

Hyperons, Quarks and Compact Stars

Isaac Vidaña



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Outline of the lectures

Introduction, Motivation & Generalities

Hyperons in Neutron Stars

Relativistic Mean Field approach of hyperonic matter

Brueckner theory of hyperonic matter

EoS, composition and neutron star structure

Hyperon Stars at birth

Quark Matter in Neutron Stars

Generalities

The hadron matter to quark matter phase transition: Hybrid Stars

The Strange Quark Matter hypothesis: Strange Stars

Observational signals of deconfinement

Strange Quark Matter & Gamma ray bursts

Well known facts about Neutron Stars

■ Formed from the collapse remnant of a massive star after a Type II, Ib or Ic supernova.

■ Baryonic number: $N_b \sim 10^{57}$ (“giant nuclei”)

■ Mass: $M \sim 1-2 M_\odot$

$$M_{\text{PSR1913+16}} = (1.4411 \pm 0.0035) M_\odot$$

Recently: $M_{\text{PSR0751+1807}} = (2.1 \pm 0.2) M_\odot$ *Nice et al., ApJ 634, 1242 (2005)*

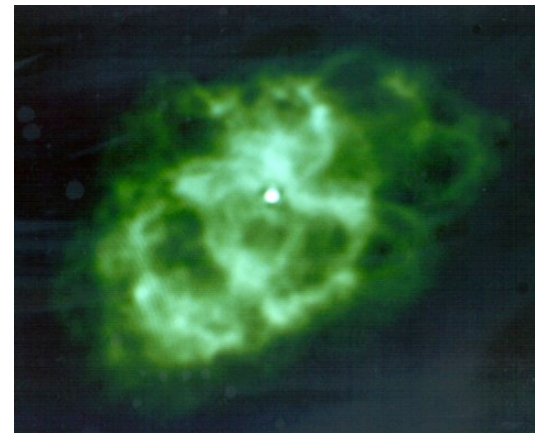
■ Radius: $R \sim 10-12$ km

■ Density: $\rho \sim 10^{15}$ g/cm³

$$\rho_{\text{universe}} \sim 10^{-30} \text{ g/cm}^3$$

$$\rho_{\text{sun}} \sim 1.4 \text{ g/cm}^3$$

$$\rho_{\text{earth}} \sim 5.5 \text{ g/cm}^3$$



■ Magnetic field: $B \sim 10^8 \dots 10^{16}$ G

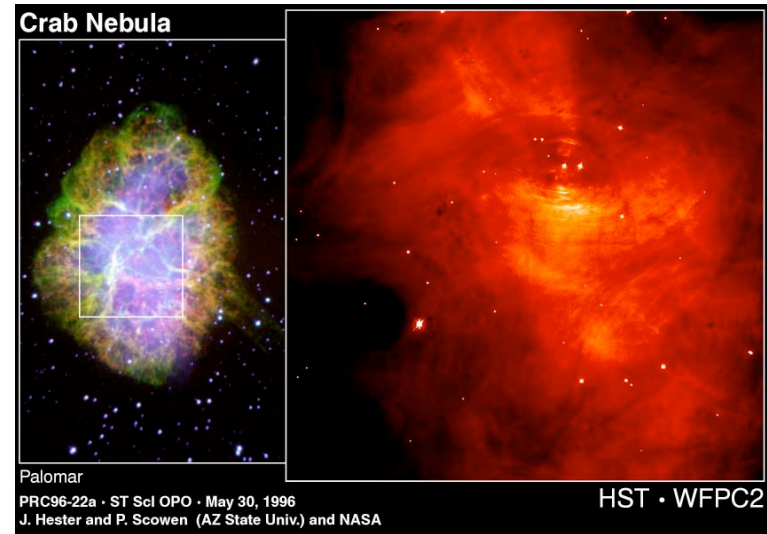
■ Electric field: $E \sim 10^{18}$ V/cm

■ Temperature: $T \sim 10^6 \dots 10^{11}$ K

■ Shortest rotational period: $P_{\text{B1937+2}} = 1.58$ ms

Latest discovery: PSR in Terzan 5: $P_{\text{J1748-244ad}} = 1.39$ ms

■ Accretion rates: 10^{-10} to 10^{-8} M_{\odot} /year



Let's play a little bit

Let's assume that a neutron star is a **sphere of uniform density $\sim 10^{14}$ g/cm³ completely made of neutrons**. We can get a first simple estimation of its radius and the number of neutrons in it. The idea is to generalize the **Weizsäcker mass formula**, which parametrizes the binding energy of nuclei, to include the gravitational energy. For a our **sphere containing N neutrons** we can write:

$$B = a_v N - a_s N^{2/3} - a_{sym} N + \frac{3}{5} \frac{G(NM_n)^2}{R}$$

we can neglect the surface term since $N \gg N^{2/3}$. We can express the radius R of the neutron star as a function of the number of neutrons N

$$R = r_0 N^{1/3} = 1.2 \times 10^{-15} N^{1/3} \text{ m}$$

Then, we can write

$$B \sim (a_v - a_{sym})N + \frac{3}{5} \frac{GN^{5/3}}{r_0} M_n^2$$

For small N , the first term dominates and the binding energy is negative ($a_v \sim 16$ MeV, $a_{sym} \sim 23$ MeV). In that case we expect no nuclei exclusively composed of neutrons. However, when N increases, the second term becomes important and at some given value $N=N_c$, the total binding energy becomes zero. This value defines the minimum number of neutrons in order to obtain a bound system and from it, we can estimate the minimum size of a neutron star:

$$0 = (a_v - a_{sym})N_c + \frac{3}{5} \frac{GN_c^{5/3}}{r_0} M_n^2 \Rightarrow N_c = \left(\frac{5 (a_v - a_{sym}) r_0}{3 GM_n^2} \right)^{3/2}$$

which yields $N_c \sim 5 \times 10^{55}$ and $R \sim 5$ km

Let's now consider that the neutrons form a giant **free Fermi sea**, enclosed by gravity in a uniform sphere of radius R . The **Fermi momentum** is given by

$$k_F = \left(3\pi^2 n\right)^{1/3} = \left(\frac{9\pi}{4}\right)^{1/3} \frac{N^{1/3}}{R}; \quad n = \frac{N}{V}$$

and the **total energy** is given by the average kinetic energy of the **free Fermi sea** plus the **gravitational energy**

$$E = N \frac{3}{5} \frac{k_F^2}{2M_n} - \frac{3}{5} \frac{G(NM_n)^2}{R} = \frac{3}{10M_n} \left(\frac{9\pi}{4}\right)^{2/3} \frac{N^{5/3}}{R^2} - \frac{3}{5} \frac{G(NM_n)^2}{R}$$

Now, for a given number of neutrons, we can ask for the size of the sphere which defines the equilibrium configuration, that is:

$$\frac{dE}{dR} = 0$$

which yields:

$$R = \frac{1}{GM_n^3} \left(\frac{9\pi}{4} \right)^{2/3} N^{-1/3}$$

In this case, it is the **degeneracy pressure of the neutrons**, direct consequence of the **Pauli principle**, which sustains the star against gravitational collapse.

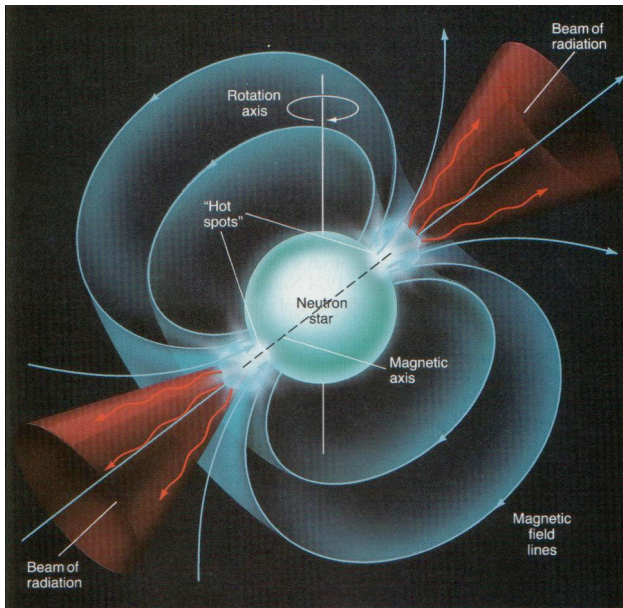
If we take a neutron star with a mass $M = 1.5 M_\odot$, the number of neutrons can be roughly estimated as

$$N \sim \frac{M}{M_n} \sim 1.8 \times 10^{57}$$

Consequently

$$R \sim 10.5 \text{ km} \quad \text{and} \quad n \sim 4.3 \times 10^{14} \text{ g/cm}^3$$

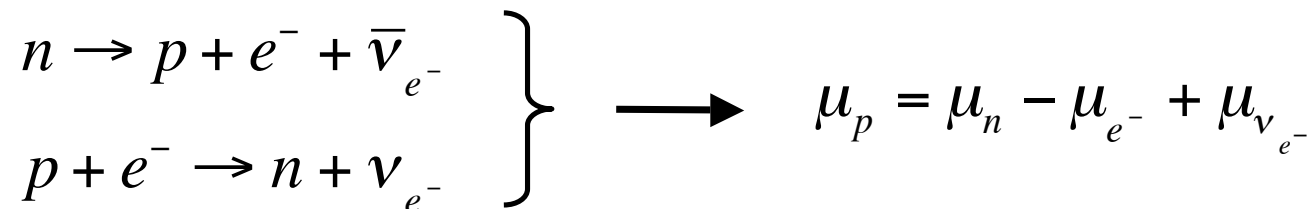
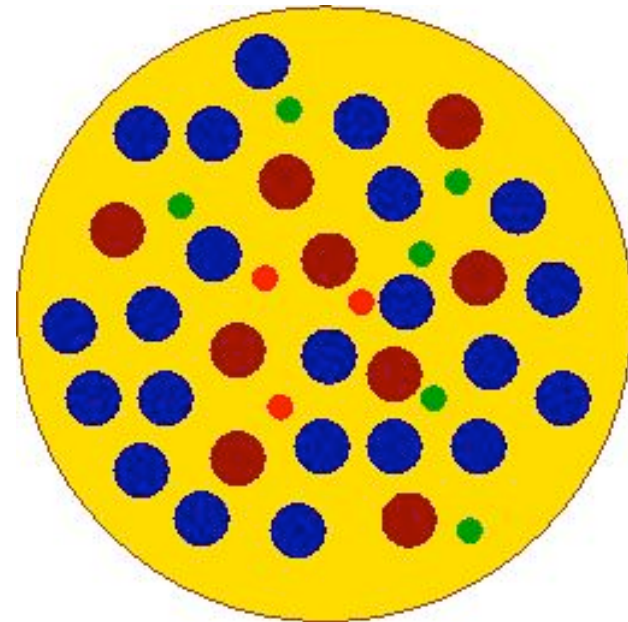
One of the most fascinating enigmas in modern astrophysics concerns the true nature of the ultradense compact objects called **Neutron Stars**



Let's have a look into the Neutron Star interior



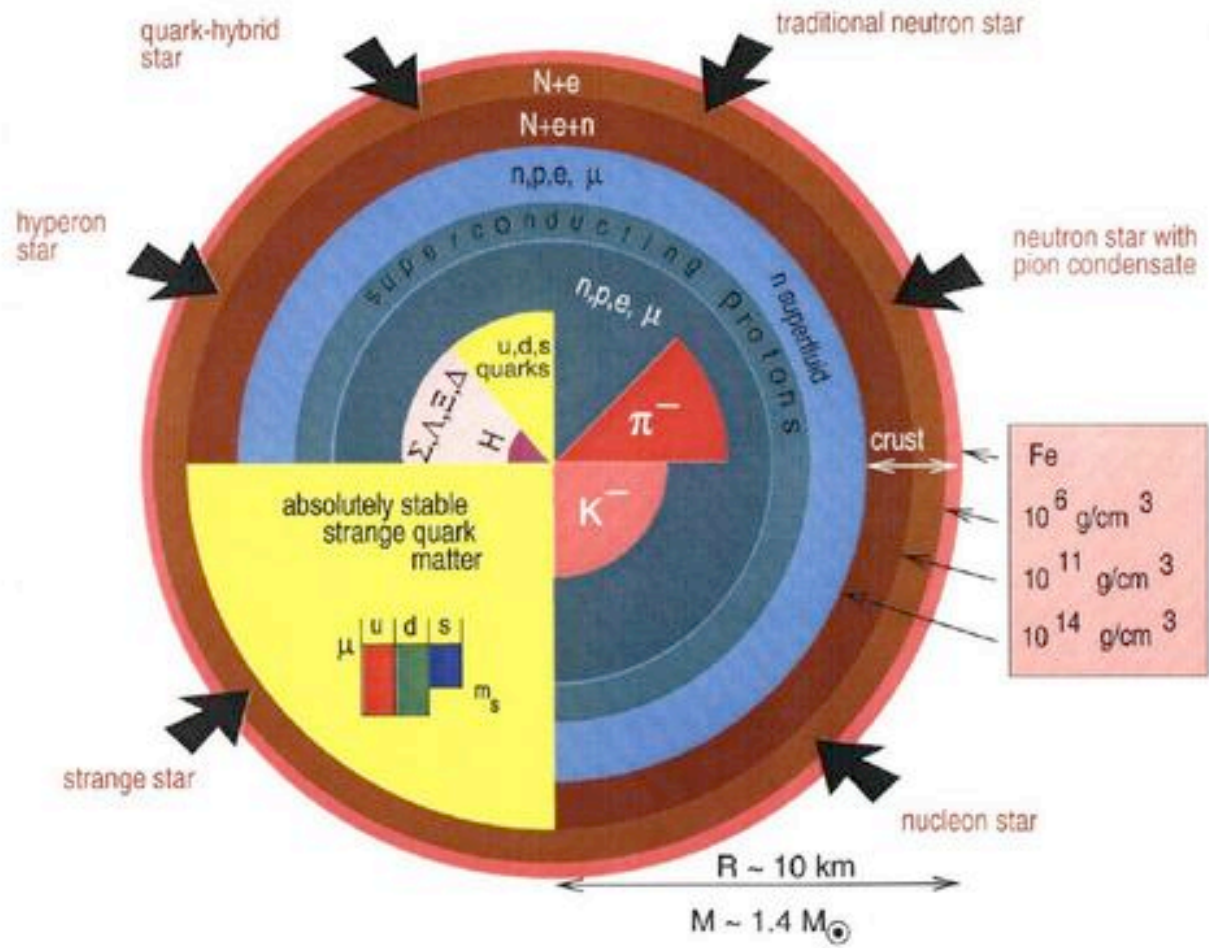
In a traditional and conservative picture the **internal composition of a Neutron Star** has been modelled by a uniform fluid of neutron rich nuclear matter in equilibrium with respect to weak interactions



But because of

- The value of the central density is high: $\rho_c \sim (4-8)\rho_0$
($\rho_0 = 0.17 \text{ fm}^{-3} = 2.8 \times 10^{14} \text{ g/cm}^3$)
- The rapid increase of the nucleon chemical potential with density

More exotic degrees of freedom are expected
in the Neutron Star interior

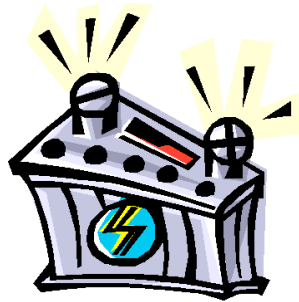


(Taken from F. Weber)

β -stable Neutron Star Matter

The **equilibrium composition** of the neutron star material is determined by the requirement of:

Charge neutrality



Equilibrium with respect to weak interacting processes



$$b_1 \rightarrow b_2 + l + \bar{\nu}_l$$

$$b_2 + l \rightarrow b_1 + \nu_l$$

To derive such **equilibrium conditions** and the **equilibrium composition** of matter in a neutron star one should:

👉 write all possible processes conserving baryon number and electric charge. For instance in our case, *i.e.*, *hyperonic matter*.



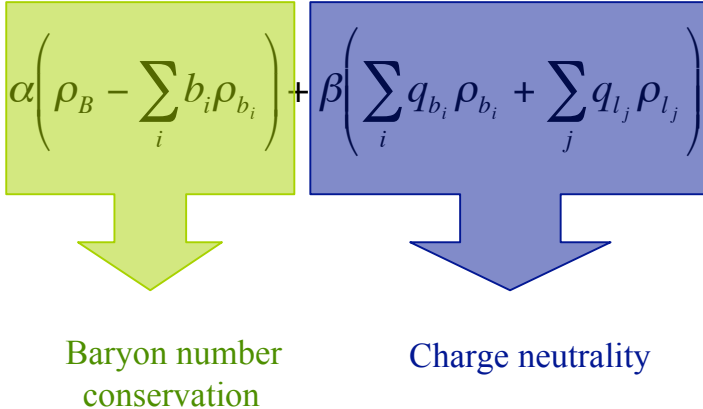
...

👉 write the relation between the chemical potentials associated to each reaction, and, finally, identify the independent ones and write the others in terms of them

$$\mu_{\Sigma^-} = \mu_n + \mu_{e^-} - \mu_{\nu_{e^-}}$$

$$\mu_{\Lambda} = \mu_n$$

Another equivalent way to derive the **equilibrium conditions** and the **equilibrium composition** is obtained through the **minimization of the energy density under the constraints of baryon number conservation and charge neutrality**

$$F(\rho_{b_1}, \dots, \rho_{b_B}; \rho_{l_1}, \dots, \rho_{l_L}) = \varepsilon(\rho_{b_1}, \dots, \rho_{b_B}; \rho_{l_1}, \dots, \rho_{l_L}) + \alpha \left(\rho_B - \sum_i b_i \rho_{b_i} \right) + \beta \left(\sum_i q_{b_i} \rho_{b_i} + \sum_j q_{l_j} \rho_{l_j} \right)$$


Baryon number conservation Charge neutrality

The minimization condition requires

$$\frac{\partial F}{\partial \rho_{b_i}} = 0, \dots, \frac{\partial F}{\partial \rho_{b_B}} = 0, \frac{\partial F}{\partial \rho_{l_i}} = 0, \dots, \frac{\partial F}{\partial \rho_{l_L}} = 0, \frac{\partial F}{\partial \alpha} = 0, \frac{\partial F}{\partial \beta} = 0,$$

Remembering that

$$\mu_i = \frac{\partial \mathcal{E}}{\partial \rho_i}$$

the previous conditions on F yield a set of equations of the type

$$\mu_{b_i} - b_i \alpha + q_{b_i} \beta = 0, \quad i = 1, \dots, B$$

for the baryons and

$$\mu_{l_j} + q_{l_j} \beta = 0, \quad j = 1, \dots, L$$

for the leptons

In general, there are as many independent chemical potentials as conserved charges, and all the others can be written in terms of them.

In the case of neutron stars there are only two conserved charges, and their corresponding chemical potentials are:

• μ_n , associated with baryon number conservation

• μ_e , associated with charge neutrality

Eliminating the Lagrange multipliers α and β , one can obtain all the set of relations among the chemical potentials:

$$\mu_i = b_i \mu_n - q_i \mu_e$$

β -stable nuclear matter

☛ (n, p, e⁻) system in neutral β -equilibrium at T=0

To begin with we assume that the constituents of the neutron star are neutrons, protons and electrons. The **equilibrium conditions** for the weak reactions



reduce to

$$\mu_p = \mu_n - \mu_e$$

and charge neutrality requires $\rho_p = \rho_e$. Therefore one can write

$$3\pi^2 \rho_B x_p = \left(\mu_n(\rho_B, x_p) - \mu_p(\rho_B, x_p) \right)^3$$

which **defines implicitly the proton fraction at equilibrium.**

- *Exercise:* Show that for a system of non interacting (n,p,e⁻), the proton fraction at equilibrium is

$$x_p^{eq} = \left[\left(\frac{m}{m_e} + 1 \right)^{3/2} + 1 \right]^{-1} \approx \left(\frac{m_e}{m} \right)^{3/2}$$

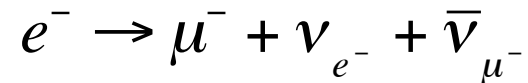
in the case of non-relativistic particles (assuming $m_p = m_n = m$); and

$$x_p^{eq} = \frac{1}{9}$$

in the case all particle species are ultra-relativistic. Notice that in these two limiting situations the result is density independent.

🍌 (n, p, e⁻, μ⁻) system in neutral β-equilibrium at T=0

As soon as $\mu_e > m_\mu c^2 \sim 105.6 \text{ MeV}$ it is energetically favorable for the electrons to convert to muons via the weak process



The following processes are also allowed



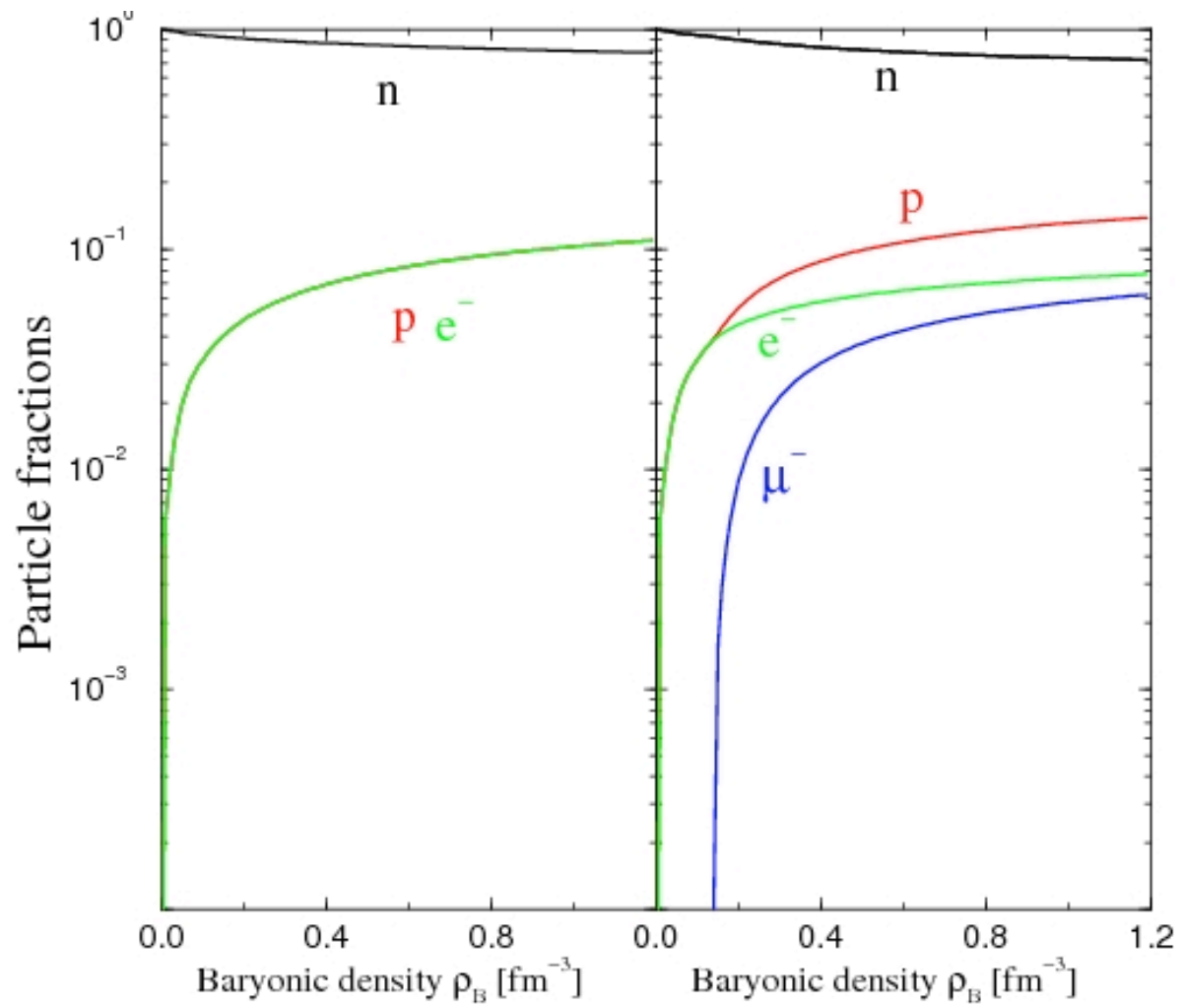
Now, the **equilibrium conditions** read

$$\mu_p = \mu_n - \mu_e; \quad \mu_\mu = \mu_e$$

which together with charge neutrality, $\rho_p = \rho_e + \rho_\mu$, give

$$3\pi^2 \rho_B x_p = \left(\mu_n(\rho_B, x_p) - \mu_p(\rho_B, x_p) \right)^3 + \left[\left(\mu_n(\rho_B, x_p) - \mu_p(\rho_B, x_p) \right)^2 - m_\mu^2 \right]^{3/2} \Theta(\mu_e - m_\mu)$$

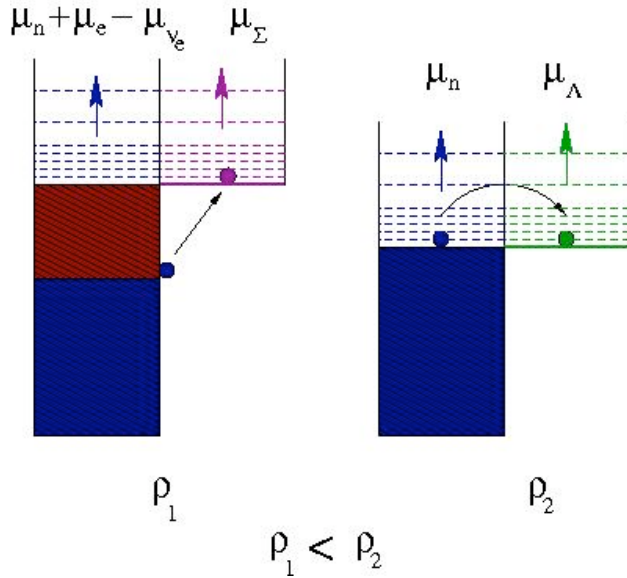
which can be solved numerically



Hyperonic degrees of freedom

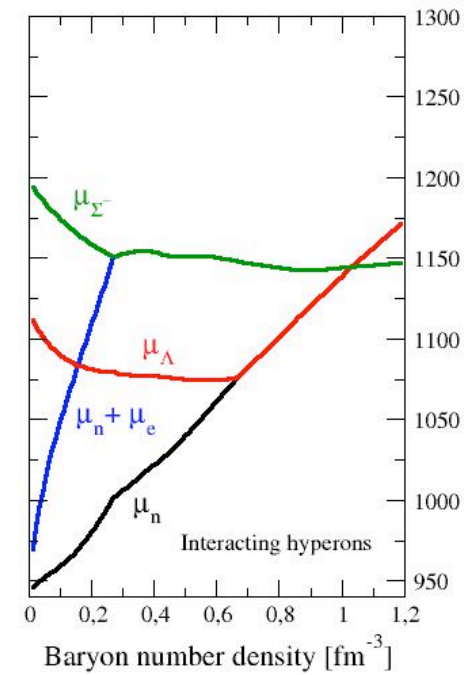
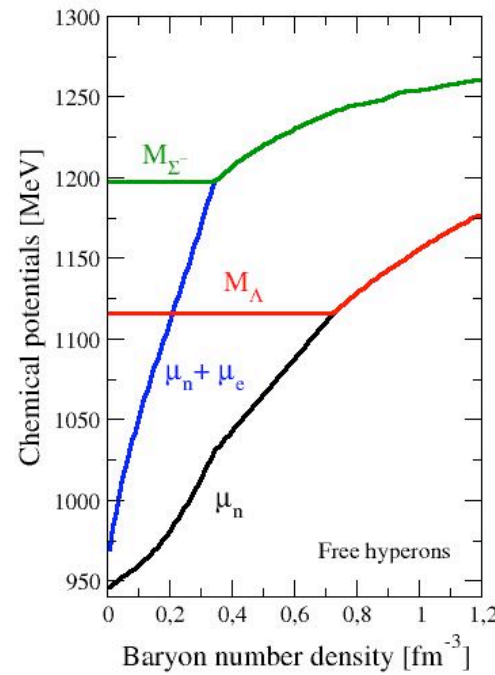
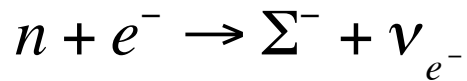
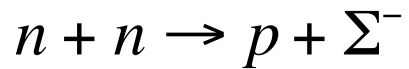
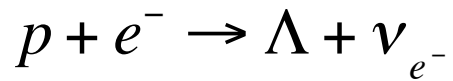
Hyperon	Quarks	I(J ^P)	Mass
Λ	uds	0(1/2 ⁺)	1115
Σ^+	uus	1(1/2 ⁺)	1189
Σ^0	uds	1(1/2 ⁺)	1193
Σ^-	dds	1(1/2 ⁺)	1197
Ξ^0	uss	1/2(1/2 ⁺)	1315
Ξ^-	dss	1/2(1/2 ⁺)	1321

Hyperons are expected to appear at $\rho \sim (2-3)\rho_0$



$$\mu_{\Sigma^-} = \mu_n + \mu_{e^-} - \mu_{\nu_{e^-}}$$

$$\mu_{\Lambda} = \mu_n$$



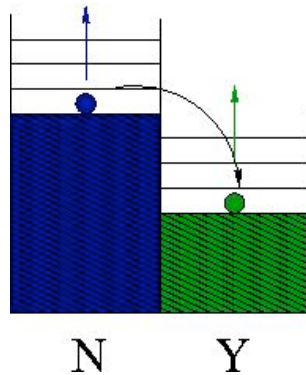
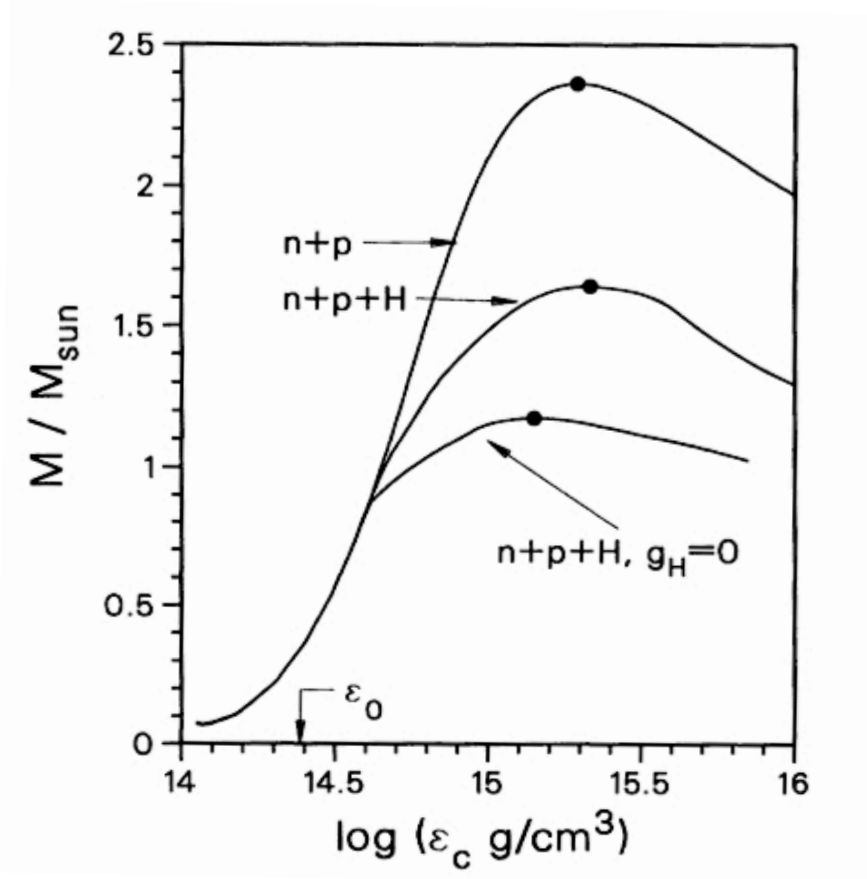
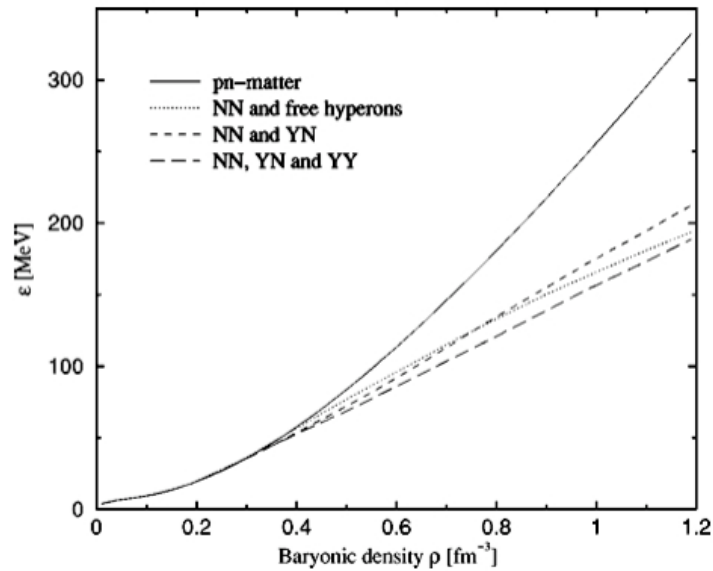
- *Exercise:* Consider a system of non interacting baryons (n, p, Λ and Σ^-) and leptons (e^-, μ^-). Determine the onset density of the two hyperons. Consider the electrons ultra-relativistic and take $m_n = m_p = m$ for simplicity

Hyperons in Neutron Stars

Since the pioneering work of Ambartsumyan & Saakyan (1960) ...

