BEC-BCS crossover in the NJL model of QCD

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Outline

- first prediction by Einstein and Bose in 1924[1]

- first experiment where BEC was observed by Cornell, Wieman and Ketterle in 1995[2] (nobel price in 2001)

- Rubidium atoms at 170nK

[1] Quantentheorie des einatomigen idealen Gases
[2] Bose-Einstein Condensation in Alcali Gases

Basis formulas

2 Color NJL Lagrangian hep-ph/0703159v2 (2007)

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\gamma^\mu\partial_\mu - m_0 + \mu\gamma_0)\psi \\ &+G_s\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2\right] \\ &+G_d(\bar{\psi}i\gamma_5\tau_2 t_2 C\bar{\psi}^T)(\psi^T C i\gamma_5\tau_2 t_2\psi)\end{aligned}$$

τ_i, t_i...Pauli matrices in flavor and color, C...charge conjugation operator

Partition Function

$$Z = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{\int^\beta d^4x\ \mathcal{L}}$$

Hubbard-Stratonovich Trick

Introduce auxiliary fields and Nambu-Gorkov spinors

$$\sigma,\ \pi,\ \phi, \phi^*; \qquad \Psi = \begin{pmatrix} \psi \\ C\bar{\psi}^T \end{pmatrix}; \quad \bar{\Psi} = (\bar{\psi}\ ,\ \psi^T C)$$

$\longrightarrow$ quadratic form in exponent by the exact transformation

$$\exp\left[G(\bar{\psi}\mathcal{O}\psi)^2\right] = \mathcal{N}\int \mathcal{D}\sigma \exp\left[\frac{\sigma^2}{4G} + \bar{\psi}\mathcal{O}\psi\sigma\right]$$

$$Z = \int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi\mathcal{D}\sigma\mathcal{D}\pi\mathcal{D}\phi\mathcal{D}\phi^* e^{\int^\beta d^4x\ \mathcal{L}_{eff}}$$

$$\mathcal{L}_{eff} = \frac{1}{2}\bar{\Psi}\mathcal{K}[\sigma,\pi,\phi]\Psi - \frac{\sigma^2+\pi^2+\mid\phi\mid^2}{4G}$$

$$\mathcal{K}[\sigma,\pi,\phi] = \begin{pmatrix} \mathcal{M}_+ & i\gamma_5\phi\tau_2 t_2 \\ i\gamma_5\phi^*\tau_2 t_2 & \mathcal{M}_- \end{pmatrix}$$

$$\mathcal{M}_\pm = i\gamma^\mu\partial_\mu - m_0 \pm \mu\gamma_0 - (\sigma \pm i\gamma_5\tau\cdot\pi)$$

integrate out quark degrees of freedom

$$Z = \int \mathcal{D}\sigma\mathcal{D}\pi\mathcal{D}\phi\mathcal{D}\phi^* e^{-S_{eff}[\sigma,\pi\phi]}$$

$$S_{eff}[\sigma,\pi\phi] = \int^\beta d^4x\, \frac{\sigma^2+\pi^2+\mid\phi\mid^2}{4G} - \frac{1}{2}\mathrm{Tr}\,\ln\mathcal{K}[\sigma,\pi\phi]$$

$$\Omega_{MF} = \frac{1}{\beta V} S^0_{eff} = \frac{(m - m_0)^2}{4G} + \frac{1}{2\beta V} \ln \det \mathcal{S}^{-1}$$

where $m = m_0 + \langle\sigma\rangle$ is the effective quark mass and $\mathcal{S}$ the quark propagator at mean field level

$$\mathcal{S}^{-1} = (\gamma^\mu \partial_\mu - m + \mu\gamma_0\sigma_3)$$

which gives the gap equation for the chiral condensate $(\partial\Omega/\partial m = 0)$

$$\frac{m - m_0}{16Gm} = \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{1 - f(E^+_{\mathbf{p}}) - f(E^-_{\mathbf{p}})}{E_{\mathbf{p}}}$$

where $E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$ and $E^\pm_{\mathbf{p}} = E_{\mathbf{p}} \pm \mu_B/2$.

