \pi^0(q_\pi)\,\ell^+(q_+)\,\ell^-(q_-), \qquad (10)$$

where $p_\pm^2 = 0$, $q_\pm^2 = m_\ell^2$, $q_\pi^2 = M^2$,
$s = q^2 = (p_+ + p_-)^2$, $s_1 = q_1^2 = (q_+ + q_-)^2$.
The matrix element of this process is:

$$\begin{aligned} M &= \frac{(4\pi\alpha)^2}{q_1^2 q^2} J^\mu J^{\nu(\ell)} V(s)\,\epsilon_{\mu\nu\alpha\beta} q^\alpha q_1^\beta, \\ J^\mu &= \bar{v}(p_+)\gamma_\mu u(p_-), \\ J^{\nu(\ell)} &= \bar{v}_\ell(q_+)\gamma_\nu u_\ell(q_-), \end{aligned} \qquad (11)$$



where quantity $V(s)$ describes conversion of two off mass shell photons to the neutral pion (pion transition formfactor, $V(Q^2) = \frac{M_q^2}{2\pi^2 F_\pi Q^2} F\left(\frac{Q^2}{M_q^2}\right)$).
The total cross section have a form:

$$\sigma^{e\bar{e}\to\pi_0\ell\bar{\ell}} = \frac{\pi\alpha^4 V(s)^2}{6}\left(1 - \frac{M^2}{s}\right)^3\left[\ln\frac{s}{m_\ell^2} - \frac{5}{3}\right]. \qquad (12)$$

Conclusions

In conclusion we should emphasize once again that **applying Sudakov radiative corrections to quark vertex function we imply rather large value of virtualities of one of the photons** (i.e. $|q^2| \geq 5$ GeV^2). Thus our approach differs from the ones based on pion wave function modification A. V. Radyushkin, Phys. Rev. **D80**, 094009 (2009) as well as ones based on instanton model A. E. Dorokhov, Phys. Part. Nucl. Lett. **7**, 229 (2010); A. E. Dorokhov, JETP Lett. **91**, 163 (2010); A. E. Dorokhov, Nucl. Phys. Proc. Suppl. **198**, 190 (2010); A. E. Dorokhov, arXiv:1003.4693 which impose some restriction in loop momentum integration.

We remind as well the possibility to measure the transition pion form factor in electro-proton scattering $ep \to e\pi_0 p$. The relevant cross section will be

$$\frac{d\sigma^{ep\to e\pi_0 p}}{dQ^2} = \left(\frac{\alpha g_{\rho qq} g_{\rho NN}}{8\pi(Q^2+M_\rho^2)}\right)^2 \frac{V^2(Q^2)}{Q^2}\left[F_1^2(Q^2)+\frac{Q^2}{4M_p^2}F_2^2(Q^2)\right] J(Q^2), \quad (13)$$

where F_1, F_2 – are Dirac and Pauli proton form factors and

$$J(Q^2) = \frac{1}{2}L_s^2 + L_s(L_e-1) - (L_e+1), \qquad L_s = \ln\frac{s}{Q^2+M^2}, \qquad L_e = \ln\frac{Q^2}{m_e^2}.$$

Here instead of virtual photon the virtual vector meson takes place; $g_{\rho qq}$, $g_{\rho NN}$ are the ρ meson couplings with quarks and nucleons correspondingly. In this case a problem with background ($ep \to e\Delta^+ \to e\pi^0 p$) must be overcomed.