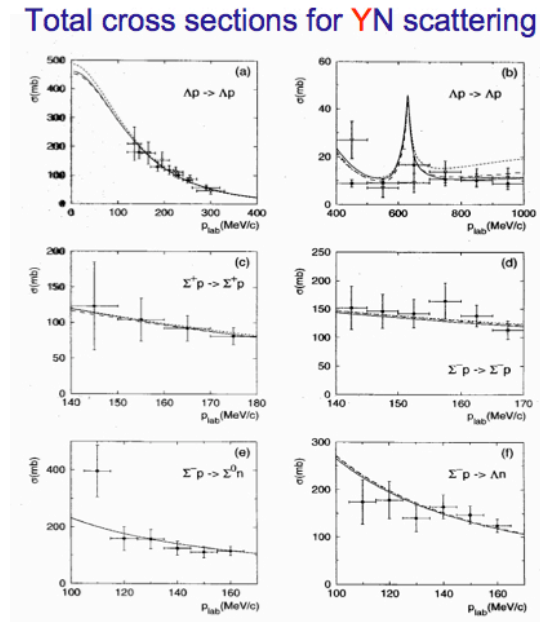
- ☛ Relativistic Mean Field Models: Glendenning, 1985; Knorren, Prakash & Ellis, 1995; Shaffner-Bielich & Mishustin, 1996
- ☛ Non-relativistic potential model: Balberg & Gal, 1997
- ☛ Quark-meson coupling model: Pal *et al.*, 1999
- ☛ Brueckner-Hartree-Fock theory: Baldo, Burgio & Schulze, 2000; Engvik, Hjorth-Jensen, Polls, Ramos & Vidaña, 2000
- ☛ Chiral Effective Lagrangians: Hanauske *et al.*, 2000
- ☛ Density dependent hadron field models: Hofmann, Keil & Lenske, 2001

Effect of Hyperons in the EoS and Mass of Neutron Stars



Hyperons make the EoS softer \rightarrow reduction of the mass

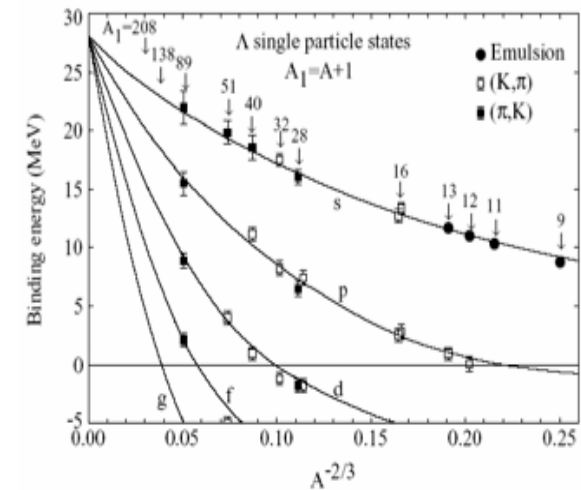
What do we know about YN and YY interactions ?



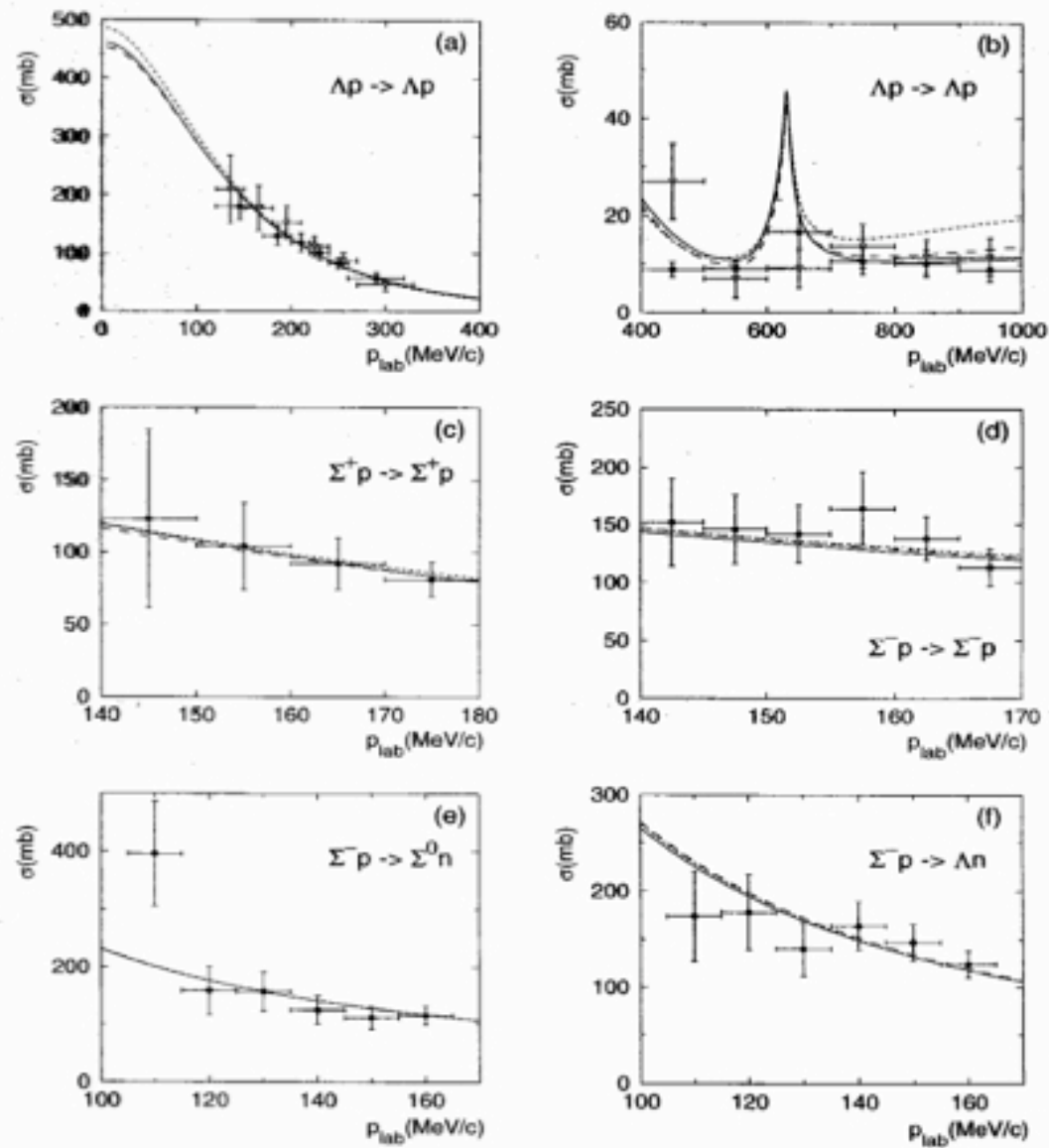
Scattering data

Binding energy of hypernuclei (mainly Λ -hypernuclei)

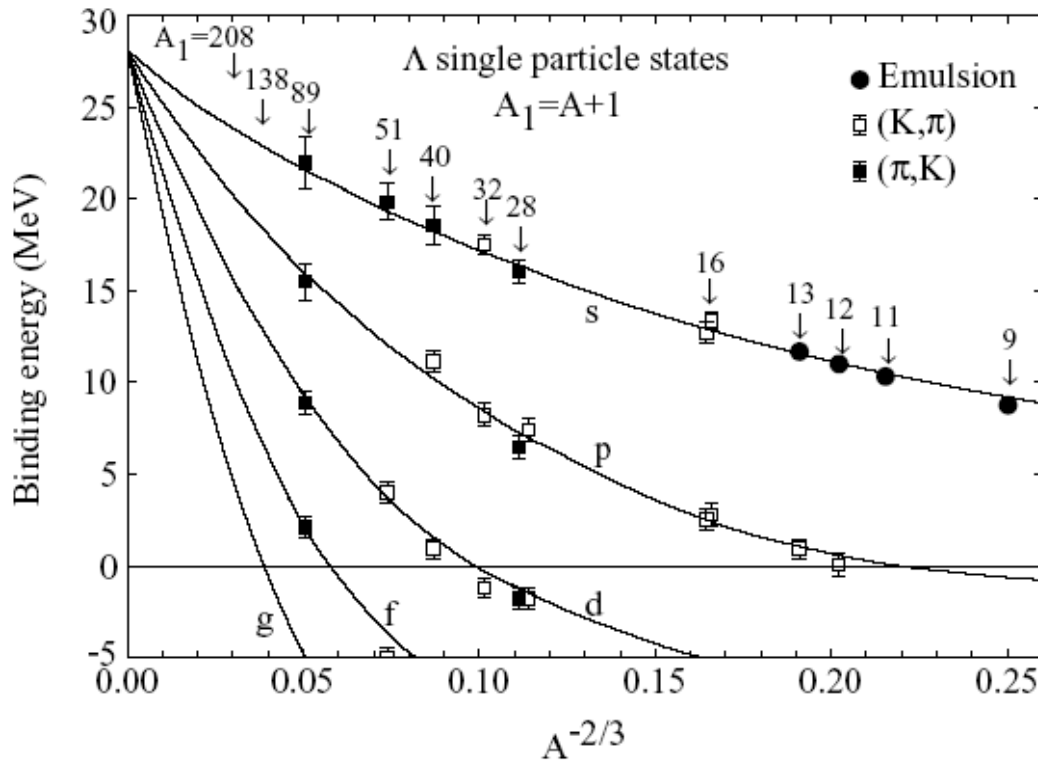
Binding energies of s.p. Λ states



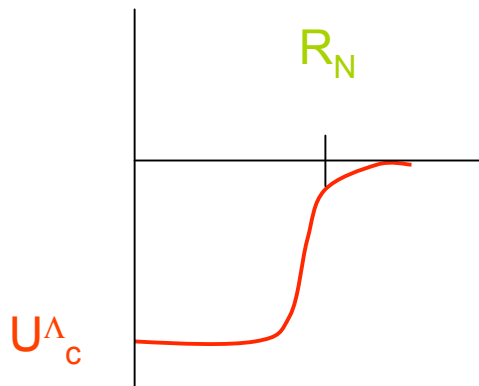
Total cross sections for ΥN scattering



Binding energies of s.p. Λ states



See e.g. D.J. Millener, C.B. Dover & A. Gal, PRC 38 (1988) 2700

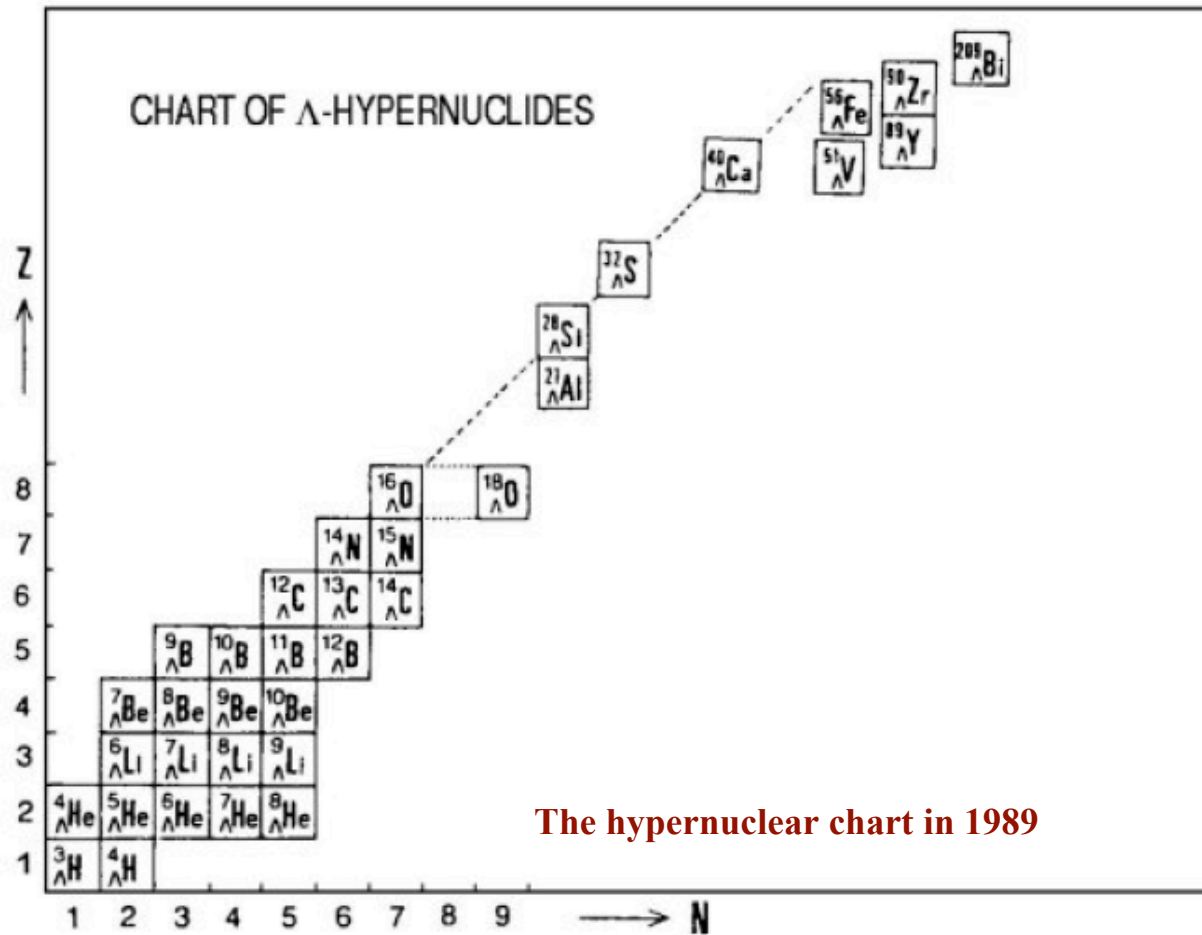


$$U(r) = U_c^\Lambda f(r) + U_{s.o.}^\Lambda \vec{l} \cdot \vec{s} \frac{1}{r} \frac{df(r)}{dr} \quad \left(f(r) = \frac{1}{1 + e^{\frac{r-R}{a}}} \right)$$

$$U_c^\Lambda = -28 \text{ MeV}$$

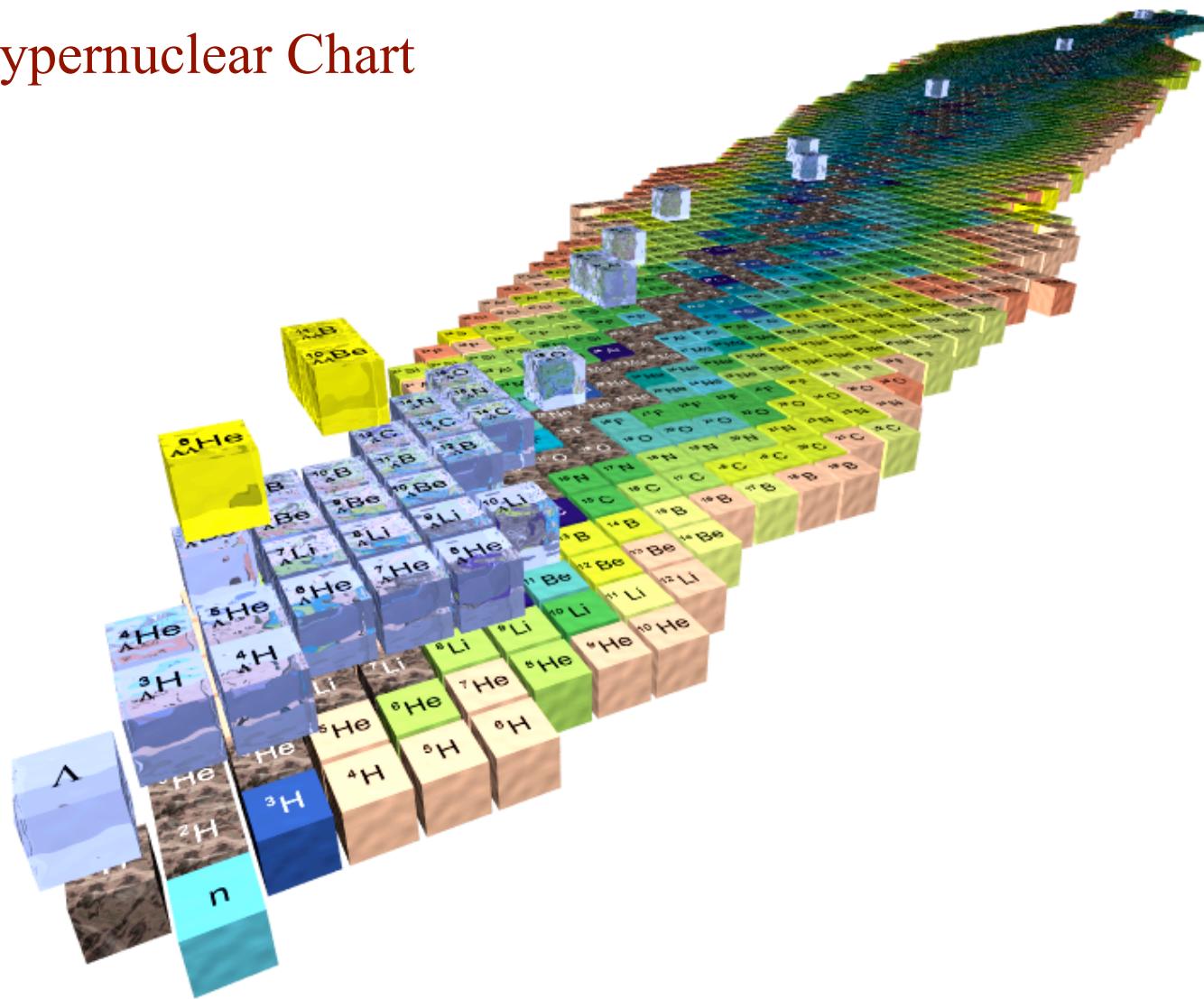
$$(U_c^N = -55 \text{ MeV})$$

Hypernuclei



And almost 20 years later: basically the same hypernuclei ... but measured with better statistics and energy resolution → excited hypernuclear states are now available!

Hypernuclear Chart



In summary ...

☛ $N\Lambda$: attractive \rightarrow Λ -hypernuclei for $A=3-209$; $U_\Lambda \sim -30$ MeV at ρ_0

☛ $N\Sigma$: ${}^4\text{He}_\Sigma$ hypernucleus bound by isospin forces
 Σ^- atoms: repulsive potential

☛ $N\Xi$: attractive \rightarrow 7 hypernuclear events; $U_\Xi \sim -28$ MeV at ρ_0
quasi-free production of Ξ : $U_\Xi \sim -18$ MeV

☛ $\Lambda\Lambda$: attractive \rightarrow 5 double Λ hypernuclear events
 $\Delta B_{\Lambda\Lambda}({}^A Z_{\Lambda\Lambda}) \sim 4-5$ MeV (old data)
 $\Delta B_{\Lambda\Lambda}({}^6\text{He}_{\Lambda\Lambda}) \sim 1$ MeV (NAGARA event)

Reproduced by most recent BB forces (NSC97) and consistent with SU(3) expectations

☛ YY : $Y=\Lambda,\Sigma,\Xi$ unknown !

Relativistic Mean Field approach of hyperonic matter

📌 Lagrangian density

$$\begin{aligned} L = & \sum_B \bar{\psi}_B \left(i\gamma_\mu \partial^\mu - m_B + g_{\sigma B} \sigma - g_{\omega B} \gamma_\mu \omega^\mu - \frac{1}{2} g_{\rho B} \gamma_\mu \vec{\tau} \cdot \vec{\rho}^\mu \right) \psi_B \\ & + \frac{1}{2} \left(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 \right) - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu \\ & - \frac{1}{4} \vec{\rho}_{\mu\nu} \cdot \vec{\rho}^{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu - \frac{1}{3} b m_N (g_{\sigma N} \sigma)^3 - \frac{1}{4} c (g_{\sigma N} \sigma)^4 \\ & + \sum_\lambda \bar{\psi}_\lambda \left(i\gamma_\mu \partial^\mu - m_\lambda \right) \psi_\lambda \end{aligned}$$

$$\omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu; \quad \vec{\rho}_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu$$

$$B = n, p, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0; \quad \lambda = e^-, \mu^-$$

Euler-Lagrange Equations

Baryon field equations

$$\left[\gamma_\mu \left(i \partial^\mu - g_{\omega B} \omega^\mu - \frac{1}{2} g_{\rho B} \vec{\tau} \cdot \vec{\rho}^\mu \right) - (m_B - g_{\sigma B} \sigma) \right] \psi_B = 0$$

Meson field equations

$$(\partial_\nu \partial^\nu + m_\sigma^2) \sigma = \sum_B g_{\sigma B} \bar{\psi}_B \psi_B$$

$$(\partial_\nu \partial^\nu + m_\omega^2) \omega_\mu - \partial_\mu \partial^\nu \omega_\nu = \sum_B g_{\omega B} \bar{\psi}_B \gamma_\mu \psi_B$$

$$(\partial_\nu \partial^\nu + m_\rho^2) \rho_\mu^i - \partial_\mu \partial^\nu \rho_\nu^i = \sum_B g_{\rho B} \bar{\psi}_B \gamma_\mu \tau^i \psi_B$$

Mean Field Approximation

The **Euler-Lagrange equations** derived from the previous Lagrangian density are solved by replacing the meson fields by their mean values in uniform matter, and the **baryon currents** are replaced by the ground state expectations generated in the presence of the mean meson fields.

Baryon mean field equations

$$\left[\gamma_\mu \left(k^\mu - g_{\omega B} \omega^\mu - \frac{1}{2} g_{\rho B} \vec{\tau} \cdot \vec{\rho}^\mu \right) - (m_B - g_{\sigma B} \sigma) \right] \psi_B = 0$$

The eigenvalues of particle and antiparticle can be found as

$$e_B(k) = g_{\omega B} \omega_0 + g_{\rho B} \rho_{03} I_{3B} + \sqrt{k^2 + (m_B - g_{\sigma B} \sigma)^2}$$

$$\bar{e}_B(k) = -g_{\omega B} \omega_0 - g_{\rho B} \rho_{03} \bar{I}_{3B} + \sqrt{k^2 + (m_B - g_{\sigma B} \sigma)^2}$$

The **onset density** of each baryon specie is determined by the condition

$$\mu_B = \mu_N - q_B \mu_e \geq g_{\omega B} \omega_0 + g_{\rho B} \rho_{03} I_{3B} + m_B - g_{\sigma B} \sigma$$

Meson mean field equations

$$\omega_0 = \sum_B \frac{g_{\omega B}}{m_\omega^2} \frac{(2J_B + 1)}{6\pi^2} b_B k_{F_B}^3; \quad \omega_k = 0$$

$$\rho_{03} = \sum_B \frac{g_{\rho B}}{m_\rho^2} I_{3B} \frac{(2J_B + 1)}{6\pi^2} b_B k_{F_B}^3; \quad \rho_{k3} = 0$$

$$m_\sigma^2 \sigma = -b m_N g_{\sigma N} (g_{\sigma N} \sigma)^2 - c g_{\sigma N} (g_{\sigma N} \sigma)^3 \\ + \sum_B \frac{(2J_B + 1)}{2\pi^2} g_{\sigma B} \int_0^{k_F} \frac{m_B - g_{\sigma B} \sigma}{\sqrt{k^2 + (m_B - g_{\sigma B} \sigma)^2}} k^2 dk$$

Equation of State

The **energy density** and **pressure** are obtained from the energy-momentum tensor

$$T^{\mu\nu} = \frac{\partial L}{\partial(\partial_\mu \phi_i)} \partial^\nu \phi_i - \eta^{\mu\nu} L$$

whose expectation value in the rest mass frame is diagonal

$$T^{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

From the above expressions we find

$$\varepsilon = -\langle L \rangle + \langle \bar{\psi} \gamma_0 k^0 \psi \rangle$$

$$p = \langle L \rangle + \frac{1}{3} \langle \bar{\psi} \gamma_i k^i \psi \rangle$$

Using the **Lagrangian density** of the present theory, we have

$$\begin{aligned} \varepsilon = & \frac{1}{3} b m_N (g_{\sigma N} \sigma)^3 + \frac{1}{4} c (g_{\sigma N} \sigma)^4 + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_{03}^2 \\ & + \sum_B \frac{2J_B + 1}{2\pi^2} \int_0^{k_{FB}} \sqrt{k^2 + (m_B + g_{\sigma B} \sigma)^2} k^2 dk + \sum_\lambda \frac{1}{\pi^2} \int_0^{k_{F\lambda}} \sqrt{k^2 + m_\lambda^2} k^2 dk \end{aligned}$$

$$\begin{aligned} p = & -\frac{1}{3} b m_N (g_{\sigma N} \sigma)^3 - \frac{1}{4} c (g_{\sigma N} \sigma)^4 - \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_{03}^2 \\ & + \frac{1}{3} \sum_B \frac{2J_B + 1}{2\pi^2} \int_0^{k_{FB}} \frac{k^4 dk}{\sqrt{k^2 + (m_B + g_{\sigma B} \sigma)^2}} + \frac{1}{3} \sum_\lambda \frac{1}{\pi^2} \int_0^{k_{F\lambda}} \frac{k^4 dk}{\sqrt{k^2 + m_\lambda^2}} \end{aligned}$$

In summary, the equation of state of β -stable hadronic matter inside a neutron star can be obtained by solving in a self-consistent way the following set of coupled non linear equations:

- 3 equations for the meson fields
- 1 equation impose by charge neutrality
- 1 equation impose by baryon number conservation
- N-2 relations among the chemical potentials of the N species considered

Coupling Constants

The nucleon coupling constants $g_{\sigma N}$, $g_{\omega N}$, $g_{\rho N}$, b and c are constrained by the empirical values of density ρ_0 , energy per particle E/A , compression modulus K , symmetry energy a_{sym} and effective mass m^* at nuclear saturation.

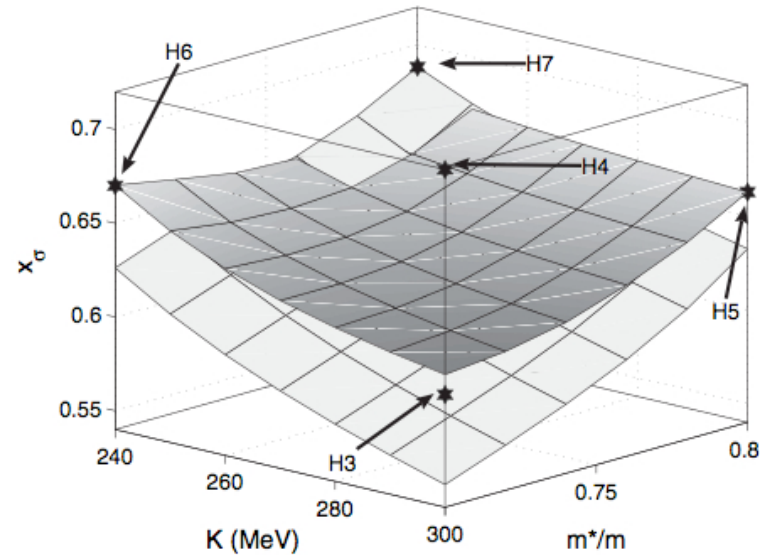
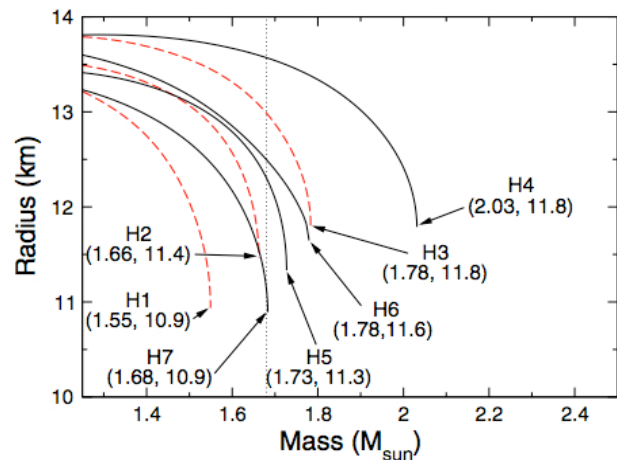
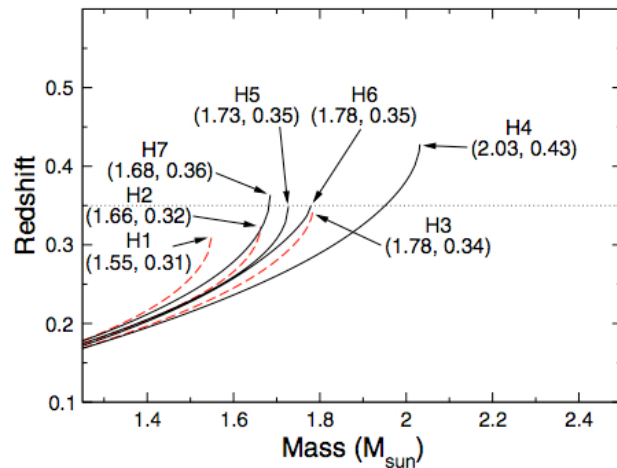
The hyperon coupling constants $g_{\sigma Y}$, $g_{\omega Y}$ and $g_{\rho Y}$ are constrained by: the binding of Λ hyperon in nuclear matter, hypernuclear levels and neutron star masses.

