The QCD phase transition probed by fermionic boundary conditions

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Bogoliubov readings
Dubna, September 2010

partly with E. Bilgici, C. Gattringer, C. Hagen, Z. Fodor, K. Szabo, B. Zhang

QCD

- need to understand confinement and chiral symmetry breaking

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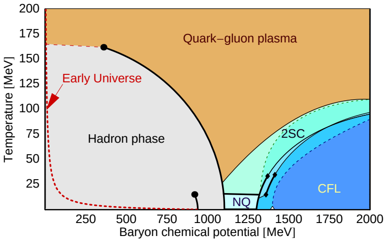
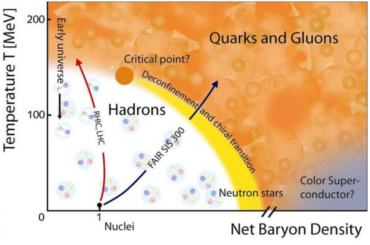
- but also deconfinement and chiral symmetry restoration at finite temperature and/or density

  $\Rightarrow$ new phases of matter





QCD and its phase diagram

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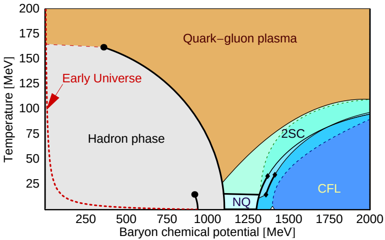
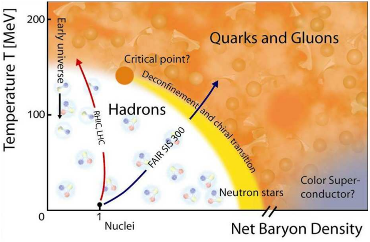


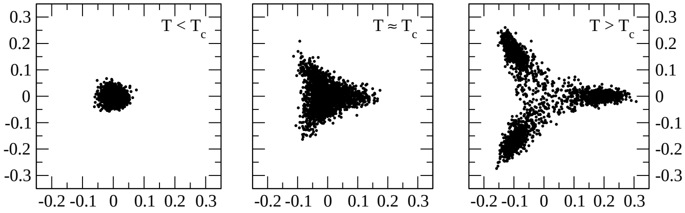


here finite temperature: $x_0 \in S^1_\beta$ . . . Eucl. and compact, $\beta \equiv 1/T$

- both effects related? Dual quantities
- generic? Random Matrix Theory

- Polyakov loop: $\mathcal{P}(\vec{x}) = \mathcal{P}\exp\left(i\int_0^\beta dx_0 A_0(x_0,\vec{x})\right) \in SU(3)$

  $\langle \mathrm{tr}\mathcal{P}\rangle_{\vec{x}}$ in complex plane [one point per configuration]



  order parameter like magnetization, but inverse behavior

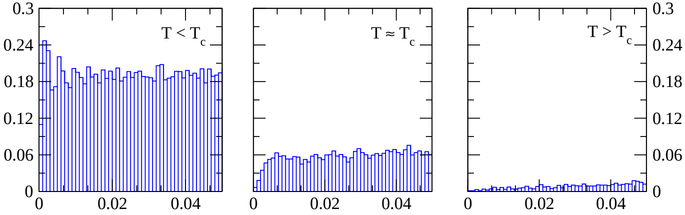
  free energy of infinitely heavy quarks

$$\langle \mathrm{tr}\,\mathcal{P}\rangle \sim e^{-\beta F_{\mathrm{quark}}} = \begin{cases} e^{-\infty} = 0 & T < T_c \\ e^{-\#} \neq 0 & T > T_c \end{cases}$$

  breaks center symmetry

Theory challenge: Chiral symmetry restoration

- spectral density $\rho(\lambda)$ of the Dirac operator:



