Two-loop resummation in (F)APT

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OUTLINE

- **Intro: Analytic Perturbation Theory (APT) in QCD**
- **Problems of APT and their resolution in FAPT:**
- **Technical development of FAPT: thresholds**
- **Resummation in APT and FAPT**
- **Applications: Higgs decay $H^0 \to b\bar{b}$**
- **Conclusions**

Collaborators & Publications

Collaborators:

S. Mikhailov (Dubna) and N. Stefanis (Bochum)

Publications:

- **A. B., Mikhailov, Stefanis — PRD 72 (2005) 074014**
- **A. B., Karanikas, Stefanis — PRD 72 (2005) 074014vspace*2mm**
- **A. B., Mikhailov, Stefanis — PRD 75 (2007) 056005**
- **Mikhailov — JHEP 0706 (2007) 009 [arXiv:hep-ph/0411397]**
- **A. B.&Mikhailov — arXiv:0803.3013 [hep-ph]**
- **A. B. — Phys. Part. Nucl. 40 (2009) 715**
- **A. B., Mikhailov, Stefanis — JHEP 1006 (2010) 085**

Analytic Perturbation Theory

in

QCD

History of APT

Euclidean	Minkowskian
$Q^2 = \vec{q}^2 - q_0^2 \geq 0$	$s = q_0^2 - \vec{q}^2 \geq 0$

RG+Analyticity
ghost-free $\overline{\alpha}_{\text{QED}}(Q^2)$
Bogoliubov et al. 1959

pQCD+RG: resum π^2-terms
Arctg(s), UV Non-Power Series
Radyush., Krasn. & Pivov. 1982

DispRel+renormalons
IR finite $\alpha_s^{\text{eff}}(Q^2)$
Dokshitzer et al. 1995

pQCD+renormalons
Arctg(s) at LE region
Ball, Beneke & Braun 1994-95

RG+Analyticity
ghost-free $\alpha_{\text{E}}(Q^2)$
Shirkov & Solovtsov 1996

Integral Transformation:
$\mathcal{R}\left[\overline{\alpha}_s\right] \to$ Arctg(s)
Jones & Solovtsov 1995

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Jones & Solovtsov 1995

pQCD+RG+Analyticity

Transforms: $\hat{\mathcal{D}} = \hat{\mathcal{R}}^{-1}$

Couplings: $\alpha_{\mathrm{E}}(Q^2) \Leftrightarrow \alpha_{\mathrm{M}}(s)$

Milton & Solovtsov 1996–97

Analytic (global) pQCD+Analyticity

Global couplings: $\mathcal{A}_n(Q^2) \Leftrightarrow \mathfrak{A}_n(s)$

Non-Power perturbative expansions

Shirkov 1999–2001

History of F(ractional)APT

Euclidean
$Q^2 = \vec{q}^2 - q_0^2 \geq 0$

Minkowskian
$s = q_0^2 - \vec{q}^2 \geq 0$

Global Fractional APT (FAPT)

Analytization of α_s^ν**:** $\mathcal{A}_\nu(Q^2) \Leftrightarrow \mathfrak{A}_\nu(s)$

A. B. & Mikhailov & Stefanis 2005–2006

History of F(ractional)APT

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Analytization of α_s^ν: $\mathcal{A}_\nu(Q^2) \Leftrightarrow \mathfrak{A}_\nu(s)$

Analytization of $\alpha_s^\nu \times \mathbf{Log}^m$: $\mathcal{L}_{\nu,m}(Q^2) \Leftrightarrow \mathfrak{L}_{\nu,m}(s)$

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A. B. & Mikhailov & Stefanis 2005–2006

Resummation in 1-loop APT

S. Mikhailov 2004

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A. B. & Mikhailov & Stefanis 2005–2006

Resummation in 1-loop global FAPT

A. B. & Mikhailov 2008

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Analytization of $\alpha_s^\nu \times$ **Log**^{m}: $\mathcal{L}_{\nu,m}(Q^2) \Leftrightarrow \mathfrak{L}_{\nu,m}(s)$

A. B. & Mikhailov & Stefanis 2005–2006

Resummation in 1-loop global FAPT

A. B. & Mikhailov 2008

Analytization of $\alpha_s^\nu(1 + c_1\alpha_s)^{\nu'}$: $\mathcal{B}_{\nu,\nu'}(Q^2) \Leftrightarrow \mathfrak{B}_{\nu,\nu'}(s)$

A. B. 2008–2009

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A. B. & Mikhailov & Stefanis 2005–2006

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A. B. & Mikhailov 2008

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A. B. 2008–2009

Resummation in 2-loop global FAPT

with 2-loop evolution factors $\mathcal{B}_{\nu,\nu'}(Q^2) \Leftrightarrow \mathfrak{B}_{\nu,\nu'}(s)$

A. B. & Mikhailov & Stefanis 2010

Intro: PT in QCD

- **coupling** $\alpha_s(\mu^2) = (4\pi/b_0)\, a_s[L]$ **with** $L = \ln(\mu^2/\Lambda^2)$
- **RG equation** $\dfrac{d\, a_s[L]}{d\, L} = -a_s^2 - c_1\, a_s^3 - \ldots$
- **1-loop solution generates Landau pole singularity:** $a_s[L] = 1/L$

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- PT series: $D[L] = 1 + d_1 a_s[L] + d_2 a_s^2[L] + \ldots$

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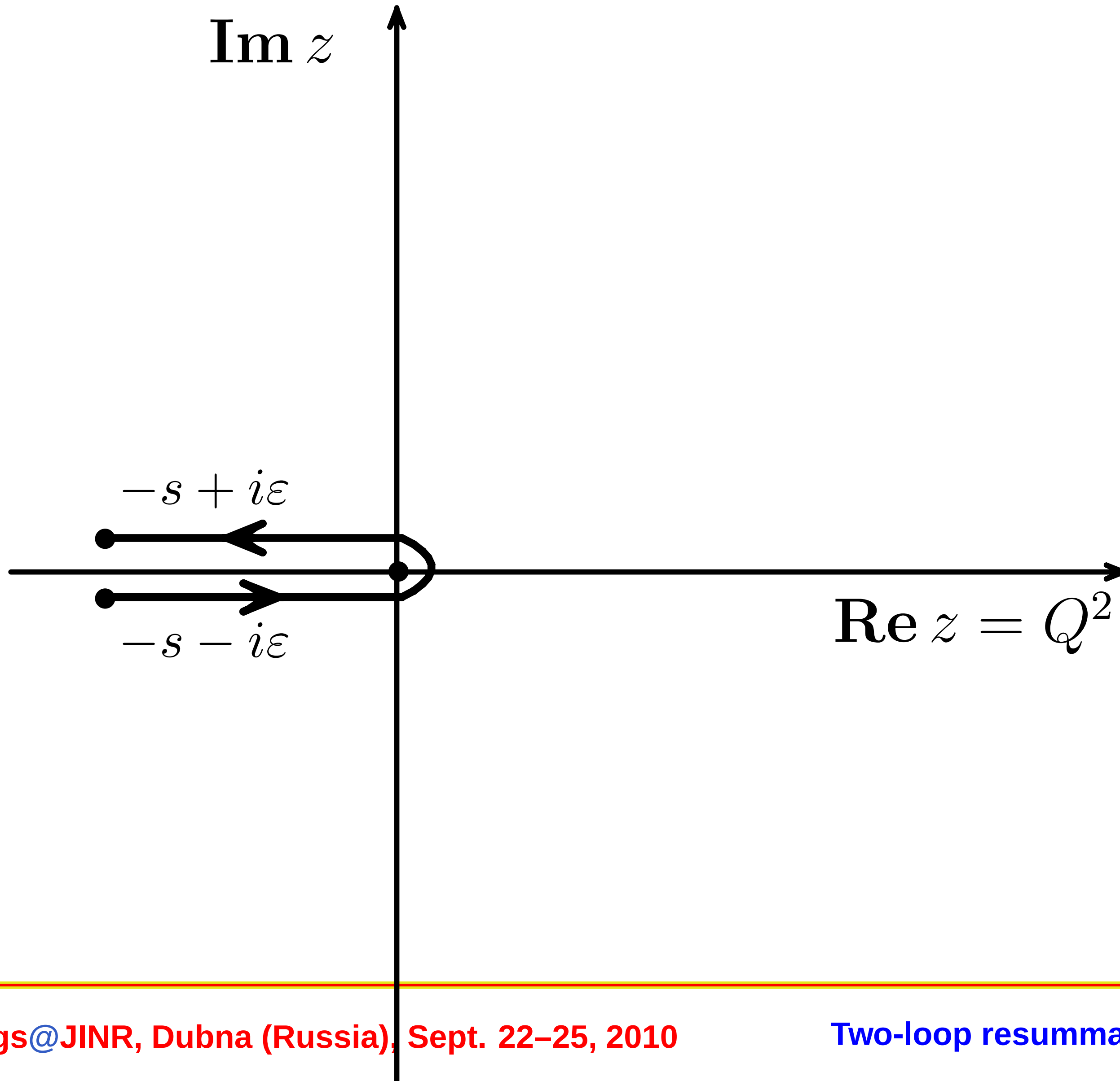
- **PT series:** $D[L] = 1 + d_1 a_s[L] + d_2 a_s^2[L] + \ldots$
- **RG evolution:** $B(Q^2) = \left[Z(Q^2)/Z(\mu^2)\right]\, B(\mu^2)$ **reduces in 1-loop approximation to**

$$Z \sim a^{\nu}[L]\Big|_{\nu\,=\,\nu_0\,\equiv\,\gamma_0/(2b_0)}$$

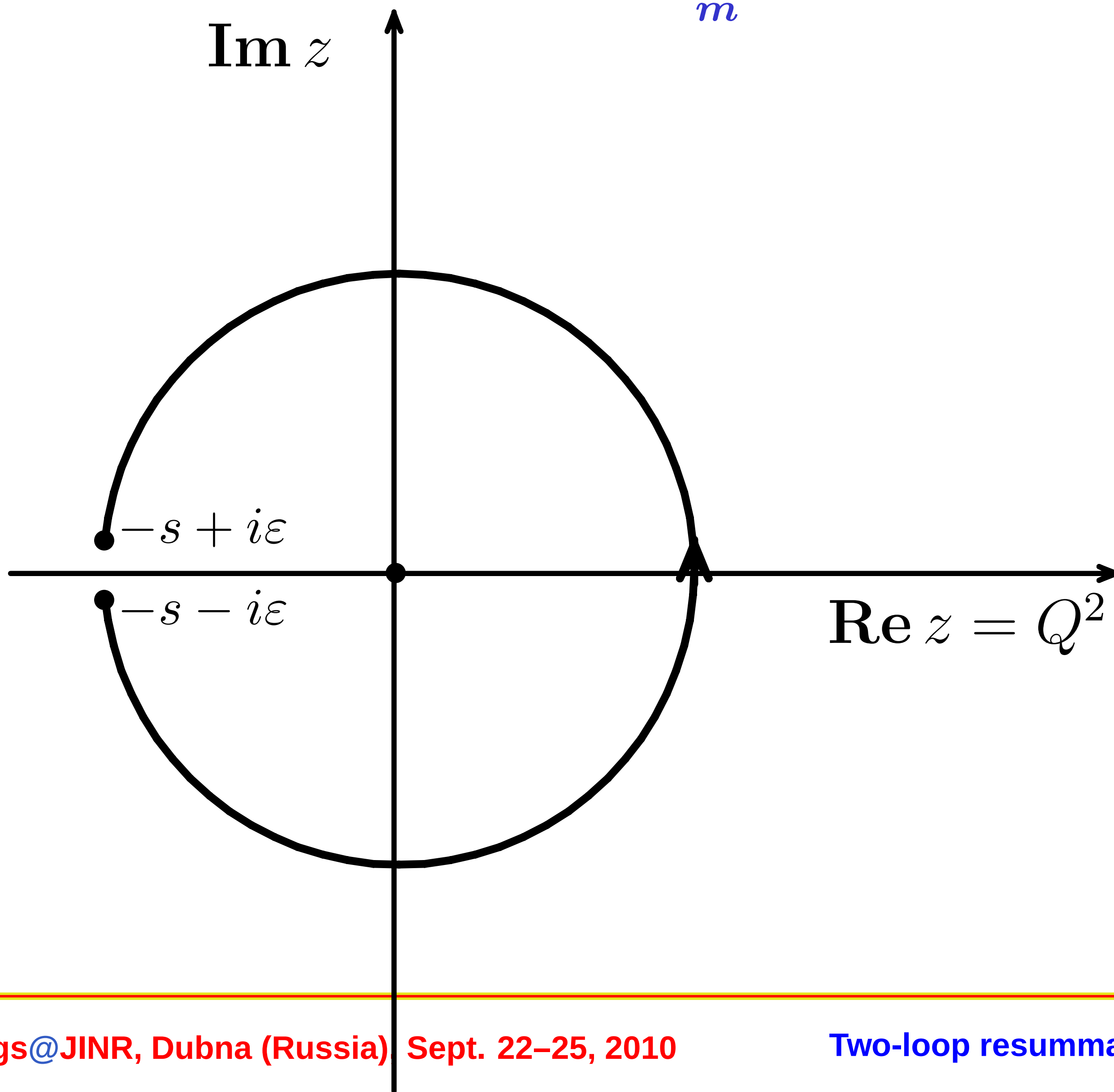
Problem in QCD PT: Minkowski region?

Quantities in Minkowski region $= \oint f(z) D(z) dz$.