Assuming that all hyperons in the octet have the same coupling, the hyperon couplings are expressed as a ratio to the above mentioned nucleon couplings

$$x_{\sigma} = \frac{g_{\sigma Y}}{g_{\sigma N}}; \quad x_{\omega} = \frac{g_{\omega Y}}{g_{\omega N}}; \quad x_{\rho} = \frac{g_{\rho Y}}{g_{\rho N}};$$

Two astrophysical constraints to the hyperon couplings

- ☛ Red shift of EXO0748-676, $z \sim 0.35$
- ☛ Mass of Ter 5 I $M \sim 1.68 M_{\odot}$



Below dark: EoS compatible with red shift

Above light: EoS compatible with mass

Brueckner theory for hyperonic matter

☛ The Nuclear Many-Body Problem

Consider a system of A fermions described by

$$H = \sum_{i=1}^A K_i + \sum_{i<j}^A V_{ij} \quad \rightarrow \quad \text{Ground State:} \quad H|\psi\rangle = E|\psi\rangle$$

UNSOLVABLE !!

$$H = \underbrace{\sum_{i=1}^A (K_i + U_i)}_{H_0 \text{ unperturbed}} + \underbrace{\sum_{i<j}^A V_{ij} - \sum_{i=1}^A U_i}_{H_1 \text{ perturbation}}$$

$$E = E_0 + \Delta E$$

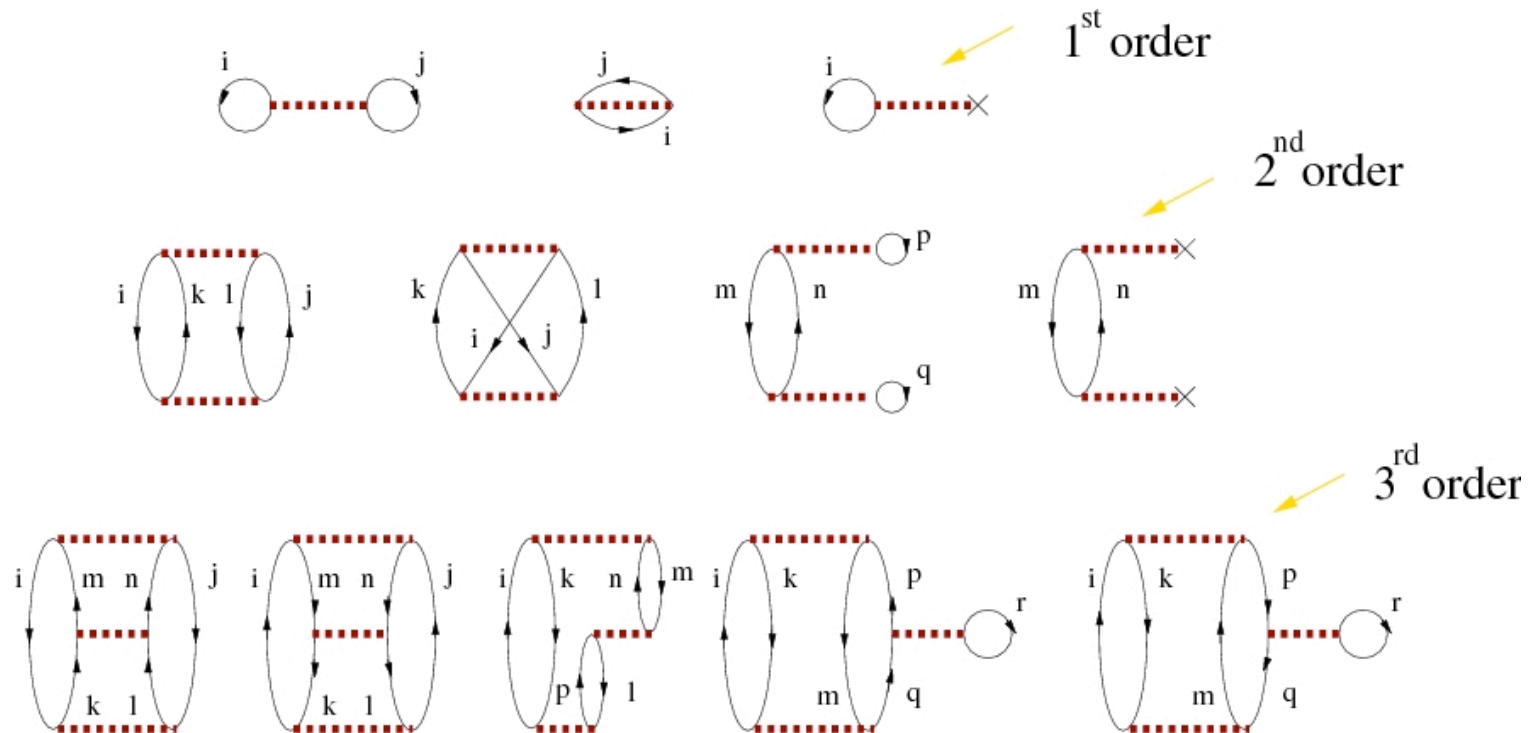
$$H_0|\phi_0\rangle = E_0|\phi_0\rangle$$

$\Delta E \rightarrow$ perturbation theory

$\Delta E \rightarrow$ perturbation theory

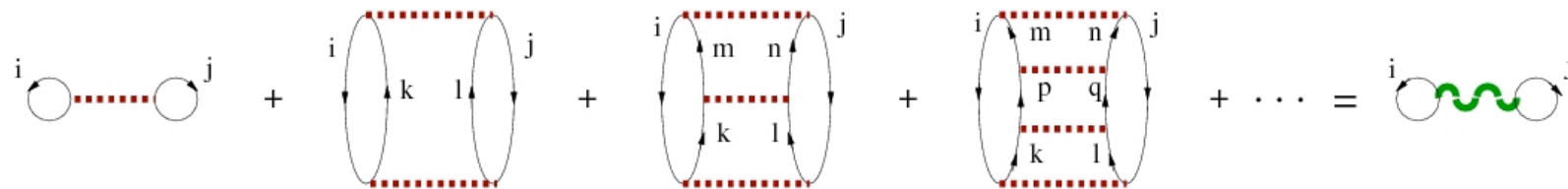
$$\Delta E = \langle \phi_0 | H_1 \sum_{n=0}^{\infty} \left[\frac{1 - |\phi_0\rangle\langle\phi_0|}{E_0 - H_0} H_1 \right]^n | \phi_0 \rangle$$

diagrammatically \rightarrow Goldstone expansion



Brueckner's reaction matrix (G-matrix)

Consider the partial summation of the set of ladder diagrams



It allows us to obtain the so-called **G-matrix**, by solving the well known **Bethe-Goldstone Equation**

$$\begin{aligned}
 G &= V + V \frac{Q}{\omega - H_0 + i\eta} V + V \frac{Q}{\omega - H_0 + i\eta} V \frac{Q}{\omega - H_0 + i\eta} V + \dots \\
 &= V + V \frac{Q}{\omega - H_0 + i\eta} \left[\underbrace{V + V \frac{Q}{\omega - H_0 + i\eta} V + V \frac{Q}{\omega - H_0 + i\eta} V \frac{Q}{\omega - H_0 + i\eta} V + \dots}_{G} \right]
 \end{aligned}$$

Then

$$G = V + V \frac{Q}{\omega - H_0 + i\eta} G$$

Note that the **Bethe-Goldstone equation** is formally identical to the **Lippman-Schwinger equation** for the scattering of two particles in the vacuum.

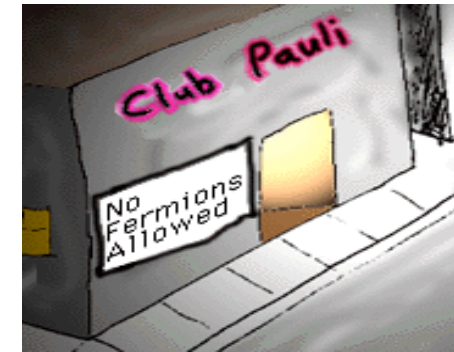
$$T = V + V \frac{1}{\omega - K + i\eta} T$$

In fact the **G-matrix** can be considered as a generalization of the **T-matrix** to the **medium**, when one takes into account the presence of other particles.

Medium effects are taken into account through

- Pauli blocking of the intermediate states

The Pauli operator Q prevents the scattering to any occupied state, limiting the phase space of the intermediate states.



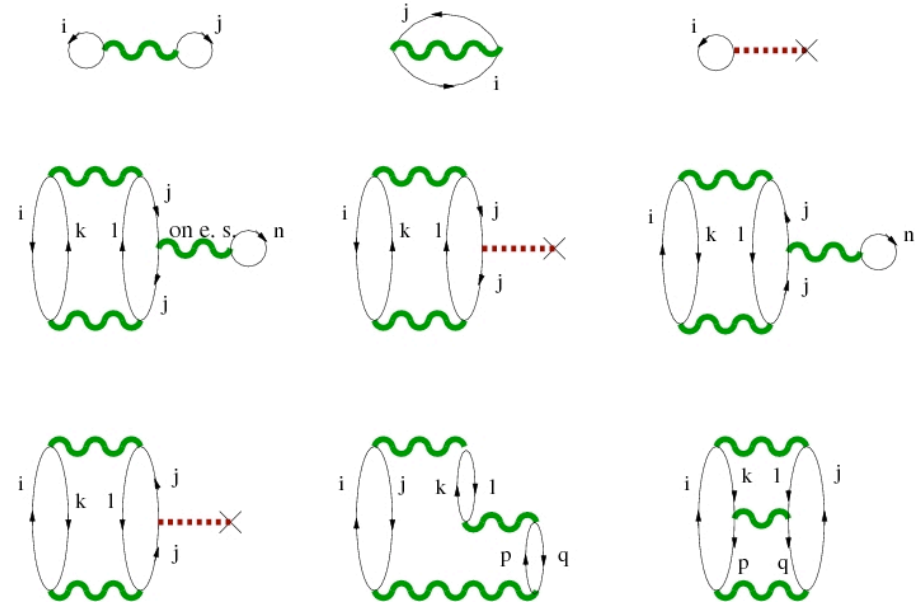
- Dressing of the intermediate particles

The modification of the single-particle spectrum due to the inclusion of the averaged potential U “felt” by a particle due to its interaction with the others must be taken into account in the propagator.



Hole-line expansion and BHF approximation

Goldstone expansion in terms of G
 → Brueckner-Goldstone expansion



Grouping by number of hole-lines ($c/r_0 < 1$) → hole-line expansion or Brueckner-Bethe-Goldstone expansion. Leading term: BHF approximation

$$E_{BHF} = \sum_{i \leq A} \langle \alpha_i | K | \alpha_i \rangle + \frac{1}{2} \text{Re} \left[\sum_{i, j \leq A} \langle \alpha_i \alpha_j | G(\omega) | \alpha_i \alpha_j \rangle \right]$$

Extended BHF approach: Hyperonic Matter

Single-particle properties

$$G(\omega)_{B_1 B_2; B_3 B_4} = V_{B_1 B_2; B_3 B_4} + \sum_{B_5 B_6} V_{B_1 B_2; B_5 B_6} \frac{Q_{B_5 B_6}}{\omega - E_{B_5} - E_{B_6} + i\eta} G(\omega)_{B_5 B_6; B_3 B_4}$$
$$E_{B_i}(k) = M_{B_i} c^2 + \frac{\hbar^2 k^2}{2M_{B_i}^2} + \text{Re}[U_{B_i}(k)]$$
$$U_{B_i}(k) = \sum_{B_j} \sum_{k \leq k_{FB_j}} \langle \vec{k}_i \vec{k}_j | G(\omega = E_{B_i} + E_{B_j}) | \vec{k}_i \vec{k}_j \rangle$$

Equation of State

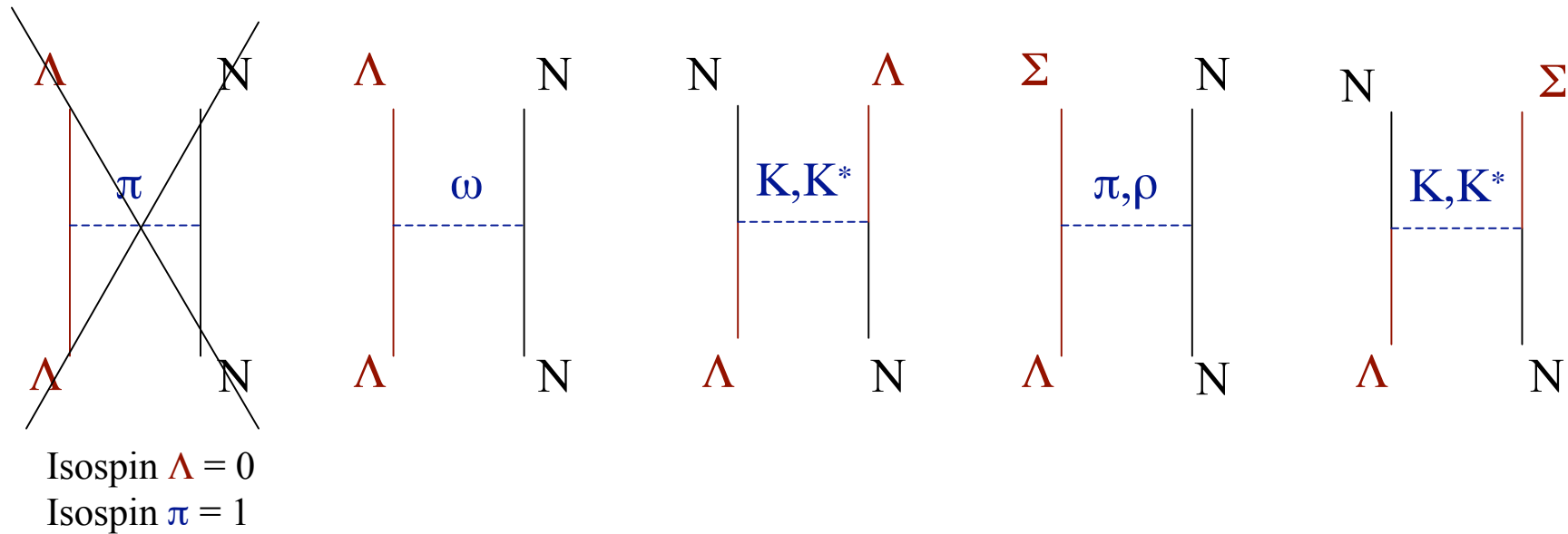
$$\varepsilon = 2 \sum_{B_i} \int_0^{k_{FB_i}} \frac{d^3 k}{(2\pi)^3} \left[M_{B_i} c^2 + \frac{\hbar^2 k^2}{2M_{B_i}^2} + \frac{1}{2} \text{Re}[U_{B_i}^N] + \frac{1}{2} \text{Re}[U_{B_i}^Y] \right]; \quad p = \rho \frac{\partial \varepsilon}{\partial \rho} - \varepsilon$$

🍷 Isospin and Strangeness channels

	S = 0	S = -1	S = -2	S = -3	S = -4
I = 0	$(NN \rightarrow NN)$		$\begin{pmatrix} \Lambda\Lambda \rightarrow \Lambda\Lambda & \Lambda\Lambda \rightarrow \Xi N & \Lambda\Lambda \rightarrow \Sigma\Sigma \\ \Xi N \rightarrow \Lambda\Lambda & \Xi N \rightarrow \Xi N & \Xi N \rightarrow \Sigma\Sigma \\ \Sigma\Sigma \rightarrow \Lambda\Lambda & \Sigma\Sigma \rightarrow \Xi N & \Sigma\Sigma \rightarrow \Sigma\Sigma \end{pmatrix}$		$(\Xi\Xi \rightarrow \Xi\Xi)$
I = 1/2		$\begin{pmatrix} \Lambda N \rightarrow \Lambda N & \Lambda N \rightarrow \Sigma N \\ \Sigma N \rightarrow \Lambda N & \Sigma N \rightarrow \Sigma N \end{pmatrix}$		$\begin{pmatrix} \Lambda\Xi \rightarrow \Lambda\Xi & \Lambda\Xi \rightarrow \Sigma\Xi \\ \Sigma\Xi \rightarrow \Lambda\Xi & \Sigma\Xi \rightarrow \Sigma\Xi \end{pmatrix}$	
I = 1	$(NN \rightarrow NN)$		$\begin{pmatrix} \Xi N \rightarrow \Xi N & \Xi N \rightarrow \Lambda\Sigma & \Xi N \rightarrow \Sigma\Sigma \\ \Lambda\Sigma \rightarrow \Xi N & \Lambda\Sigma \rightarrow \Lambda\Sigma & \Lambda\Sigma \rightarrow \Sigma\Sigma \\ \Sigma\Sigma \rightarrow \Xi N & \Sigma\Sigma \rightarrow \Lambda\Sigma & \Sigma\Sigma \rightarrow \Sigma\Sigma \end{pmatrix}$		$(\Xi\Xi \rightarrow \Xi\Xi)$
I = 3/2		$(\Sigma N \rightarrow \Sigma N)$		$(\Sigma\Xi \rightarrow \Sigma\Xi)$	
I = 2			$(\Sigma\Sigma \rightarrow \Sigma\Sigma)$		

The hyperon-nucleon interaction

One Boson exchange potential

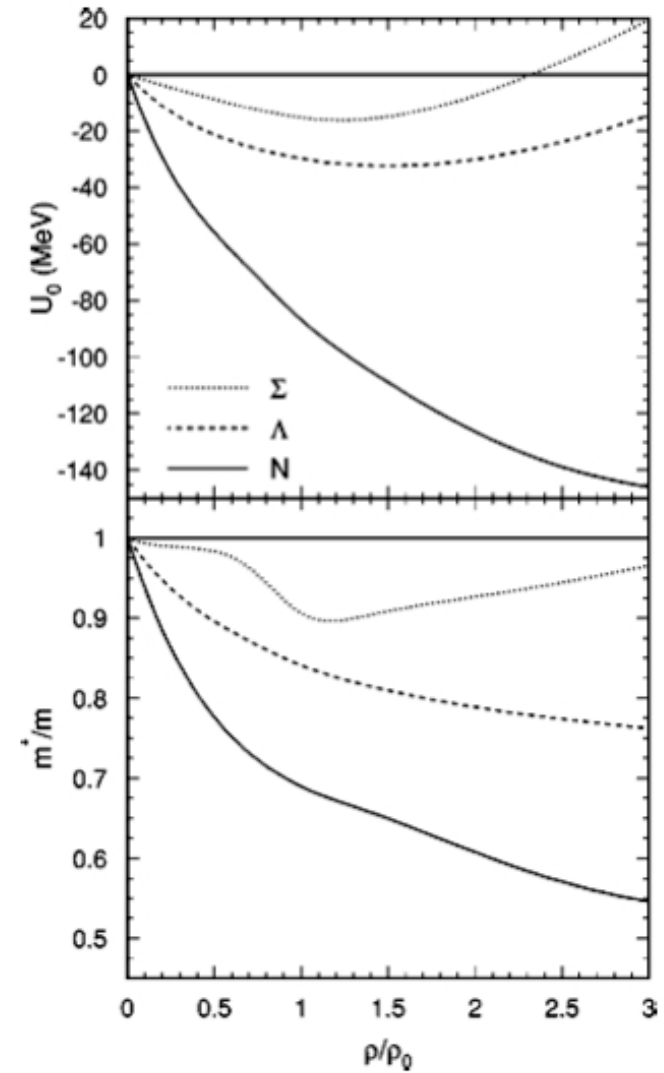
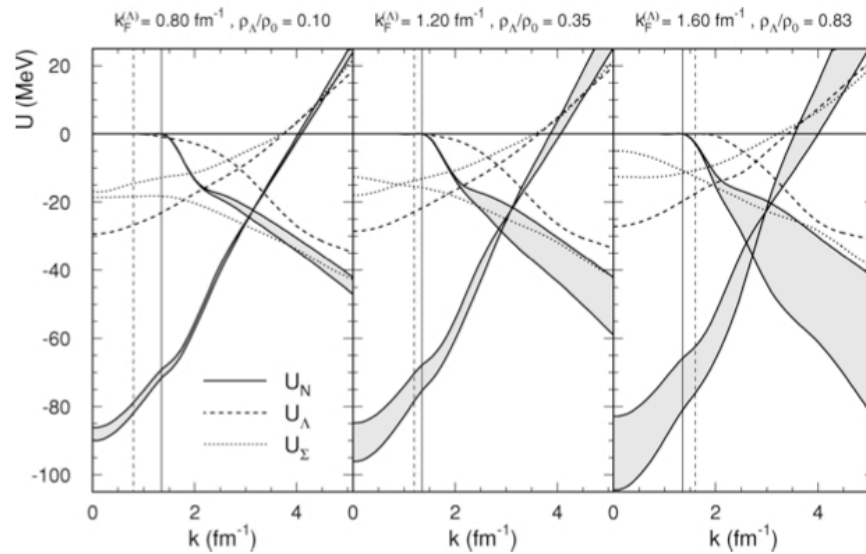
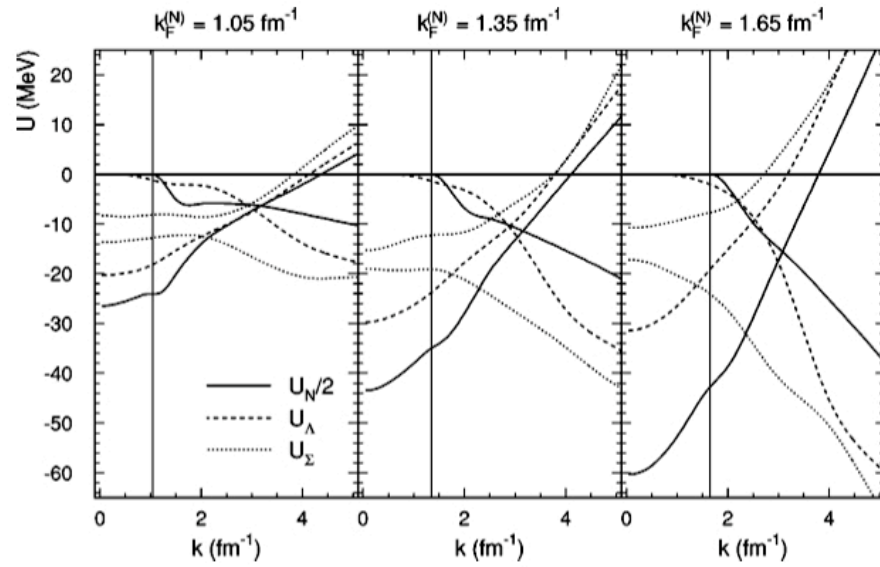


$$V_{YN} = V_C + V_s \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T S_{12} + V_{LS} \vec{L} \cdot \vec{S}^+ + V_{ALS} \vec{L} \cdot \vec{S}^-$$

- ☛ Jülich A and B: B. Hozelkamp, K. Holinde & J. Speth, NPA500, 485 (1989)
J. Haidenbauer & U. G. Meissner, PRC72, 044005 (2005)
- ☛ Jülich (EFT): H. Polinder, J. Haidenbauer & U. G. Meissner,
NPA779, 244 (2006)
- ☛ Nijmegen NSC89: M. M. Nagels, Th. A. Rijken & J. J. de Swart,
PRC40, 2226 (1988)
- ☛ Nijmegen NSC97: Th. A. Rijken, Y. Yamamoto & V. G. J. Stoks,
(YN & YY) PRC59, 21 (1999)
- ☛ Nijmegen ESC04: Th. A. Rijken, Y. Yamamoto & V. G. J. Stoks,
(YN & YY) PRC73, 044008 (2006)

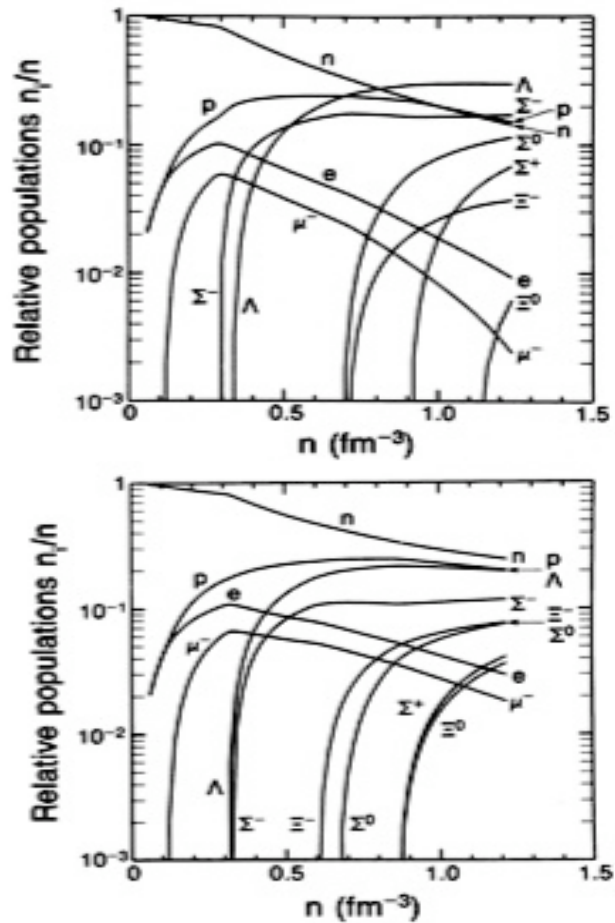
Fitted to 35 scattering YN data + SU(3) or SU(6)
to be compared with the 4300 scattering NN data

Hyperons in nuclear matter

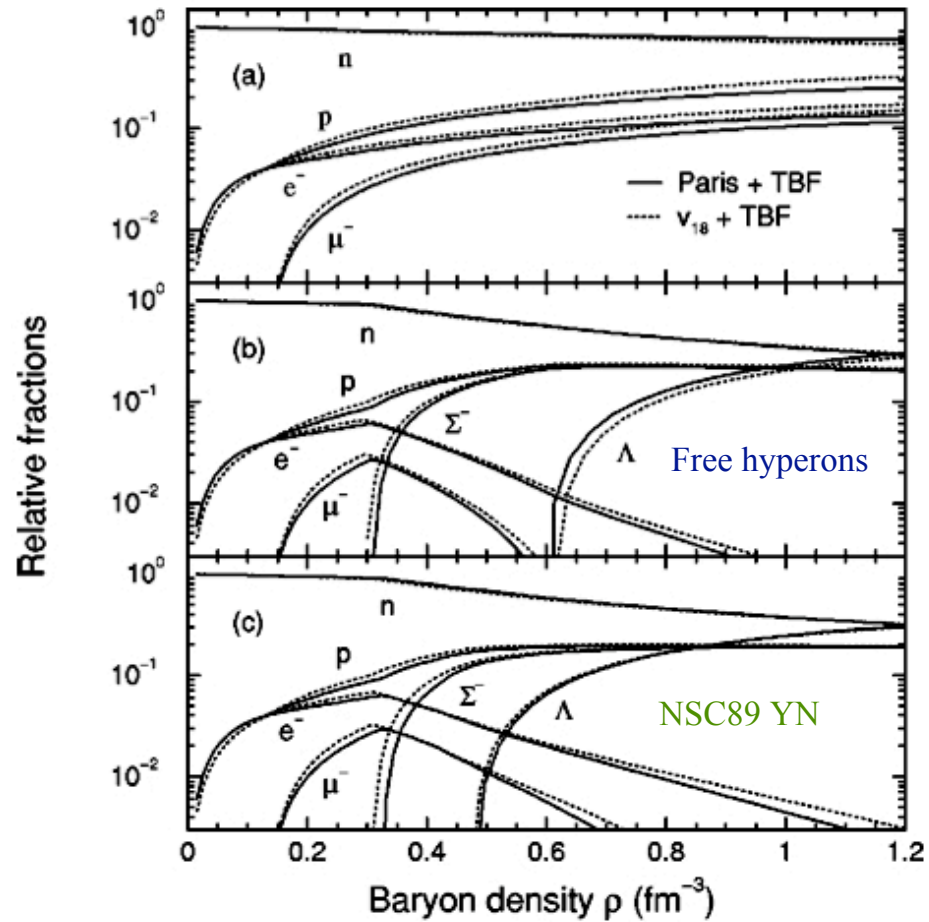


Neutron Star Matter Composition

RMFT



BHF

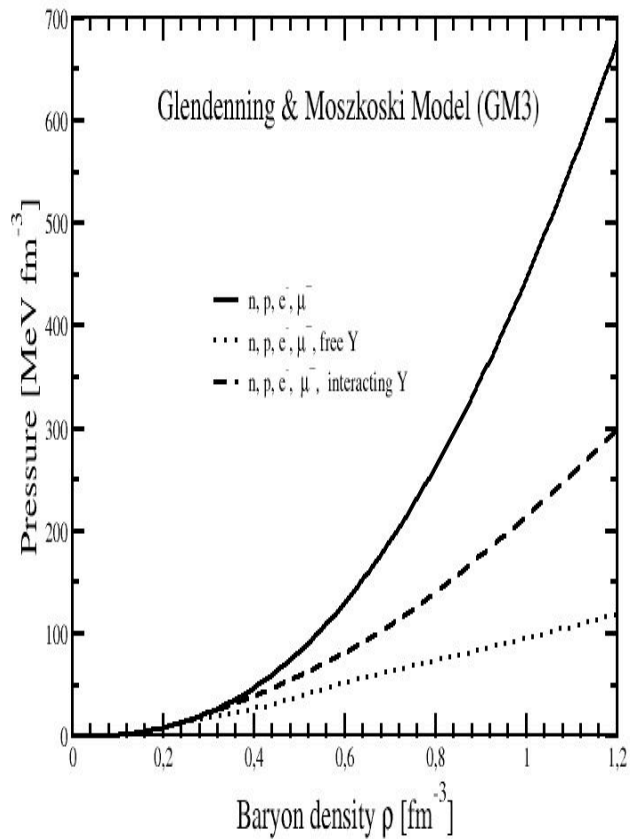


N.K. Glendenning, ApJ 293, 470 (1985)

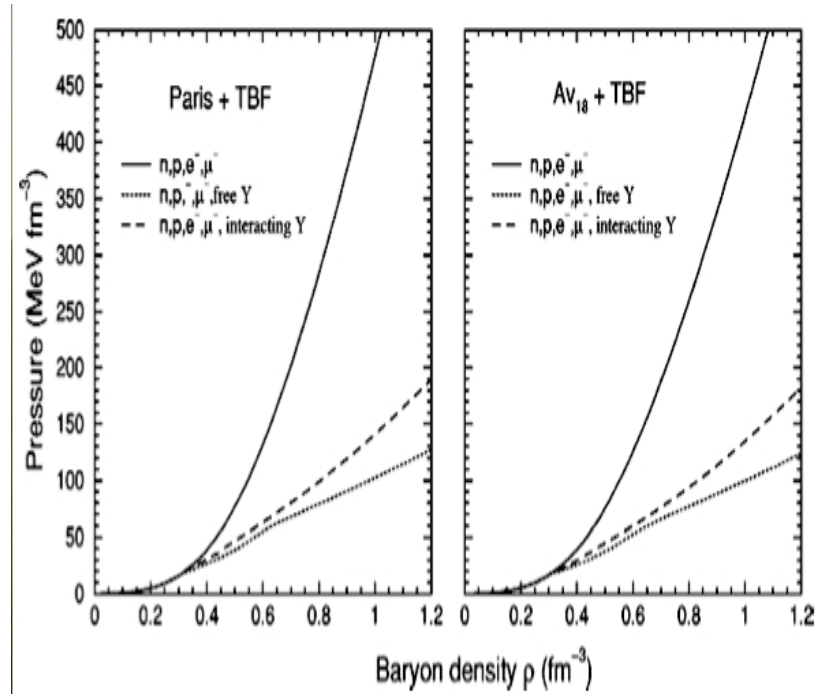
M. Baldo *et.al.*, Phys. Rev. C 61, 055801 (2000)

Neutron Star Matter EoS (I)

RMFT



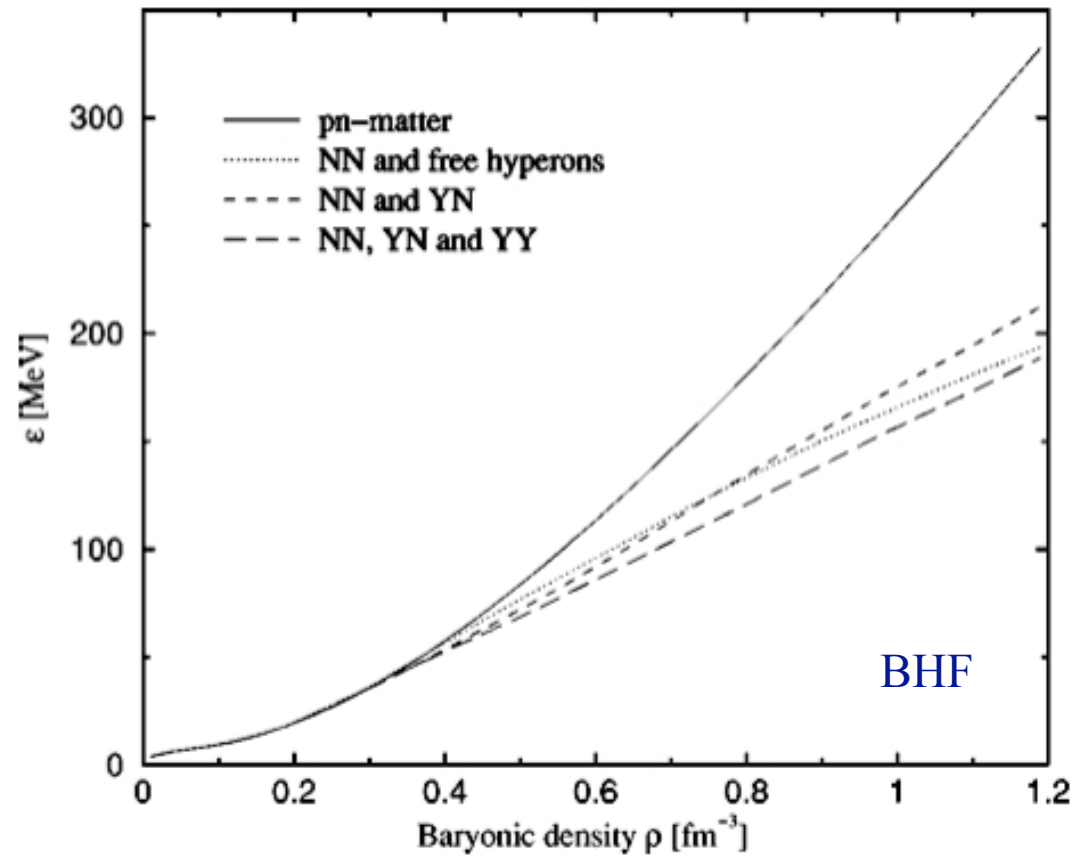
BHF



M. Baldo *et al.*, Phys. Rev. C 61, 055801 (2000)

No YY interaction !