2nd order terms (Gaussian approximation):

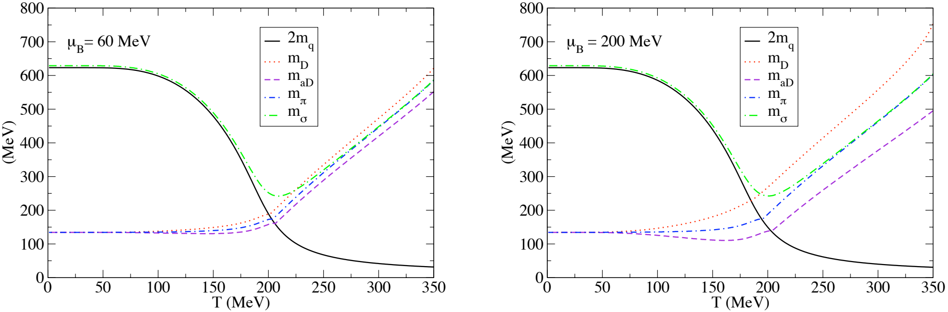
$$S^{(2)}_{fluc}[\sigma,\pi,\phi] = \int^{\beta} d^4x \, \frac{\sigma^2+\pi^2+|\,\phi\,|^2}{4G} + \frac{1}{4}\mathrm{Tr}\{\mathcal{S}\Sigma\mathcal{S}\Sigma\}$$

$$\Sigma[\sigma,\pi,\phi] = \begin{pmatrix} \sigma + i\gamma_5\tau\cdot\pi & i\gamma_5\phi\tau_2 t_2 \\ i\gamma_5\phi^*\tau_2 t_2 & \sigma - i\gamma_5\tau\cdot\pi \end{pmatrix}$$

which can be evaluated as

$$S^{(2)}_{fluc}[\sigma,\pi,\phi] = \frac{1}{2} iT \sum_n \int \frac{d\mathbf{k}}{(2\pi)^3} \left[\left[\frac{1}{2G} - \Pi_\sigma(k)\right] \sigma(-k)\sigma(k) + \left[\frac{1}{2G} - \Pi_\pi(k)\right] \pi(-k)\pi(k) \right. \\ \left. + \left[\frac{1}{2G} - \Pi_d(k)\right] \phi^*(-k)\phi(k) + \left[\frac{1}{2G} - \Pi_{\bar{d}}(k)\right] \phi(-k)\phi^*(k) \right]$$

The energy dispersions and masses arise from the poles of the propagators $1 - 2G\Pi_i(\omega(\mathbf{k}), \mathbf{k}) = 0$, $i = \pi, \sigma, d, \bar{d}$



$$\Omega_{MF} = \frac{1}{\beta V} S^0_{eff} = \frac{(m - m_0)^2 + \Delta^2}{4G} + \frac{1}{2\beta V} \ln \det \mathcal{S}_\Delta^{-1}$$

where $\mathcal{S}_\Delta$ the quark propagator at mean field level

$$\mathcal{S}_\Delta^{-1} = (\gamma^\mu \partial_\mu - m + \mu\gamma_0\sigma_3 + i\gamma_5 \Delta \tau_2 t_2 \sigma_1)$$

and after Matsubara summation we finally obtain

$$\Omega_{MF} = \frac{1}{\beta V} S^0_{eff} = \frac{(m - m_0)^2 + \Delta^2}{4G} - 8 \int \frac{d\mathbf{p}}{(2\pi)^3} \left[\zeta(E^+_\Delta) + \zeta(E^-_\Delta)\right]$$

where $\zeta(x) = x/2 + \beta^{-1} \ln(1 + e^{-\beta x})$

This gives the gap equation for the effective quark mass m and the diquark condensate Δ at $T = 0$ ($\partial\Omega/\partial m = 0, \; \partial\Omega/\partial\Delta = 0$)

$$m - m_0 = 8Gm \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{1}{E_\mathbf{p}} \left(\frac{E_\mathbf{p}^-}{E_\Delta^-} + \frac{E_\mathbf{p}^+}{E_\Delta^+} \right)$$

$$\Delta = 8G\Delta \int \frac{d\mathbf{p}}{(2\pi)^3} \left(\frac{1}{E_\Delta^-} + \frac{1}{E_\Delta^+} \right)$$

where $E_\mathbf{p} = \sqrt{\mathbf{p}^2 + m^2}$, $E_\mathbf{p}^\pm = E_\mathbf{p} \pm \mu_B/2$
and $E_\Delta^\pm = \sqrt{(E_\mathbf{p}^\pm)^2 + \Delta^2}$.