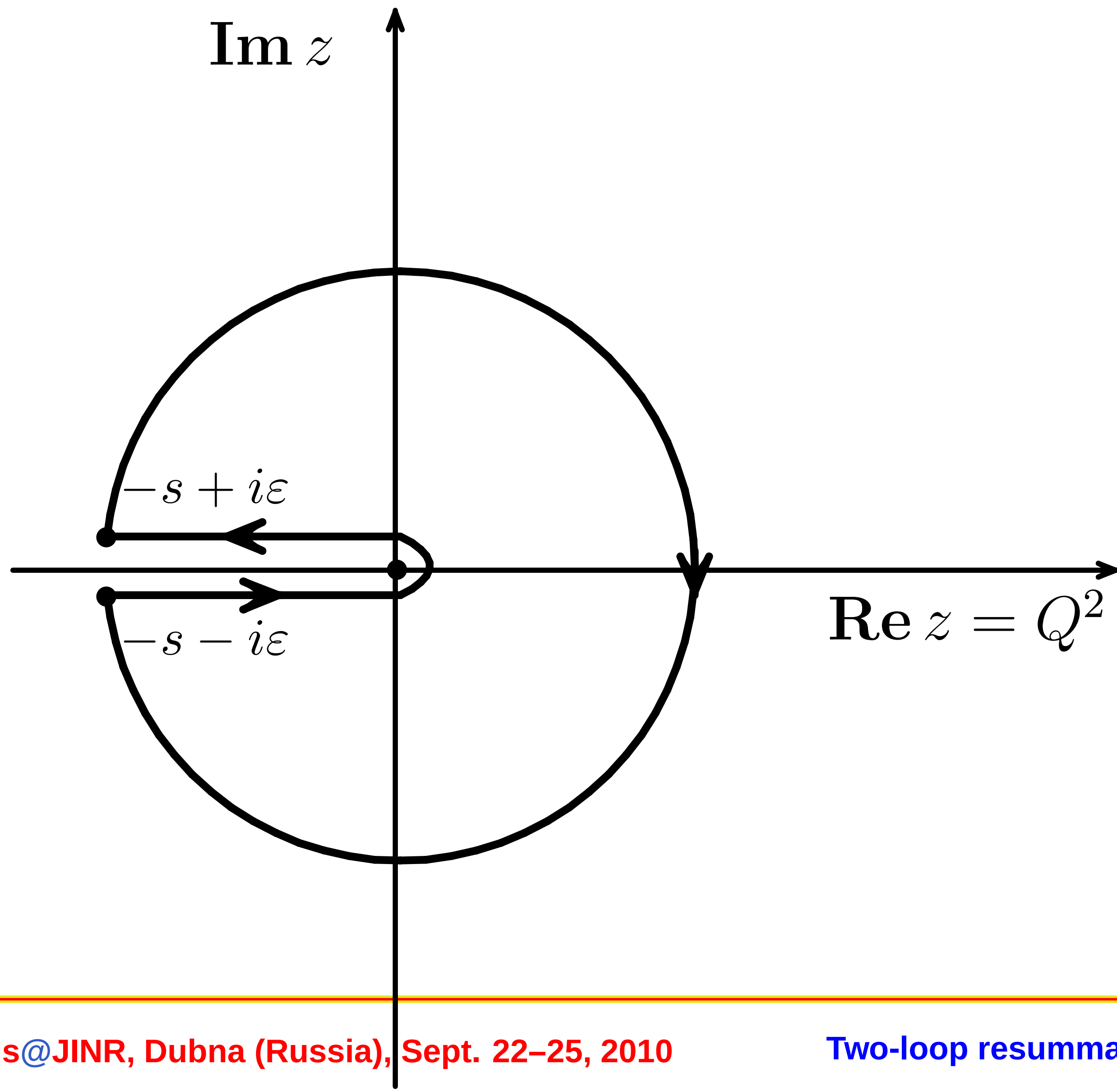
Problem in QCD PT: Minkowski region?

In $\oint f(z)D(z)dz$ one uses $D(z) = \sum_m d_m \alpha_s^m(z)$.



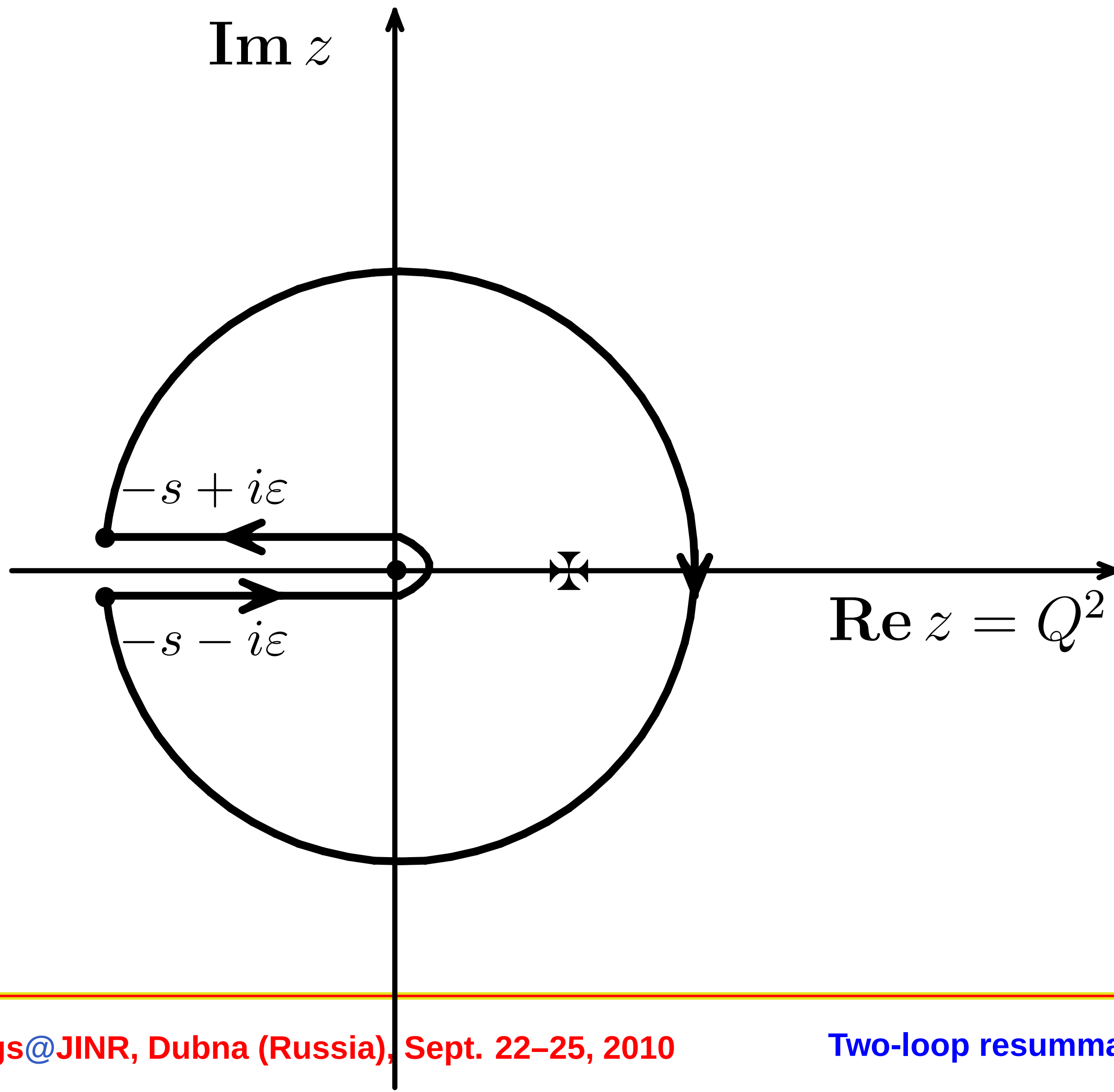
Problem in QCD PT: Minkowski region?

This change of integration contour is legitimate if $D(z)\,f(z)$ is analytic inside



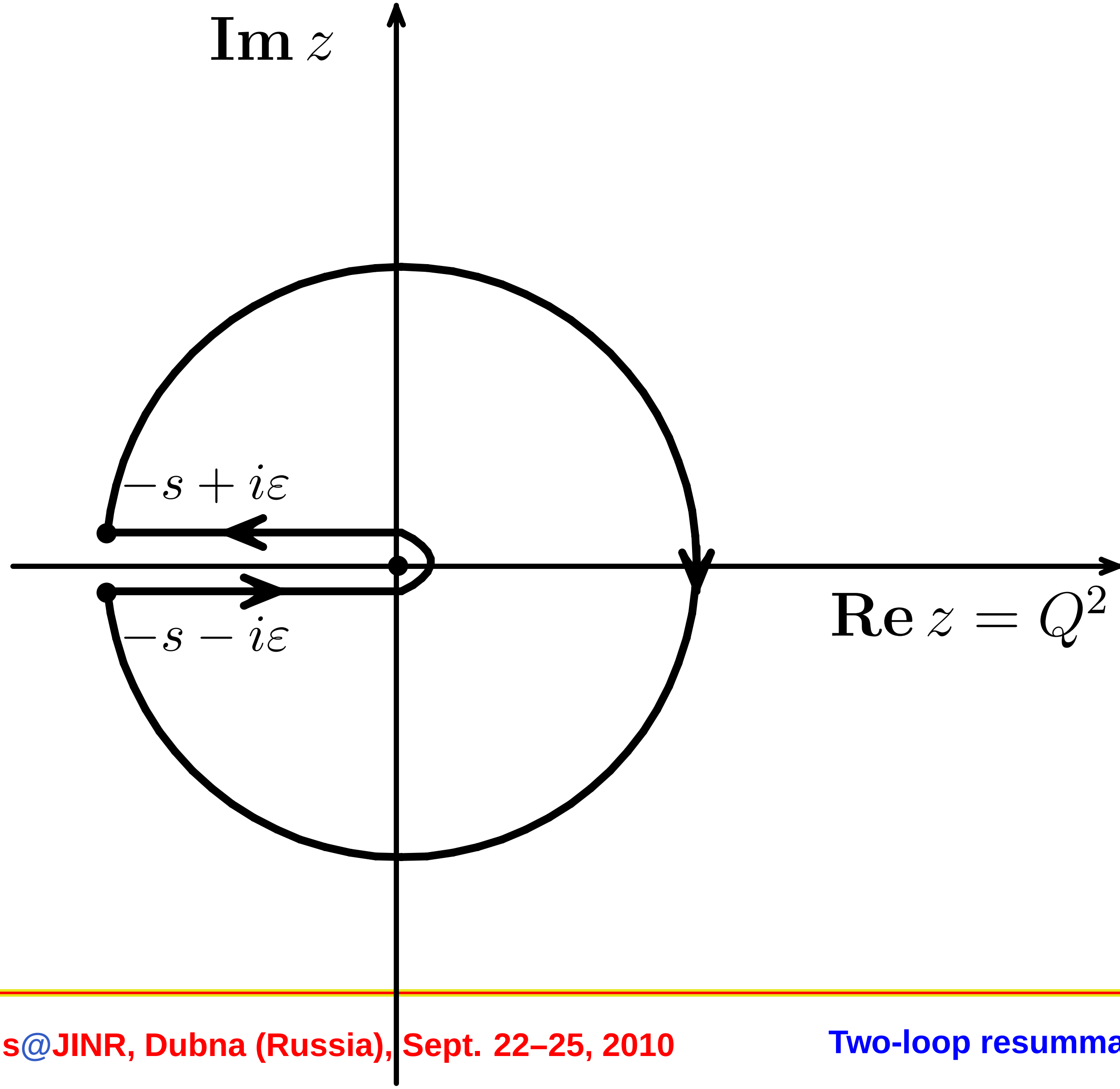
Problem in QCD PT: Minkowski region?

But $\alpha_s(z)$ and hence $D(z)f(z)$ have Landau pole singularity just inside!



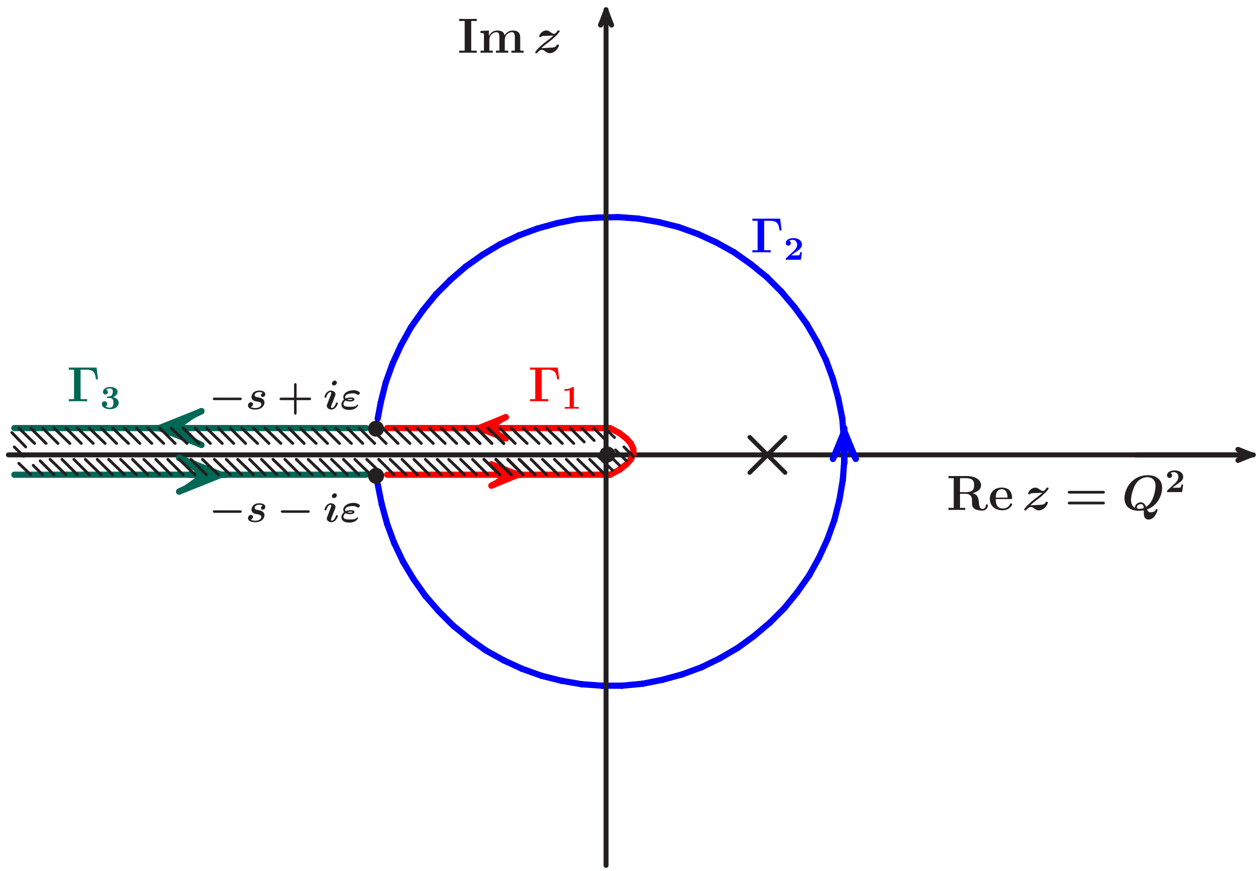
Problem in QCD PT: Minkowski region?