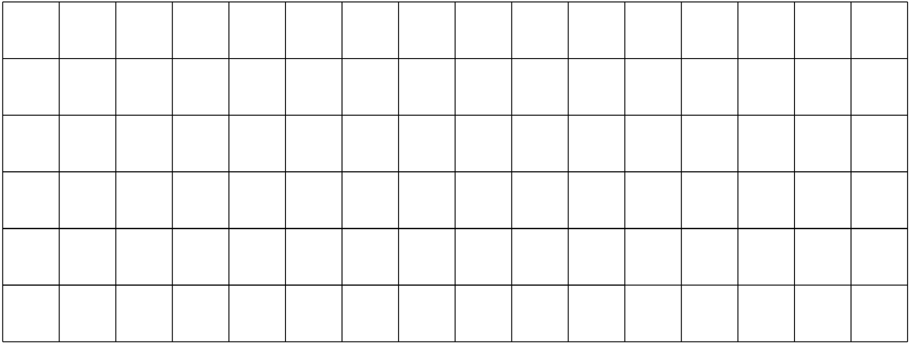
order parameter of chiral symmetry: $\rho(0) \sim \langle\bar{\psi}\psi\rangle$ Banks-Casher

i.e. for massless quarks [mass breaks chiral symmetry explicitly]

Confinement and chiral symmetry related? Dual quantities

Dual quantities: idea and definition

lattice: gauge invariant quantities $\rightleftharpoons$ link products along closed loops



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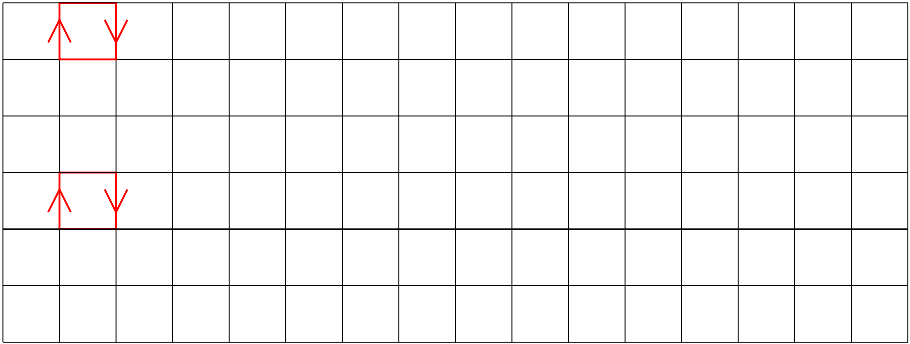

plaquettes

($\rightarrow$ action)

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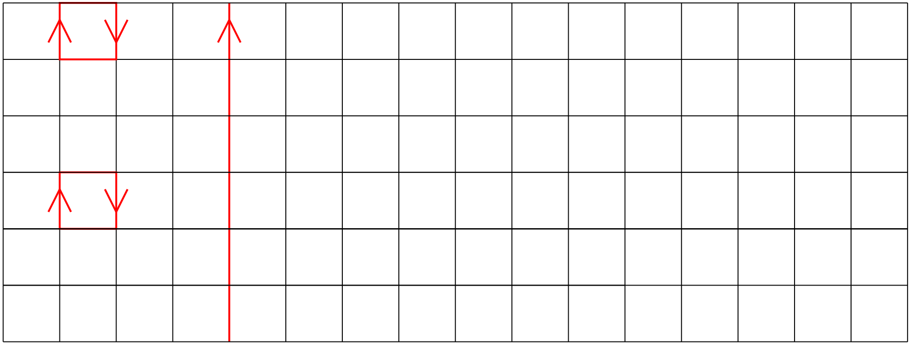

plaquettes (→ action)

Polyakov loop $U_0(0,\vec{x})U_0(a,\vec{x})\ldots$

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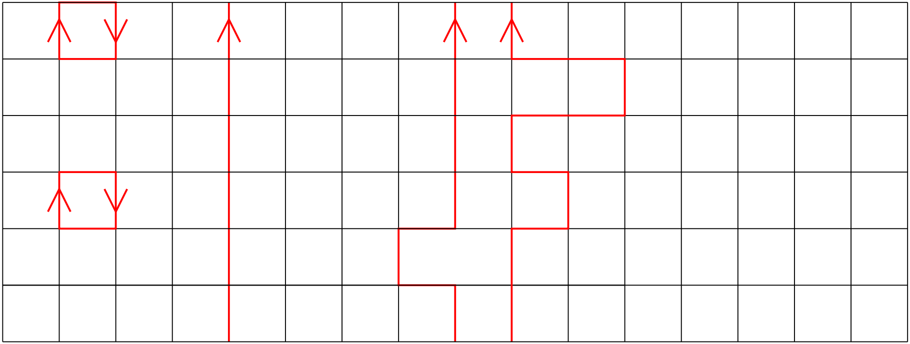

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Polyakov loop $U_0(0,\vec{x})U_0(a,\vec{x})\ldots$