$T < T_c$: $\sigma \longrightarrow \langle\sigma\rangle + \sigma$, $\phi \longrightarrow \Delta + \phi$,

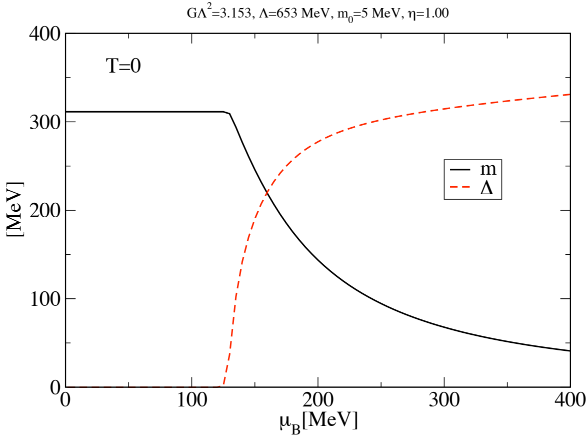
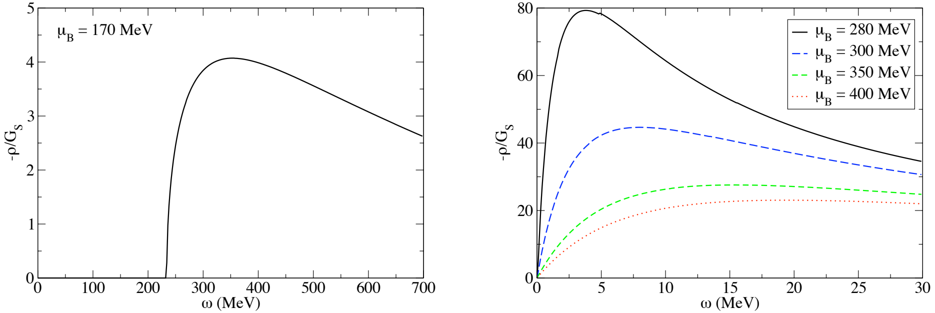


Figure: 2 Color: effective quark mass and Diquark mass for T=0

diquark spectral function

$$\rho(\omega, \mathbf{k}) = \frac{-8G^2 \mathbf{Im}\Pi_d(\omega + i\eta, \mathbf{k})}{[1 - 2G\mathbf{Re}\Pi_d(\omega + i\eta, \mathbf{k})]^2 + [2G\mathbf{Im}\Pi_d(\omega + i\eta, \mathbf{k})]^2}$$



3 Color Lagrangian

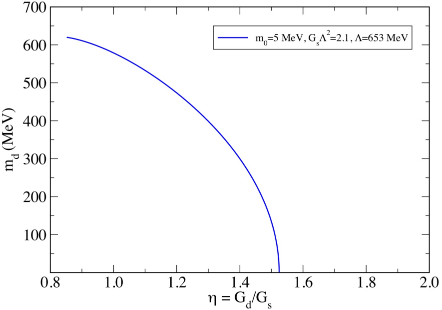
3 Color NJL Lagrangian

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\gamma^\mu\partial_\mu - m_0 + \mu\gamma_0)\psi \\ &+G_s\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2\right] \\ &+G_d\sum_{a=2,5,7}(\bar{\psi}i\gamma_5\tau_2\lambda_a C\bar{\psi}^T)(\psi^T C i\gamma_5\tau_2\lambda_a\psi)\end{aligned}$$

which gives equations for the effective quark mass and m_d

$$m - m_0 = 24 G_s m \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{1}{E_\mathbf{p}}$$

$$1 = 8 G_d \int \frac{d\mathbf{p}}{(2\pi)^3} \left(\frac{1}{E_\mathbf{p} + m_d/2} + \frac{1}{E_\mathbf{p} - m_d/2} \right)$$