In APT effective couplings $\mathcal{A}_n(z)$ are analytic functions $\Rightarrow$ Problem does not appear! Equivalence to CIPT for $R(s)$.



Equivalence CIPT and APT for $R(s)$

$$\text{CIPT}\left\{\oint_{\Gamma_2}\frac{D(z)dz}{z}\right\} = \text{APT}\left\{\oint_{\Gamma_3}\frac{D(z)dz}{z}\right\}$$



Basics of APT

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RG $\Downarrow$ **UV asymptotics**

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  RG $\Downarrow$ UV asymptotics

  Causality $\Downarrow$ Spectrality

- Euclidean: $-q^2 = Q^2, \;\; L = \ln Q^2/\Lambda^2, \;\; \{\mathcal{A}_n(L)\}_{n\in\mathbb{N}}$
- Minkowskian: $q^2 = s, \;\; L_s = \ln s/\Lambda^2, \;\; \{\mathfrak{A}_n(L_s)\}_{n\in\mathbb{N}}$

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- Different effective couplings in Euclidean (S&S) and Minkowskian (R&K&P) regions
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  Causality $\Downarrow$ **Spectrality**

- Euclidean: $-q^2 = Q^2, \; L = \ln Q^2/\Lambda^2, \; \{\mathcal{A}_n(L)\}_{n\in\mathbb{N}}$
- Minkowskian: $q^2 = s, \; L_s = \ln s/\Lambda^2, \; \{\mathfrak{A}_n(L_s)\}_{n\in\mathbb{N}}$
- **PT** $\sum_m d_m a_s^m(Q^2) \quad \Rightarrow \quad \sum_m d_m \mathcal{A}_m(Q^2)$ **APT**

  m is power $\quad \Rightarrow \quad$ m is index

Spectral representation

By analytization we mean “Källen–Lehmann” representation

$$\left[f(Q^2)\right]_{\text{an}} = \int_0^\infty \frac{\rho_f(\sigma)}{\sigma + Q^2 - i\epsilon}\, d\sigma$$

Then (note here pole remover):

$$\begin{aligned}
\rho(\sigma) &= \frac{1}{L_\sigma^2 + \pi^2} \\
\mathcal{A}_1[L] &= \int_0^\infty \frac{\rho(\sigma)}{\sigma + Q^2}\, d\sigma = \frac{1}{L} - \frac{1}{e^L - 1} \\
\mathfrak{A}_1[L_s] &= \int_s^\infty \frac{\rho(\sigma)}{\sigma}\, d\sigma = \frac{1}{\pi} \arccos \frac{L_s}{\sqrt{\pi^2 + L_s^2}}
\end{aligned}$$

Spectral representation

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$$\left[f(Q^2)\right]_{\text{an}} = \int_0^\infty \frac{\rho_f(\sigma)}{\sigma + Q^2 - i\epsilon}\, d\sigma$$

with spectral density $\rho_f(\sigma) = \text{Im}\left[f(-\sigma)\right]/\pi$. Then:

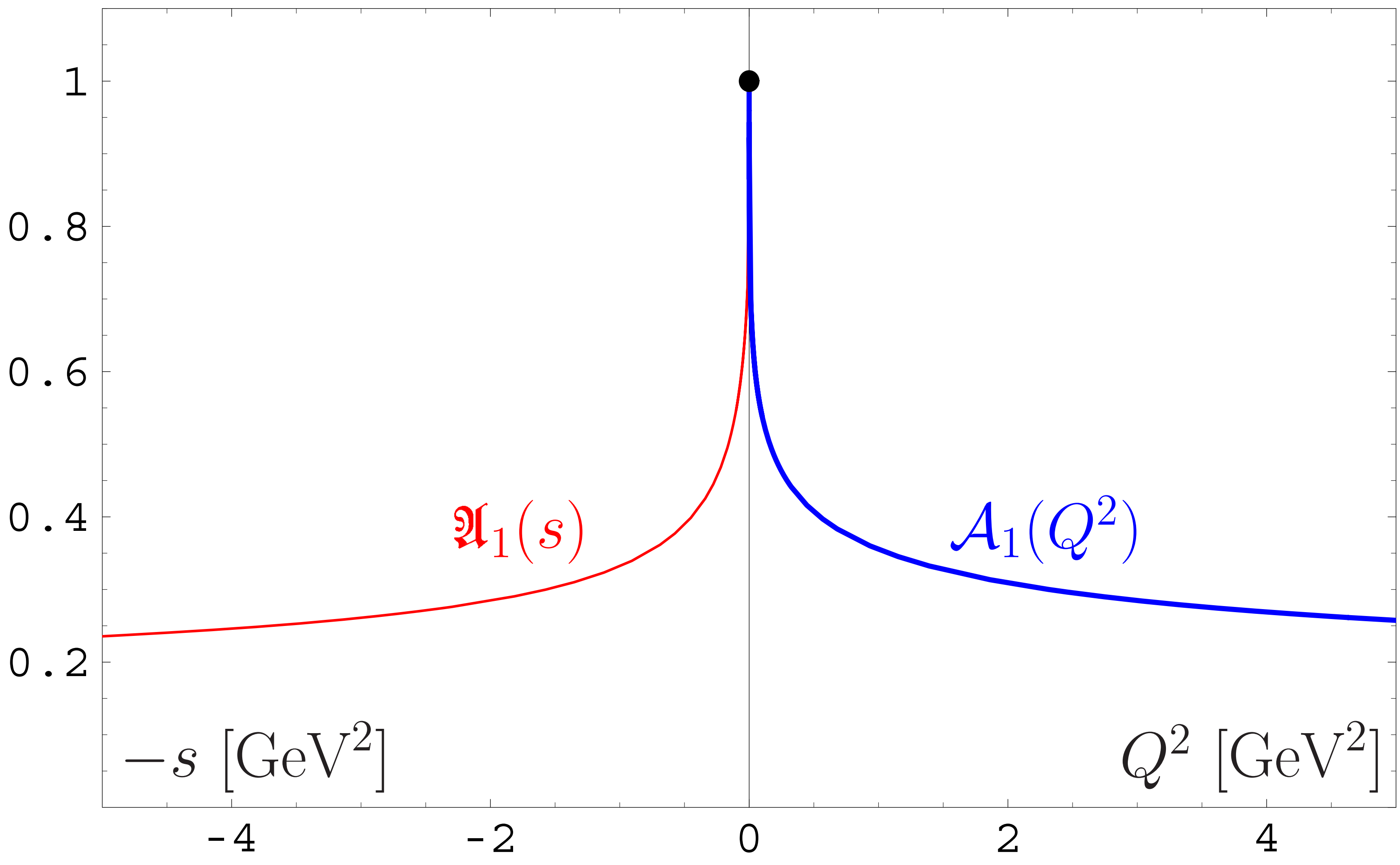
$$\mathcal{A}_n[L] = \int_0^\infty \frac{\rho_n(\sigma)}{\sigma + Q^2}\, d\sigma = \frac{1}{(n-1)!}\left(-\frac{d}{dL}\right)^{n-1} \mathcal{A}_1[L]$$

$$\mathfrak{A}_n[L_s] = \int_s^\infty \frac{\rho_n(\sigma)}{\sigma}\, d\sigma = \frac{1}{(n-1)!}\left(-\frac{d}{dL_s}\right)^{n-1} \mathfrak{A}_1[L_s]$$

$$a_s^n[L] = \frac{1}{(n-1)!}\left(-\frac{d}{dL}\right)^{n-1} a_s[L]$$

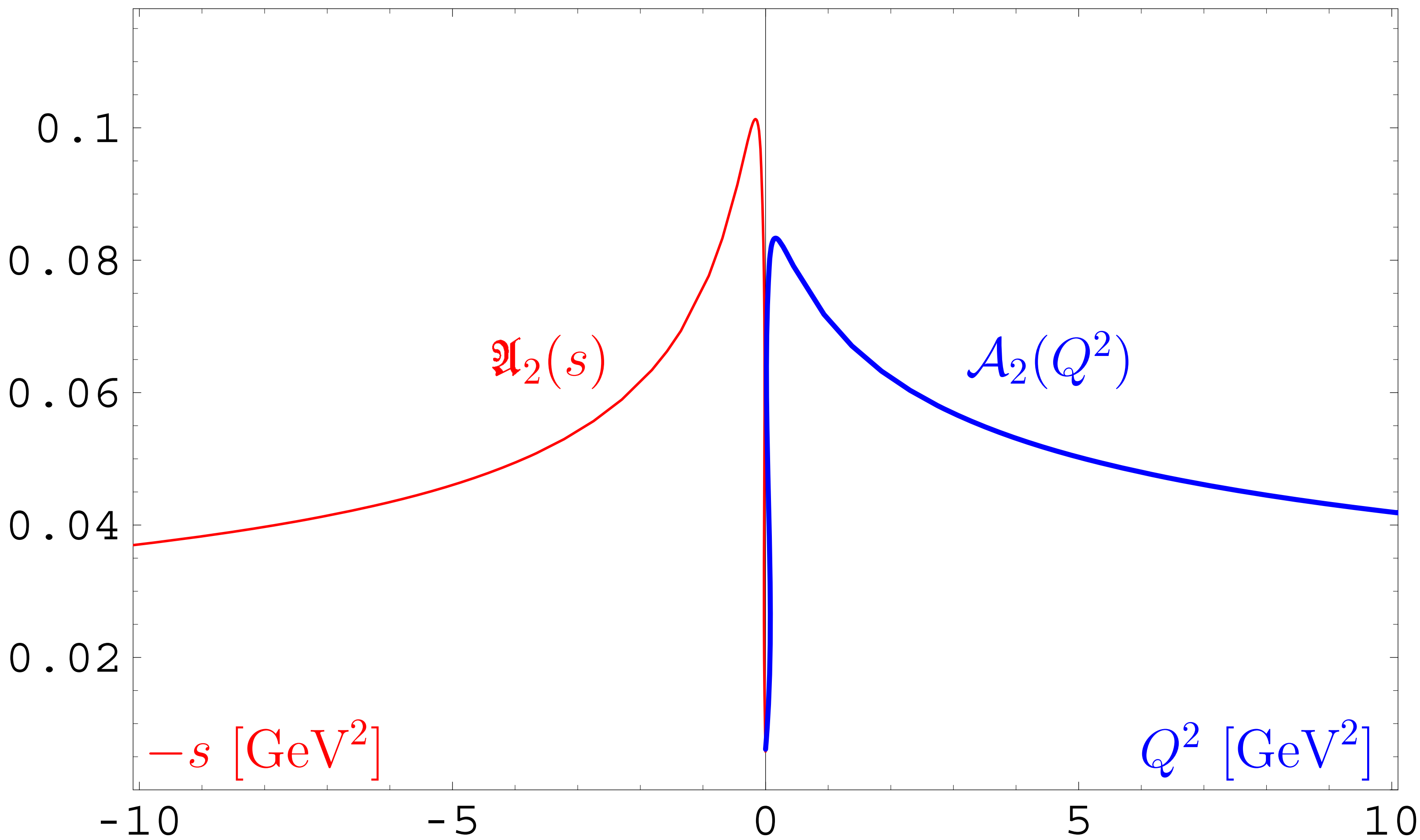
APT graphics: Distorting mirror

First, couplings: $\mathfrak{A}_1(s)$ **and** $\mathcal{A}_1(Q^2)$



APT graphics: Distorting mirror

Second, square-images: $\mathfrak{A}_2(s)$ **and** $\mathcal{A}_2(Q^2)$



Problems of APT.
Resolution:
Fractional APT

Problems of APT

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Problems of APT

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- **RG-improvement to account for higher-orders** $\to$

$$Z[L] = \textbf{exp}\left\{\int^{a_s[L]} \frac{\gamma(a)}{\beta(a)}\, da\right\} \xrightarrow{\textbf{1-loop}} \left[a_s[L]\right]^{\gamma_0/(2\beta_0)}$$

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New functions: $(a_s)^\nu\,,\ (a_s)^\nu \ln(a_s),\ (a_s)^\nu\, L^m,\ \ldots$

Constructing one-loop FAPT

In one-loop APT we have a very nice recurrence relation

$$\mathcal{A}_n[L] = \frac{1}{(n-1)!}\left(-\frac{d}{dL}\right)^{n-1}\mathcal{A}_1[L]$$

and the same in Minkowski domain

$$\mathfrak{A}_n[L] = \frac{1}{(n-1)!}\left(-\frac{d}{dL}\right)^{n-1}\mathfrak{A}_1[L]\,.$$

We can use it to construct FAPT.

FAPT(E): Properties of $\mathcal{A}_\nu[L]$

First, Euclidean coupling ($L = L(Q^2)$):

$$\mathcal{A}_\nu[L] = \frac{1}{L^\nu} - \frac{F(e^{-L}, 1-\nu)}{\Gamma(\nu)}$$

Here $F(z,\nu)$ is reduced Lerch transcendent. function. It is analytic function in ν.

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Here $F(z,\nu)$ is reduced Lerch transcendent. function. It is analytic function in ν. Properties:

- $\mathcal{A}_0[L] = 1$;
- $\mathcal{A}_{-m}[L] = L^m$ for $m \in \mathbb{N}$;
- $\mathcal{A}_m[L] = (-1)^m \mathcal{A}_m[-L]$ for $m \geq 2\,,\ m \in \mathbb{N}$;
- $\mathcal{A}_m[\pm\infty] = 0$ for $m \geq 2\,,\ m \in \mathbb{N}$;

FAPT(M): Properties of $\mathfrak{A}_\nu[L]$

Now, Minkowskian coupling ($L = L(s)$):

$$\mathfrak{A}_\nu[L] = \frac{\sin\left[(\nu-1)\arccos\left(L/\sqrt{\pi^2+L^2}\right)\right]}{\pi(\nu-1)\left(\pi^2+L^2\right)^{(\nu-1)/2}}$$

Here we need only elementary functions.