Neutron Star Matter EoS (II)



Additional softening
From YY interaction

I. V. *et.al.*, Phys. Rev. C 62, 035801 (2000)

Structure equations for Neutron Stars: TOV Equations

Since neutron stars have masses $M \sim 1-2 M_{\odot}$, and radii $R \sim 10-20$ km, the value of the gravitational potential on the neutron star surface is of the order 1

$$\frac{GM}{c^2 R} \sim 1$$

with escape velocities of the order of $c/2$. Therefore, general relativistic effects become very important and thus the structure equations read

$$\frac{dp}{dr} = -G \frac{m(r)\epsilon(r)}{r^2} \left(1 + \frac{p(r)}{c^2 \epsilon(r)} \right) \left(1 + \frac{4\pi r^3 p(r)m(r)}{c^2} \right) \left(1 - \frac{Gm(r)}{c^2 r} \right)^{-1}$$

$$\frac{dm}{dr} = 4\pi r^2 \epsilon(r)$$

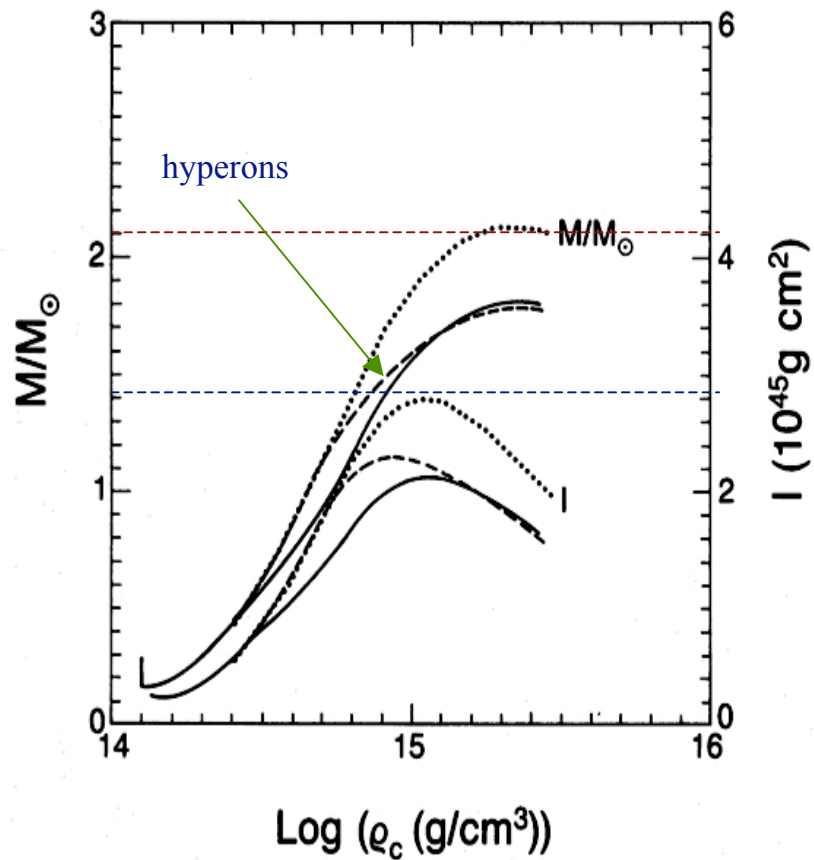
with the boundary conditions

$$m(r=0) = 0$$

$$p(r=R) = p_{surf}$$

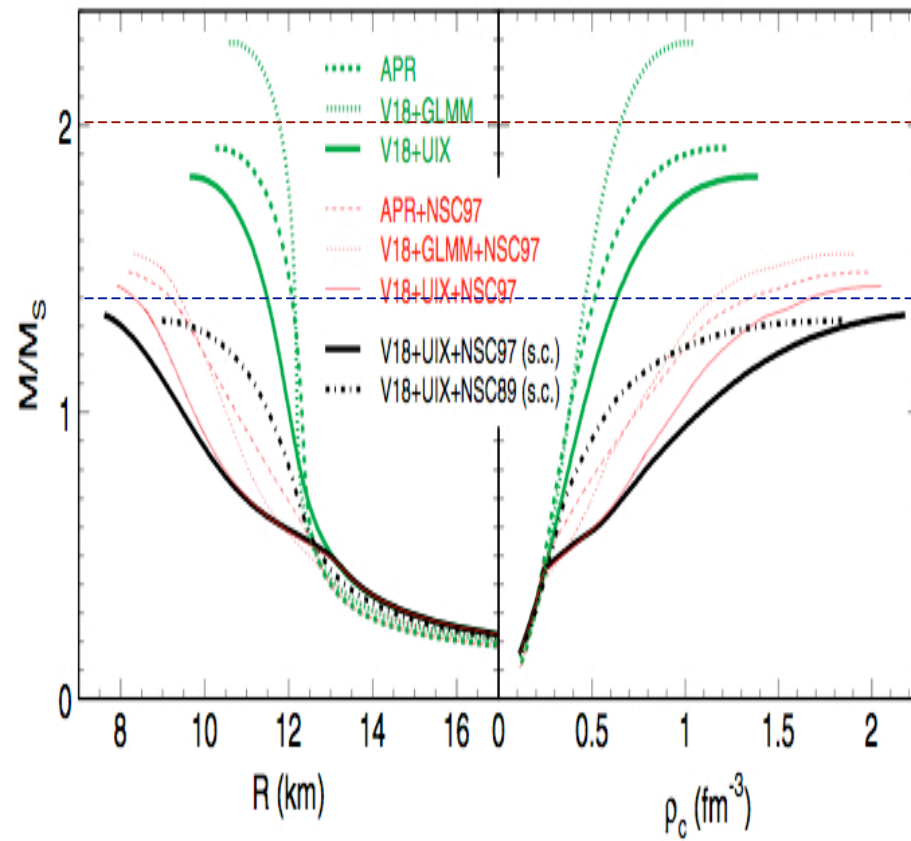
Neutron Star Structure

RMFT



N.K. Glendenning, ApJ 293, 470 (1985)

BHF



H.-J. Schulze, I.V., A. Polls & A. Ramos Phys. Rev. C 73, 08801 (2006)

Implications for Neutron Star Structure

- ☛ The presence of hyperons reduces the maximum mass of Neutron Stars by an amount $\Delta M_{\max} \sim (0.5-0.8)M_{\odot}$
- ☛ Microscopic EoS “very soft EoS” non compatible with measured masses of NS

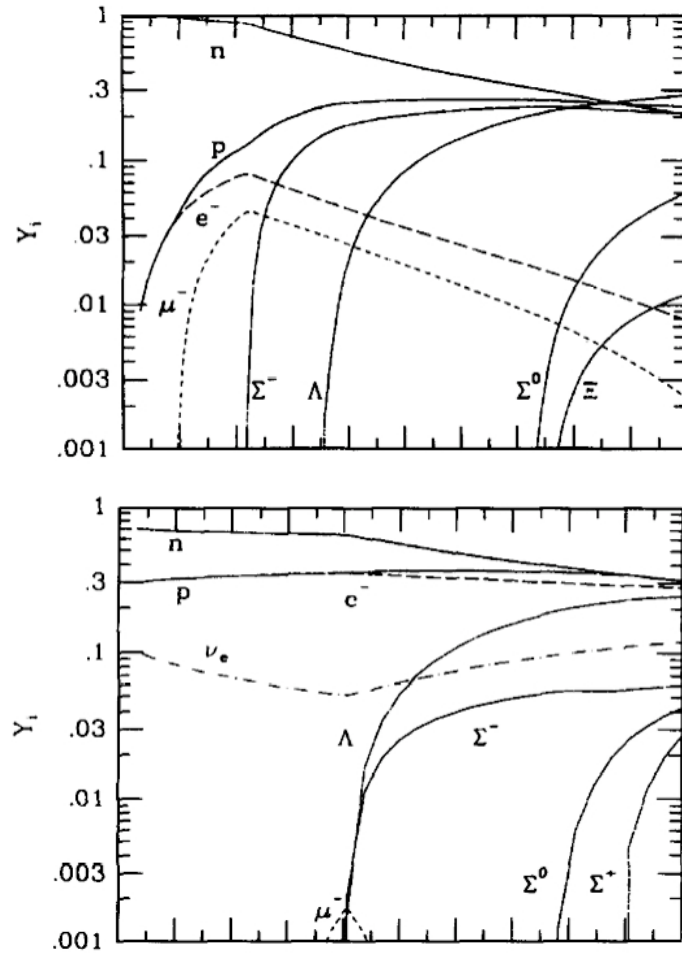
Need for extra pressure at high densities

Two-body forces: Improved YN and YY

Three-body forces: NNY, NYY and YYY

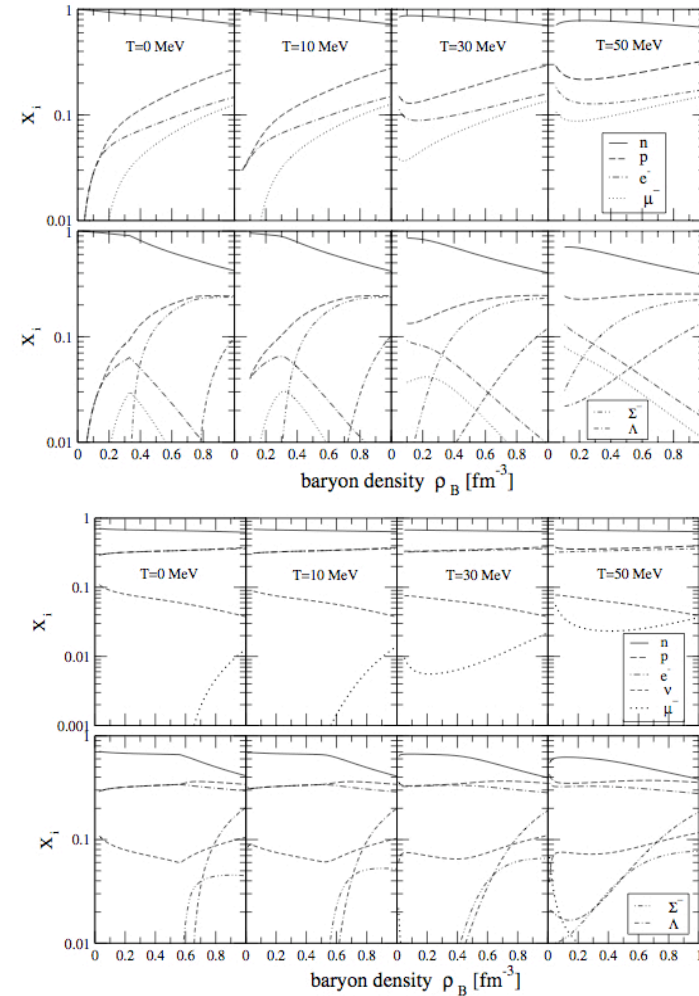
Hyperon Stars at birth: Composition

RMFT



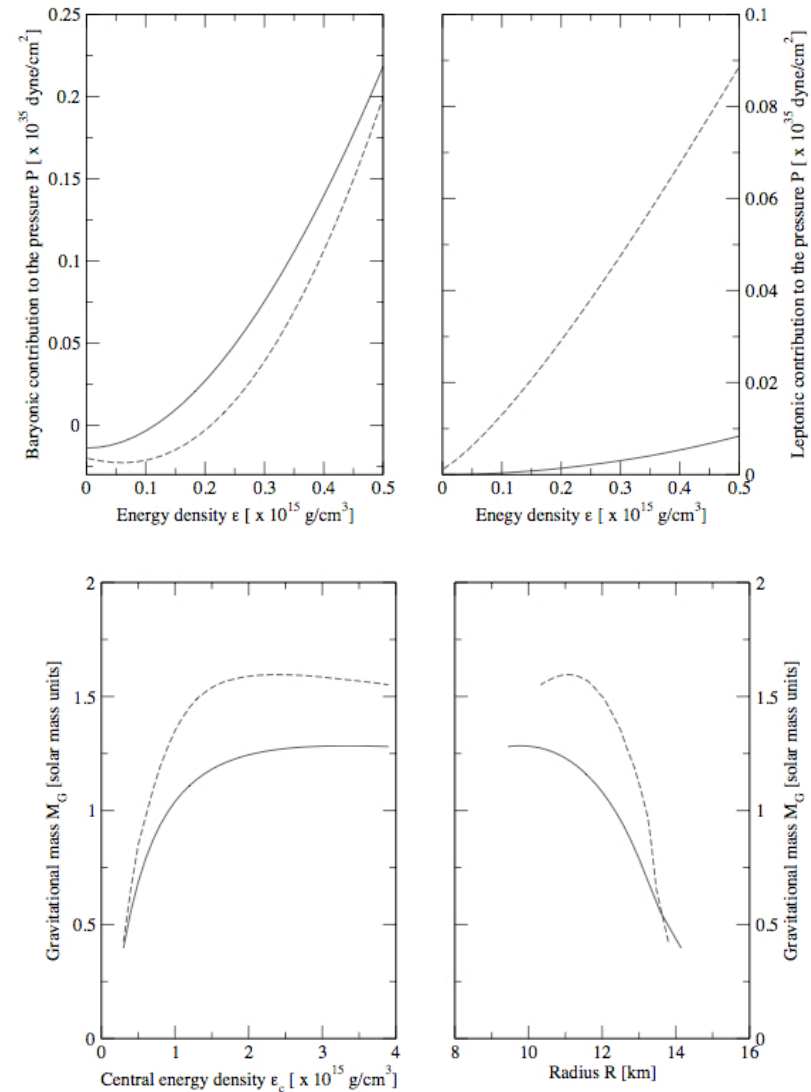
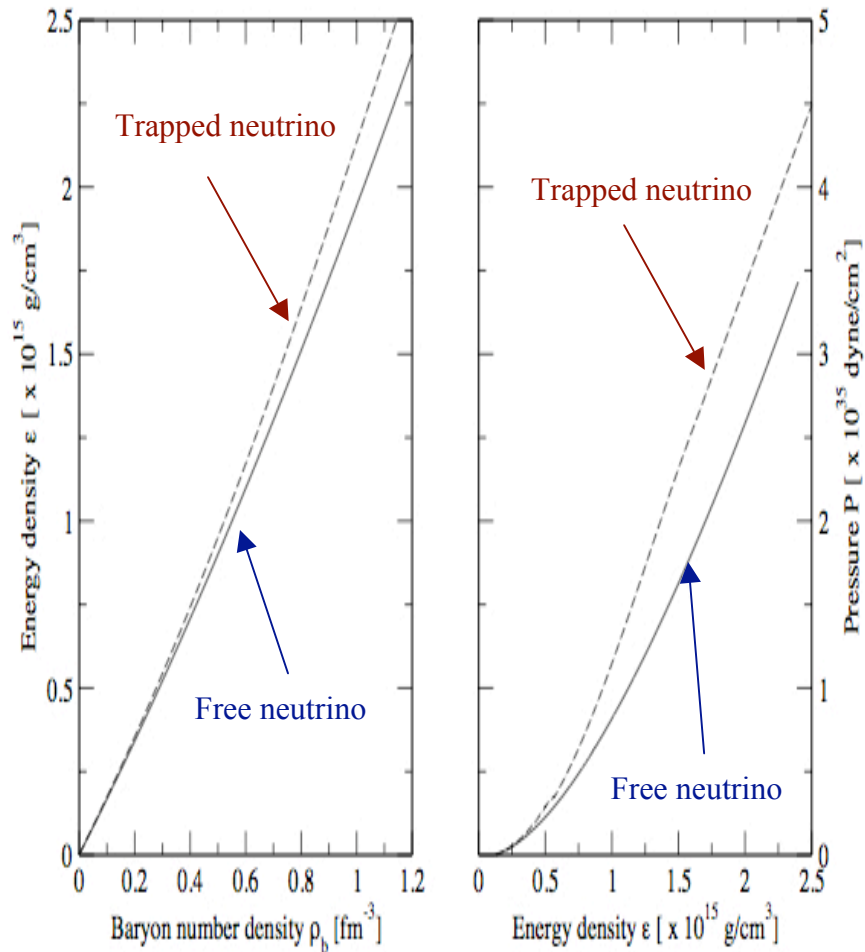
M. Prakash et al., Phys. Rep. 280, 1 (1997)

BHF



G. E. Nicotra et al., A&A 451, 213 (2006)

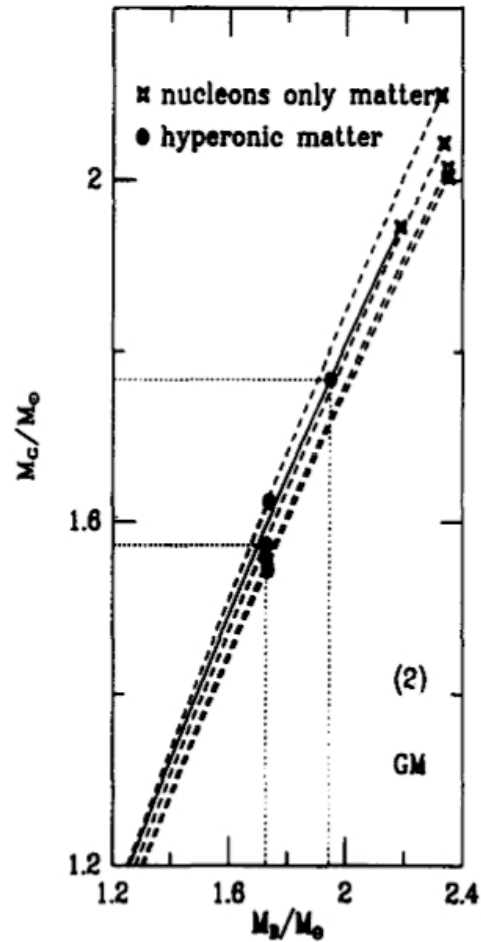
Hyperon Stars at birth: EoS and Mass



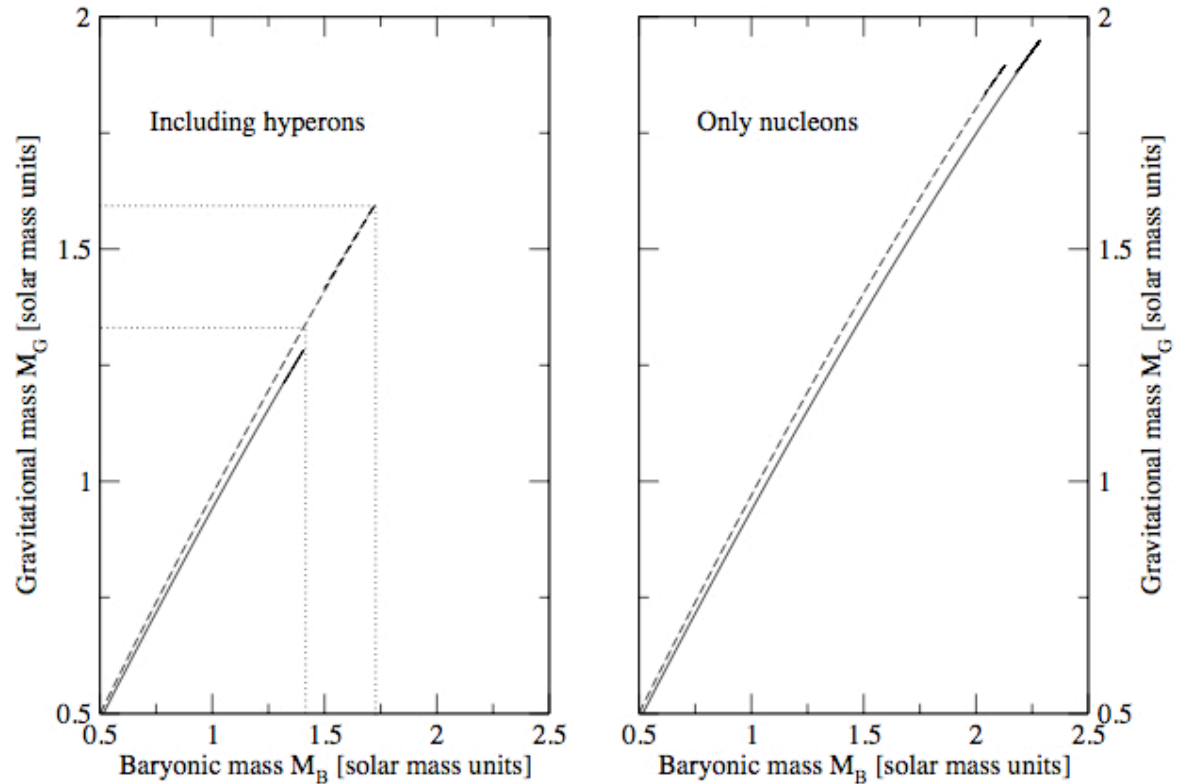
I. V., I. Bombaci, A. Polls & A. Ramos, A&A 399, 687 (2003)

Hyperon Stars at birth: Metastable Configurations

RMFT



BHF



Hyperon Superfluidity

Hyperon superfluidity is interesting for:

🗨️ Cooling of Neutron Stars: Hyperon URCA processes

$$\Lambda \rightarrow p + e^- + \bar{\nu}_e$$

$$\Sigma^- \rightarrow n + e^- + \bar{\nu}_e$$

...

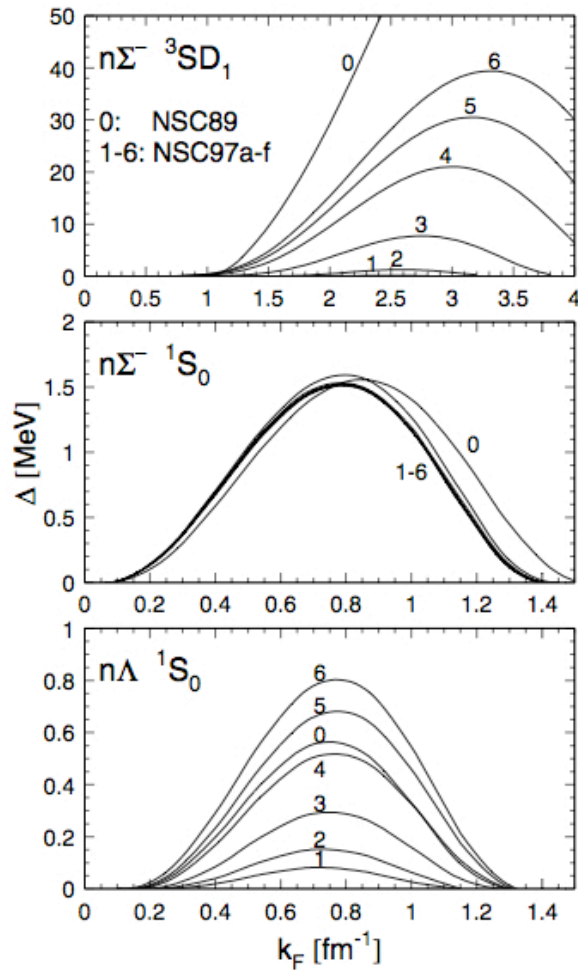
happens immediately when hyperons are present.

Only suppressed by hyperon gaps.

🗨️ Hyperon bulk viscosity: rotational modes

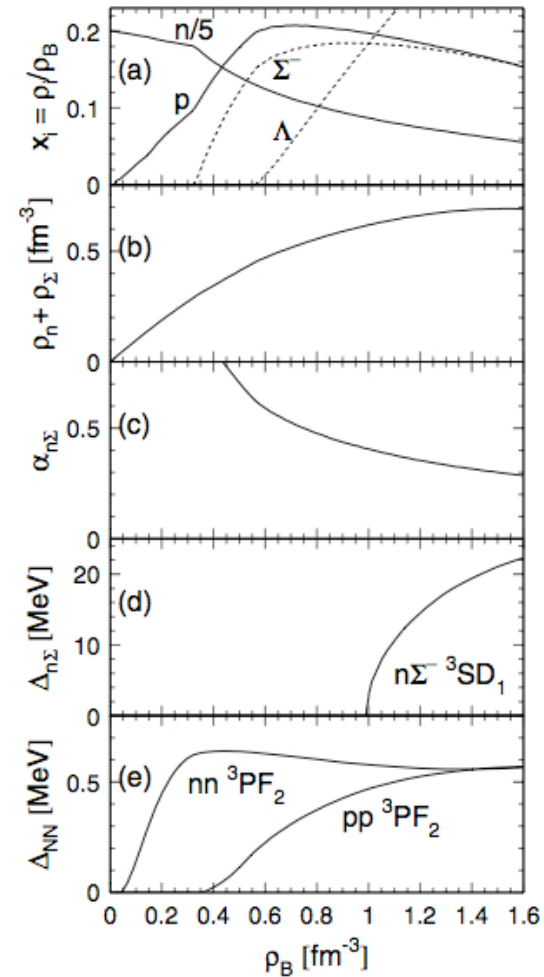
Hyperon-Nucleon Superfluidity

Symmetric Matter



X.-R. Xou et al., PRL 95, 051101 (2005)

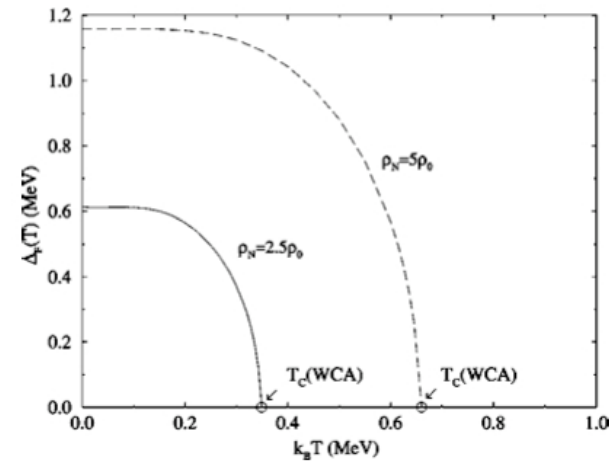
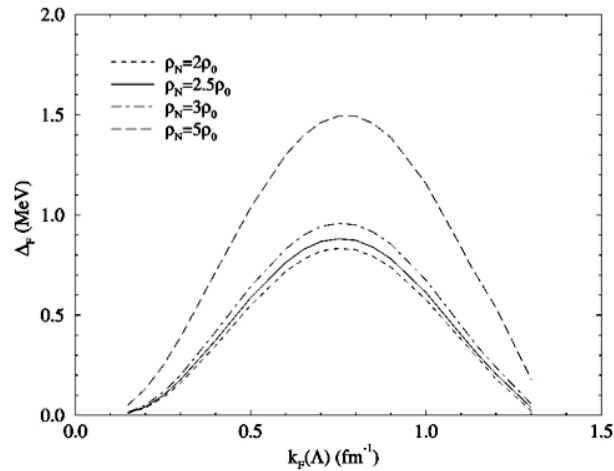
β -stable Matter



Hyperon-Hyperon Superfluidity

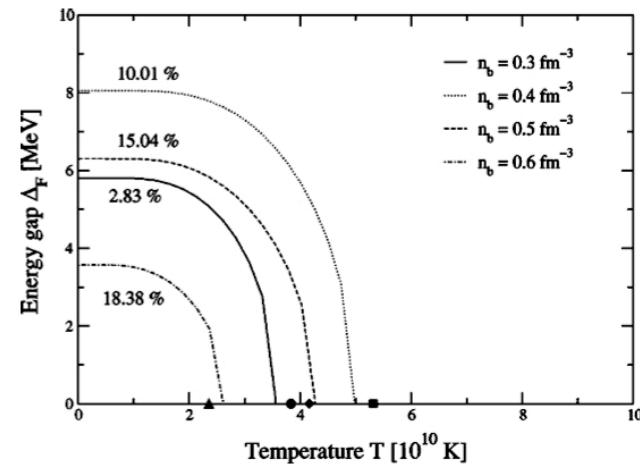
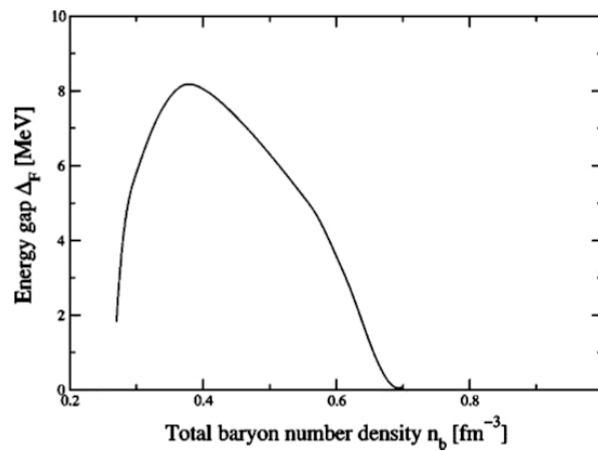
1S_0 $\Lambda\Lambda$ gap: S. Balberg et al., PRC 57, 409 (1998)

Parametrized G-matrix



1S_0 $\Sigma\Sigma$ gap, no $\Lambda\Lambda$ gap: I. V. et al., PRC 70, 028802 (2004)

Realistic interaction



Summary & Conclusions I



Hyperons in Neutron Stars

- 💡 The presence of hyperons reduces the maximum mass of Neutron Stars by an amount $\Delta M_{\max} \sim (0.5-0.8)M_{\odot}$
- 💡 Microscopic EoS “very soft EoS” non compatible with measured masses of NS

Need for extra pressure at high densities

Two-body forces: Improved YN and YY

Three-body forces: NNY, NYY and YYY

Hyperon Stars at birth

- 💡 The presence of neutrinos change the composition of matter:
higher number of protons,
hyperon thresholds shifted to higher densities
- 💡 The maximum mass of hyperon stars decreases as soon as neutrinos diffuse out, contrary to what happens when only nucleonic degrees of freedom are considered
- 💡 Window of metastable configurations. Stable only during neutrino trapping time, collapsing afterwards into low-mass black holes.