"Polyakov loops with detours"

Dual quantities: idea and definition

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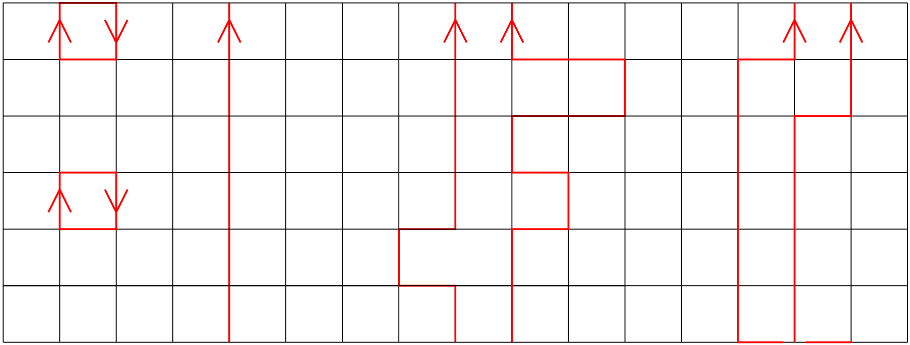

plaquettes ($\rightarrow$ action)

Polyakov loop $U_0(0,\vec{x})U_0(a,\vec{x})\ldots$

"Polyakov loops with detours"

loops winding twice

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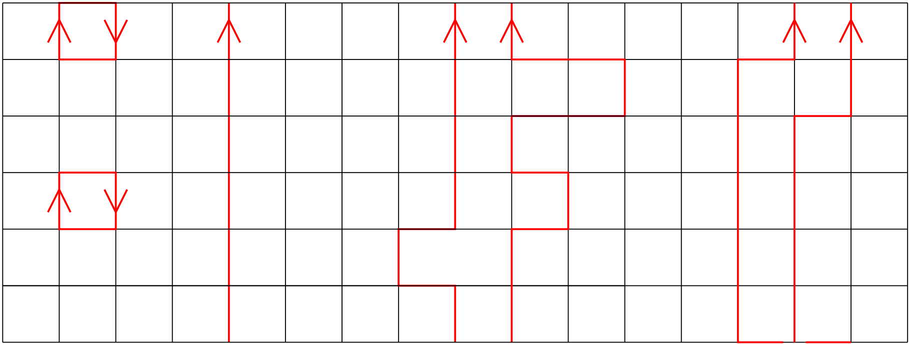

plaquettes	Polyakov loop	"Polyakov loops	loops
($\rightarrow$ action)	$U_0(0,\vec{x})U_0(a,\vec{x})\ldots$	with detours"	winding twice

how to distinguish these classes of loops?

Dual quantities: idea and definition

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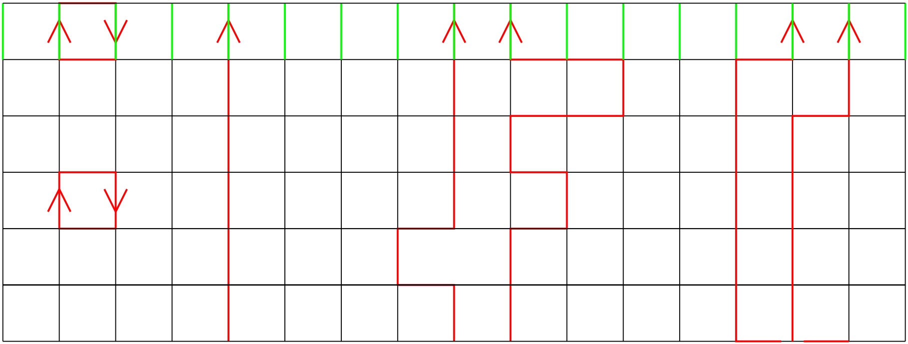


plaquettes ($\to$ action) | Polyakov loop $U_0(0,\vec{x})U_0(a,\vec{x})\ldots$ | "Polyakov loops with detours" | loops winding twice

how to distinguish these classes of loops?