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Here we need only elementary functions. Properties:

- $\mathfrak{A}_0[L] = 1$;
- $\mathfrak{A}_{-1}[L] = L$;
- $\mathfrak{A}_{-2}[L] = L^2 - \frac{\pi^2}{3}, \quad \mathfrak{A}_{-3}[L] = L\left(L^2-\pi^2\right), \ \ldots$;
- $\mathfrak{A}_m[L] = (-1)^m \mathfrak{A}_m[-L]$ **for** $m \geq 2\,,\ m \in \mathbb{N}$;
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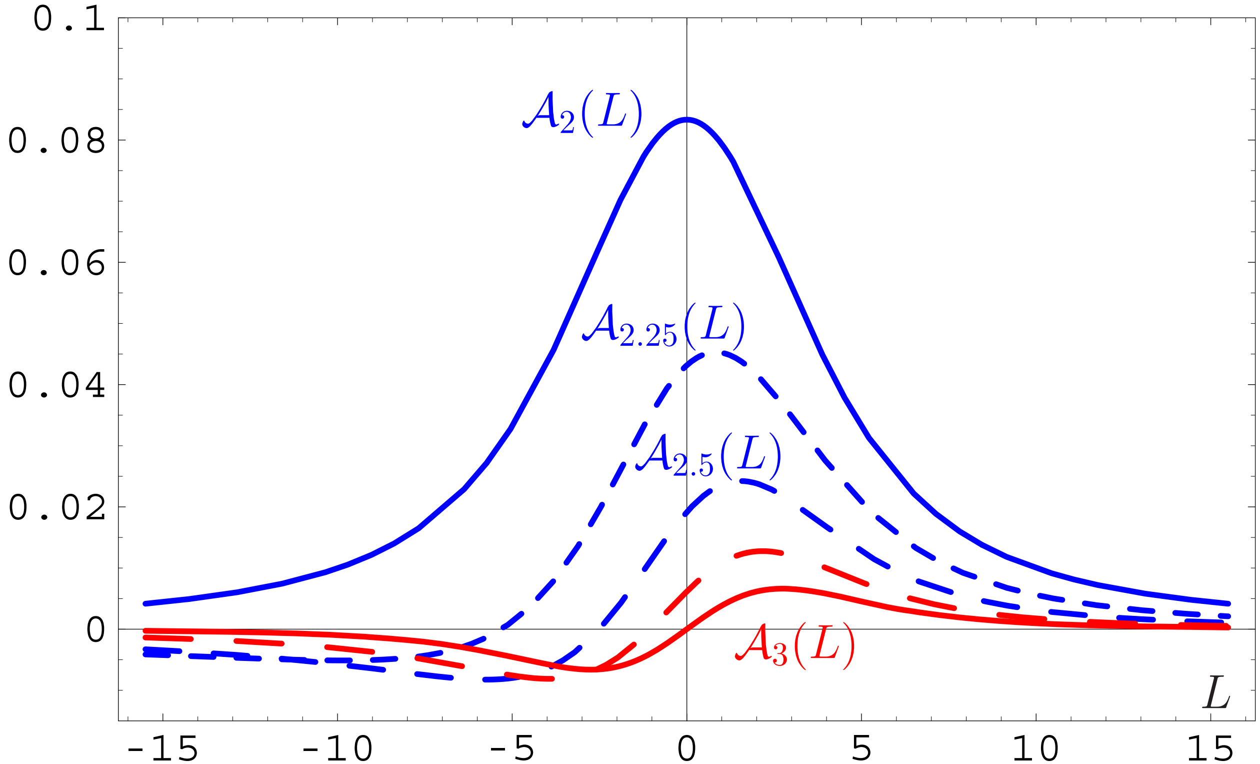
FAPT(E): Graphics of $\mathcal{A}_\nu[L]$ vs. L

$$\mathcal{A}_\nu[L] = \frac{1}{L^\nu} - \frac{F(e^{-L}, 1-\nu)}{\Gamma(\nu)}$$

Graphics for fractional $\nu \in [2, 3]$ **:**



FAPT(M): Graphics of $\mathfrak{A}_\nu[L]$ vs. L

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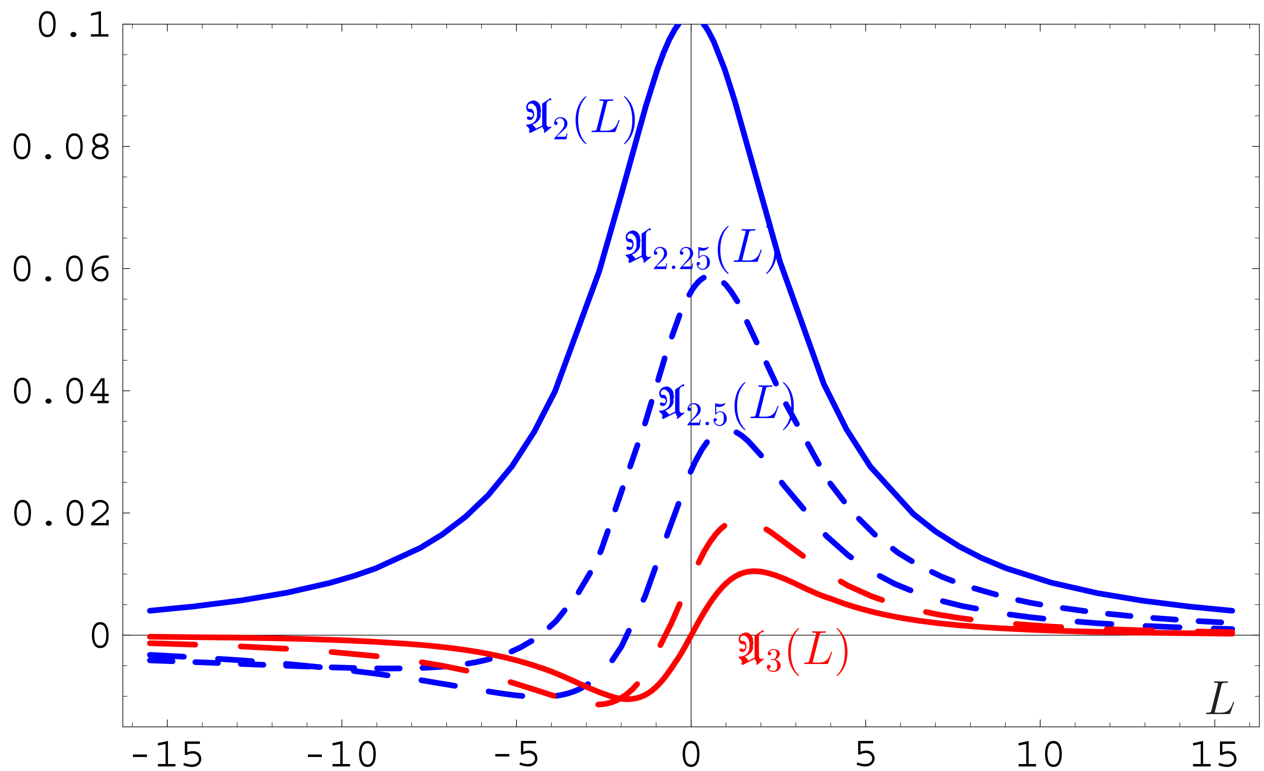
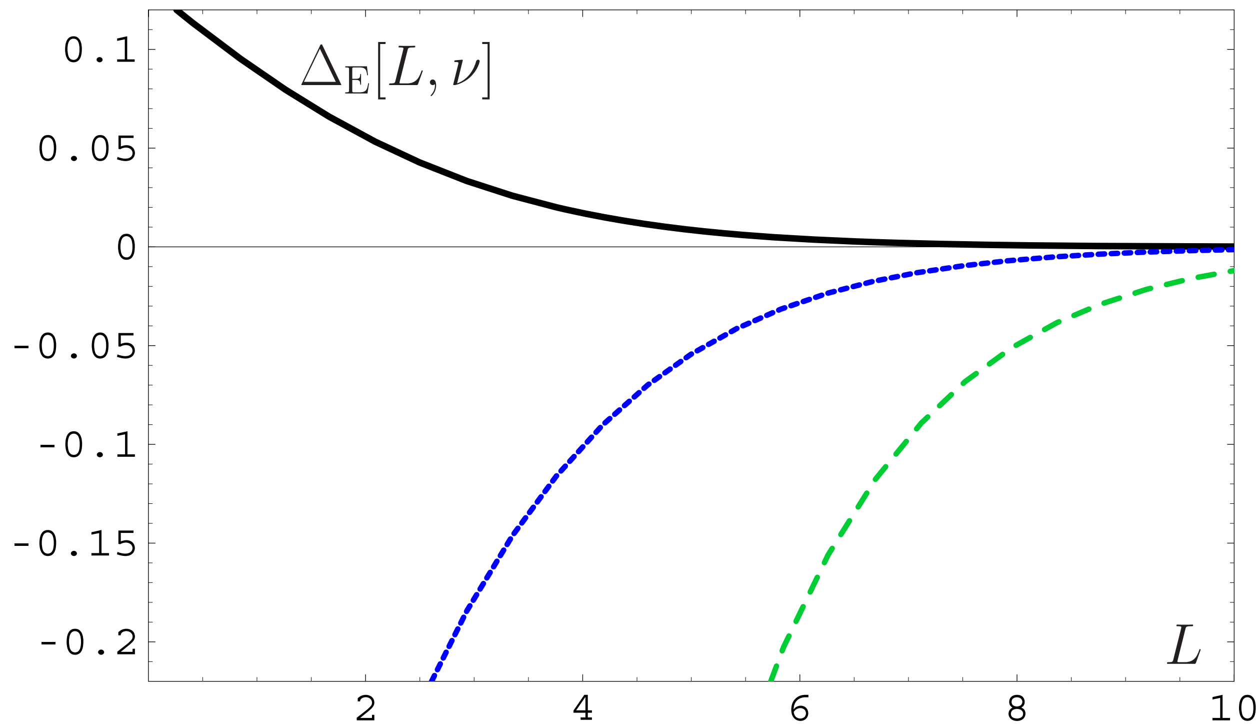
Compare with graphics in Minkowskian region :



FAPT(E): Comparing $\mathcal{A}_\nu$ with $(\mathcal{A}_1)^\nu$

$$\Delta_{\mathbf{E}}(L,\nu) = \frac{\mathcal{A}_\nu[L] - (\mathcal{A}_1[L])^\nu}{\mathcal{A}_\nu[L]}$$

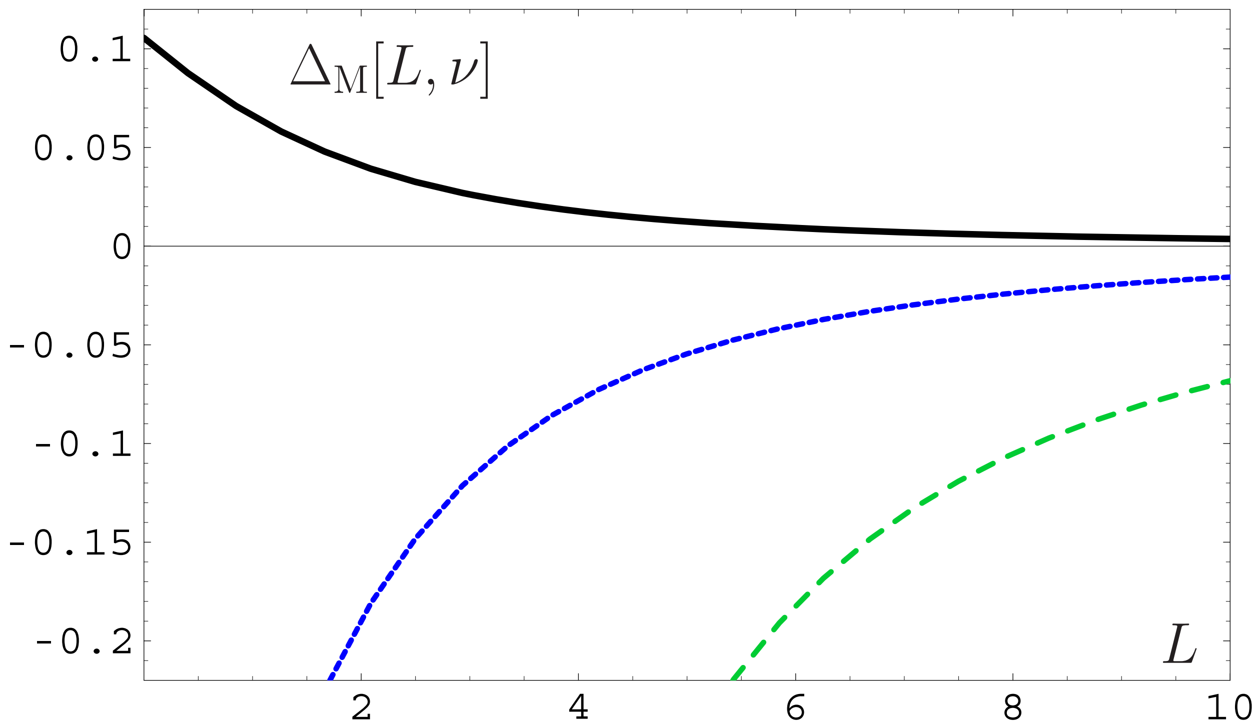
Graphics for fractional $\nu =$0.62, 1.62 and 2.62:



FAPT(M): Comparing $\mathfrak{A}_\nu$ with $\left(\mathfrak{A}_1\right)^\nu$

$$\Delta_{\mathsf{M}}(L,\nu) = \frac{\mathfrak{A}_\nu[L] - \left(\mathfrak{A}_1[L]\right)^\nu}{\mathfrak{A}_\nu[L]}$$

Minkowskian graphics for $\nu =$0.62, 1.62 and 2.62:



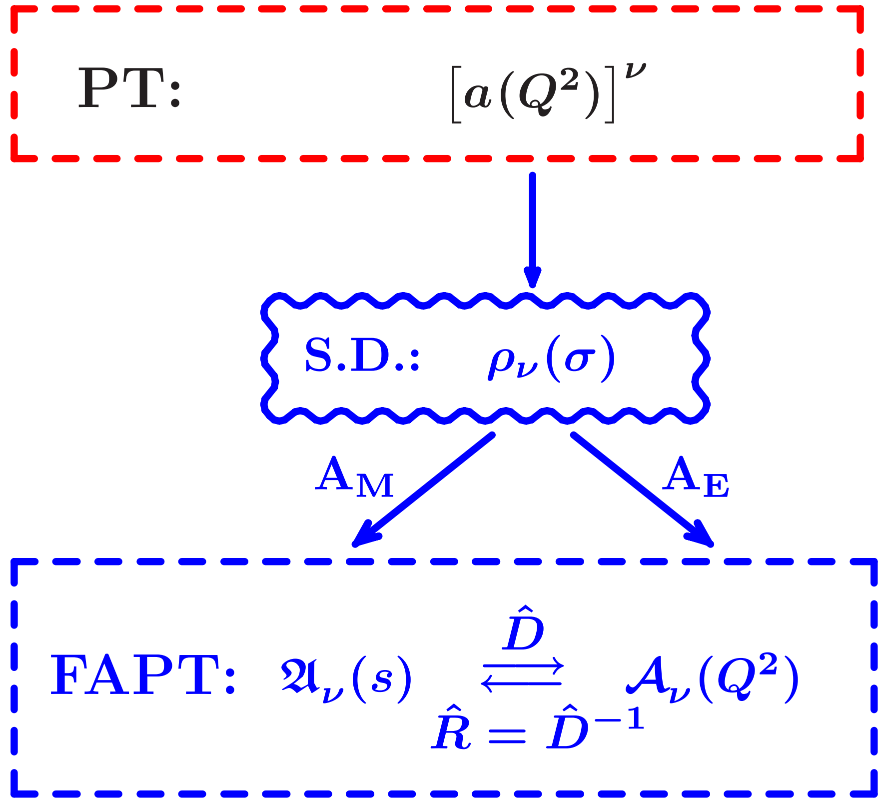
Comparison of PT, APT, and FAPT

Theory	PT	APT	FAPT
Set	$\left\{a^{\nu}\right\}_{\nu\in\mathbb{R}}$	$\left\{\mathcal{A}_m, \mathfrak{A}_m\right\}_{m\in\mathbb{N}}$	$\left\{\mathcal{A}_\nu, \mathfrak{A}_\nu\right\}_{\nu\in\mathbb{R}}$
Series	$\sum_m f_m\, a^m$	$\sum_m f_m\, \mathcal{A}_m$	$\sum_m f_m\, \mathcal{A}_m$
Inv. powers	$(a[L])^{-m}$	—	$\mathcal{A}_{-m}[L] = L^m$
Products	$a^\mu a^\nu = a^{\mu+\nu}$	—	—
Index deriv.	$a^\nu \ln^k a$	—	$\mathcal{D}^k \mathcal{A}_\nu$
Logarithms	$a^\nu L^k$	—	$\mathcal{A}_{\nu-k}$

Development of FAPT:

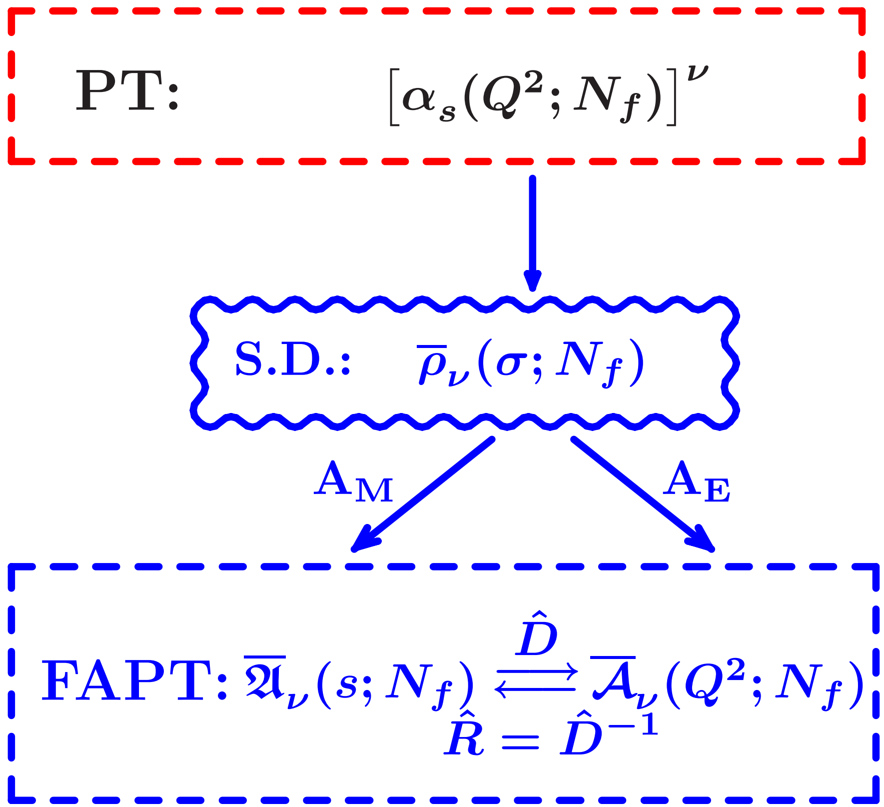
Heavy-Quark Thresholds

Conceptual scheme of FAPT



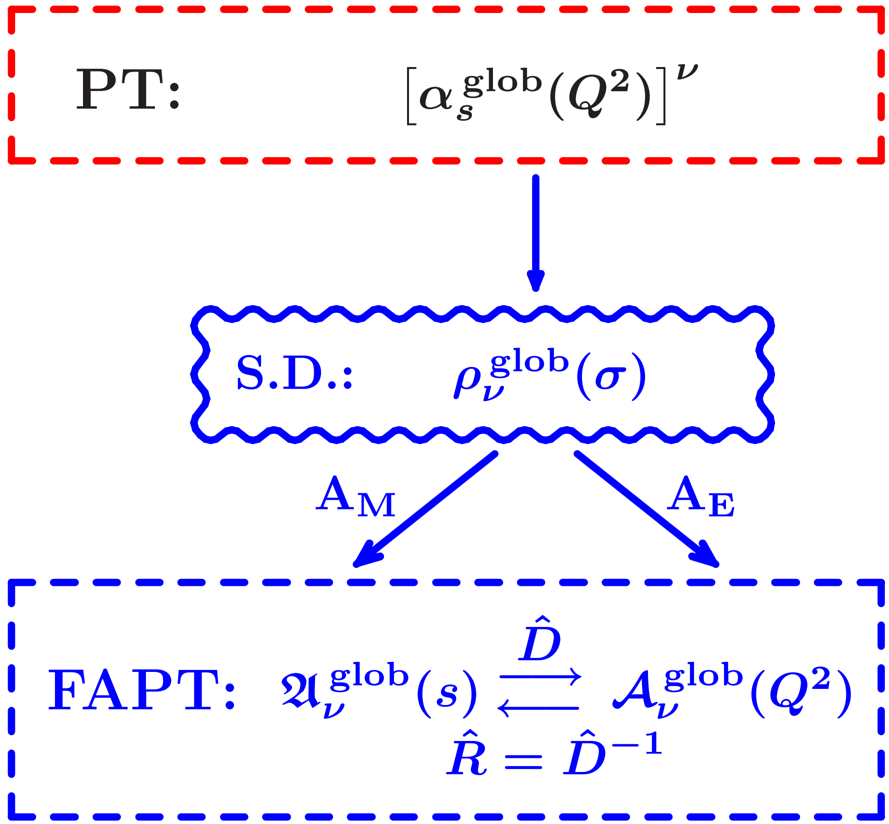
Here N_f is fixed and factorized out.

Conceptual scheme of FAPT



Here N_f is fixed, but not factorized out.

Conceptual scheme of FAPT



Here we see how "analytization" takes into account N_f-dependence.

Global FAPT: Single threshold case

- **Consider for simplicity only one threshold at $s = m_c^2$ with transition $N_f = 3 \to N_f = 4$.**
- **Denote: $L_4 = \ln\left(m_c^2/\Lambda_3^2\right)$ and $\lambda_4 = \ln\left(\Lambda_3^2/\Lambda_4^2\right)$.**

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Then:

$$\mathfrak{A}_\nu^{\text{glob}}[L] = \theta\,(L < L_4)\left[\overline{\mathfrak{A}}_\nu[L;3] - \overline{\mathfrak{A}}_\nu[L_4;3] + \overline{\mathfrak{A}}_\nu[L_4 + \lambda_4;4]\right] + \theta\,(L \geq L_4)\,\overline{\mathfrak{A}}_\nu[L + \lambda_4;4]$$

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$$+\,\theta\left(L \geq L_4\right)\overline{\mathfrak{A}}_\nu[L+\lambda_4;4]$$

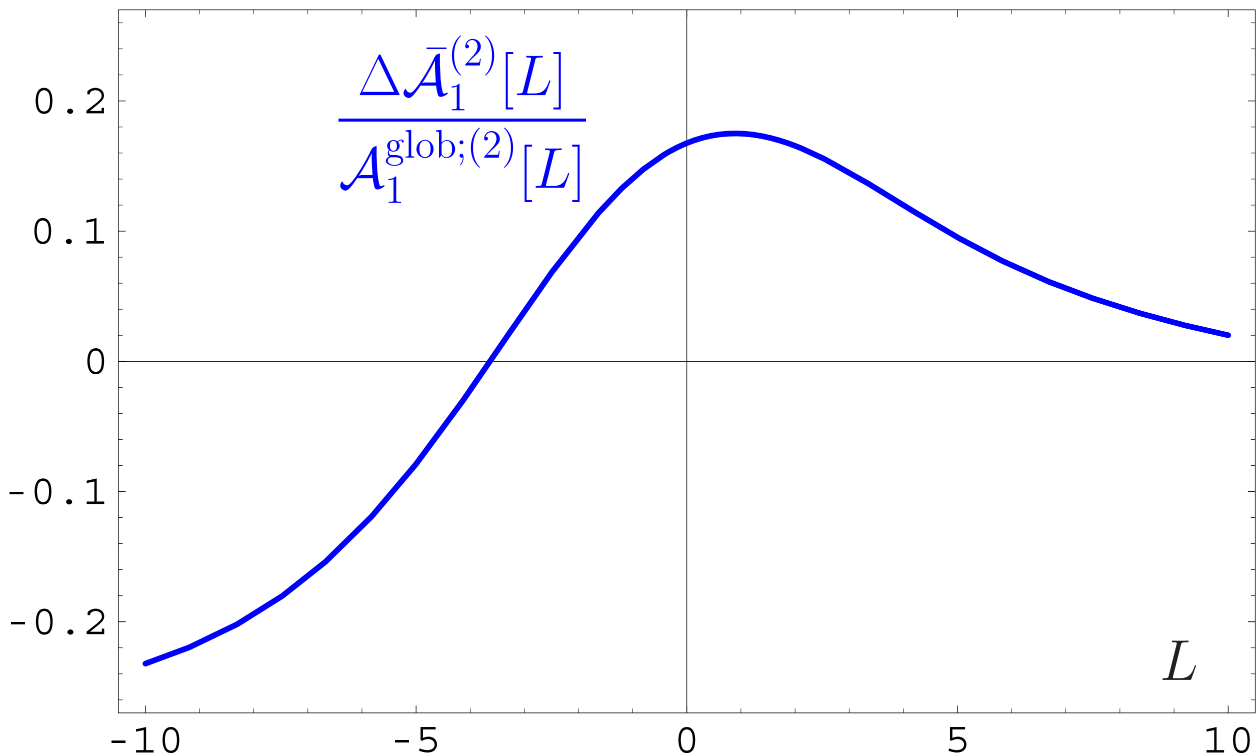
and

$$\mathcal{A}_\nu^{\text{glob}}[L] = \overline{\mathcal{A}}_\nu[L+\lambda_4;4] + \int\limits_{-\infty}^{L_4} \frac{\overline{\rho}_\nu\left[L_\sigma;3\right] - \overline{\rho}_\nu\left[L_\sigma+\lambda_4;4\right]}{1+e^{L-L_\sigma}}\,dL_\sigma$$

Graphical comparison: Fixed-N_f—Global

$$\mathcal{A}_\nu^{\text{glob}}[L] = \overline{\mathcal{A}}_\nu[L + \lambda_4; 4] + \Delta\overline{\mathcal{A}}_\nu[L]\,;$$

$\Delta\overline{\mathcal{A}}_1[L]/\mathcal{A}_1^{\text{glob}}[L]$ — **solid**:



Resummation
in
one- and two-loop (F)APT

Resummation in one-loop APT

Consider series $$\mathcal{D}[L] = d_0 + \sum_{n=1}^{\infty} d_n \, \mathcal{A}_n[L]$$

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Let exist the generating function $P(t)$ for coefficients:

$$d_n = d_1 \int_0^\infty P(t)\, t^{n-1} dt \quad \text{with} \quad \int_0^\infty P(t)\, dt = 1\,.$$

We define a shorthand notation

$$\langle\langle f(t) \rangle\rangle_{P(t)} \equiv \int_0^\infty f(t)\, P(t)\, dt\,.$$

Then coefficients $d_n = d_1 \, \langle\langle t^{n-1} \rangle\rangle_{P(t)}$.

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with coefficients $d_n = d_1 \,\langle\langle t^{n-1} \rangle\rangle_{P(t)}$.

We have one-loop recurrence relation:

$$\mathcal{A}_{n+1}[L] = \frac{1}{\Gamma(n+1)} \left(-\frac{d}{dL}\right)^n \mathcal{A}_1[L]\,.$$

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and for Minkowski region:

$$\mathcal{R}[L] = d_0 + d_1 \, \langle\langle \mathfrak{A}_1[L-t] \rangle\rangle_{P(t)}$$

Models for perturbative coefficients

Coefficients d_n of the PT series:

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$(t/c)^{\gamma+1}e^{-t/c}$	$n^{\gamma}\,c^{n}\Gamma(n+1)$

Resummation in Global Minkowskian APT

Consider series $\mathcal{R}[L] = d_0 + \sum_{n=1}^{\infty} d_n\, \mathfrak{A}_n^{\textsf{glob}}[L]$

with coefficients $d_n = d_1 \,\langle\langle t^{n-1} \rangle\rangle_{P(t)}$.