Hyperon Gaps

- 💡 Strong $n\Sigma^-$ 3SD_1 gaps that would dominate any direct NN pairing and could have important implications for cooling behaviour (nucleonic & hyperonic URCA)
- 💡 $\Lambda\Lambda$ 1S_0 gaps \sim few tenths of MeV. No gap is predicted by more “realistic” YY interactions (e.g., NSC97a-f)
- 💡 $\Sigma^-\Sigma^-$ 1S_0 gaps ~ 8 MeV at 0.37 fm^{-3} and $x_{\Sigma^-} \sim 8$ %. Questionable because of the uncertainty of the SS interaction



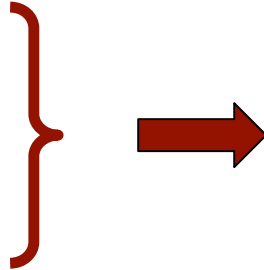
Quark Matter
in
Neutron Stars

Quark Matter in Neutron Stars

QCD

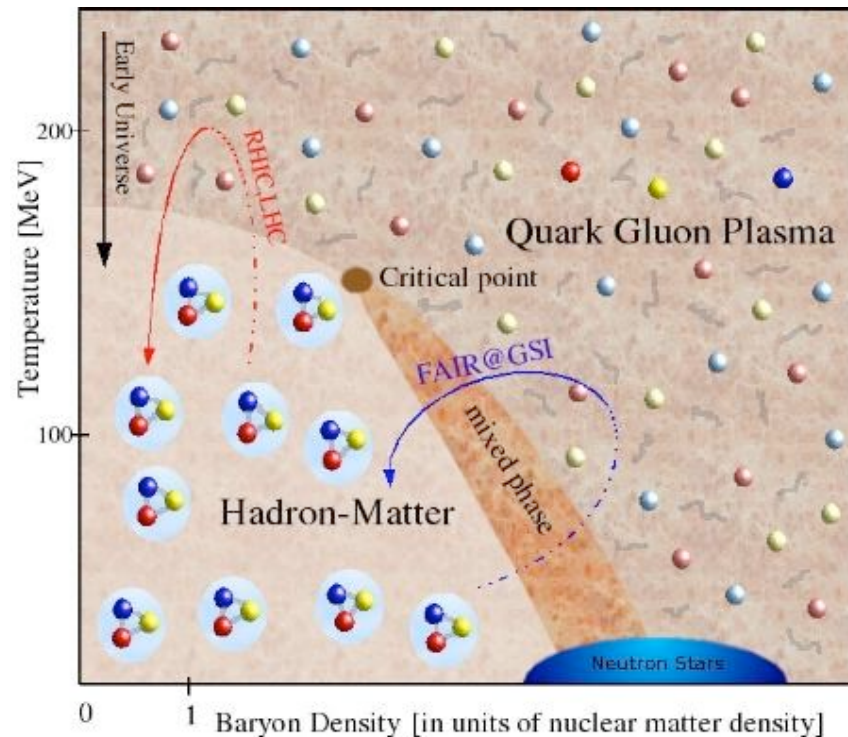
Ultra-Relativistic

Heavy Ion Collisions



Quark-deconfinement phase transition expected at

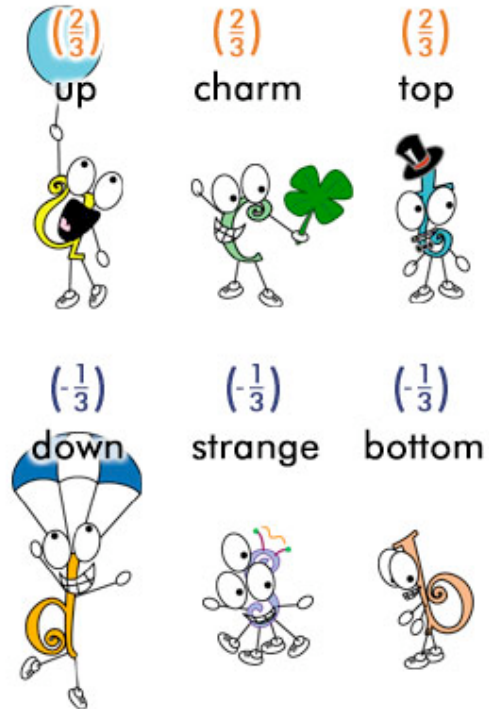
$$\rho_c \approx (3 - 5) \rho_0$$



The core of the most massive **Neutron Star** is one of the best candidates in the Universe where such a deconfined phase of Quark Matter can be found

What **quark flavours** are expected in a **Neutron Star** ?

flavor	Mass	Q/ e
<i>u</i>	$5 \pm 3 \text{ MeV}$	$2/3$
<i>d</i>	$10 \pm 5 \text{ MeV}$	$-1/3$
<i>s</i>	$200 \pm 100 \text{ MeV}$	$-1/3$
<i>c</i>	$1.3 \pm 0.3 \text{ GeV}$	$2/3$
<i>b</i>	$4.3 \pm 0.2 \text{ GeV}$	$-1/3$
<i>t</i>	$175 \pm 6 \text{ GeV}$	$2/3$



Suppose: $\left. \begin{array}{l} \bullet \text{ u,d,s non-interacting} \\ \bullet m_u = m_d = m_s = 0 \end{array} \right\} \text{i.e., ideal ultrarelativistic Fermi gas } (*)$

○ Threshold density for the c quark

$$s \rightarrow c + e^- + \bar{\nu}_e \quad \Rightarrow \quad \mu_s = \mu_c + \mu_{e^-} - \mu_{\bar{\nu}_e}$$

but u, d, s in β -equilibrium $\left. \begin{array}{l} Q_{\text{tot}} = 0 \end{array} \right\} \xrightarrow{(*)} \begin{array}{l} n_B = n_u = n_d = n_s \\ n_e = n_{\bar{\nu}_e} = 0 \end{array}$

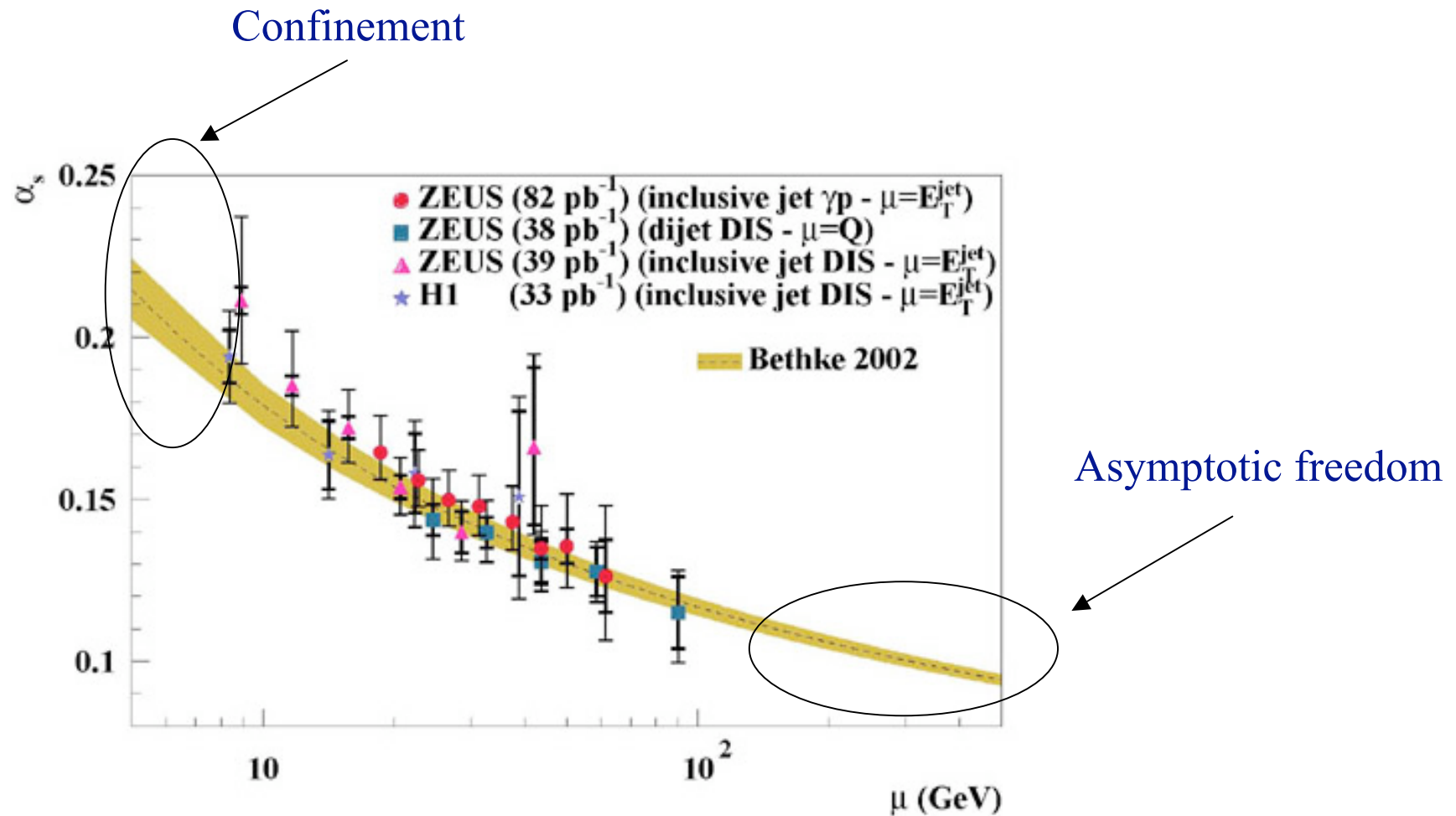
then

$$\begin{aligned} \mu_s = E_{F_s} &= \hbar c (\pi^2 n_s)^{1/3} = \hbar c (\pi^2 n_B)^{1/3} \geq m_c = 1.3 \text{ GeV} \\ \Rightarrow n_B &\geq 29 \text{ fm}^{-3} \sim 180 n_0 \end{aligned}$$

and similarly for the b and t quarks

Only u, d, s quark flavors are expected in Neutron Stars.

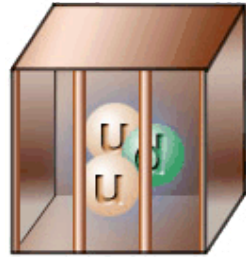
There are a few **observational facts** which establish some **basic features of strong interactions**





Asymptotic freedom

“The momentum distribution of quarks inside a hadron can be measured by **deep inelastic experiments**. These experiments demonstrate that **quarks inside the hadron behave as if they were almost free**. Or, in other words, at **short distances quarks are almost non-interacting**. The small interaction between quarks can be treated in terms of a perturbative expansion in powers of the QCD structure constant $\alpha_c = g^2/4\pi$ (**perturbative regime**).”



Confinement

“The absence of observations of a single isolated quark suggest that the interaction between quarks and gluons must be very strong on large distance scales. This leads to the concept of *confinement* of quarks inside hadrons. In this case a perturbative treatment in powers of α_c is no longer applicable (non-perturbative regime).”

A simple EoS of Quark Matter

Grand canonical potential per unit volume $\Omega = \Omega_{free} + \Omega_{int}$

Non interacting term

$$\Omega_{free} = \Omega_{free}^u + \Omega_{free}^d + \Omega_{free}^s$$

In the following we assume:

$$m_u = m_d = 0, \quad m_s \neq 0$$

$$\Omega_{free}^q = -\frac{\mu_q^4}{4\pi^2(\hbar c)^3}, \quad (q = u, d)$$

$$\Omega_{free}^s = -\frac{1}{4\pi^2(\hbar c)^3} \left\{ \mu_s \mu_s^* \left(\mu_s^2 - \frac{5}{2} m_s^2 \right) + \frac{3}{2} m_s^4 \ln \left(\frac{\mu_s + \mu_s^*}{m_s} \right) \right\}$$

$$\mu_s^* \equiv \left(\mu_s^2 - m_s^2 \right)^{1/2} = \hbar c k_{Fs}$$

$\mu_u \quad \mu_d \quad \mu_s$: quarks chemical potentials

Interacting term

$$\Omega_{\text{int}} = \Omega_{\text{long}} + \Omega_{\text{short}}$$

- The long range contribution Ω_{long} is very hard to evaluate because of the difficulties involved in solving nonperturbative QCD. A very promising approach to deal with the nonperturbative regime of strong interactions is to solve QCD equations on a discrete lattice of space-time. In the present approach, inspired by the MIT bag model, Ω_{long} is approximated by the bag pressure B , which is responsible for confinement

$$\Omega_{\text{long}} \approx B$$

- This is a crude approximation, which is expected to be reasonable at very high density, but which is not appropriate in the density region where quarks clusterize to form hadrons, *i.e.*, in the region of the phase transition between hadronic and quark matter.

- The short range contribution Ω_{short} can be evaluated using **perturbative QCD**. Keeping only the linear terms in the QCD structure constant α_c

$$\Omega_{short} \approx \Omega^{(1)} = \Omega_u^{(1)} + \Omega_d^{(1)} + \Omega_s^{(1)}$$

$$\Omega_q^{(1)} = \frac{1}{4\pi^2 (\hbar c)^3} \frac{2\alpha_c}{\pi} \mu_q^4, \quad (q = u, d)$$

$$\Omega_s^{(1)} = \frac{1}{4\pi^2 (\hbar c)^3} \frac{2\alpha_c}{\pi} \left\{ 3 \left[\mu_s \mu_s^* - m_s^2 \ln \left(\frac{\mu_s + \mu_s^*}{\mu_s} \right) \right]^2 - 2\mu_s^{*4} \right. \\ \left. + 3m_s^4 \ln^2 \left(\frac{m_s}{\mu_s} \right) + 6 \ln \left(\frac{\rho_{ren}}{\mu_s} \right) \left[\mu_s \mu_s^* m_s^2 - m_s^4 \ln \left(\frac{\mu_s + \mu_s^*}{m_s} \right) \right] \right\}$$

where $\rho_{ren} = M_N/3 = 313$ MeV is the so-called *renormalization point*

To summarize, **the EoS of Strange Quark Matter at T=0** in the approximation of the grand canonical potential we are considering is

$$P(\mu_u, \mu_d, \mu_s) = -\Omega \approx -\Omega_{free} - \Omega^{(1)} - B$$

$$\rho(\mu_u, \mu_d, \mu_s) = \frac{1}{c^2} \left(\Omega + \sum_f \mu_f n_f \right) \approx \frac{1}{c^2} \left(\Omega_{free} + \Omega^{(1)} + B + \sum_f \mu_f n_f \right)$$

Number densities n_f

$$n_f = - \left(\frac{\partial \Omega_f}{\partial \mu_f} \right), \quad (f = u, d, s)$$

Total baryon number density

$$n = \frac{1}{3} (n_u + n_d + n_s)$$

using the previous expressions for Ω_f one has

for massless quarks like u and d

$$n_q = n_q^{free} + n_q^{(1)} = \frac{\mu_q^3}{(\hbar c)^3 \pi^2} \left(1 - \frac{2\alpha_c}{\pi} \right)$$

for massive quarks like s

$$n_s = \frac{\mu_q^{*3}}{(\hbar c)^3 \pi^2} - \frac{1}{(\hbar c)^3 \pi^3} \frac{\alpha_c}{2} \left\{ \left[4\mu_s \mu_s^{*2} - 12\mu_s^* m_s^2 \ln \left(\frac{\mu_s + \mu_s^*}{\rho_{ren}} \right) \right. \right. \\ \left. \left. + 6 \frac{m_s^4}{\mu_s} \ln \left(\frac{\mu_s + \mu_s^*}{\mu_s} \right) + 12\mu_s^* m_s^2 \ln \left(\frac{m_s}{\mu_s} \right) - 6\mu_s^* m_s^2 \right] \right\}$$

with $\mu_s^* \equiv (\mu_s^2 - m_s^2)^{1/2} = \hbar c k_{Fs}$

β -stable Strange Quark Matter

The composition of β -stable strange quark matter, as the one of β -stable hadronic matter, is determined by **charge neutrality** and **equilibrium with respect to the weak processes**

$$u + e^- \rightarrow d + \nu_e$$

$$u + e^- \rightarrow s + \nu_e$$

$$d \rightarrow u + e^- + \bar{\nu}_e \quad \longrightarrow$$

$$s \rightarrow u + e^- + \bar{\nu}_e$$

$$s + u \rightarrow d + u$$

$$e^- \leftrightarrow \mu^- + \nu_e + \bar{\nu}_\mu$$

...

$$\mu_d = \mu_s \equiv \mu$$

$$\mu = \mu_u + \mu_e - \mu_{\nu_e}$$

$$\mu_u - \mu_{\nu_\mu} = \mu_e - \mu_{\nu_e}$$

$$\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e = 0$$

To be solved for any given value of the total baryon number density n_B

- *Exercise:* Prove that in the case of massless quarks, the charge neutrality and β -equilibrium conditions implies:

$$n_u = n_d = n_s$$

$$n_e = 0$$

- *Exercise:* Consider massless quarks, interacting perturbatively to lowest order in α_c . Show that under these circumstances the EoS of β -stable strange quark matter can be written in the parametrical form

$$\varepsilon = Kn^{4/3} + B$$

$$P = \frac{1}{3}K^{4/3}n - B = \frac{\varepsilon - 4B}{3}$$

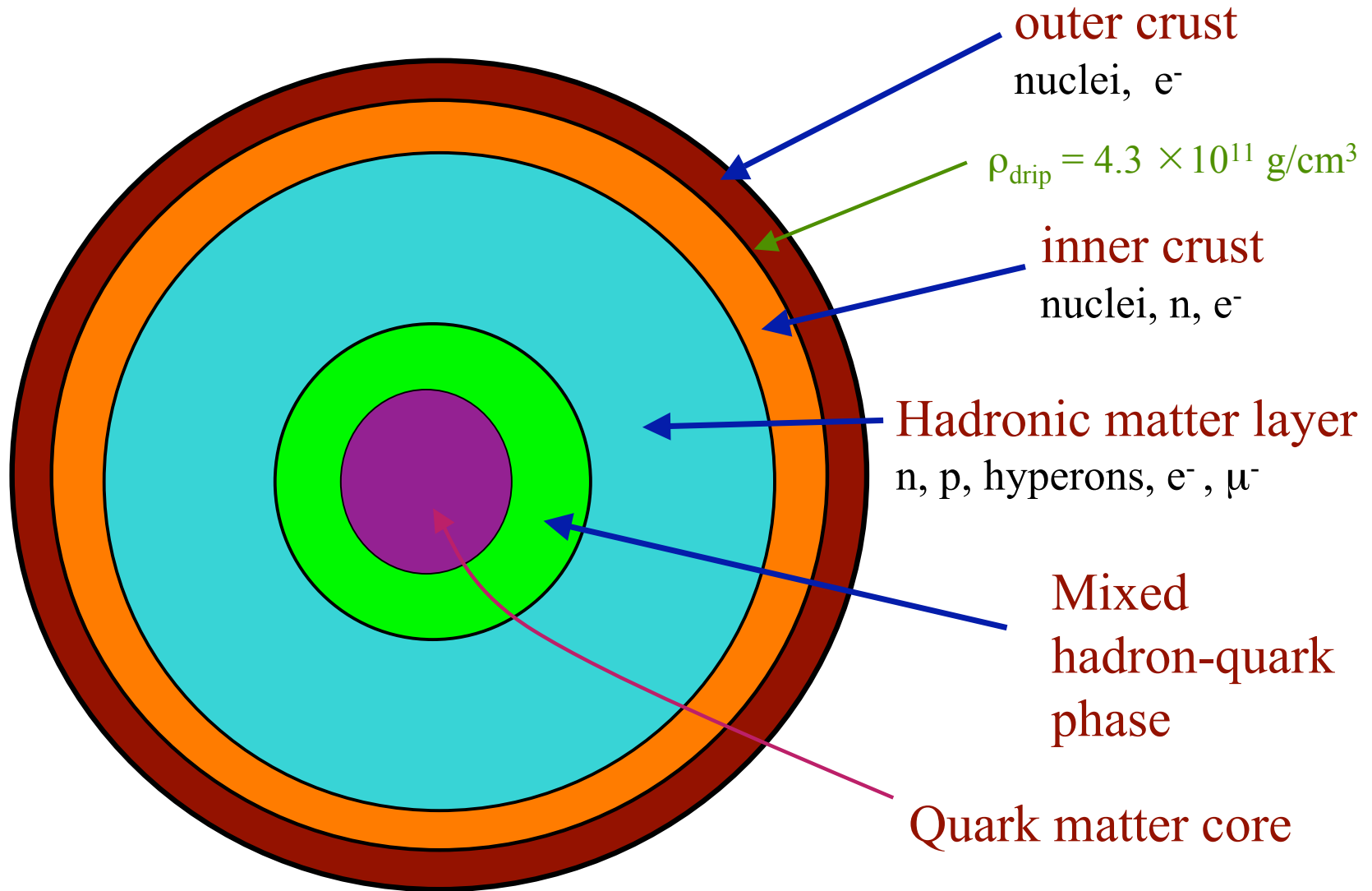
being

$$K = \frac{9}{4}\pi^{2/3}\left(1 + \frac{2\alpha_c}{3\pi}\right)\hbar c$$

Types of Quark Stars

- 🍌 **Hybrid Stars:** Neutron Stars with a Quark Matter Core
- 🍌 **Strange Stars:** Consequence of the **Bomer-Witten-Terezawa** hypotesis. Completely made of deconfined Strange Quark Matter

Hybrid Star Cross Section



The Equation of State for Hybrid Stars

Hadronic phase :

RMF Models
Microscopic BHF

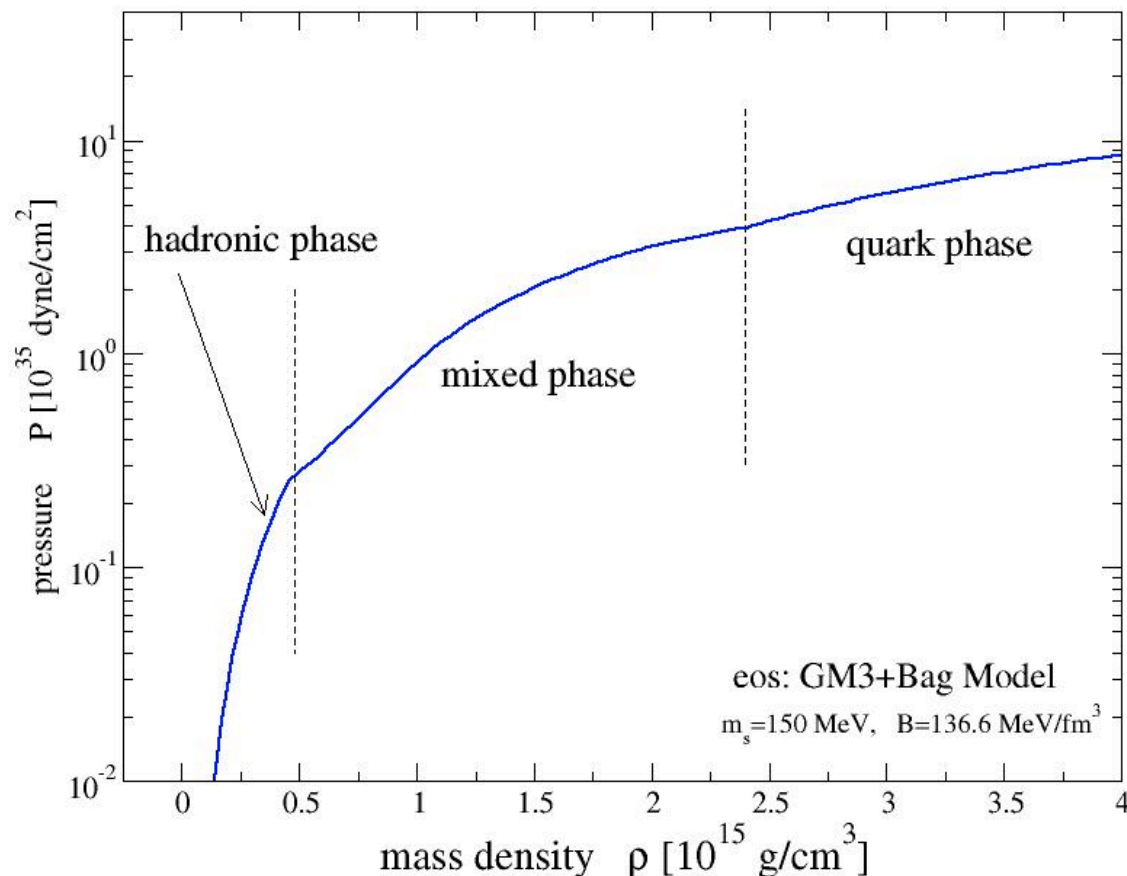
Quark phase :

EOS based on the MIT bag model for hadrons.

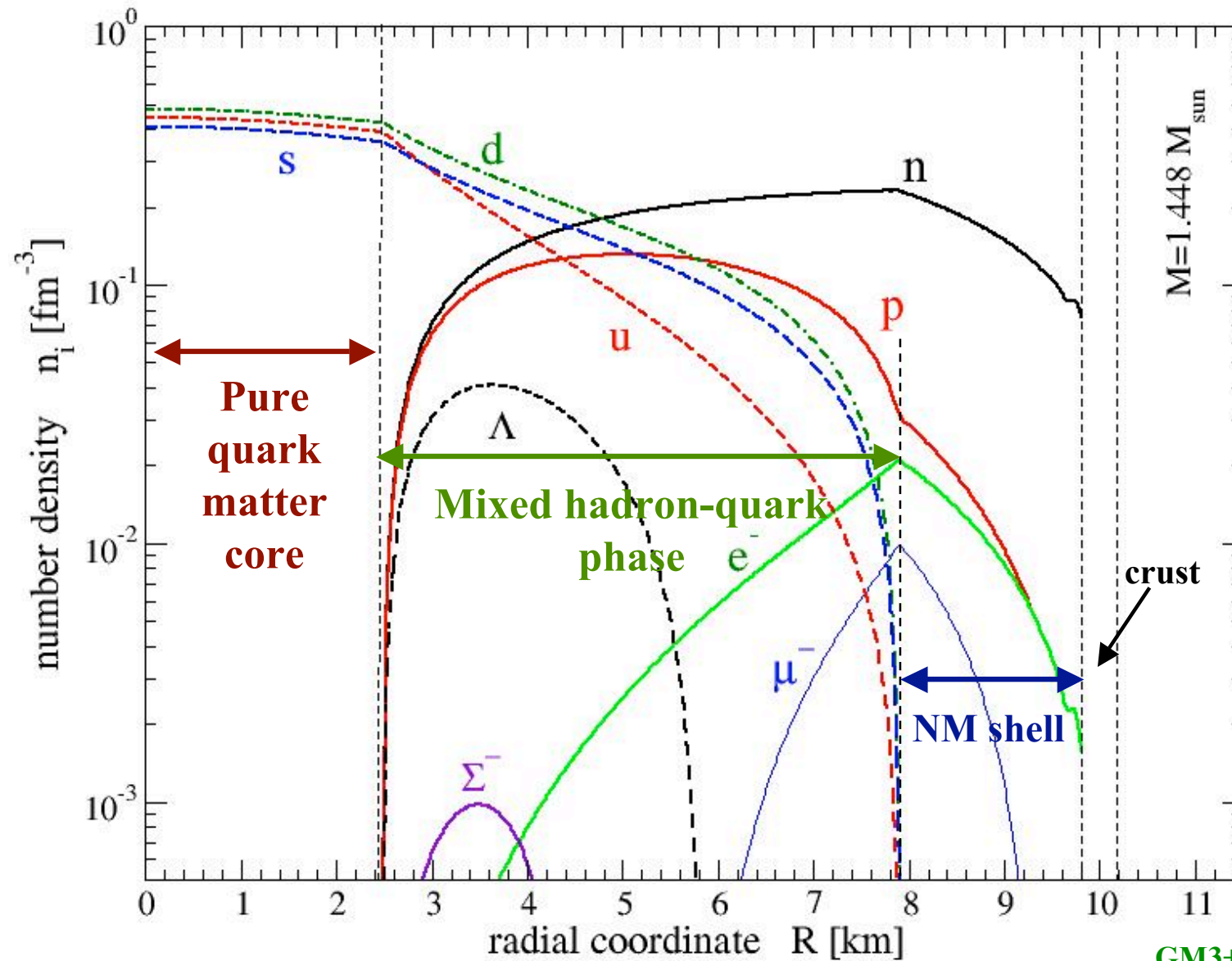
[Farhi, Jaffe, Phys. Rev. D46(1992)]

Mixed phase :

Gibbs construction for a multicomponent system with two conserved “charges”. [Glendenning, Phys. Rev. D46 (1992)]

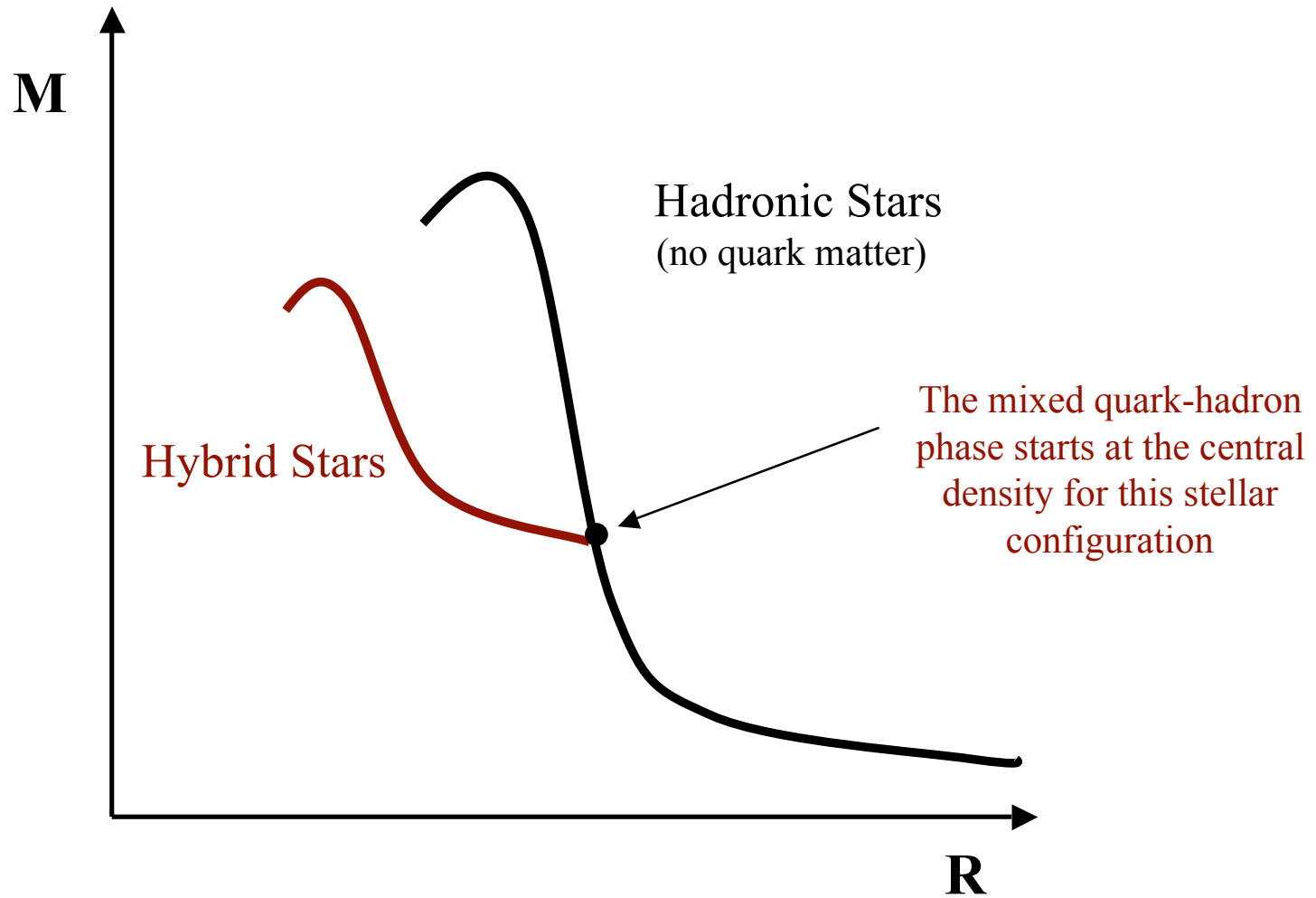


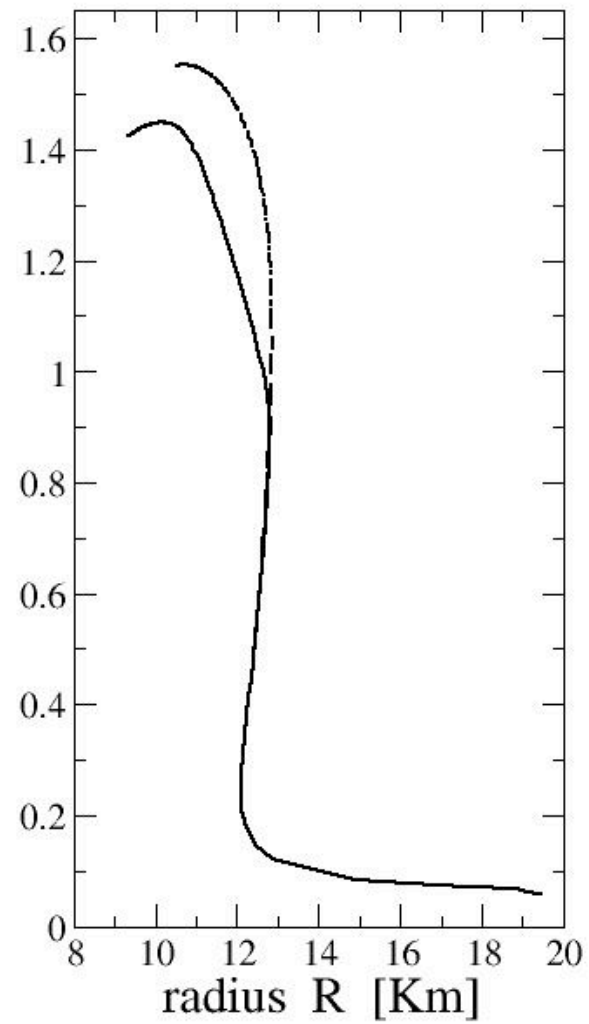
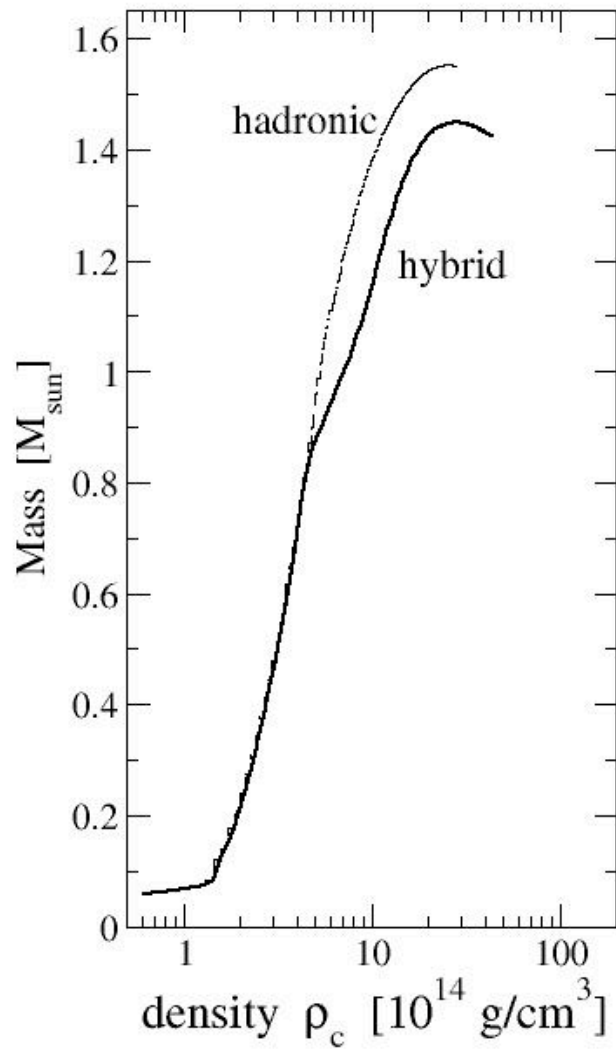
Hybrid Star Composition



GM3+Bag model
 $m_s = 150 \text{ MeV}$, $B = 13.6.6 \text{ MeV}/\text{fm}^3$

The mass-radius relation for hybrid stars





EOS: GM3 + Bag model
 ($B=136 \text{ MeV/fm}^3$, $m_s=150 \text{ MeV}$)

The hadron matter to quark matter phase transition

As a simple substance (i.e., with only one conserved charge) is **compressed at constant T** or **cooled at constant pressure**, it begins condensing the other phase at certain point. This phase of initial transformation from one pure phase to another, assumed to be in equilibrium is referred to as the **mixed or coexistence phase**, and consist of **the two phases in their respective equilibrium states**, no matter what their proportions.