In this case the thermodynamic potential reads

$$\Omega = \frac{(m-m_0)^2}{4G_s} + \frac{\Delta^2}{4G_d} - 4\int \frac{d\mathbf{p}}{(2\pi)^3} \left[2\left(\zeta(E_\Delta^+) + \zeta(E_\Delta^-)\right) + \zeta(E_b^+) + \zeta(E_b^-)\right]$$

with the quark energies

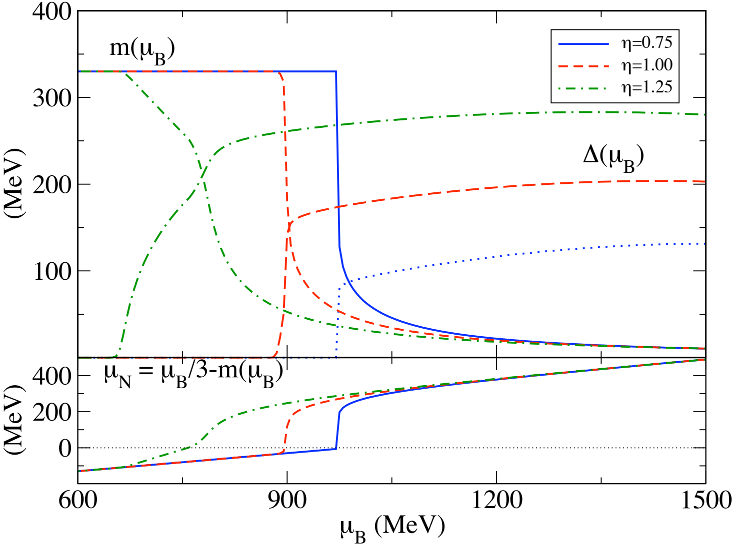
$$E_\Delta^\pm = \sqrt{(E_\mathbf{p} \pm \mu_B/3)^2 + \Delta^2}$$
$$E_b^\pm = E_\mathbf{p} \pm \mu_B/3$$

So, the gap equations read

$$
\begin{aligned}
m - m_0 &= 8Gm \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{1}{E_{\mathbf{p}}} \left(\frac{E_{\mathbf{p}}^-}{E_\Delta^-} + \frac{E_{\mathbf{p}}^+}{E_\Delta^+} + \Theta(E_b^-) \right) \\
\Delta &= 8G\Delta \int \frac{d\mathbf{p}}{(2\pi)^3} \left(\frac{1}{E_\Delta^-} + \frac{1}{E_\Delta^+} \right)
\end{aligned}
$$

$T = 0; \mu \neq 0$

Set M=330 MeV: m_0=5.27697 MeV, $G_s\Lambda^2$=2.17576, Λ=629.54 MeV



3 Color Lagrangian

3 Color NJL Lagrangian including Vector channel

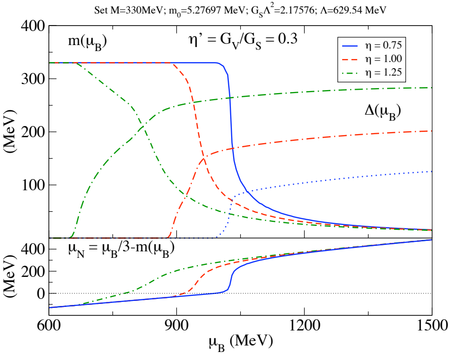
$$
\begin{aligned}
\mathcal{L} \quad = \quad & \bar{\psi}(i\gamma^\mu\partial_\mu - m_0 + \mu\gamma_0)\psi \\
& + G_s\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2\right] \\
& + G_d \sum_{a=2,5,7} (\bar{\psi}i\gamma_5\tau_2\lambda_a C\bar{\psi}^T)(\psi^T C i\gamma_5\tau_2\lambda_a\psi) \\
& - G_v\left[(\bar{\psi}\gamma_\mu\psi)^2 + (\bar{\psi}\gamma_\mu\gamma_5\tau\psi)^2\right]
\end{aligned}
$$

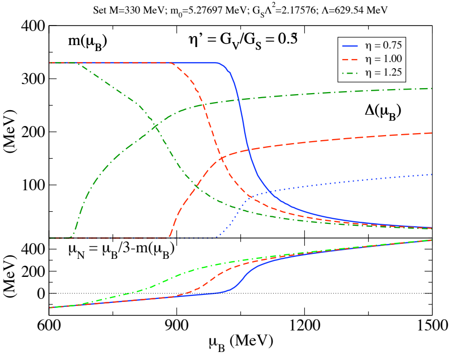
This results in another term in the therodynamic potential $-\rho_v^2/4G_v$ and finally shifts the chemical potential