Result:

$$\begin{aligned}\mathcal{R}[L] = d_0 \;&+\; d_1 \langle\langle \theta\,(L < L_4) \Big[\Delta_4 \overline{\mathfrak{A}}_1[t] + \overline{\mathfrak{A}}_1\Big[L - \frac{t}{\beta_3}; 3\Big]\Big] \rangle\rangle_{P(t)} \\ &+\; d_1 \langle\langle \theta\,(L \geq L_4) \overline{\mathfrak{A}}_1 \Big[L + \lambda_4 - \frac{t}{\beta_4}; 4\Big] \rangle\rangle_{P(t)}\,.\end{aligned}$$

where

$$\Delta_4 \overline{\mathfrak{A}}_1[t] = \overline{\mathfrak{A}}_1\Big[L_4 + \lambda_4 - \frac{t}{\beta_4}; 4\Big] - \overline{\mathfrak{A}}_1\Big[L_3 - \frac{t}{\beta_3}; 3\Big]\,.$$

Resummation in Global Euclidean APT

In Euclidean domain the result is more complicated:

$$\mathcal{D}[L] = d_0 + d_1 \langle\langle \int\limits_{-\infty}^{L_4} \frac{\overline{\rho}_1\,[L_\sigma;3]\; dL_\sigma}{1+e^{L-L_\sigma-t/\beta_3}} \rangle\rangle_{P(t)}$$

$$+ \langle\langle \Delta_4[L,t] \rangle\rangle_{P(t)} + d_1 \langle\langle \int\limits_{L_4}^{\infty} \frac{\overline{\rho}_1\,[L_\sigma+\lambda_4;4]\; dL_\sigma}{1+e^{L-L_\sigma-t/\beta_4}} \rangle\rangle_{P(t)}\,.$$

where

$$\Delta_4[L,t] = \int\limits_0^1 \frac{\overline{\rho}_1\,[L_4+\lambda_4-tx/\beta_4;4]\; t}{\beta_4\,\left[1+e^{L-L_4-t\bar{x}/\beta_4}\right]}\, dx$$

$$- \int\limits_0^1 \frac{\overline{\rho}_1\,[L_3-tx/\beta_3;3]\; t}{\beta_3\,\left[1+e^{L-L_4-t\bar{x}/\beta_3}\right]}\, dx.$$

Resummation in FAPT

Consider seria $$\mathcal{R}_\nu[L] = d_0\,\mathfrak{A}_\nu[L] + \sum_{n=1}^{\infty} d_n\,\mathfrak{A}_{n+\nu}[L]$$

and $$\mathcal{D}_\nu[L] = d_0\,\mathcal{A}_\nu[L] + \sum_{n=1}^{\infty} d_n\,\mathcal{A}_{n+\nu}[L]$$

with coefficients $d_n = d_1\,\langle\langle t^{n-1}\rangle\rangle_{P(t)}$.

Result:

$$\begin{aligned}
\mathcal{R}_\nu[L] &= d_0\,\mathfrak{A}_\nu[L] + d_1\,\langle\langle \mathfrak{A}_{1+\nu}[L-t]\rangle\rangle_{P_\nu(t)}\,;\\
\mathcal{D}_\nu[L] &= d_0\,\mathcal{A}_\nu[L] + d_1\,\langle\langle \mathcal{A}_{1+\nu}[L-t]\rangle\rangle_{P_\nu(t)}\,.
\end{aligned}$$

where $P_\nu(t) = \int\limits_0^1 P\left(\frac{t}{1-z}\right)\nu\,z^{\nu-1}\frac{dz}{1-z}\,.$

Resummation in Global Minkowskian FAPT

Consider series $\mathcal{R}_\nu[L] = d_0\,\mathfrak{A}_\nu^{\textsf{glob}} + \sum_{n=1}^{\infty} d_n\,\mathfrak{A}_{n+\nu}^{\textsf{glob}}[L]$

with coefficients $d_n = d_1\,\langle\langle t^{n-1}\rangle\rangle_{P(t)}$.

Then result is complete analog of the Global APT(M) result with natural substitutions:

$$\overline{\mathfrak{A}}_1[L] \to \overline{\mathfrak{A}}_{1+\nu}[L] \quad \text{and} \quad P(t) \to P_\nu(t)$$

with $P_\nu(t) = \int\limits_0^1 P\left(\frac{t}{1-z}\right)\nu\, z^{\nu-1}\frac{dz}{1-z}\,.$

Resummation in Global Euclidean FAPT

Consider series $\mathcal{D}_{\nu}[L] = d_0\,\mathcal{A}_{\nu}^{\text{glob}} + \sum_{n=1}^{\infty} d_n\,\mathcal{A}_{n+\nu}^{\text{glob}}[L]$

with coefficients $d_n = d_1\,\langle\langle t^{n-1}\rangle\rangle_{P(t)}$.

Then result is complete analog of the Global APT(E) result with natural substitutions:

$$\overline{\rho}_1[L] \to \overline{\rho}_{1+\nu}[L] \quad \textbf{and} \quad P(t) \to P_{\nu}(t)$$

with $P_{\nu}(t) = \int_0^1 P\left(\frac{t}{1-z}\right)\nu\, z^{\nu-1}\frac{dz}{1-z}\,.$

Resummation in two-loop (global) FAPT

Consider series $\mathcal{S}_\nu[L] = \sum_{n=1}^{\infty} \langle\langle t^{n-1} \rangle\rangle_{P(t)} \mathcal{F}_{n+\nu}[L]$**.**

Here $\mathcal{F}_\nu[L] = \mathcal{A}_\nu^{(2)}[L]$ **or** $\mathfrak{A}_\nu^{(2)}[L]$ **(or** $\rho_\nu^{(2)}[L]$ **— for global).**

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We have two-loop recurrence relation ($c_1 = b_1/b_0^2$**):**

$$-\frac{1}{n+\nu}\,\frac{d}{dL}\,\mathcal{F}_{n+\nu}[L] = \mathcal{F}_{n+1+\nu}[L] + c_1\,\mathcal{F}_{n+2+\nu}[L]\,.$$

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Result (with $\tau(t) = t - c_1 \ln(1 + t/c_1)$**):**

$$\mathcal{S}[L] = \Big\langle\!\!\Big\langle \mathcal{F}_{1+\nu}[L] - \frac{t^2}{c_1 + t}\int_0^1 z^\nu dz\, \dot{\mathcal{F}}_{1+\nu}[L + \tau(t\,z) - \tau(t)]$$

$$+ \frac{c_1\,t}{c_1 + t}\left\{\mathcal{F}_{2+\nu}[L] - \int_0^1 dz\,\frac{t^2\,z^{\nu+1}}{c_1 + t\,z}\,\dot{\mathcal{F}}_{2+\nu}[L + \tau(t\,z) - \tau(t)]\right\}\Big\rangle\!\!\Big\rangle_{P(t)}$$

Resummation in two-loop (global) FAPT

Consider series $$\mathcal{S}_{\nu_0,\nu_1}[L] = \sum_{n=1}^{\infty} \langle\langle t^{n-1} \rangle\rangle_{P(t)}\, \mathcal{F}_{n+\nu_0,\nu_1}[L].$$

Here $\mathcal{F}_{n+\nu_0,\nu_1}[L] = \mathcal{B}^{(2)}_{n+\nu_0,\nu_1}[L]$ **or** $\mathfrak{B}^{(2)}_{n+\nu_0,\nu_1}[L]$

(or $\rho^{(2)}_{n+\nu_0,\nu_1}[L]$ **— for global),**

where

$$\mathcal{B}_{\nu;\nu_1}[L] = \mathbf{A}_{\mathbf{E,M}}\left[a^{\nu}_{(2)}[L]\left(1 + c_1\, a_{(2)}\right)^{\nu_1}[L]\right]$$

is the analytic image of the two-loop evolution factor.

We have constructed formulas of resummation for $\mathcal{S}_{\nu_0,\nu_1}[L]$ **as well.**

Higgs boson decay

$$H^0 \to b\bar{b}$$

Higgs boson decay into $b\bar{b}$-pair

This decay can be expressed in QCD by means of the correlator of quark scalar currents $J_{\mathsf{S}}(x) = :\bar{b}(x)b(x):$

$$\Pi(Q^2) = (4\pi)^2 i \int dx e^{iqx} \langle 0|\, T[\, J_{\mathsf{S}}(x) J_{\mathsf{S}}(0)\,]\, |0\rangle$$

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in terms of discontinuity of its imaginary part

$$R_{\mathsf{S}}(s) = \mathrm{Im}\,\Pi(-s - i\epsilon)/(2\pi\, s)\,,$$

so that

$$\Gamma_{\mathsf{H}\to b\bar{b}}(M_{\mathsf{H}}) = \frac{G_F}{4\sqrt{2}\pi} M_{\mathsf{H}}\, m_b^2(M_{\mathsf{H}})\, R_{\mathsf{S}}(s = M_{\mathsf{H}}^2)\,.$$

FAPT(M) analysis of R_S

Running mass $m(Q^2)$ is described by the RG equation

$$m^2(Q^2) = \hat{m}^2 \alpha_s^{\nu_0}(Q^2) \left[1 + \frac{c_1 b_0 \alpha_s(Q^2)}{4\pi^2}\right]^{\nu_1} .$$

with RG-invariant mass $\hat{m}^2$ (for b-quark $\hat{m}_b \approx 8.53$ GeV) and $\nu_0 = 1.04$, $\nu_1 = 1.86$.

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$$\left[3\, \hat{m}_b^2\right]^{-1} \widetilde{D}_S(Q^2) = \alpha_s^{\nu_0}(Q^2) + \sum_{m>0} \frac{d_m}{\pi^m}\, \alpha_s^{m+\nu_0}(Q^2) .$$

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In 1-loop FAPT(M) we obtain

$$\widetilde{\mathcal{R}}_S^{(1);N}[L] = 3\hat{m}^2 \left[\mathfrak{A}_{\nu_0}^{(1);\text{glob}}[L] + \sum_{m>0}^{N} \frac{d_m}{\pi^m}\, \mathfrak{A}_{m+\nu_0}^{(1);\text{glob}}[L]\right]$$

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In 2-loop FAPT(M) we obtain

$$\tilde{\mathcal{R}}_S^{(2);N}[L] = 3\hat{m}^2 \left[\mathfrak{B}_{\nu_0,\nu_1}^{(2);\text{glob}}[L] + \sum_{m>0}^{N} \frac{d_m}{\pi^m} \mathfrak{B}_{m+\nu_0,\nu_1}^{(2);\text{glob}}[L]\right]$$

Model for perturbative coefficients

Coefficients of our series, $\tilde{d}_m = d_m/d_1$, with $d_1 = 17/3$:

Model	$\tilde{d}_1$	$\tilde{d}_2$	$\tilde{d}_3$	$\tilde{d}_4$	$\tilde{d}_5$
pQCD	1	7.42	62.3		—

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"PMS" model	—	—	64.8	547	7782

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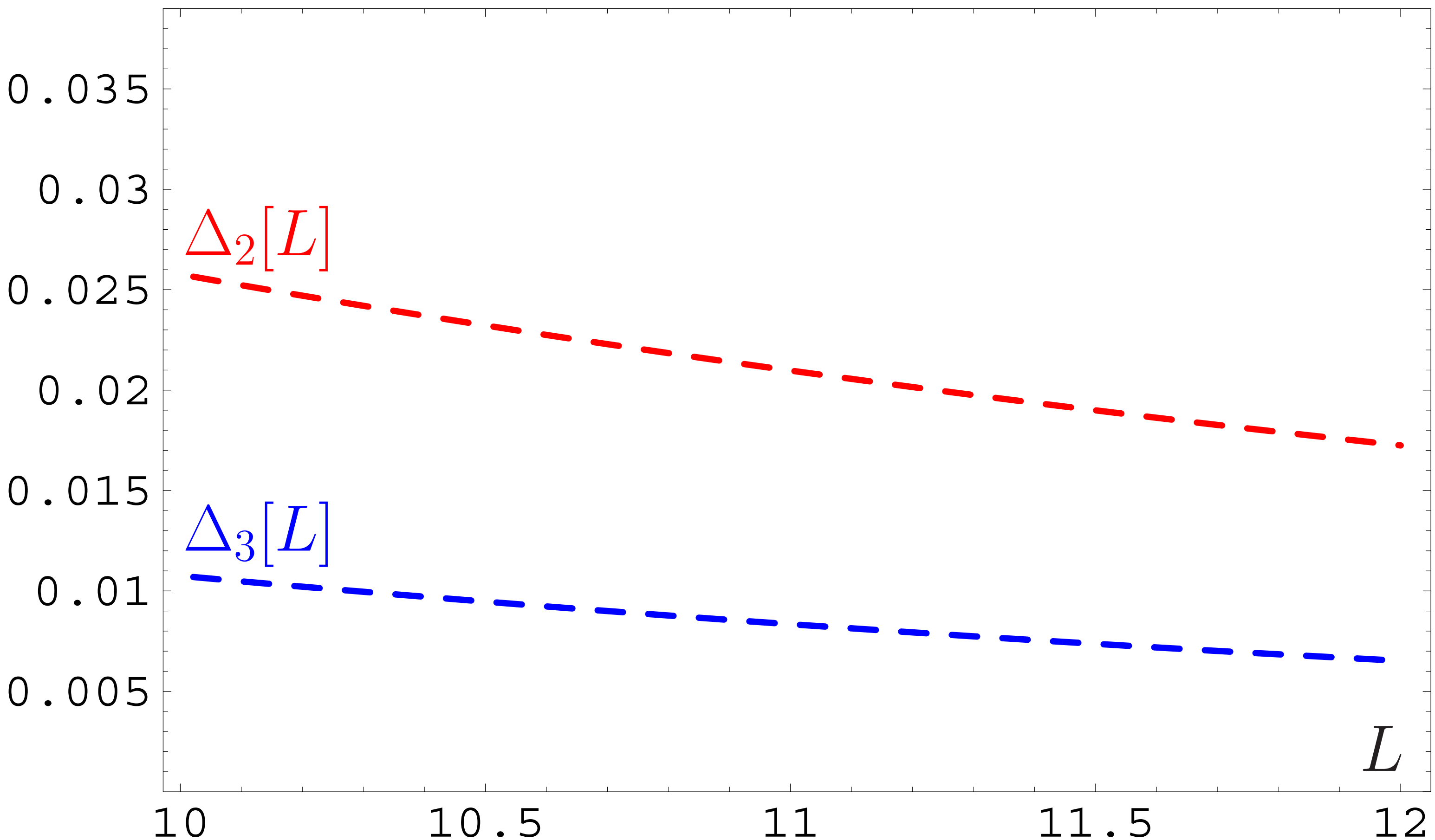
FAPT(M) for $\Gamma_{H\to\bar{b}b}(m_H)$: Truncation errors