$\Rightarrow$ phase factor $e^{i\varphi}$ multiplying U_0 at fixed x_0-slice & Fourier component

$\simeq$ quarks with general boundary conditions — Gattringer '06

$$\psi(x_0+\beta)=e^{i\varphi}\psi(x_0)$$

physical quarks are antiperiodic: $\varphi=\pi$

- general quark propagator: — cf. Synatschke, Wipf, Wozar, '07

$$\frac{1}{\gamma_\mu D^\mu_\varphi+m}$$

- (physical) chiral condensate:

$$\rho(0)=\langle\bar\psi\psi\rangle=\lim_{m\to0}\lim_{V\to\infty}\frac{1}{V}\Big\langle \mathrm{tr}\frac{1}{\gamma_\mu D^\mu_{\varphi=\pi}+m}\Big\rangle\equiv\Sigma_{\varphi=\pi}$$

- dual condensate: — Bilgici, FB, Gattringer, Hagen '08

$$\tilde\Sigma_1\equiv\frac{1}{2\pi}\int_0^{2\pi}d\varphi\, e^{-i\varphi}\frac{1}{V}\Big\langle \mathrm{tr}\frac{1}{\gamma_\mu D^\mu_\varphi+m}\Big\rangle$$

Fourier component picks out all contributions that wind once

$\equiv$ dressed Polyakov loop: chiral symmetry connected to confinement

Dual quantities and center symmetry

- center: commutes with all group elements
  for $SU(3)$: $\{1, e^{2\pi i/3} \equiv z, e^{-2\pi i/3} \equiv z^*\} \cdot 1_3$
- center transformation: non-periodic gauge transformation, e.g.
  $U_0 \to zU_0$ in some time slice
- invariance: action invariant, Polyakov loop: $\mathrm{tr}\,\mathcal{P} \to z\,\mathrm{tr}\,\mathcal{P}$
- center symmetric = confined phase:

$$\mathrm{tr}\,\mathcal{P} = 0 \quad \text{at low } T$$

- center broken = deconfined phase:

$$\mathrm{tr}\,\mathcal{P} \approx \{1, z, z^*\} \neq 0 \quad \text{at high } T \quad \text{[transform into each other]}$$

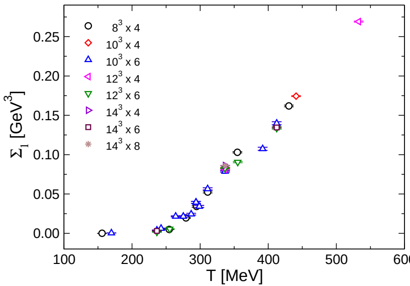
dual quantities like dual condensate $\tilde{\Sigma}_1$:

- same behaviour under center: $\tilde{\Sigma}_1 \to z\,\tilde{\Sigma}_1$ Synatschke, Wipf, Langfeld '08

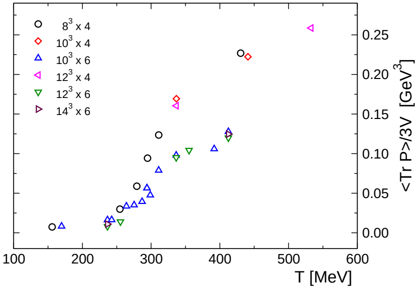
SU(3) quenched:

Bilgici, FB, Gattringer, Hagen '08

(bare) $\tilde{\Sigma}_1$ with $m = 100\text{MeV}$



(bare) Polyakov loop



less renormalisation ← detours = dressing

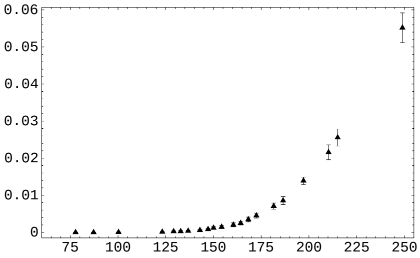
Dual condensate: order parameter II

$SU(3)$ with dynamical fermions: FB, Fodor, Gattringer, Szabo, Zhang preliminary

$N_f = 2+1$ staggered fermions at phys. masses Aoki et al. '06

$\Rightarrow$ crossover with $T_c^{\langle\bar{\psi}\psi\rangle} = 155(2)(3)$MeV and $T_c^{\mathcal{P}} = 170(4)(3)$MeV