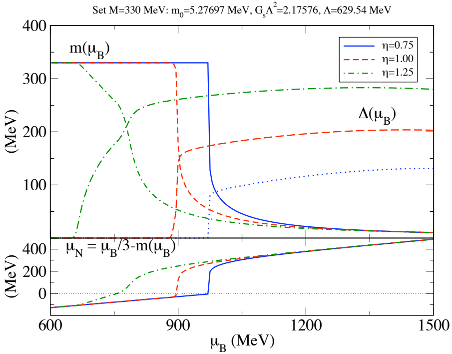
$$\mu_B/3 \rightarrow \mu_B/3 - \rho_v$$

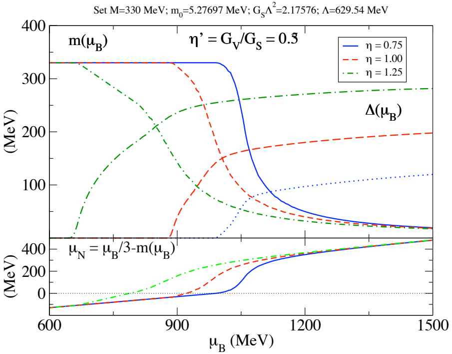
The gap equation at zero temperature reads

$$\rho_\mathrm{v} = 8G_\mathrm{v} \int \frac{d\mathbf{p}}{(2\pi)^3} \left[\frac{E_\mathbf{p}^+}{E_\Delta^+} - \frac{E_\mathbf{p}^-}{E_\Delta^-} + \Theta\left(-E_b^-\right) \right].$$


Set M=330MeV; m_0=5.27697 MeV; G_SΛ^2=2.17576; Λ=629.54 MeV
m(μ_B)
η' = G_V/G_S = 0.3
η = 0.75
η = 1.00
η = 1.25
Δ(μ_B)
(MeV)
μ_N = μ_B/3-m(μ_B)
μ_B (MeV)
400
300
200
100
0
600
900
1200
1500


Set M=330 MeV; m_0=5.27697 MeV; G_SΛ^2=2.17576; Λ=629.54 MeV
m(μ_B)
η' = G_V/G_S = 0.5
η = 0.75
η = 1.00
η = 1.25
Δ(μ_B)
(MeV)
μ_N = μ_B/3-m(μ_B)
μ_B (MeV)
400
300
200
100
0
600
900
1200
1500


Set M=330 MeV: m0=5.27697 MeV, GsΛ2=2.17576, Λ=629.54 MeV
m(μB)
η=0.75
η=1.00
η=1.25
Δ(μB)
μN = μB/3-m(μB)
(MeV)
μB (MeV)
400
300
200
100
0
600
900
1200
1500


Set M=330 MeV; m0=5.27697 MeV; GsΛ2=2.17576; Λ=629.54 MeV
m(μB)
η' = GV/GS = 0.5
η = 0.75
η = 1.00
η = 1.25
Δ(μB)
μN = μB/3-m(μB)
(MeV)
μB (MeV)
400
300
200
100
0
600
900
1200
1500

Conclusions

- Below T_c diquarks are stable bound states, but at high μ_B become unstable resonances
- BEC/BSC crossover region could be found (at large couplings), here discussed at $T = 0$
- Vector coupling affects the crossover region

Outlook

- For applications to compact stars the constraint of color- and electric neutrality needs to be considered
- For heavy-ion collisions, an extension to finite temperatures is necessary
- Calculate EoS from the thermodynamic potential
- Include Polyakov loops into the Lagrangian
- Confinement?