In the language of Gibbs, **the two phases are in equilibrium when their chemical potentials, temperatures and pressures are equal**, corresponding respectively to chemical, thermal and mechanical equilibrium:

$$\mu_1 = \mu_2 = \mu$$

$$T_1 = T_2 = T$$

$$P_1(\mu, T) = P_2(\mu, T) = P$$

At constant T the last equation can be solved for the unique value of μ independent of the proportion of the two phases in equilibrium. This value specifies the constant but unequal values of the energy density of each phase in equilibrium

$$\varepsilon_1 \equiv \varepsilon_1(\mu) \neq \varepsilon_2(\mu) \equiv \varepsilon_2$$

A neutron star, however, has two conserved charges: electric and baryonic. Therefore, there are two independent chemical potentials: μ_q and μ_b , and the condition for equal pressures reads

$$P_H(\mu_b, \mu_q, T) = P_Q(\mu_b, \mu_q, T)$$

The condition of local charge neutrality reduces the problem to the one of a simple substance

$$q_H(\mu_b, \mu_q) = 0 \qquad q_Q(\mu_b, \mu_q) = 0$$

We can solve the above two equations in the form

$$\mu_q = f(\mu_b) \quad \tilde{\mu}_q = g(\mu_b)$$

Therefore

$$P_H(\mu_b, f(\mu_b), T) = P_Q(\mu_b, g(\mu_b), T)$$

By demanding that each phase in equilibrium be separately charge-neutral, the phase transition has been made resemble that of a simple substance.

However, such treatment of the phase transition will produce a discontinuity in the energy density at the radial coordinate where the pressure is equal to that of the mixed phase.

Charge neutrality, **however**, is a **global restriction** and not a local one. **Nature requires only global neutrality**; if it is energetically favorable to have charges separated, it will be so (e.g., atomic nuclei)

Consider again the Gibbs condition

$$P_H(\mu_b, \mu_q, T) = P_Q(\mu_b, \mu_q, T)$$

and assume the **weaker** condition of **global charge neutrality**

$$\chi Q_H(\mu_b, \mu_q, T) + (1 - \chi) Q_Q(\mu_b, \mu_q, T) = 0, \quad \chi = \frac{V_H}{V_H + V_Q}$$

Both equations can be solved for a given T and volume fraction $0 < \chi < 1$

$$\mu_b = \mu_b(\chi) \quad \mu_q = \mu_q(\chi)$$

These equations prove that the properties of each phase in equilibrium of a multicomponent substance vary as the proportion of the phases, in particular the common pressure. As a consequence, the mixed phase corresponding to any phase transition in neutron star matter will occupy the radial extent spanned by the pressure variation defined by the previous equations with $0 < \chi < 1$

Once the chemical potentials have been obtained, the averaged baryon number density and the averaged energy density of the mixed phase can be computed from

$$\chi n_H(\mu_b, \mu_q, T) + (1 - \chi) n_Q(\mu_b, \mu_q, T) = \langle n \rangle$$

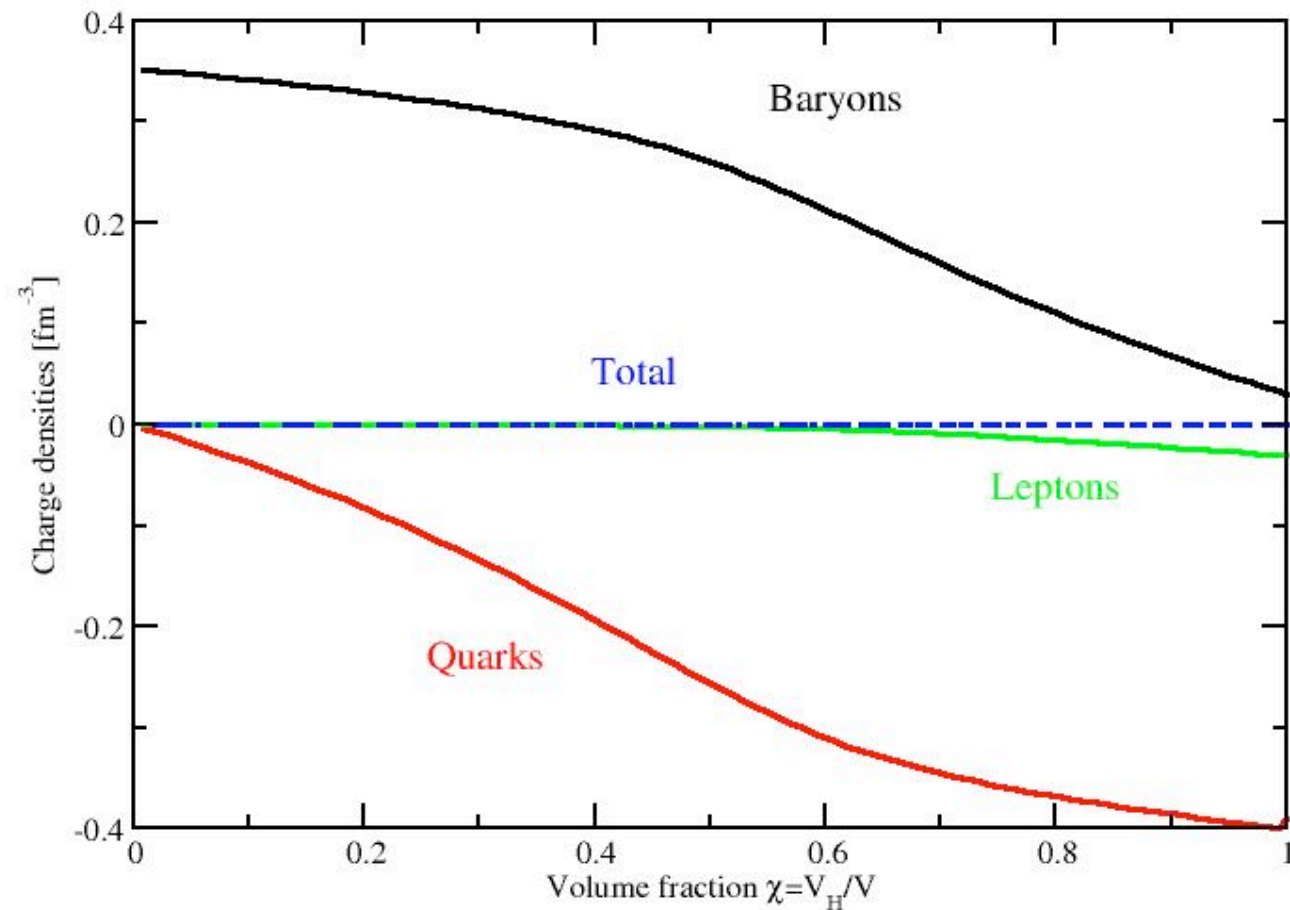
and

$$\chi \varepsilon_H(\mu_b, \mu_q, T) + (1 - \chi) \varepsilon_Q(\mu_b, \mu_q, T) = \langle \varepsilon \rangle$$

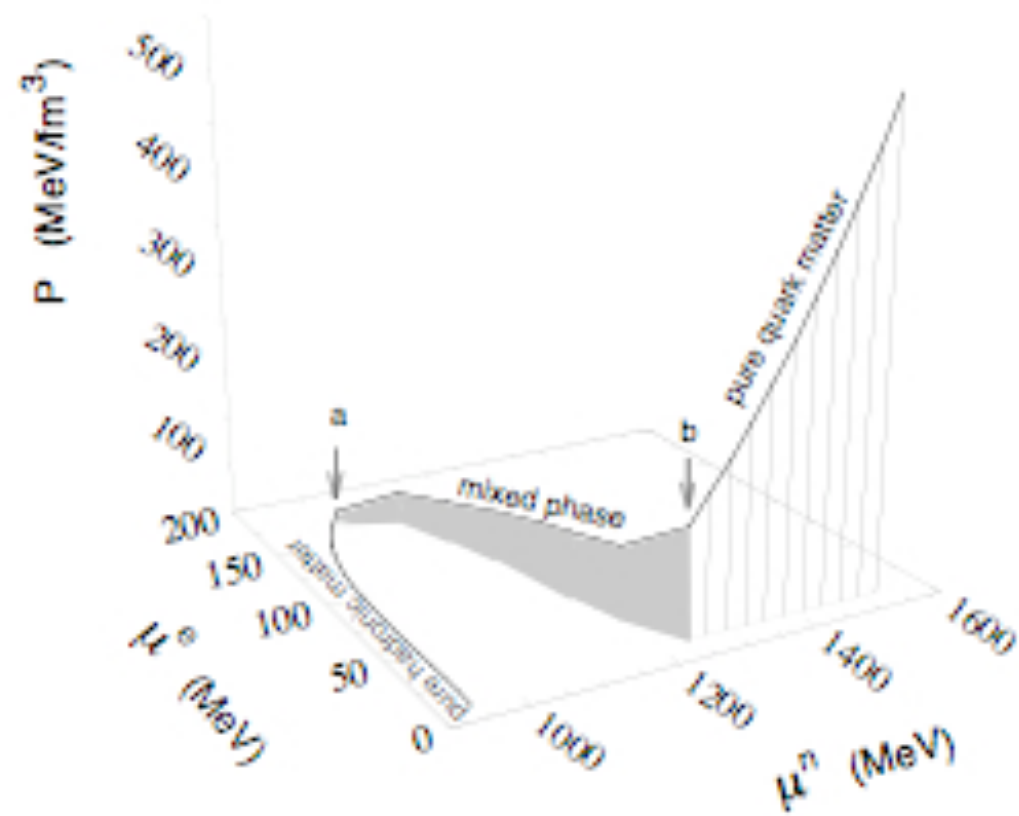
Global Charge Neutrality: $\chi Q_H + (1 - \chi)Q_Q + Q_L = 0$

$\chi = 1$ → pure hadron phase

$\chi = 0$ → pure quark phase



Phase diagram



The Strange Matter Hypothesis

Bodmer (1971), Tereza (1979) & Witten (1984)

Three-flavour **u,d,s quark matter** in equilibrium with respect to the weak interactions, could be the **true ground state of strongly interacting matter**, rather than ^{56}Fe

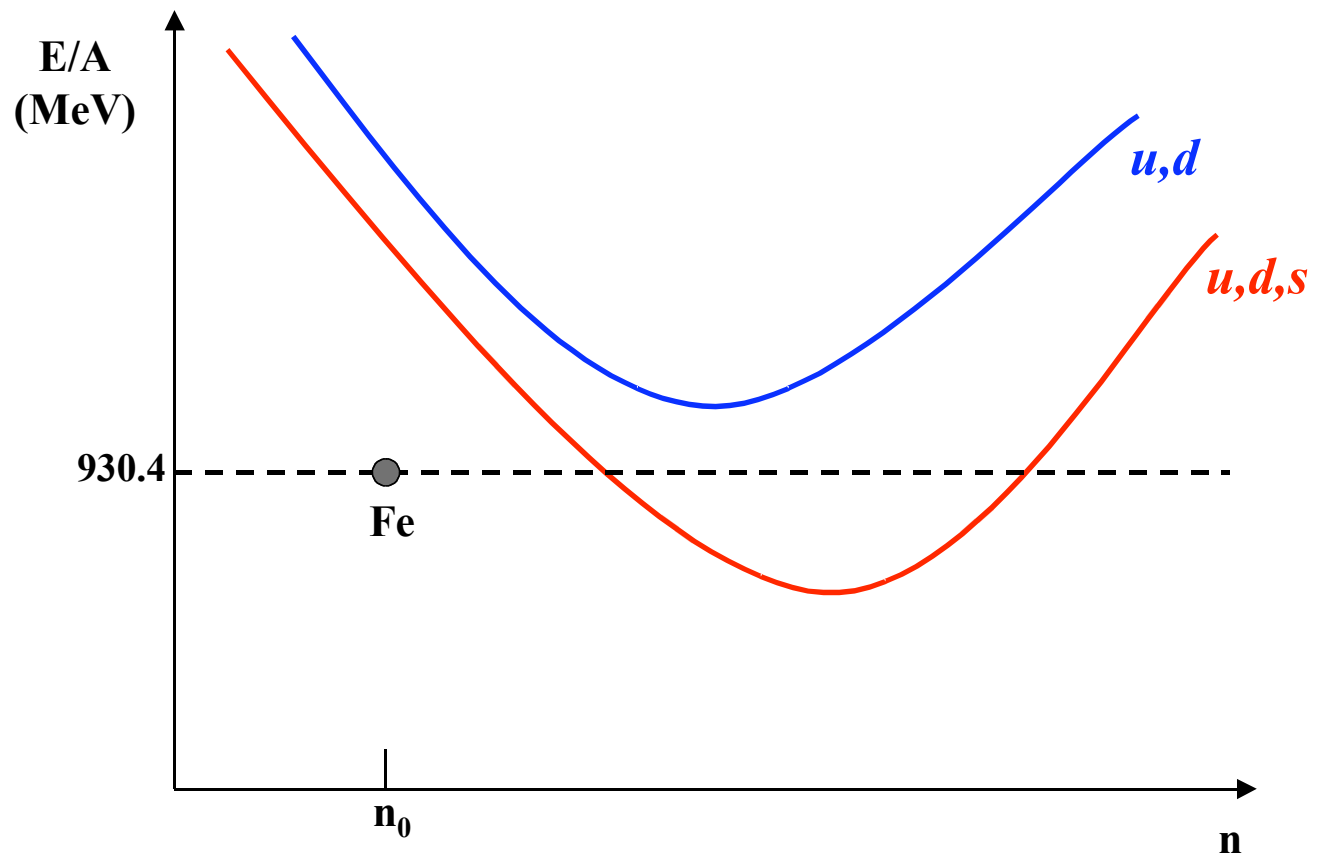
$$E/A|_{\text{SQM}} < E(^{56}\text{Fe})/56 \sim 930 \text{ MeV}$$

Stability of nuclei with respect to u,d quark matter

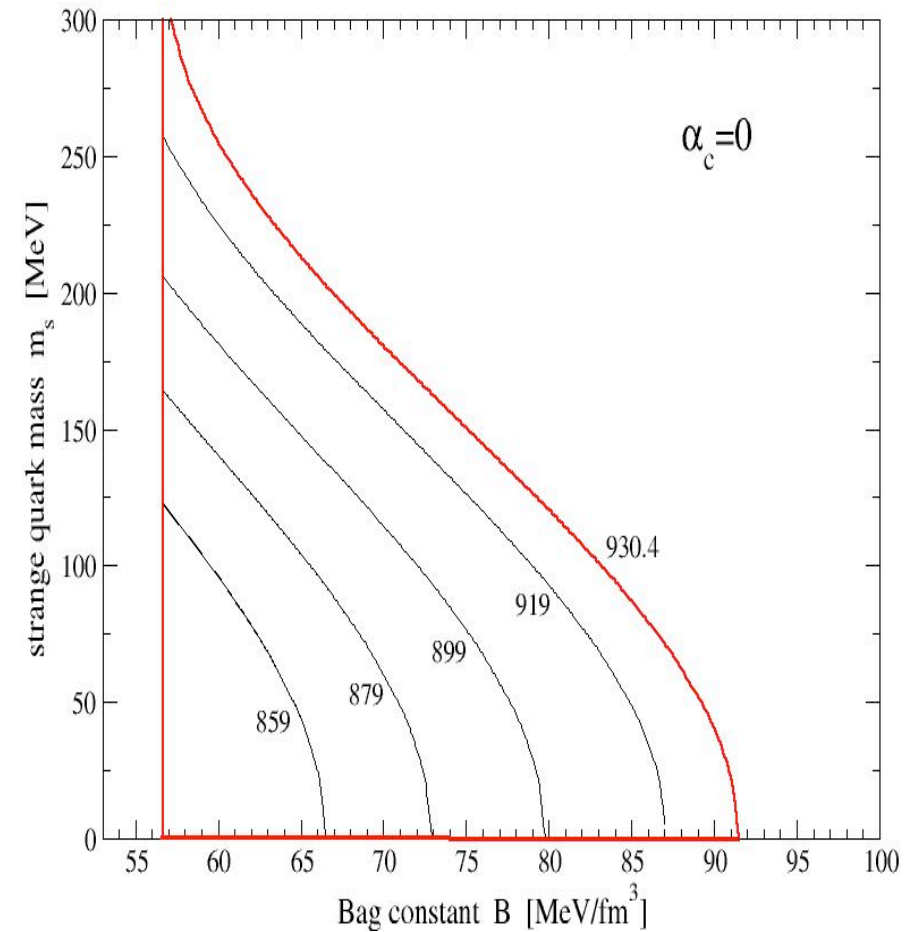
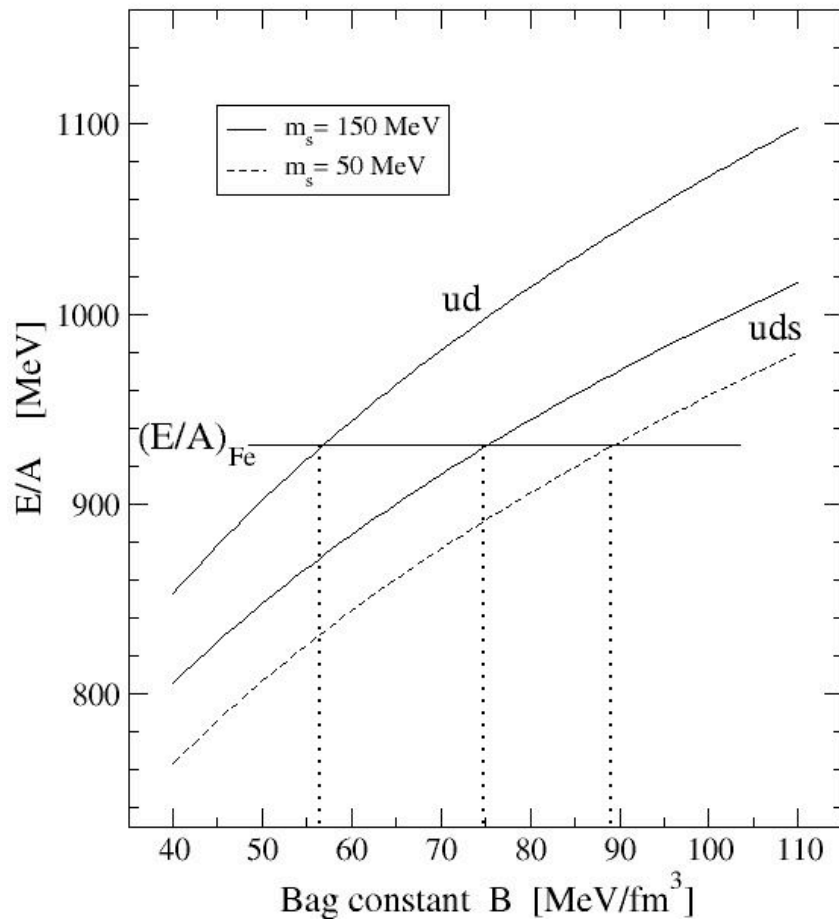
The success of traditional nuclear physics provides a clear indication that **quarks in the atomic nuclei are confined within neutrons and protons**

$$E/A|_{\text{ud}} > E(^{56}\text{Fe})/56 \sim 930 \text{ MeV}$$

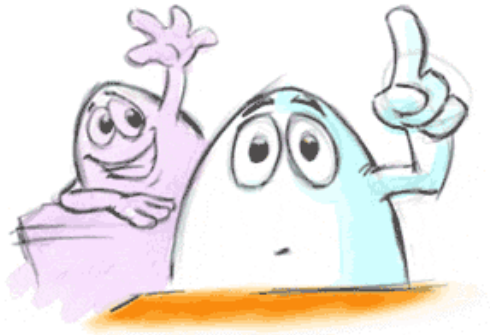
schematically



In the case of the simple EoS for strange quark matter considered, the **parameter space of validity** of the Bodmer-Terezawa-Witten hypothesis is the following

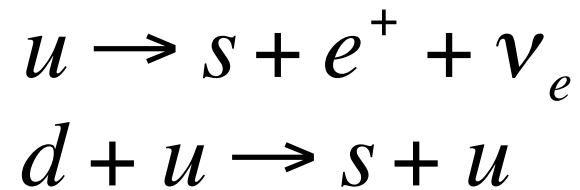
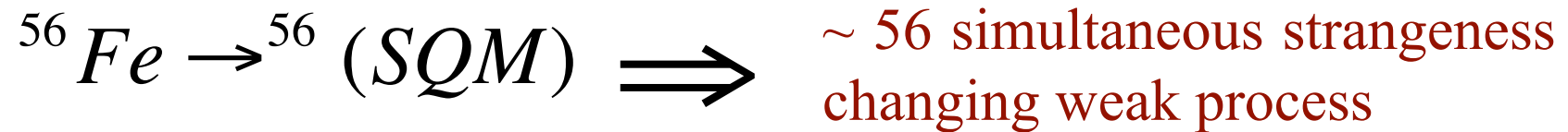


- If the SQM hypothesis is true, **why nuclei do not decay into SQM droplets (strangelets) ?**
- One should explain the **existence of atomic nuclei** in Nature



Stability of Nuclei with respect to SQM

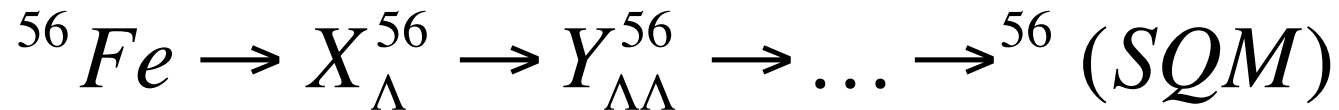
- Direct decay of ^{56}Fe to a SQM droplet



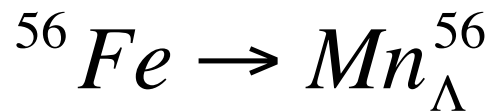
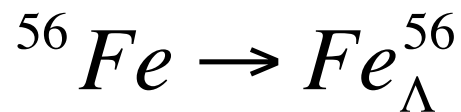
The probability for the direct decay is $P \sim (G_F^2)^{56} \sim 0$
and the **mean-life time** of ^{56}Fe with respect to the
direct decay to a drop of SQM is

$$\tau \gg \text{age of the Universe}$$

- Step by step decay of ^{56}Fe to a SQM droplet



e.g.,



These processes are not energetically possible since

$$Q = M(^{56}\text{Fe}) - M(X_{\Lambda}^{56}) < 0$$

Thus, according with the Bodmer-Terezawa-Witten hypothesis, nuclei are metastable states of strong interacting matter with a mean-life time

$$\tau \gg \text{age of the Universe}$$

Masses and binding energies of Compact Stars

● Gravitational mass: $M \equiv M_G = m(R) = \int_0^R 4\pi r^2 \rho(r) dr$

● Baryonic mass: $M_B = m_u N_B = m_u \int_0^R \frac{4\pi r^2 n(r) dr}{\sqrt{1 - 2Gm(r)/c^2 r}}$

M_B is the **rest mass** of N_B baryons (dispersed at infinity) which form the compact object.

● Proper mass: $M_P = \int_0^R \frac{4\pi r^2 \rho(r) dr}{\sqrt{1 - 2Gm(r)/c^2 r}}$

M_P is the sum of the mass elements on the whole volume of the star, it includes the contributions of the **rest mass** and **internal energy**.

○ Gravitational energy:

$$E_G = (M_G - M_P)c^2 = c^2 \int_0^R 4\pi r^2 dr \rho(r) \left[1 - \left(1 - 2Gm(r)/c^2 r \right)^{-1/2} \right] \leq 0$$

Its opposite $B_G = -E_G$ is called the **gravitational binding energy**.

In the **Newtonian limit**:
$$E_G^{Newt} = -G \int_0^R 4\pi r^2 dr \frac{m(r)\rho(r)}{r}$$

○ Internal energy:

$$E_I = (M_P - M_B)c^2 = \int_0^R \varepsilon'(r) \frac{4\pi r^2 dr}{\sqrt{1 - 2Gm(r)/c^2 r}}$$

$B_I = -E_I$ is called the **internal binding energy**.

- Total energy and total binding energy:

$$E = M_B c^2 + E_I + E_G, \quad B = B_I + B_G$$

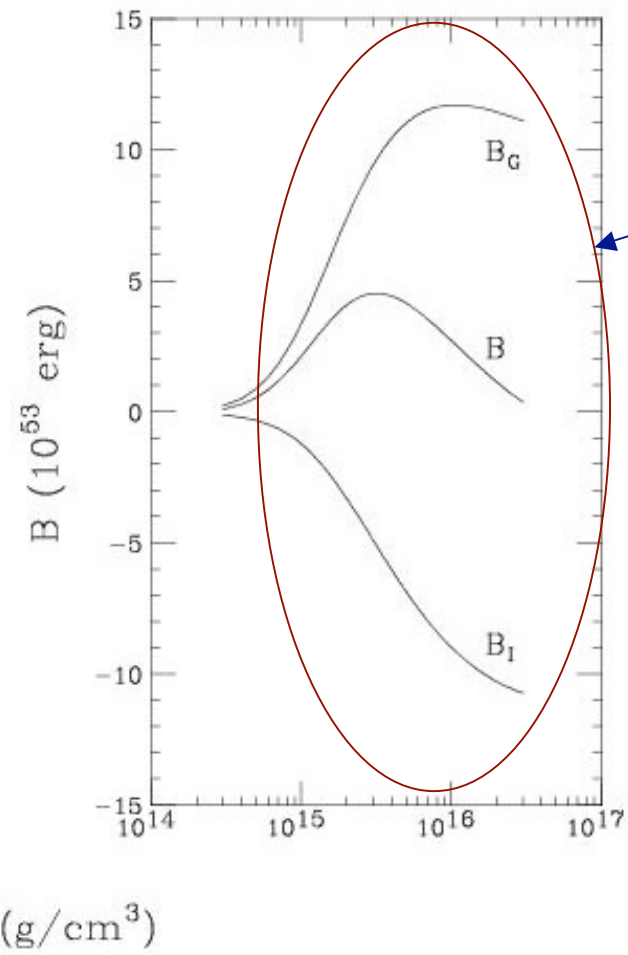
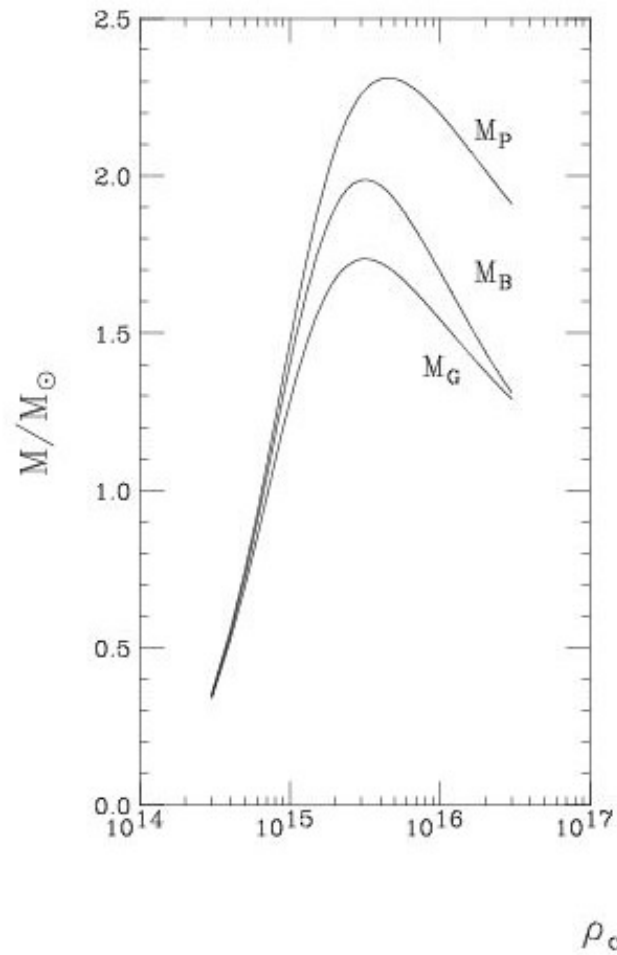
using the previous definitions

$$E = M_G c^2$$

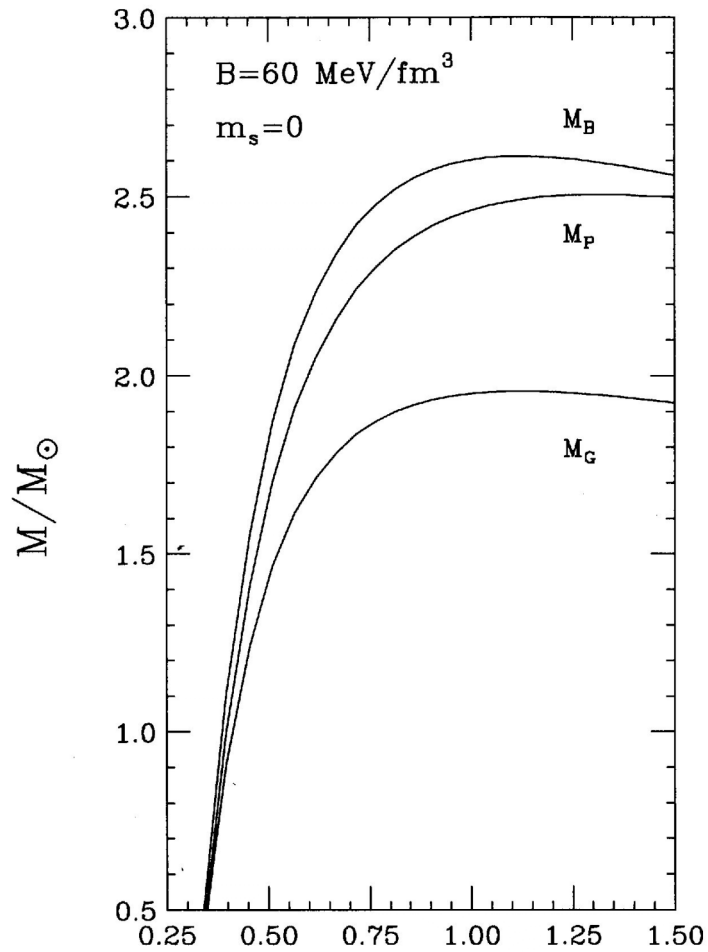
The **gravitational mass** of a compact star, represents the **total energy** ($E=M_G c^2$) of the star, including both the **rest mass energy** $M_B c^2$ of its constituents dispersed at infinity, and the mass-energy contribution coming from the **microscopic motion** and the **interactions** (including gravitation) between the star's constituents.

$$M_G c^2 = M_B c^2 + E_I + E_G$$

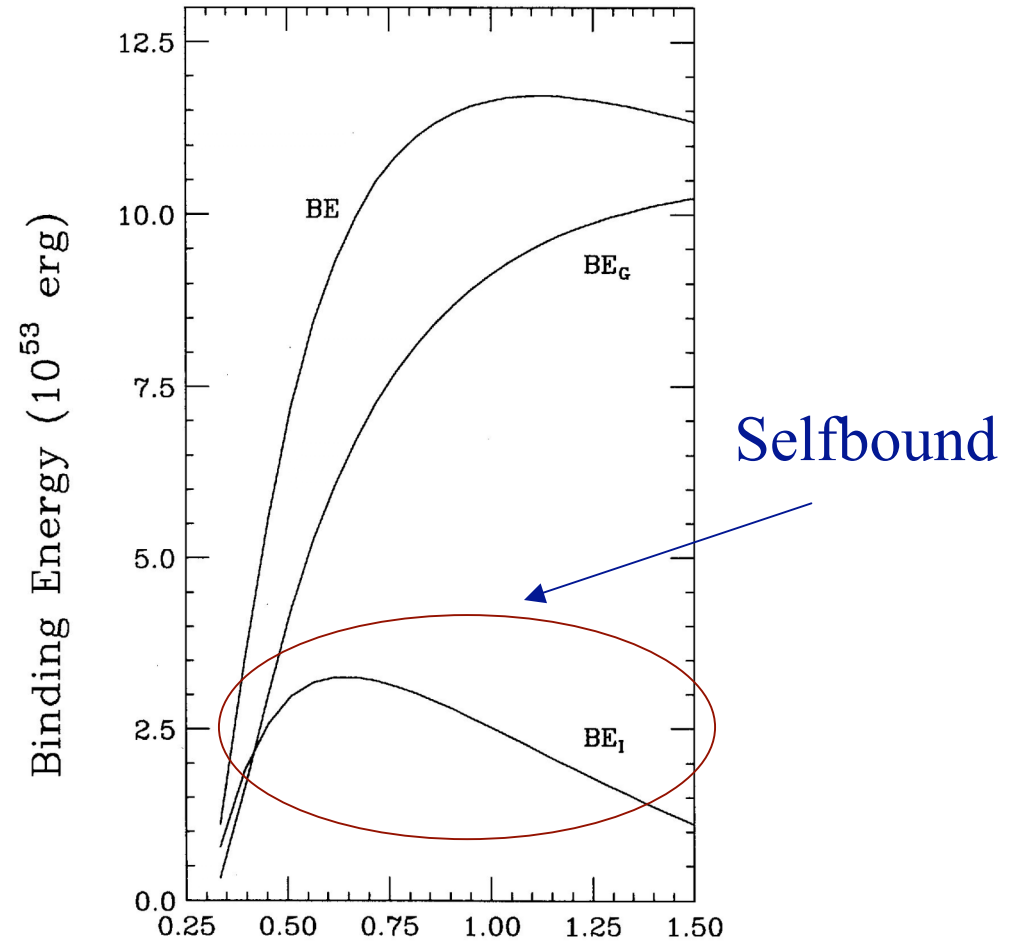
Masses and binding energies of Hadronic Stars