(bare) $\tilde{\Sigma}_1$ with $m = 60$MeV



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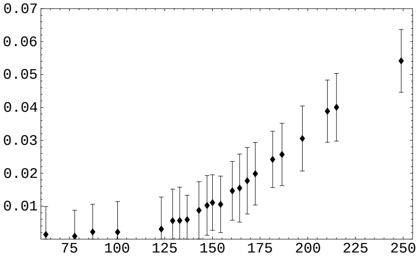
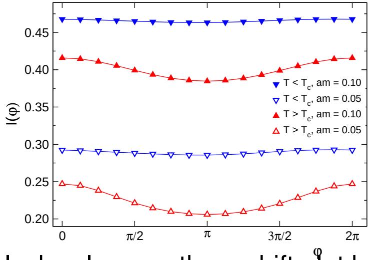


similar behaviour (center symmetry not an exact symmetry anymore)

$$\tilde{\Sigma}_1 = \frac{1}{2\pi}\int_0^{2\pi} d\varphi\, e^{-i\varphi} \cdot \quad \frac{1}{V}\left\langle \mathrm{tr}\frac{1}{\gamma_\mu D_\varphi^\mu + m}\right\rangle$$

Fourier integrand $\langle ... \rangle$ as a function of φ: Bilgici, FB, Gattringer, Hagen '08



[for real Polyakov loops, others shift plot by $2\pi/3$]

$\Rightarrow$ depends on φ only at high temperatures $\Rightarrow \tilde{\Sigma}_1 \neq 0$ ✓

in particular: chiral condensate survives at high T for periodic bc.s

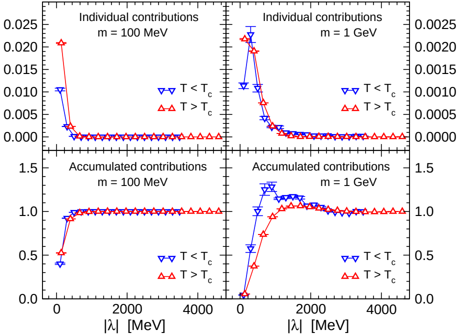
several lattice works

tr means sum over all eigenmodes:

$$\tilde{\Sigma}_1 \equiv \int_0^{2\pi} \frac{d\varphi}{2\pi}\, e^{-i\varphi} \frac{1}{V} \Big\langle \mathrm{tr} \frac{1}{\gamma_\mu D^\mu_\varphi + m} \Big\rangle = \int_0^{2\pi} \frac{d\phi}{2\pi}\, e^{-i\varphi} \frac{1}{V} \Big\langle \sum_k \frac{1}{i\lambda^k_\varphi + m} \Big\rangle$$

truncate the ev sum: IR dominance

Bilgici, FB, Gattringer, Hagen '08



expected: λ in denominator, lowest modes most sensitive to bc.s

Summary so far

the dual condensate $\tilde{\Sigma}_1$ is an order parameter under center symmetry

- $\tilde{\Sigma}_1 = 0$ at low T $\leftarrow$ similar to the Polyakov loop
  $\tilde{\Sigma}_1 > 0$ at low T
- limit of large mass: detours suppressed $\Rightarrow$ conventional (straight) Polyakov loop
- limit of small mass: Fourier component of chiral condensate wrt. fermionic boundary conditions

mechanism: lowest modes respond to boundary conditions at high T

[boundary angle $\simeq$ imag. chemical potential, but only at the level of observables, not for dynamical quarks]

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relax ... change subject! Bogoliubov

How generic are these features? Random Matrix Theory

Random matrix theory in a nutshell

$\equiv$ replace dynamics of a given physical system by random matrices ("0-dim. field theory") with the correct symmetry

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showcase: distribution of (neighbouring) level spacings $s = \Delta\lambda$

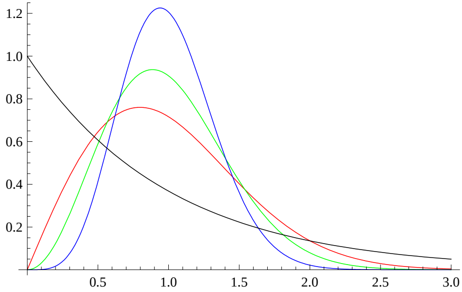
$$P(s) = \int dX \exp(-\mathrm{tr} XX^\dagger)\, \mathrm{prob.}(s)_X$$

where X is $N \times N$ and $\left\{ \begin{array}{l} \text{real} \\ \text{complex} \\ \text{quaternionic} \end{array} \right.$

$\equiv$ Gaussian $\left\{ \begin{array}{l} \text{Orthogonal} \\ \text{Unitary} \\ \text{Symplectic} \end{array} \right\}$ Ensemble $\leftarrow$ different anti-unitary symm.s

Dyson index $\beta_D = \left\{ \begin{array}{c} 1 \\ 2 \\ 4 \end{array} \right\} \sim$ number of real d.o.f.