polarization function

$$\Pi_\sigma(k;\mu) = -2\int\frac{d^3\mathbf{p}}{(2\pi)^3} \left[\left(\frac{f_\mathbf{p}^- + f_\mathbf{p}^+ - f_\mathbf{q}^- - f_\mathbf{q}^+}{k_0 - E_\mathbf{q} + E_\mathbf{p}} - \frac{f_\mathbf{p}^- + f_\mathbf{p}^+ - f_\mathbf{q}^- - f_\mathbf{q}^+}{k_0 + E_\mathbf{q} - E_\mathbf{p}}\right)\mathcal{T}_-^- + \left(\frac{2 - f_\mathbf{p}^- - f_\mathbf{p}^+ - f_\mathbf{q}^- - f_\mathbf{q}^+}{k_0 - E_\mathbf{q} - E_\mathbf{p}} - \frac{2 - f_\mathbf{p}^- - f_\mathbf{p}^+ - f_\mathbf{q}^- - f_\mathbf{q}^+}{k_0 + E_\mathbf{q} + E_\mathbf{p}}\right)\mathcal{T}_+^-\right]$$

$$\Pi_\pi(k;\mu) = -2\int\frac{d^3\mathbf{p}}{(2\pi)^3} \left[\left(\frac{f_\mathbf{p}^- + f_\mathbf{p}^+ - f_\mathbf{q}^- - f_\mathbf{q}^+}{k_0 - E_\mathbf{q} + E_\mathbf{p}} - \frac{f_\mathbf{p}^- + f_\mathbf{p}^+ - f_\mathbf{q}^- - f_\mathbf{q}^+}{k_0 + E_\mathbf{q} - E_\mathbf{p}}\right)\mathcal{T}_-^+ + \left(\frac{2 - f_\mathbf{p}^- - f_\mathbf{p}^+ - f_\mathbf{q}^- - f_\mathbf{q}^+}{k_0 - E_\mathbf{q} - E_\mathbf{p}} - \frac{2 - f_\mathbf{p}^- - f_\mathbf{p}^+ - f_\mathbf{q}^- - f_\mathbf{q}^+}{k_0 + E_\mathbf{q} + E_\mathbf{p}}\right)\mathcal{T}_+^+\right]$$

$$\Pi_d(k;\mu) = -4\int\frac{d^3\mathbf{p}}{(2\pi)^3} \left[\left(\frac{f_\mathbf{p}^+ - f_\mathbf{q}^-}{k_0 + \mu - E_\mathbf{q} + E_\mathbf{p}} - \frac{f_\mathbf{p}^- - f_\mathbf{q}^+}{k_0 + \mu + E_\mathbf{q} - E_\mathbf{p}}\right)\mathcal{T}_-^+ + \left(\frac{1 - f_\mathbf{p}^- - f_\mathbf{q}^-}{k_0 + \mu - E_\mathbf{q} - E_\mathbf{p}} - \frac{1 - f_\mathbf{p}^+ - f_\mathbf{q}^+}{k_0 + \mu + E_\mathbf{q} + E_\mathbf{p}}\right)\mathcal{T}_+^+\right],$$

$$\Pi_{\bar d}(k;\mu) = -4\int\frac{d^3\mathbf{p}}{(2\pi)^3} \left[\left(\frac{f_\mathbf{p}^- - f_\mathbf{q}^+}{k_0 - \mu - E_\mathbf{q} + E_\mathbf{p}} - \frac{f_\mathbf{p}^+ - f_\mathbf{q}^-}{k_0 - \mu + E_\mathbf{q} - E_\mathbf{p}}\right)\mathcal{T}_-^+ + \left(\frac{1 - f_\mathbf{p}^+ - f_\mathbf{q}^+}{k_0 - \mu - E_\mathbf{q} - E_\mathbf{p}} - \frac{1 - f_\mathbf{p}^- - f_\mathbf{q}^-}{k_0 - \mu + E_\mathbf{q} + E_\mathbf{p}}\right)\mathcal{T}_+^+\right],$$

where we have defined $\mathbf{q} = \mathbf{p} + \mathbf{k}$, $E_\mathbf{p} = \sqrt{\mathbf{p}^2 + m^2}$, $f_\mathbf{p}^\pm = f(E_\mathbf{p} \pm \mu/2)$ with $f(x) = \left(1 + e^{x/T}\right)^{-1}$ being the Fermi-Dirac distribution function and $\mathcal{T}_\pm^\mp = 1 \pm \left(\mathbf{p}\cdot\mathbf{q} \mp m^2\right) / \left(E_\mathbf{p} E_\mathbf{q}\right)$.