Masses and binding energies of Strange Stars

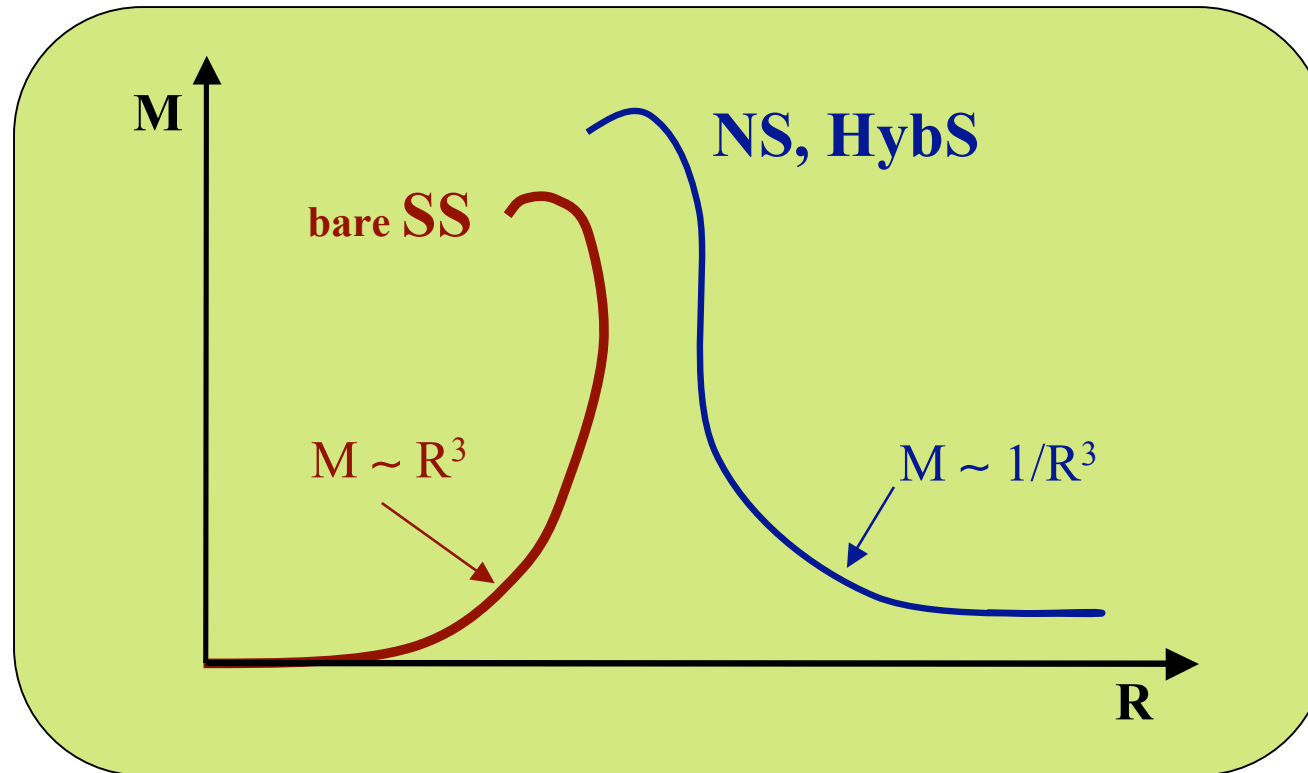


n_c (fm⁻³)



Selfbound

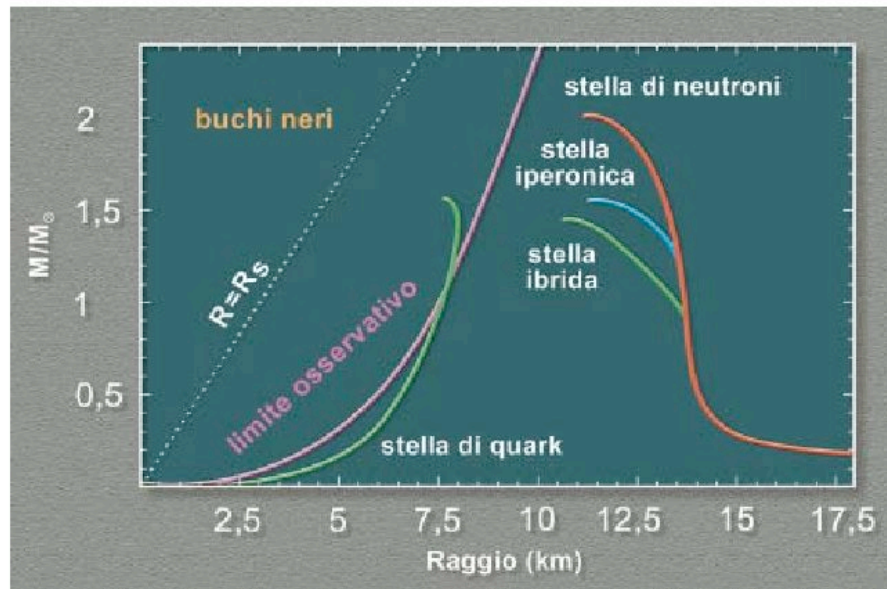
Mass-radius relation



- **Strange Stars are self-bound bodies**, at zero external pressure, they are **bound by the strong interactions**. Note that an EoS of the type $P=(\epsilon-4B)/3 \rightarrow M \sim R^3$
- Neutron Stars (Hadronic Stars) are **bound by gravity**.

One of the most likely strange star candidate is the **X-ray burster SAX J1808.4-3658**

- Discovered in September 1996 by **Beppo SAX**
- Two bright type-I X-ray burst detected ($\Delta T < 30$ s)
- Millisecond PSR: coherent pulsation with $P=2.49$ ms
- Member of a LMXB: $P_{\text{orb}}=2.01$ hours



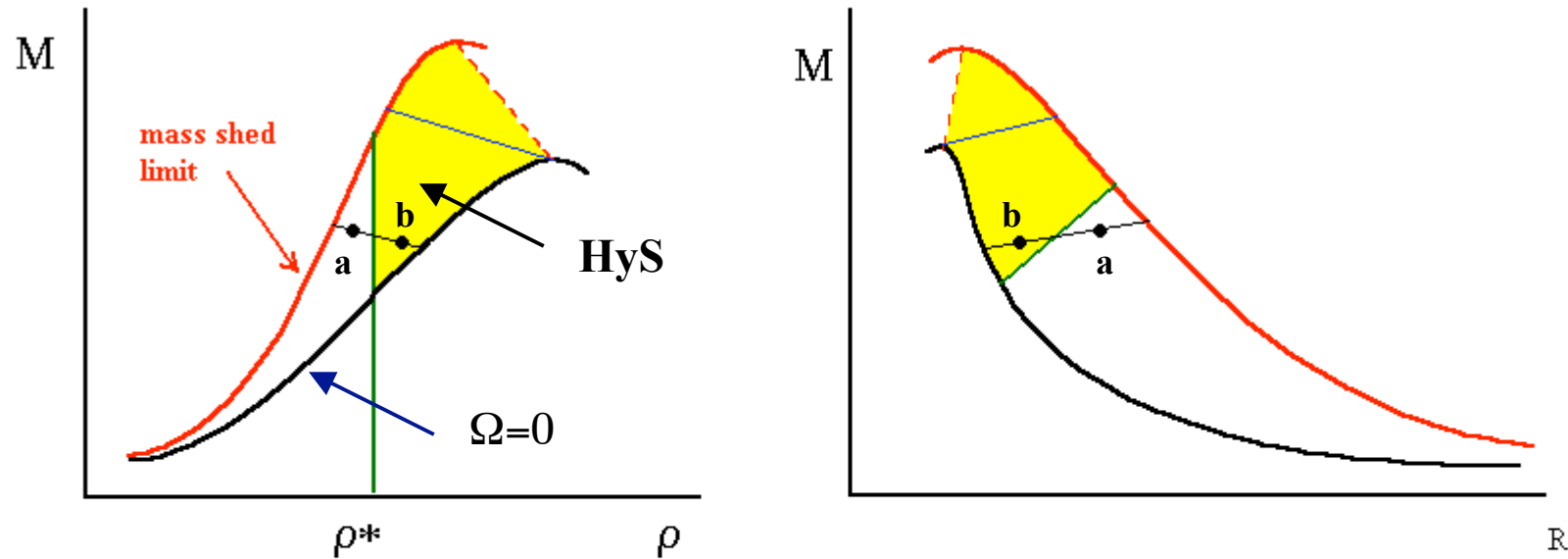
Observational limit by **Li et al., PRL 83, 3776 (1999)**

$$R < 27.5 \left(\frac{F_{\min}}{F_{\max}} \right)^{2/7} \left(\frac{P}{2.49 \text{ ms}} \right)^{2/3} \left(\frac{M}{M_{\text{sun}}} \right)$$

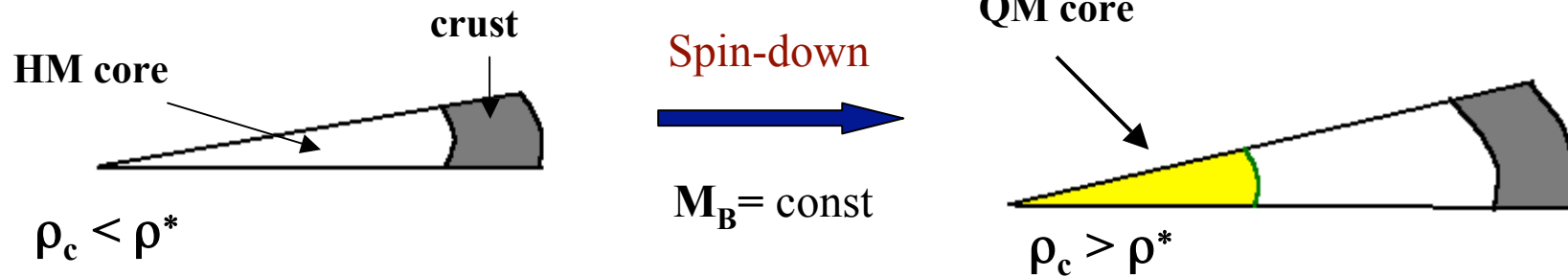
Which are the
observational signals for
the appearance of **Quark
Matter in Compact Stars ?**



Possible signature for deconfinement phase transition in isolated spinning-down neutron stars



ρ^* = critical density for quark deconfinement



Spin-down: J decreases, Ω decreases
 ρ_c increases, I decreases

Consider that the rotational energy lost follows the power law:

$$\frac{dE}{dt} = \frac{d}{dt} \left(\frac{1}{2} I(\Omega) \Omega^2 \right) = -C \Omega^{n+1}$$

n: braking index.

$$C = \frac{2}{3} \mu^2 \sin^2 \alpha \quad \text{in the case of magnetic dipole radiation}$$

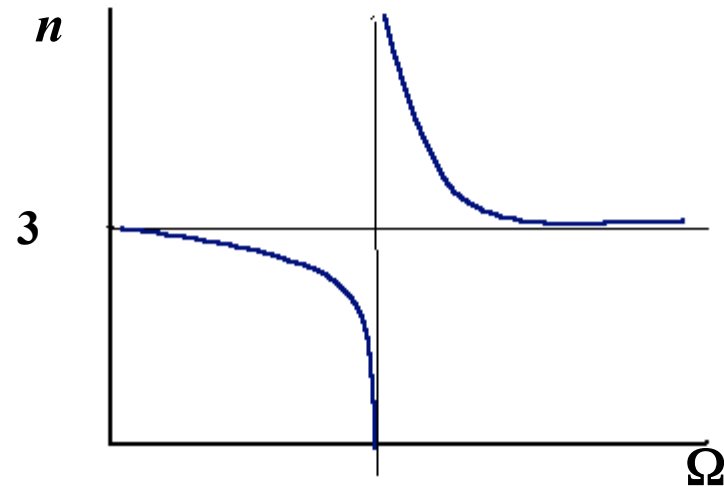
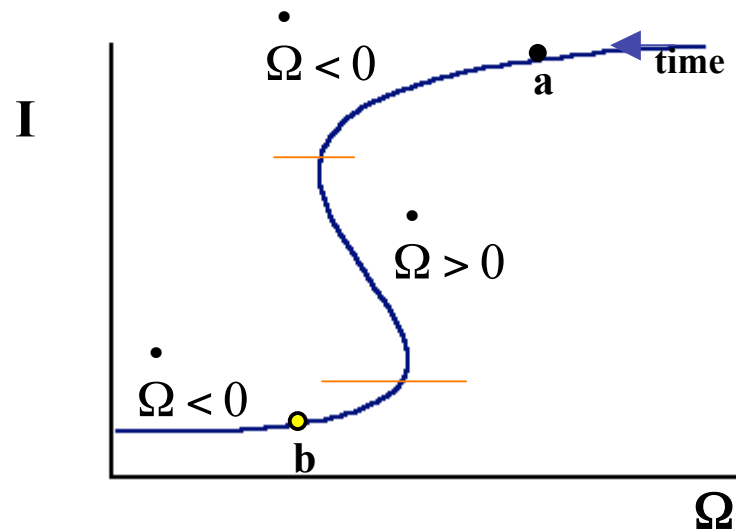
from which, we can obtain the rate of change of the pulsar frequency

$$\dot{\Omega} = -2C \Omega^n \left[2I(\Omega) + \Omega \frac{dI}{d\Omega} \right]^{-1}$$

and the frequency dependence of the braking index n

braking index

$$n(\Omega) \equiv \frac{\Omega \ddot{\Omega}}{\dot{\Omega}^2} + \frac{3I'\Omega + I''\Omega^2}{2I + I'\Omega}$$



measured large value
of the braking index
 $|n| \gg 3$



Observational signature for
quark deconfinement phase
transition in compact stars

Glendenning, Pei, Weber, 1997

Summary & Conclusions II



💡 The hadron star to quark star phase transition:

Gibbs criteria for phase equilibrium.

Global Charge Neutrality.

Hybrid Stars.

💡 The Strange Quark Matter Hypothesis:

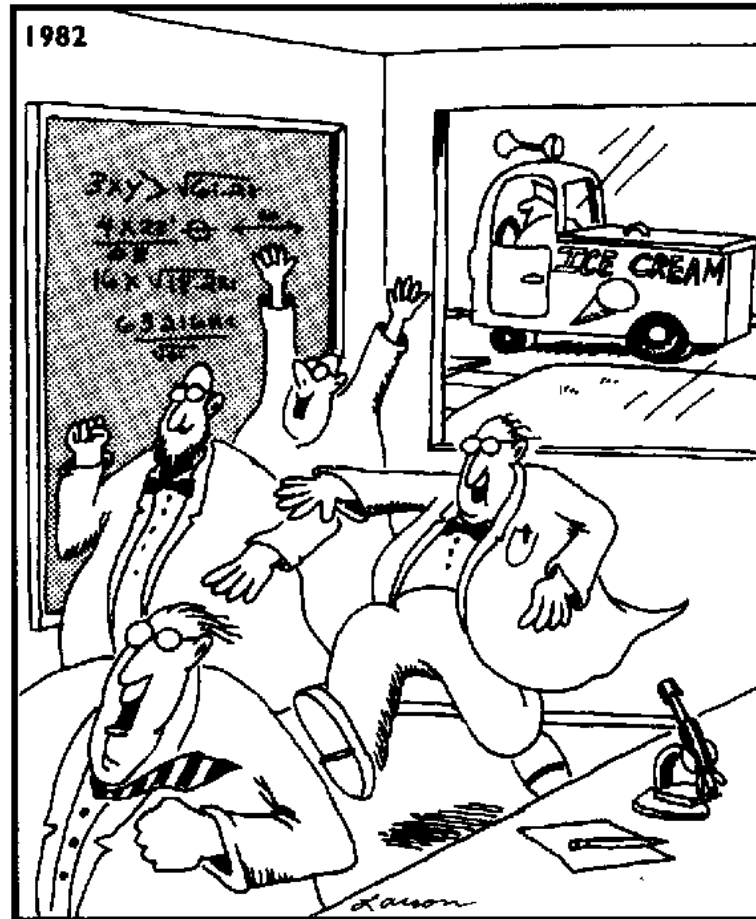
“Strange quark matter is the real ground state of strongly interacting matter”.

Strange Stars

💡 Observational Signals of deconfinement:

Spinning down of isolated rotating Neutron Stars

Time for Coffee and Cookies

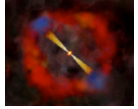


Thanks a lot for your patience

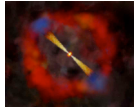


Strange Quark Matter
and
Gamma Ray Burst

Gamma Ray Bursts (GRBs)



One of the most **violent** and **mysterious phenomena** in the Universe. *T. Piran, Phys. Rep. 314, 575 (1999).*



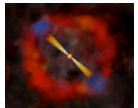
Distances: “Cosmological” $d=(1-10) 10^9$ ly



Energy range: 100 keV - a few MeV

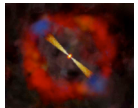


Energy emitted: $\sim 10^{51}$ erg (beamed/jets)

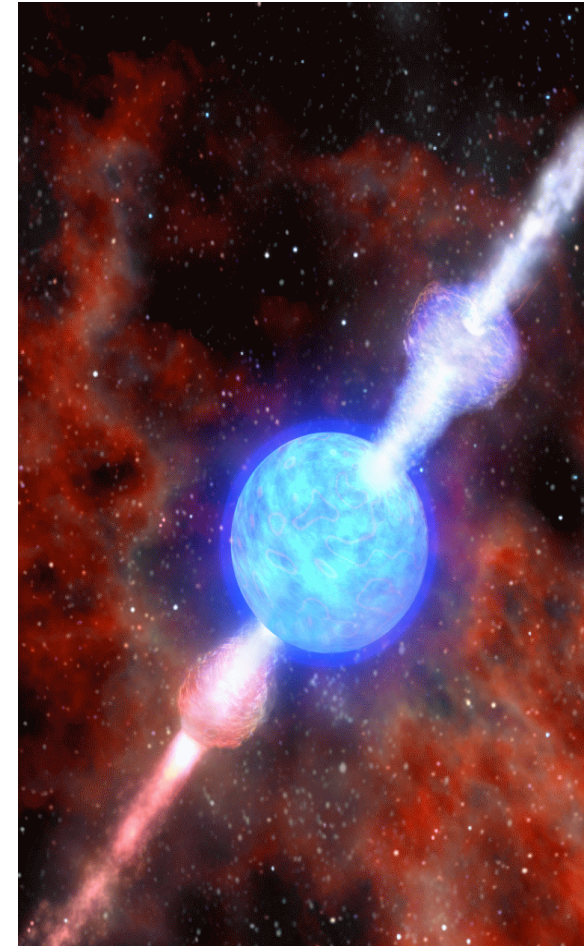


Duration: 1 - 300 s

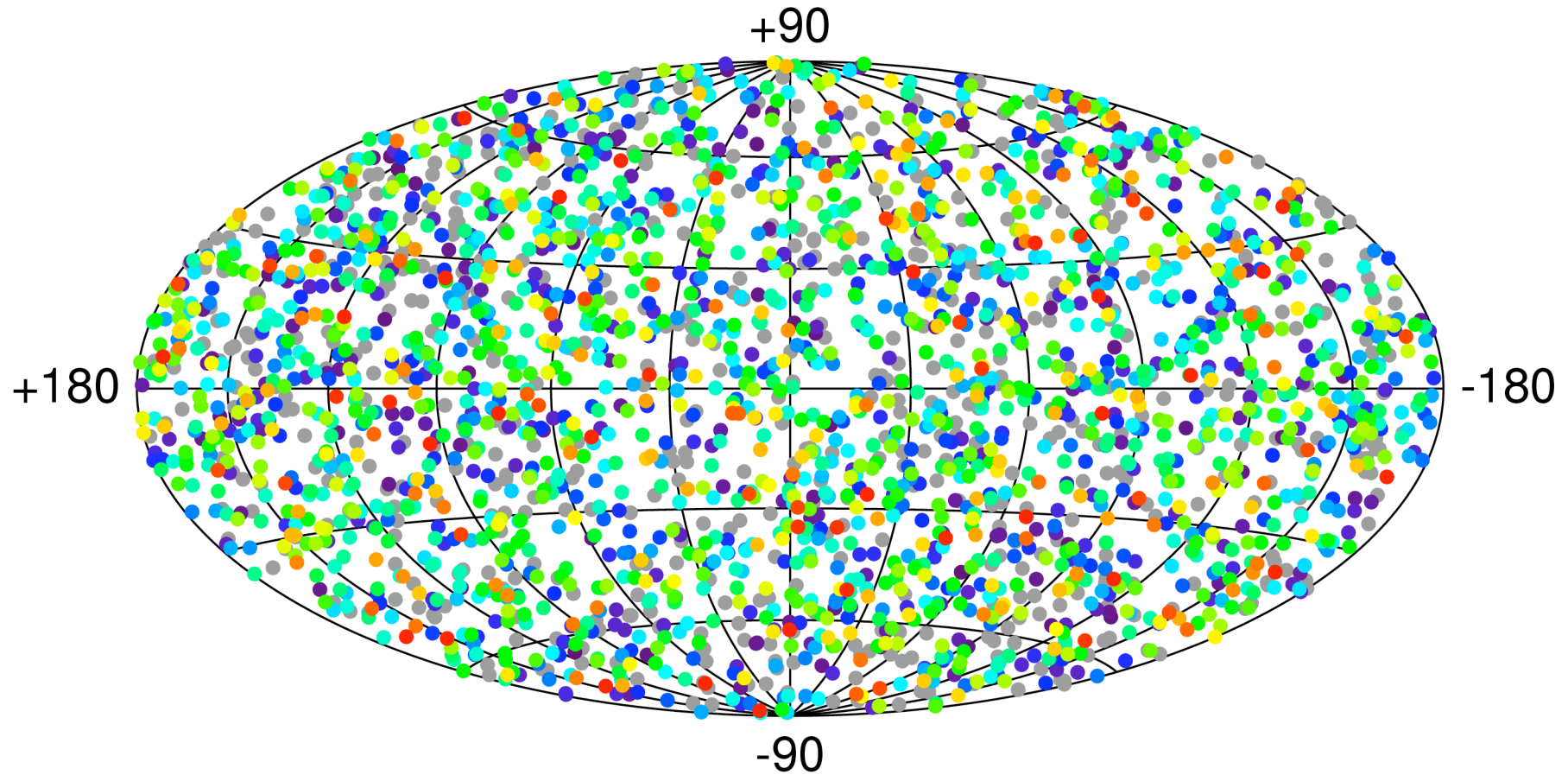
Two different types: **short** and **long** GRBs



Spatial distribution: **Isotropic**



2704 BATSE Gamma-Ray Bursts



10^{-7} 10^{-6} 10^{-5} 10^{-4}

Fluence, 50-300 keV (ergs cm^{-2})

The supernova connection

Peter Mészáros

They are the most energetic events in the Universe, but the origin of γ -ray bursts has been hard to establish. Observations of a burst close to our Galaxy now show that supernovae are, as suspected, likely culprits.

The fog surrounding the identity of the progenitors of γ -ray bursts (GRBs) is beginning to lift, at least for the class of GRBs known as 'long' bursts. This is thanks to a series of observations of a burst that began on 29 March 2003, very close to our Galaxy. On pages 843, 844 and 847 of this issue, Uemura *et al.*¹, Price *et al.*² and Hjorth *et al.*³ reveal the evolution of this burst in unprecedented detail — and show that behind the GRB is the unmistakable signature of a supernova.

The GRB population divides neatly into long ones and short ones, depending on whether the burst of γ -rays lasts more or less than a few seconds⁴. About two-thirds of all observed bursts are long, and these are the only ones for which longer-lasting 'afterglows' or X-ray, optical and radio wavelengths have also been found. These afterglows may last up to several months, and from them the distance to the GRB and the identity of its host galaxy can be determined. There is good evidence that long bursts are largely associated with active, star-forming regions in small, blue galaxies. And, in at least three cases, there has been tantalizing evidence that GRBs are associated with a particular type of supernova⁵ — although that interpretation has so far been fraught with uncertainty.

A 'usual' supernova arises when the core of a massive star collapses, ejecting the stellar outer envelope. The majority of such supernovae result from parent stars that are less than about 30 times heavier than the Sun, and the core collapse produces a neutron star. These supernovae are normally detected weeks after the collapse, because the ejected envelope only brightens sufficiently to be detected at optical wavelengths some weeks later. The only signals of the collapse that are expected to reach the Earth promptly are a flux of tiny particles called neutrinos (which was picked up for the supernova SN1987a by the Japanese neutrino detector Kamio-kande) and gravitational waves (which have yet to be detected).

For heavier stars, however, the core is thought to collapse into a black hole, and the resulting brief episode of mass accretion has been proposed as the central engine driving GRBs⁶. This kind of collapse was initially referred to as a 'failed' supernova, as it was thought that the stellar envelope would not be ejected. A GRB would instead result from

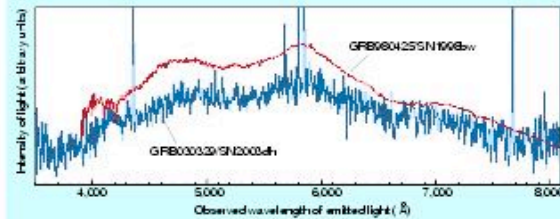


Figure 1 A good match. The spectra of the wavelengths of radiation from the γ -ray burst GRB030329, believed to be associated with the supernova SN2003dh, and from GRB980425/SN1998bw are remarkably similar in shape⁶, suggesting that in general the GRB and supernova phenomena are related. Detailed observations^{2,3} of GRB030329 offer the strongest proof yet that γ -ray bursts are indeed produced by supernovae that result when the core of a massive star collapses.

a relativistic jet of gas fed by the black hole; it would break through the stellar envelope, leading to radiative shocks in the rarified environment outside the star.

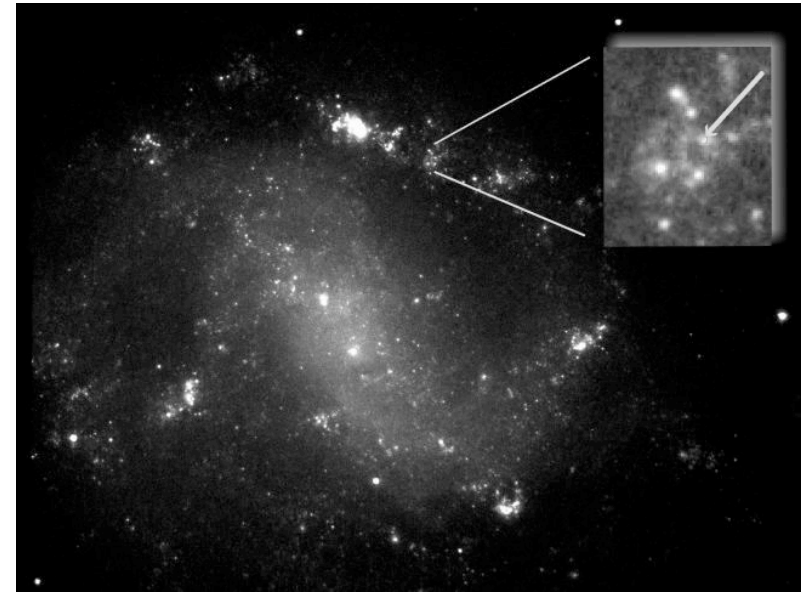
In 1998, observations of GRB980425 showed an anomalous brightening of its optical afterglow a few weeks after the burst, possibly linking it to a roughly contemporaneous supernova, known as SN1998bw, whose ejected envelope would have brightened at about that time. Suspicion grew that long GRBs might, after all, be associated with 'successful' supernovae. In fact, the few supernovae tentatively linked to GRBs appeared even more energetic than usual, and were dubbed 'hypernovae'⁶, or 'collapsars'⁷. There is also a more elaborate offshoot of the supernova idea — the 'supernova'⁸. Here, the core collapse is assumed to be a two-step affair: the first step produces a temporary neutron star and a supernova; in the second step, a few weeks or months later, the neutron star collapses into a black hole, producing a GRB.

The association of long GRBs with supernovae (or even supernovae) is based on the approximate coincidence in time of the GRB and the inferred instant of the supernova core-collapse. The latter is deduced from an extrapolation back from the peak brightness of emitted light — an extrapolation that is model-dependent and uncertain, not least because these objects are so far away and very faint. This situation changed dramatically with the observation of GRB030329 in March of this year. Its coordinates were

determined by the HETE-2 spacecraft within 90 minutes of its detection, enabling ground-based telescopes to make follow-up observations almost immediately. Although more than two billion light years away, GRB030329 may be the nearest cosmological GRB yet seen⁹. (In terms of the conventional astronomical distance measure, its 'redshift', z , is 0.169; previous GRBs have usually only been seen in the range 0.4–4.5; the exception is GRB980425, if its association with SN1998bw at $z=0.008$ is real.)

After a week, the pattern of light emitted by GRB030329 — its 'light curve' — started to show the beginnings of a slight bump. Ten days later, this bump was identified as being caused by an energetic supernova, labelled SN2003dh¹⁰. Because this GRB is relatively close to us, the identification of the light-curve peak, and its wavelength spectrum, is significantly stronger than in previous cases. The extrapolation from the peak brightness indicates that the time offset between the GRB and the collapse of the supernova is unlikely to be greater than about two days, and is comparable with the two events being simultaneous. Hjorth *et al.*³ interpret this as ruling out the supernova model: if the two-step collapse of a supernova were to happen within two days, it is unlikely that its γ -ray emission and afterglow would match the observations.

Another remarkable feature of GRB030329 is the spectrum of wavelengths it has emitted. There is a tantalizing match between the shape of the distribution for GRB030329/SN2003dh and that of



ESO 184-682 SN-GRB connection

P. Mészáros, *Nature* 423, 809 (2003)

Discovery of a Transient Absorption Edge in the X-ray Spectrum of GRB 990705

Lorenzo Amati,^{1*} Filippo Frontera,^{1,2} Mario Vietri,³
Jean J. M. in 't Zand,⁴ Paolo Soffitta,⁵ Enrico Costa,⁵
Stefano Del Sordo,⁶ Elena Pian,¹ Luigi Piro,⁵ Lucio A. Antonelli,⁷
D. Dal Fiume,¹ Marco Feroci,⁵ Giangiacomo Gandolfi,⁵
Cristiano Guidorzi,² John Heise,⁴ Erik Kuulkers,⁴ Nicola Masetti,¹
Enrico Montanari,² Luciano Nicastro,⁶ Mauro Orlandini,¹
Elia Palazzi¹

We report the discovery of a transient equivalent hydrogen column density with an absorption edge at ~ 3.8 kiloelectron volts in the spectrum of the prompt x-ray emission of gamma-ray burst (GRB) 990705. This feature can be satisfactorily modeled with a photoelectric absorption by a medium located at a redshift of ~ 0.86 and with an iron abundance of ~ 75 times the solar one. The transient behavior is attributed to the strong ionization produced in the circumburst medium by the GRB photons. The high iron abundance points to the existence of a burst environment enriched by a supernova along the line of sight.

The supernova explosion is estimated to have occurred about 70 years before the burst. Our results agree with models in which GRBs originate from the collapse of very massive stars and are preceded by a supernova event.