We define relative errors of series truncation at Nth term:

$$\Delta_N[L] = 1 - \widetilde{\mathcal{R}}_{\mathbf{S}}^{(2;N)}[L]/\widetilde{\mathcal{R}}_{\mathbf{S}}^{(2;\infty)}[L]$$



FAPT(M) for $\Gamma_{H\to\bar{b}b}(m_{\mathbf{H}})$*: Truncation errors*

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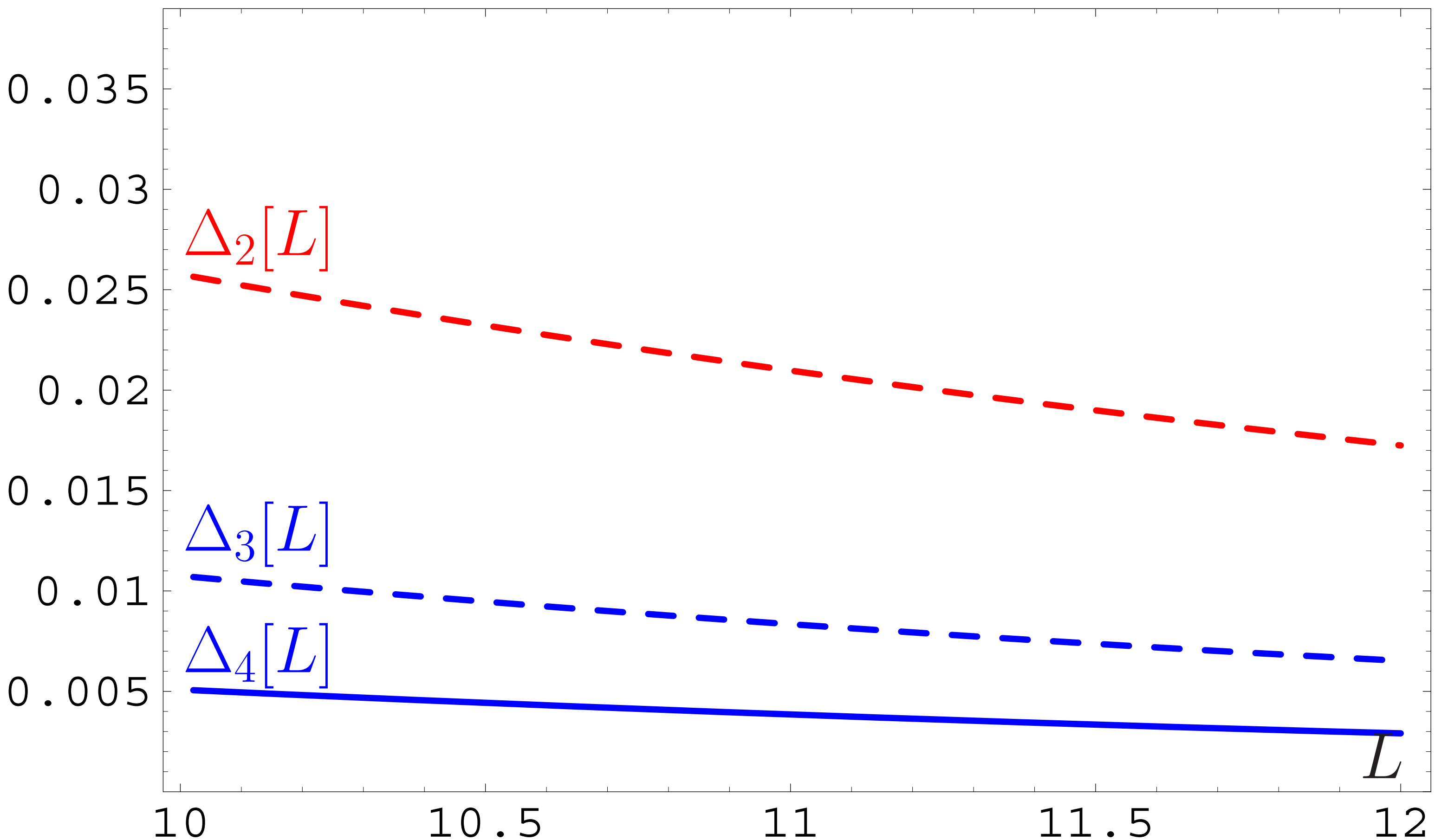

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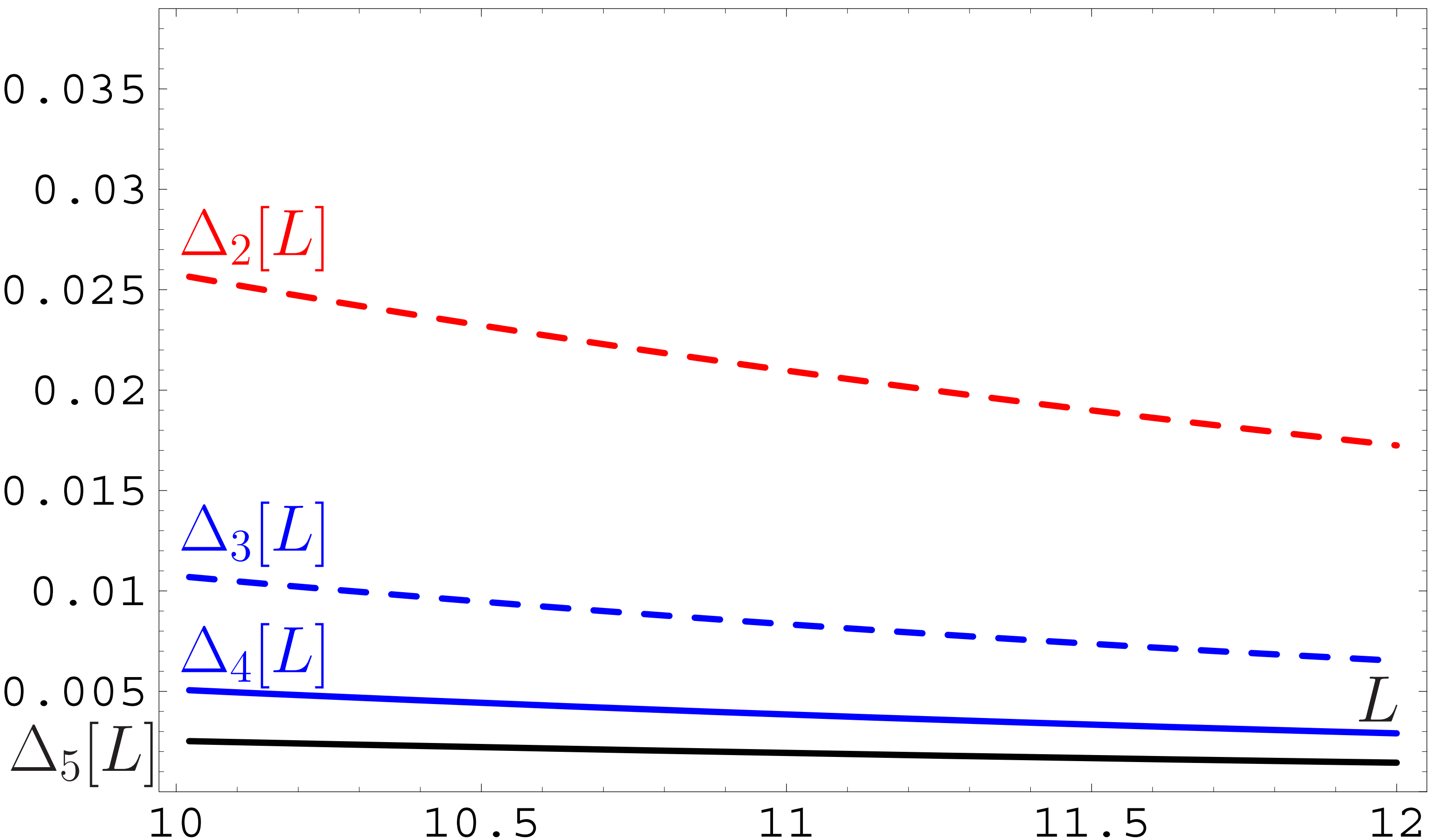

FAPT(M) for $\Gamma_{H\to\bar{b}b}(m_H)$: Truncation errors

Conclusion: **If we need accuracy better than 0.5% — only then we need to calculate the 5-th correction.**

FAPT(M) for $\Gamma_{H\to\bar{b}b}(m_H)$: Truncation errors

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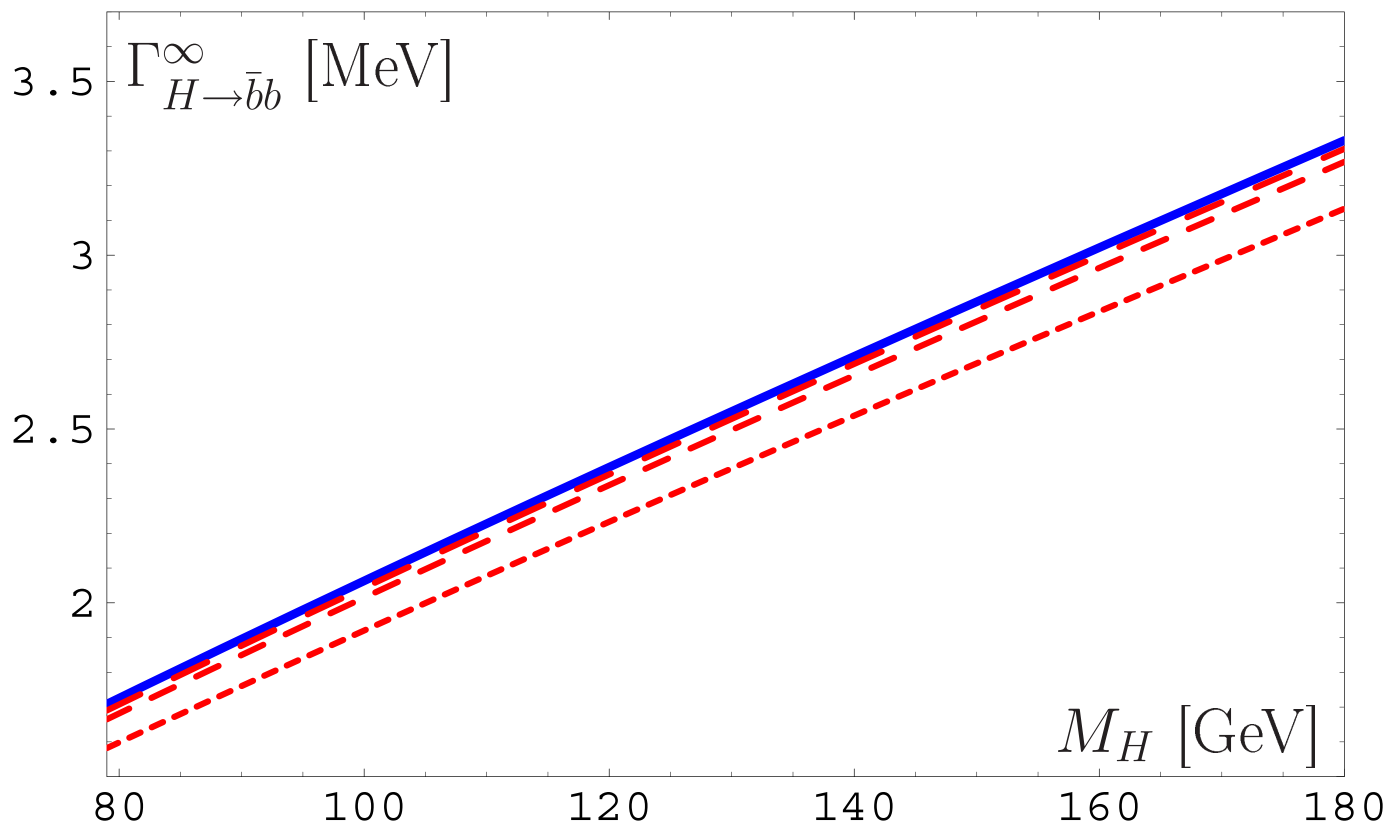
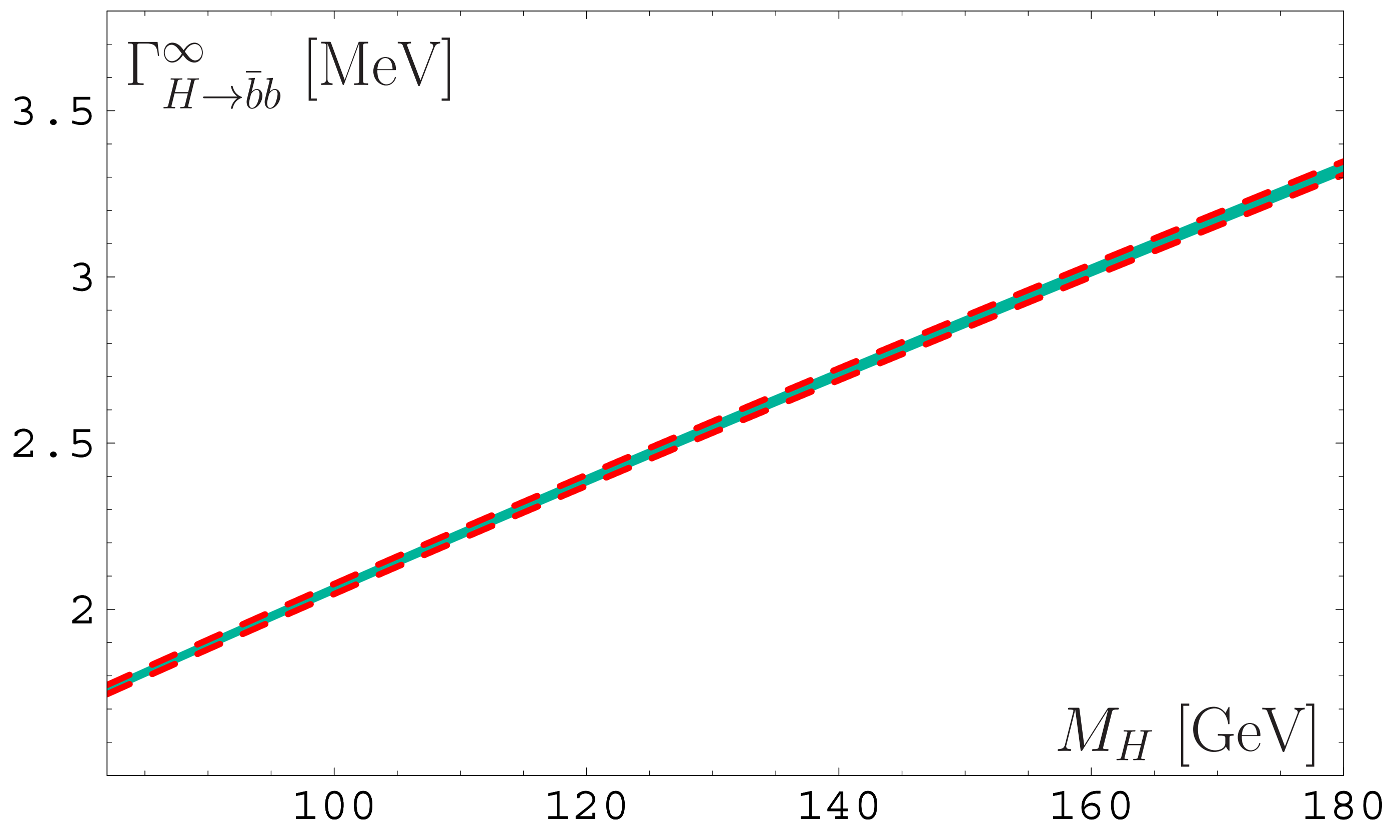
But profit will be tiny — instead of 0.5% one'll obtain 0.3%!