$P(s)$ for large matrices, $\beta_D = 1, 2, 4$:



$\Rightarrow$ typical eigenvalue repulsion depending on ensemble

well described by 2×2 matrices: Wigner

$$P(s) \sim s^{\beta_D} e^{-\# s^2}$$

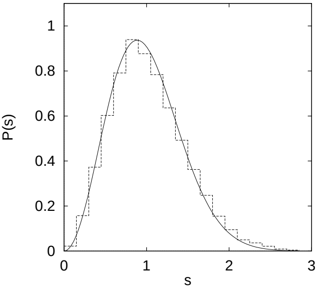
$\rightleftharpoons$ independent eigenvalues: $P(s) \sim e^{-s}$

Random matrix theory for QCD

random entries of the Dirac operator:

$$\text{ev.s}(X) \to \text{ev.s}\begin{pmatrix} m & iX \\ iX^\dagger & m \end{pmatrix}$$

mimics γ's in chiral representation: 'chiral ensembles', same $P(s)$



lattice vs. chGUE prediction ($\beta_D = 2$)

Pullirsch, Rabitsch, Wettig, Markum '98

universal 'bulk' property, exact in ϵ-regime ...

Random matrix theory for QCD at finite T

quarks are antiperiodic in $x_0 \in [0, \beta]$

$\Rightarrow$ Dirac eigenvalues shifted by Matsubara frequencies

$$\pi T \; + 2\pi n T$$

(exact in free case: waves with certain frequencies)

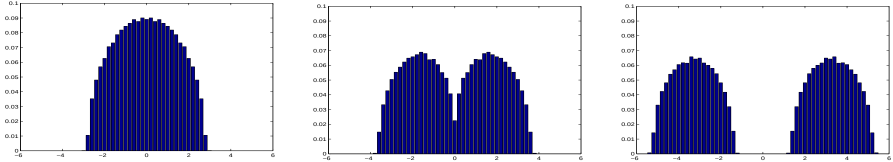
Random matrix model: Jackson, Verbaarschot '96

$$Z = \int dX_{N\times N} \exp(-NC^2 \mathrm{tr} XX^\dagger) \det \begin{pmatrix} m & iX + i\pi T \cdot \mathbb{1}_N \\ iX^\dagger + i\pi T \cdot \mathbb{1}_N & m \end{pmatrix}$$

- lowest Matsubara frequency as non-random trace part
- schematic (crit. exponents like mean field)
- model parameter: C

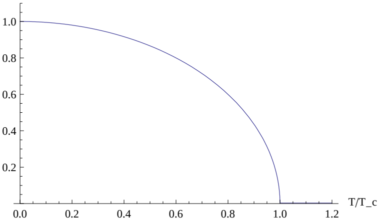
- numerical simulations: $\rho(\lambda)$ from 500 30×30 matrices

$T = 0$ $\qquad$ $T = 0.45/C$ $\qquad$ $T = 1/C$



semicircle of width $\sim 1/C$, shift $\pm T \Rightarrow \rho(0)$ vanishes for high T

- saddle point method:



$$\rho(0) = C\sqrt{1 - (\pi TC)^2}$$

vanishes above $T_c \equiv \frac{1}{\pi C}$

chiral phase transition, 2$^{\text{nd}}$ order

$\Rightarrow$ chiral condensate $\rho(0) \sim \langle\bar{\psi}\psi\rangle$ and its absence at high T “generic”

Random matrix theory for dual condensate

- general bc.s $\varphi \Rightarrow$ modified Matsubara frequencies FB in preparation

$$\omega_\varphi \equiv \min_n |(\varphi + 2\pi n)T| = \begin{cases} \varphi T & \varphi \in [0,\pi] \\ (2\pi - \varphi)T & \varphi \in [\pi, 2\pi] \end{cases}$$

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- saddle point similar to before:

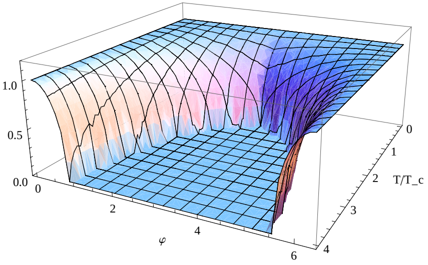
$$\rho(0)_\varphi = \Sigma_\varphi = C\sqrt{1 - (T/T_{c,\varphi})^2}$$

with

$$T_{c,\varphi} \equiv \begin{cases} \frac{1}{\varphi C} & \varphi \in [0, \pi] \\ \frac{1}{(2\pi - \varphi)C} & \varphi \in [\pi, 2\pi] \end{cases}$$