The nature of the progenitors of celestial GRBs is an open issue of key astrophysical importance. Collapse of massive fast-rotating stars—the hypernova model (1)—or delayed collapse of a rotationally stabilized neutron star—the supranova model (2)—are among the favored scenarios for the origin of these events. Both models predict that the preburst

and, ultimately, the nature of the GRB progenitor. Owing to the coalignment of two detection units of the gamma-ray burst monitor (GRBM, 40 to 700 keV) (3, 6) with the

two wide-field cameras (WFCs, 2 to 26 keV) (7), the Italian-Dutch x-ray mission BeppoSAX can provide not only arc minute localizations of GRBs, but also measurements of their spectra in a broad (2 to 700 keV) energy band (8). Among the GRBs detected by the BeppoSAX WFC and GRBM, the event of 5 July 1999 (GRB 990705) is the second brightest in γ -rays (40 to 700 keV) after GRB 990123 and ranks in the top 15% in x-rays (2 to 26 keV). This burst triggered the GRBM on 5 July at 16:01:25 universal time and was positioned with an error radius of 3 arc min at right ascension $\alpha(2000) = 05^{\text{h}}09^{\text{m}}52^{\text{s}}$ and declination $\delta(2000) = -72^{\circ}08'02''$, in a direction close to the edge of the Large Magellanic Cloud. Optical and near-infrared observations of the GRB 990705 location led to the discovery of a reddened fading counterpart and a possible host galaxy (9). Recently, Holland *et al.* (10) have imaged the GRB 990705 field with the Hubble Space Telescope, detecting a spiral galaxy at the position of the GRB. Although the distance of this galaxy is not known, its size and brightness are compatible with a redshift $z \leq 1$ (11).

A 120,000 s follow-up observation with the BeppoSAX narrow-field instruments (12) was also performed starting ~ 11 hours after the GRBM trigger. In the first 7 hours of observing time, an x-ray source of 3.3 σ significance, corresponding to 1.9 (± 0.6) \times

SN-GRB connection



Evidence for atomic lines in the spectra of the X-ray afterglow

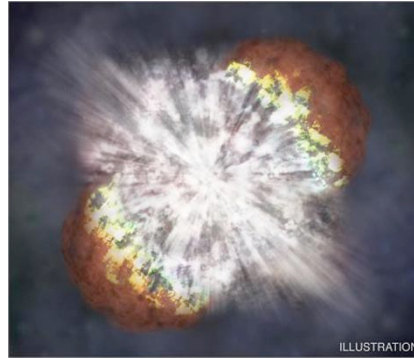


Time delay ΔT between the SN explosion and the GRB

- GRB 990705: $\Delta T \sim 10$ yr
Amati et al., Science 290, 953 (2000)
- GRB 030227: $\Delta T \sim 3 - 8$ days
Watson et al., ApJ 595, L29 (2003)
- GRB 030813: $\Delta T \sim 2$ months
Butler et al., ApJ 597, 1010 (2003)
- GRB 011211: $\Delta T \sim 4$ days
Reeves et al., Nature (2002)

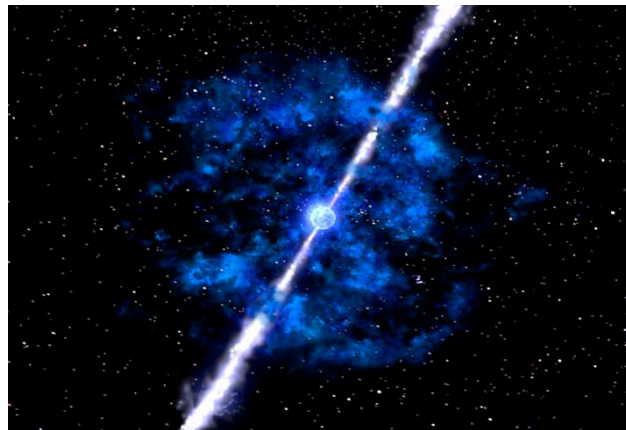
A two-step scenario

1st explosion



Supernova: Birth of the **Neutron Star**

2nd “explosion”



Associated with the **Neutron Star**: central engine of the GRB

Questions

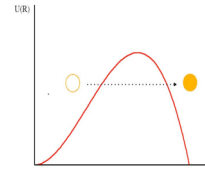
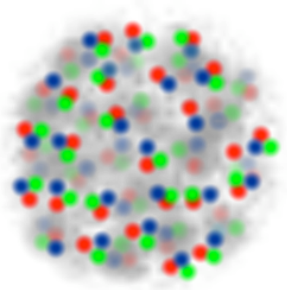


What is the origin of the 2nd “explosion” ?



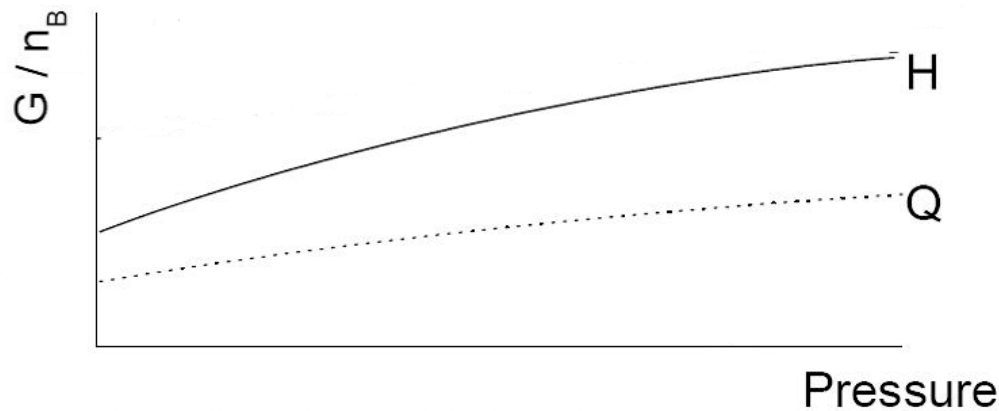
How to explain the different time delays between the two events ?

Formation of a quark matter bubble at the centre of a Neutron Star (I)



β -stable
quark-matter bubble

Direct nucleation of the β -stable quark matter: *high order weak process* \rightarrow *suppressed by a factor $\sim G_F^{2N/3}$, with $N=100-1000$.*

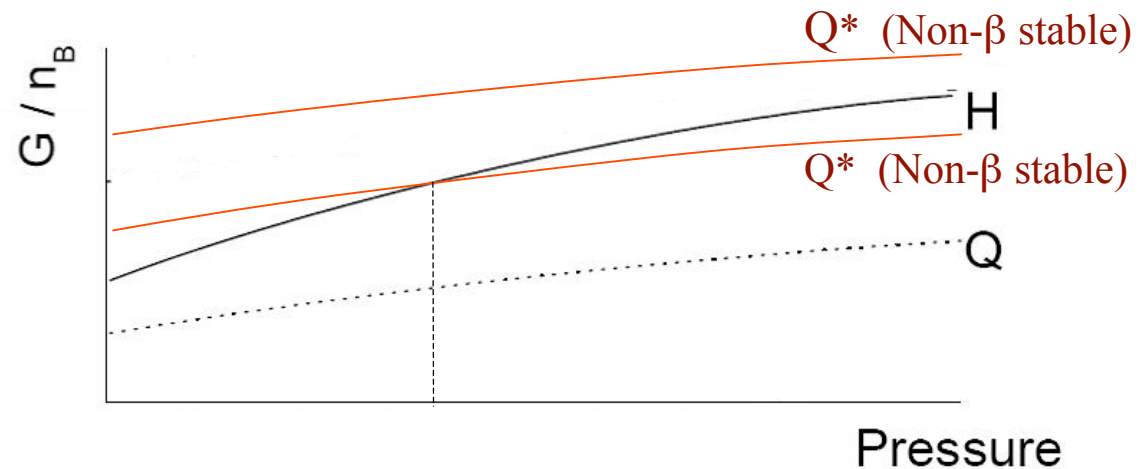
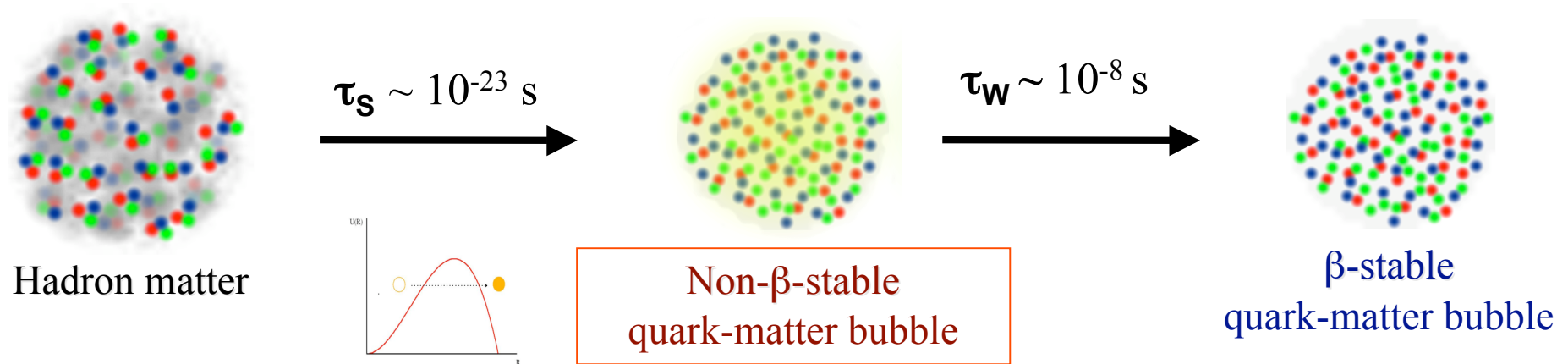


Ruled out: even when the final state has a lower energy

Berezhiani et al. 2003 (unpaired)

Drago, Lavagno & Pagliara 2004 (CFL)

Formation of a quark matter bubble at the centre of a Neutron Star (II)



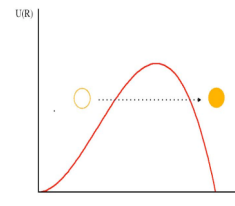
Has the **intermediate phase** lower energy than hadron matter?

Lifshitz-Kagan quantum nucleation theory

Quantum fluctuation of a **virtual drop of QM** in HM

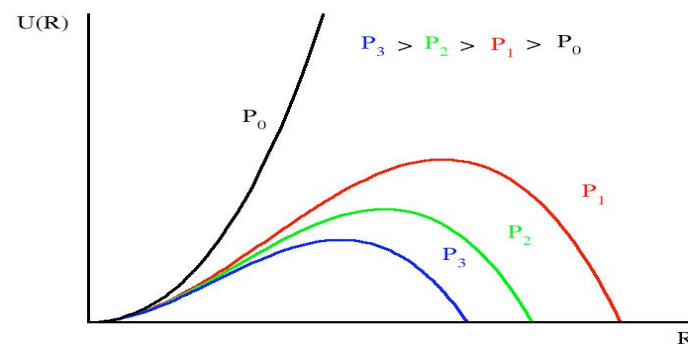
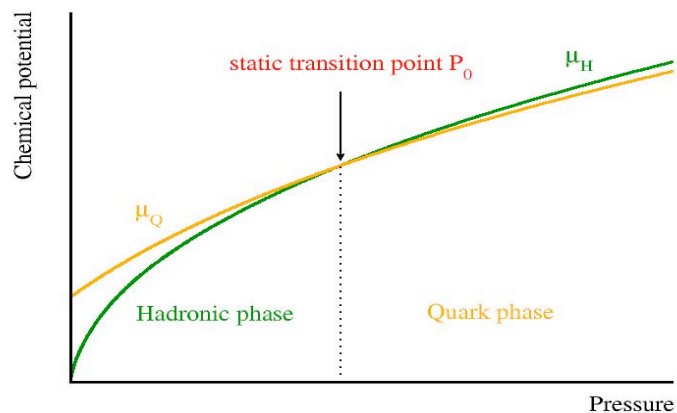
$$L = -M(R) \sqrt{1 - \left(\frac{dR}{dt}\right)^2} + M(R) - U(R)$$

$$M(R) = 4\pi\rho_H \left(1 - \frac{n_{Q^*}}{n_H}\right)^2 R^3$$



$$U(R) = \frac{4}{3}\pi n_{Q^*} (\mu_{Q^*} - \mu_H) R^3 + 4\pi\sigma R^2 + 8\pi\gamma R + E_c$$

$$\sigma = 10 - 50 \text{ MeV} / \text{fm}^2$$



Nucleation Time

Oscillation frequency of the virtual drop inside the potential well and Penetrability of the potential barrier (WKB)

$$\nu_0 = \left(\frac{dI}{dE} \right)^{-1}; E = E_0$$
$$p_0 = \exp\left(-\frac{A(E_0)}{\hbar} \right)$$

$$I(E) = 2 \int_0^{R_1} dR \sqrt{[2M(R) + E - U(R)][E - U(R)]}$$

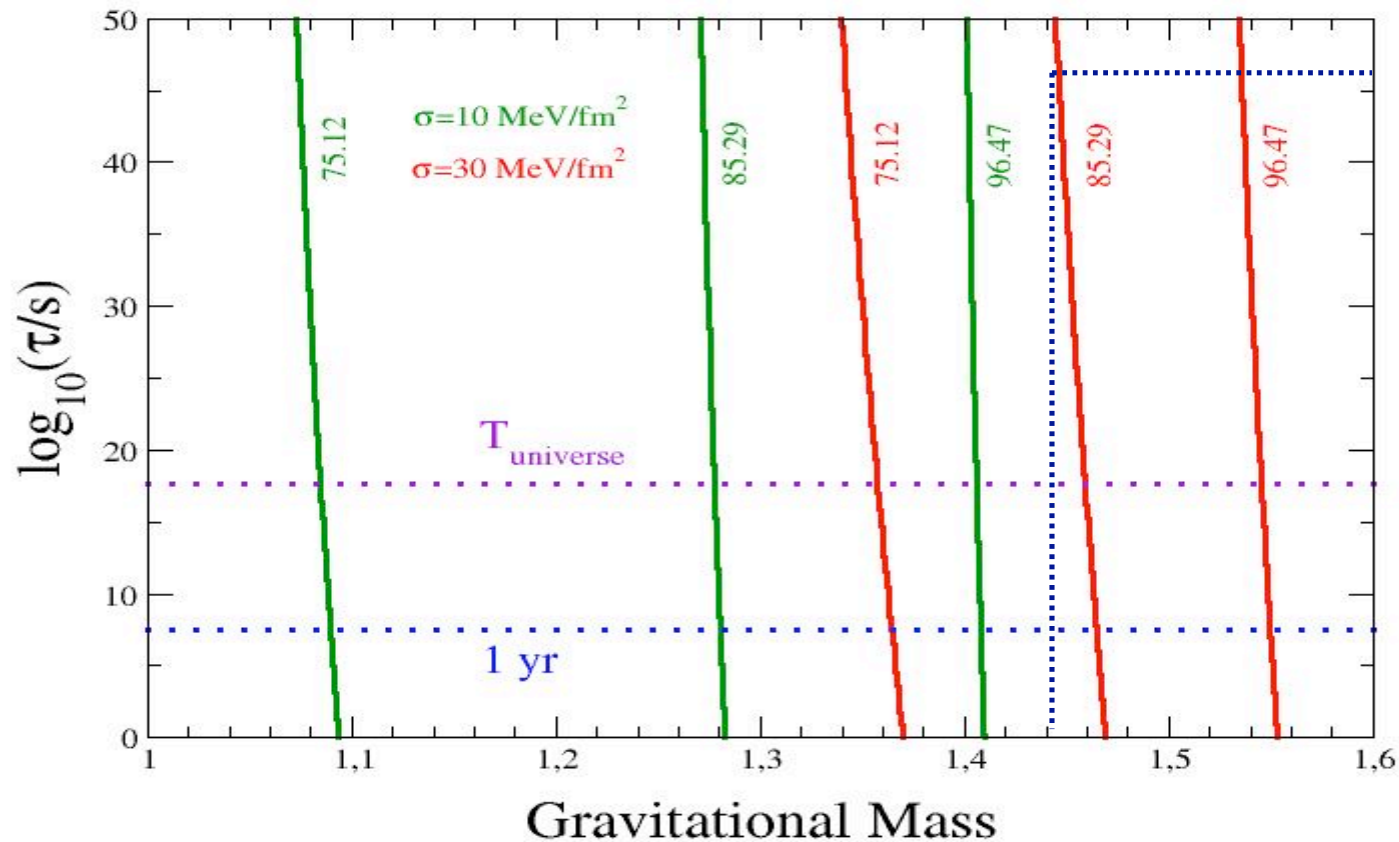
$$A(E) = 2 \int_{R_1}^{R_2} dR \sqrt{[2M(R) + E - U(R)][U(R) - E]}$$

Action over and under the barrier

Nucleation time

$$\tau = (\nu_0 p_0 N_c)^{-1}; N_c \approx 10^{48}$$

Nucleation Time .vs. gravitational mass



Increasing M_G less than $0.01 M_{\text{sun}}$ reduces τ from $\tau \gg \text{age universe}$ to a τ of few years \rightarrow *different time delays*

Critical mass of metastable hadronic stars

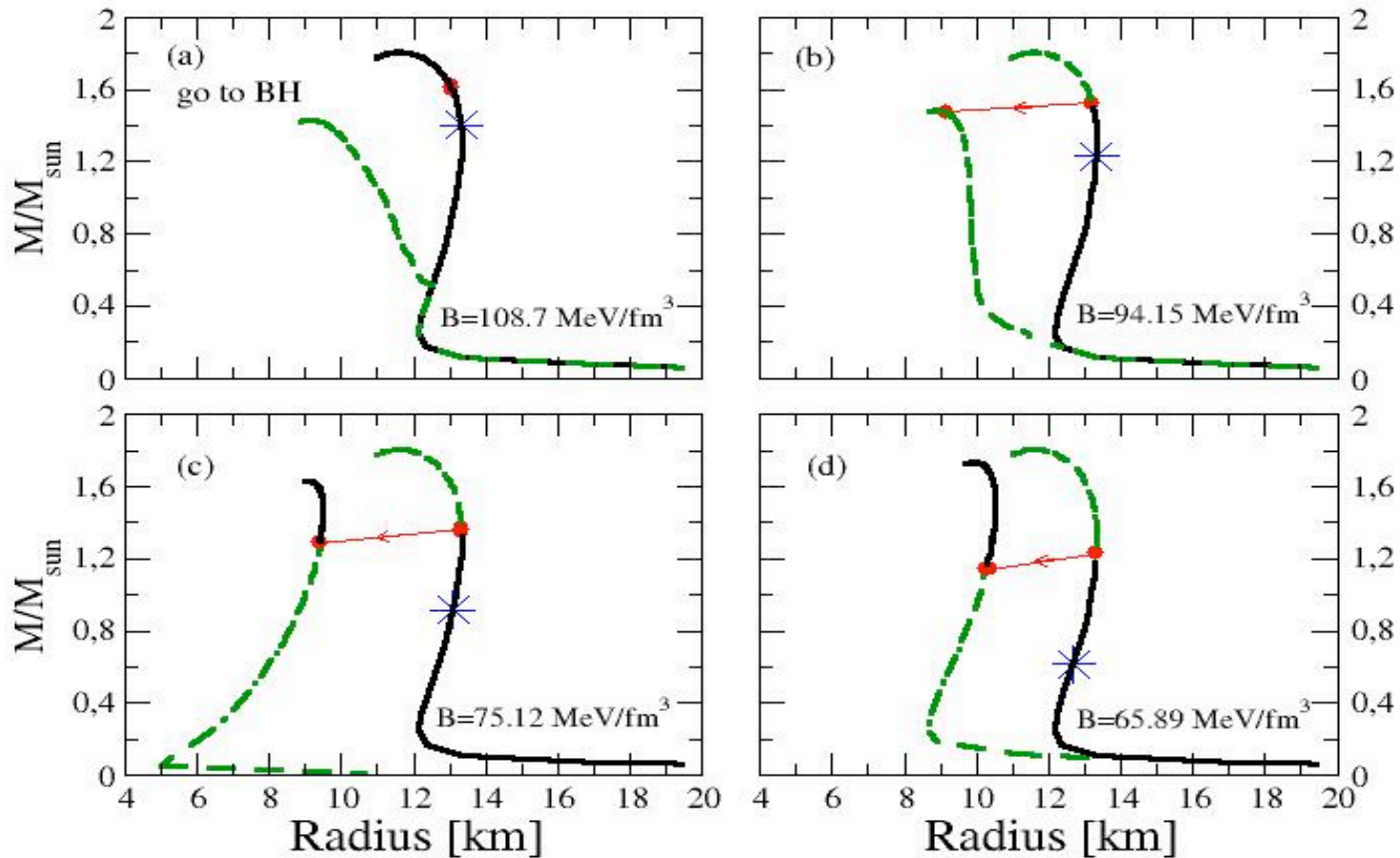
Definition: $M_{\text{cr}} = M_{\text{HS}}(\tau = 1 \text{ yr})$

Hadronic stars with $M_{\text{HS}} < M_{\text{cr}}$ are **metastable** with $\tau = 1 \text{ yr}$ to infinity

Hadronic stars with $M_{\text{HS}} > M_{\text{cr}}$ are **very unlikely** observed

“The critical mass M_{cr} plays the role of an **effective maximum mass** for the hadronic branch of compact stars”

The two families of Compact Stars



The limiting mass of compact stars

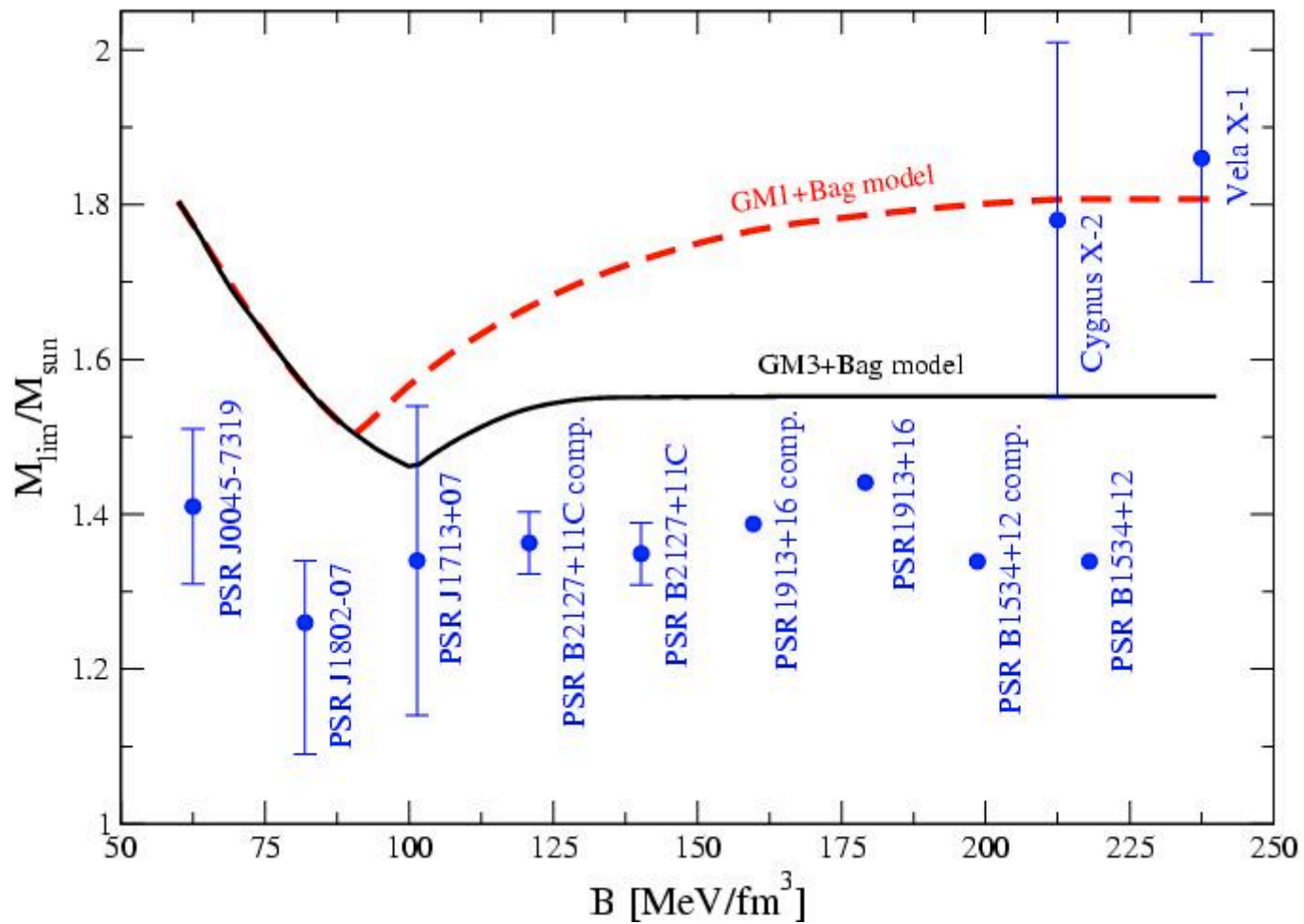
The metastability of HS and the existence of two families of compact stars demands an extension of the concept of maximum mass of a “neutron star” with respect to the *classical* one introduced by Oppenheimer, Tolman & Volkoff.

Hadronic Stars with a “short” *mean-life time* are very unlikely to be observed

A new **operational** definition of neutron star limiting mass

- If $\tau(M_{\text{HS,max}}) \sim \tau_{\text{Universe}}$ or $\tau(M_{\text{HS,max}}) \gg \tau_{\text{Universe}} \rightarrow M_{\text{lim}} = M_{\text{HS,max}}$
- If $M_{\text{crit}} < M_{\text{HS,max}}$ i.e., $\tau(M_{\text{HS,max}}) < 1 \text{ yr} \rightarrow M_{\text{lim}} = \max[M_{\text{crit}}, M_{\text{HS,max}}]$
- If $1 \text{ yr} < \tau(M_{\text{HS,max}}) < \tau_{\text{Universe}} \rightarrow M_{\text{lim}} = \max[M_{\text{HS,max}}, M_{\text{QS,max}}]$

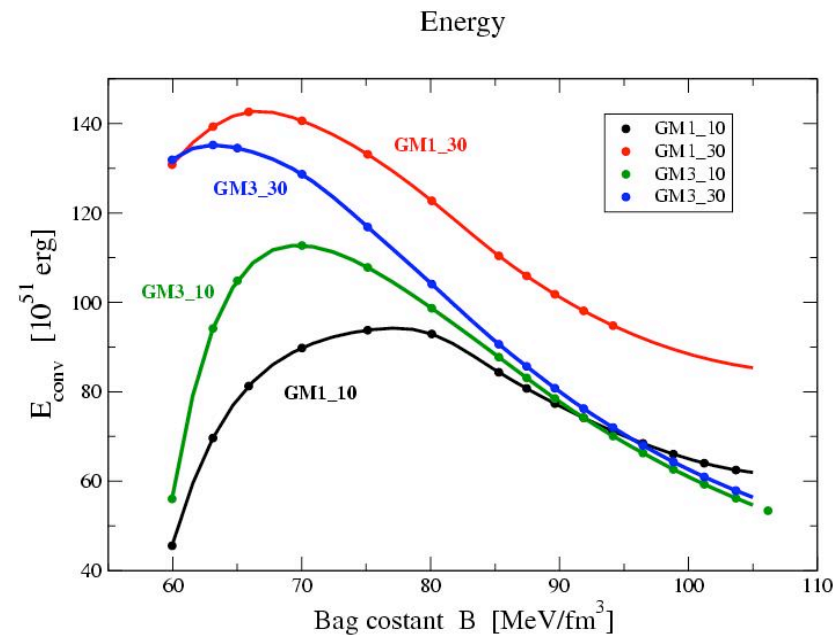
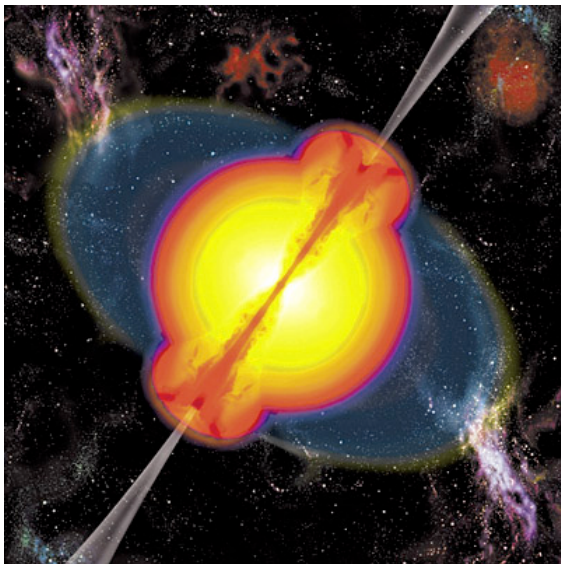
The limiting mass of compact stars



Total energy released in the conversion

Assuming that the **baryonic mass is conserved** during the conversion

$$E_{conv} = M_{crit} - M_{QS}(M_{crit}^b)$$



Production of γ rays

Total energy released from the conversion: $10^{52} - 10^{53}$ erg

$$\nu + \bar{\nu} \rightarrow e^+ + e^- \rightarrow 2\gamma$$

$$E_\gamma = \eta E_{conv}$$

(1) Ignoring **strong gravitational effects** on the cross section

$$\eta = \eta_{newt} \sim 0.01$$

(2) In a **strong gravitational field**: $\eta_{GR} = (10 - 30)\eta_{newt}$

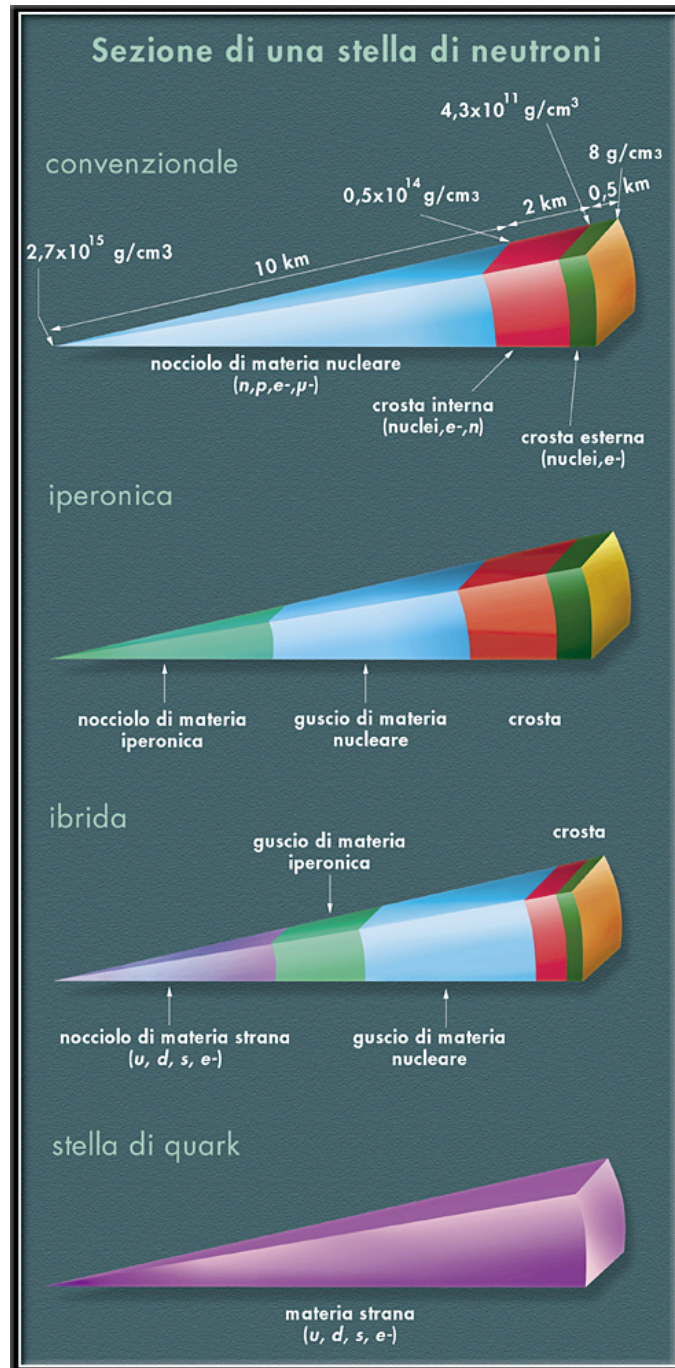
$$E_\gamma = 10^{51} - 10^{52} \text{ erg}$$

Salomon & Wilson, ApJ 517 (1999)

Summary & Conclusions III



- 💡 Existence of two families of Compact Stars: Hadron Stars (HS) and Quark Stars (QS)
- 💡 Hadron Stars are metastable with respect to HS \rightarrow QS conversion with mean life times τ ranging from $\tau \gg$ age of the universe to a t of few years
- 💡 The energy released in the conversion $E_{\text{conv}} \sim 10^{52}-10^{53}$ ergs is enough to power a GRB
- 💡 The model explains the SN-GRB connection and different delay times ΔT inferred from GRB990705, GRB030227, GRB030813 and GRB011211



Two families of Compact Stars

Hadron Stars (HS)

- “Traditional” Neutron Stars
- Hyperonic Stars

Quark Stars (QS)

- Hybrid Stars
- Strange Stars