FAPT(M) for $\Gamma_{H\to\bar{b}b}(m_H)$: Truncation errors

Conclusion: If we need accuracy of the order 0.5% — then we need to take into account up to the 4-th correction.

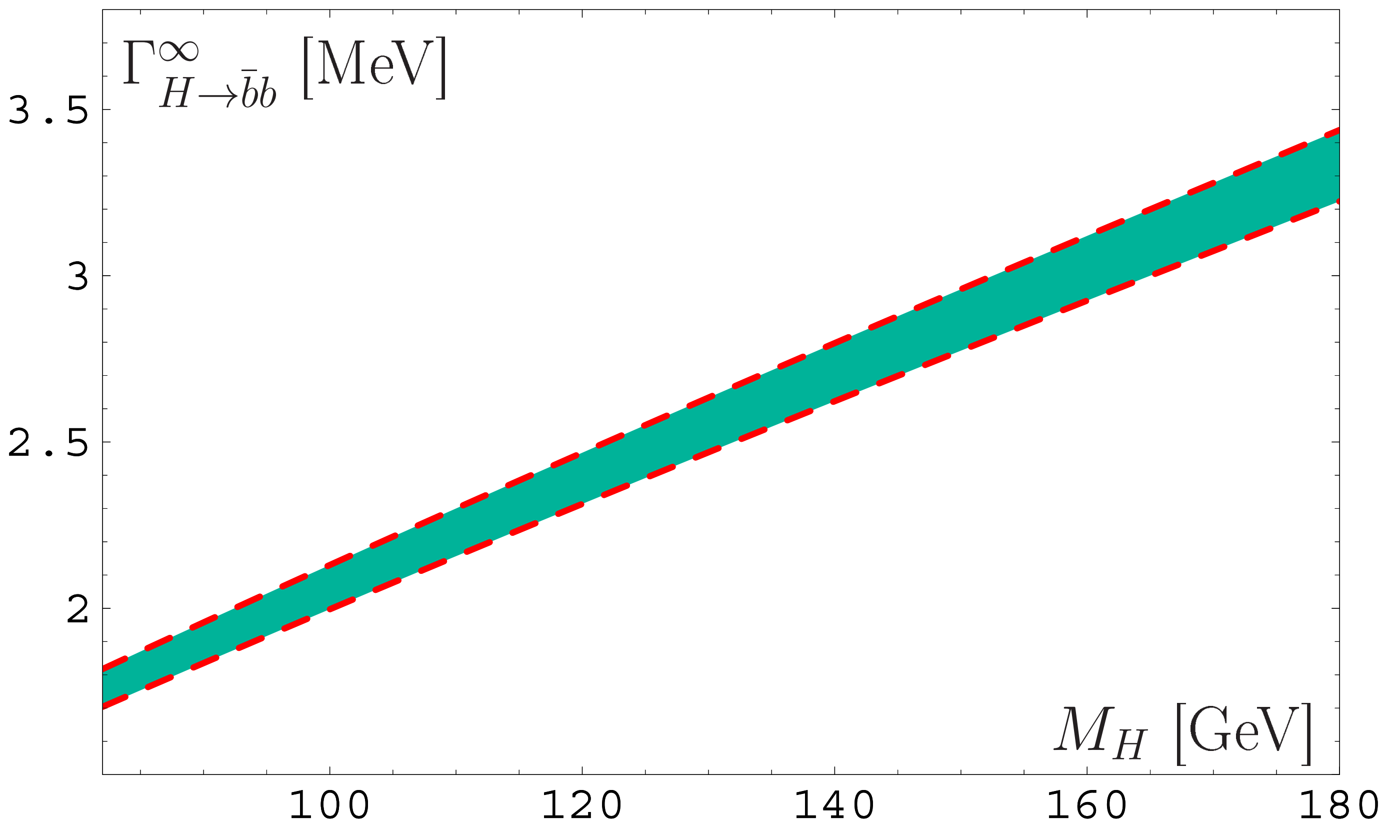
Note: uncertainty due to $P(t)$-modelling is small $\lesssim 0.6\%$.



FAPT(M) for $\Gamma_{H\to\bar{b}b}(m_H)$: Truncation errors

Conclusion: **If we need accuracy of the order 1% — then we need to take into account up to the 3-rd correction — in agreement with Kataev&Kim [0902.1442].**

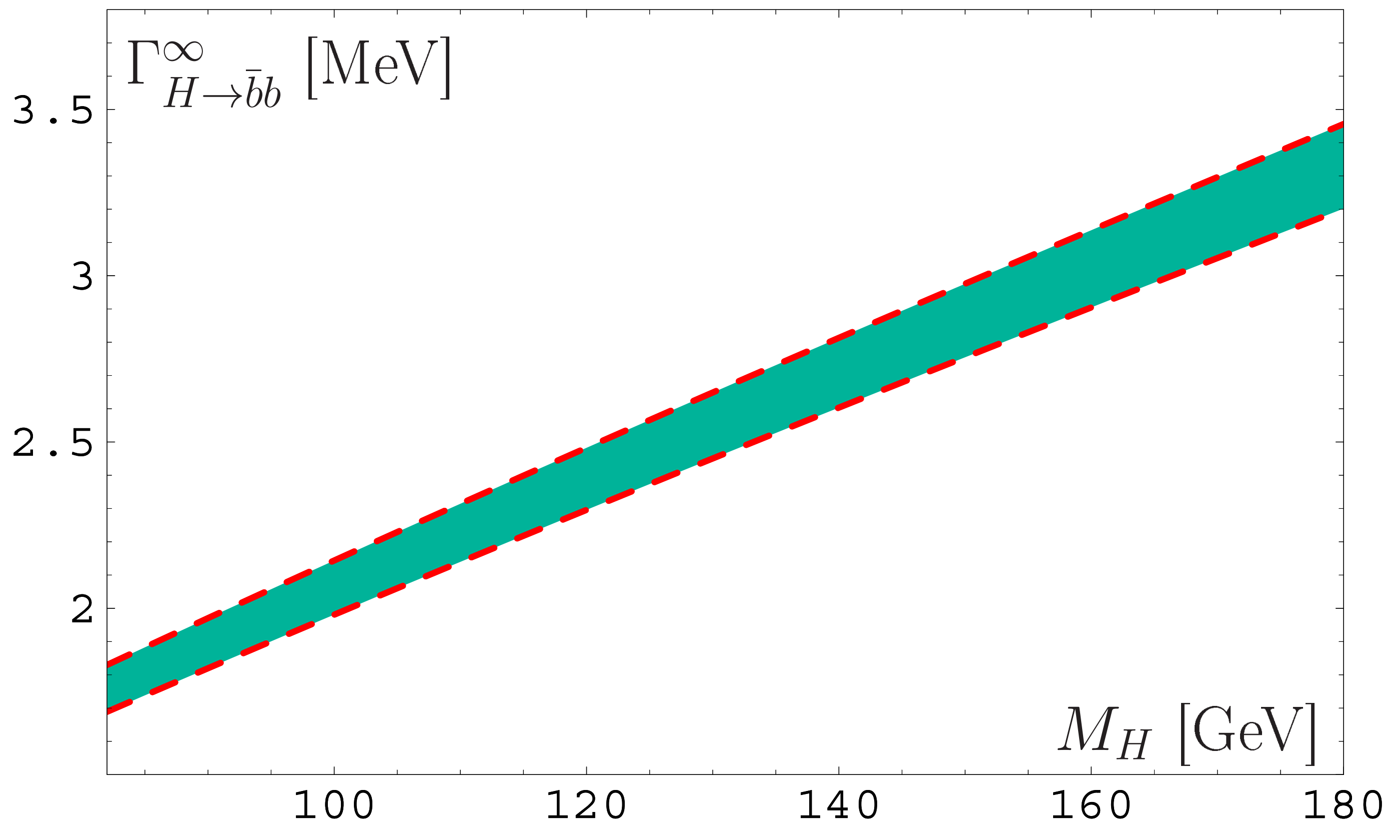
Note: **RG-invariant mass uncertainty $\sim 2\%$.**



FAPT(M) for $\Gamma_{H\to\bar{b}b}(m_H)$: Truncation errors

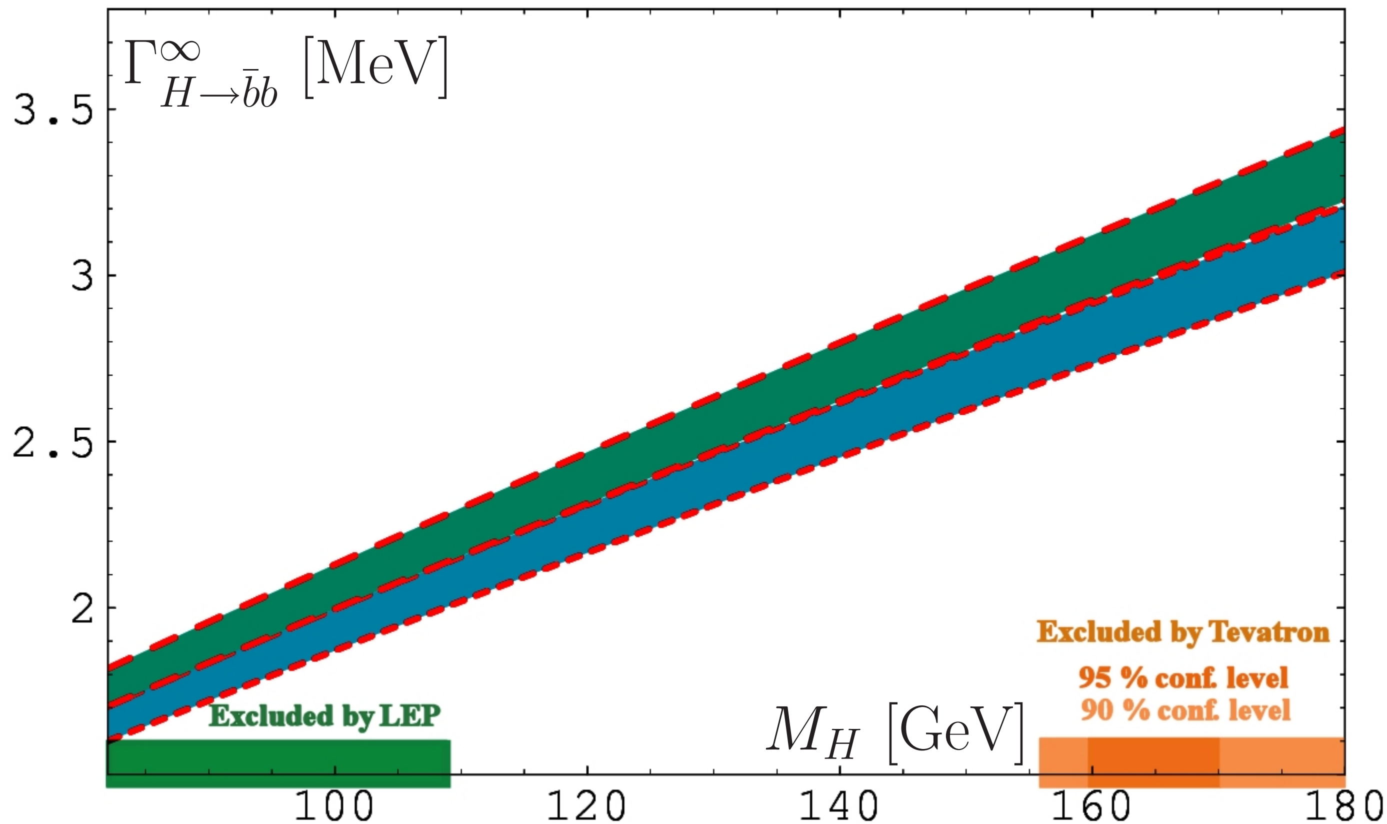
Conclusion: **If we need accuracy of the order 1% — then we need to take into account up to the 3-rd correction — in agreement with Kataev&Kim [0902.1442].**

Note: **overall uncertainty** $\sim 3\%$ **.**



Resummation for $\Gamma_{H\to\bar{b}b}(m_H)$: Loop orders

Comparison of 1- (upper strip) and 2- (lower strip) loop results. We observe a 5% reduction of the two-loop estimate.



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- **APT provides natural way to Minkowski region for coupling and related quantities.**

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- **At the two-loop level we confirm our one-loop estimations for the Higgs boson decay $H \to \bar{b}b$.**
  Already at N^{3}LO we have accuracy of the order of:
  1% — due to truncation error ;
  2% — due to RG-invariant mass uncertainty.
  Agreement with Kataev&Kim [0902.1442].