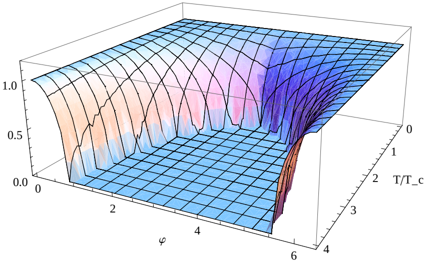
... hence survives up to higher critical temperature, $T_{c,0} = \infty$

- chiral condensate with general bc.s:



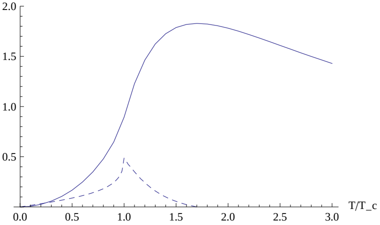
$\Sigma_\varphi(T)$ in units of C

- chiral condensate with general bc.s:



$\Sigma_\varphi(T)$ in units of C

- dual chiral condensate:



$\tilde{\Sigma}_1(T)$ plus its T-derivative

changes at the chiral phase transition

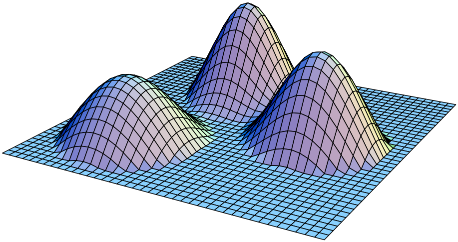
Summary

- the chiral condensate and the chiral phase transition at high T can easily be obtained in Random matrix theory: are 'generic' ✓
- the boundary condition can be incorporated in RMT by virtue of Matsubara frequencies
- the chiral condensate as a function of the boundary angle agrees qualitatively with results from lattice, functional methods and QCD models Fischer, Müller '09, Braun et al. '09, Kashiwa, Kouno, Yahiro '09, ...
- the dual condensate $\tilde{\Sigma}_1$ shows a phase transition at the chiral T_c [but no exact center symmetry ...]
- deconfinement transition 'generic' and near the chiral transition

Relevant excitations!?

calorons $\equiv$ class. solns. of Yang-Mills (instantons) at finite temperature

Harrington, Shepard '78; Kraan, van Baal; Lee, Lu '98



topological (action) density for total charge $Q = 1$ in $SU(3)$

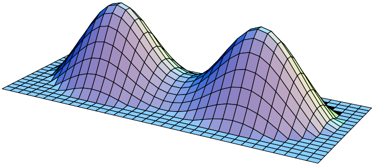
- substructure: N_c constituents = magn. monopoles/dyons
- masses governed by asymptotic Polyakov loop

$$P_\infty = \lim_{|\vec{x}| \to \infty} P(\vec{x}) \ldots \text{holonomy}$$

conjecture: holonomy tr $P_\infty \leftrightharpoons$ order parameter $\langle \operatorname{tr} P \rangle$

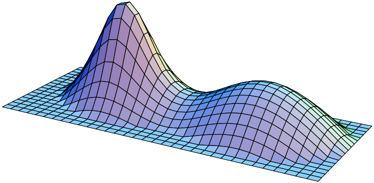
$\Rightarrow$ dyon masses sensitive to the phase of QCD dyon masses sensitive to the phase of QCD, in $SU(2)$ with 2 dyons:

confined phase



equal mass constituent dyons

deconfined phase



heavy + light constituent dyon

fermionic zero modes: $\psi_{\varphi\simeq 0}$ at light dyon, $\psi_{\varphi\simeq\pi}$ at heavy dyon
make up condensates in a caloron gas model

mechanism above T_c: heavy dyons suppressed FB '09

$\Rightarrow \langle\bar{\psi}\psi\rangle_{\varphi\simeq\pi}$ suppressed, $\langle\bar{\psi}\psi\rangle_{\varphi\simeq 0}$ stays ✓ Bornyakov et al. '09

$\Rightarrow$ top. susceptibility suppressed ✓