Nuclear Symmetry Energy from Heavy Ion Collisions

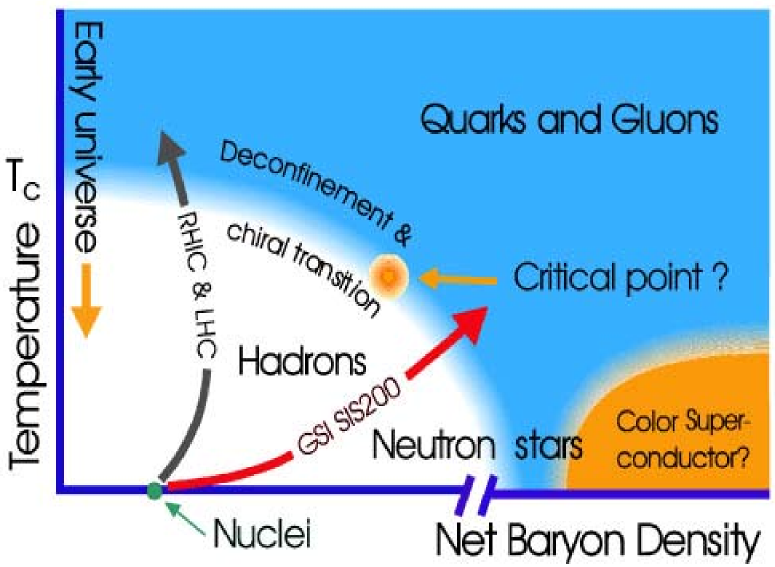
Hermann Wolter

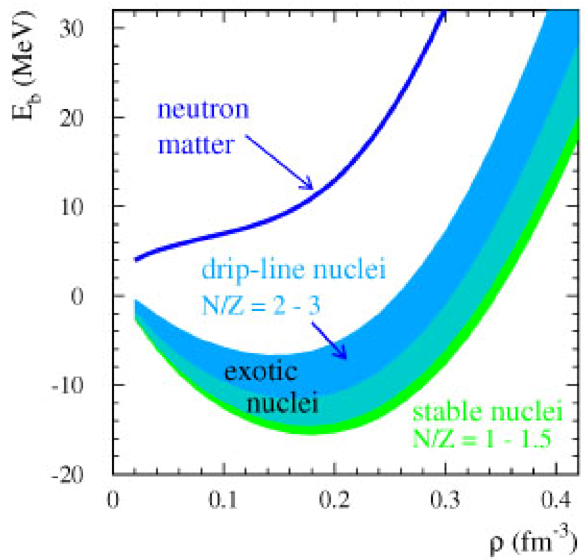
University of Munich

Outline:

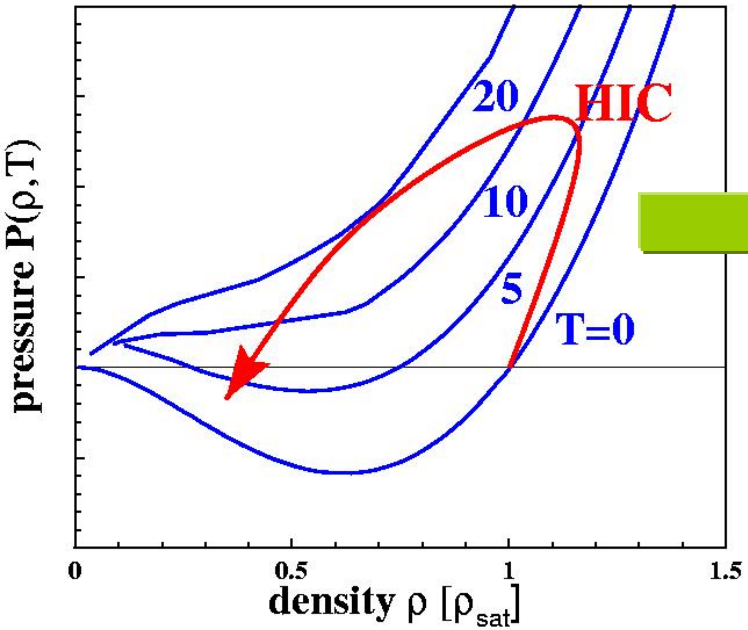
1. Motivation
2. Description of Heavy Ion Collisions in Transport Theory
3. Models of Equation-of-State (EOS)
4. Observables
5. Results for symmetric nuclear Matter
6. Results for asymmetric nuclear matter
7. Neutron star structure
8. 8.Conclusions

Determination of the Equation of State of Hadronic Matter in Heavy Ion Collisions





Motivation



E_{beam}=0.1-2 GeV/N

Heavy Ion Collisions(HIC)

Explore matter under **extreme** Conditions

ρ=(2-3)*ρ_{sat}, T>>T_c and isospin

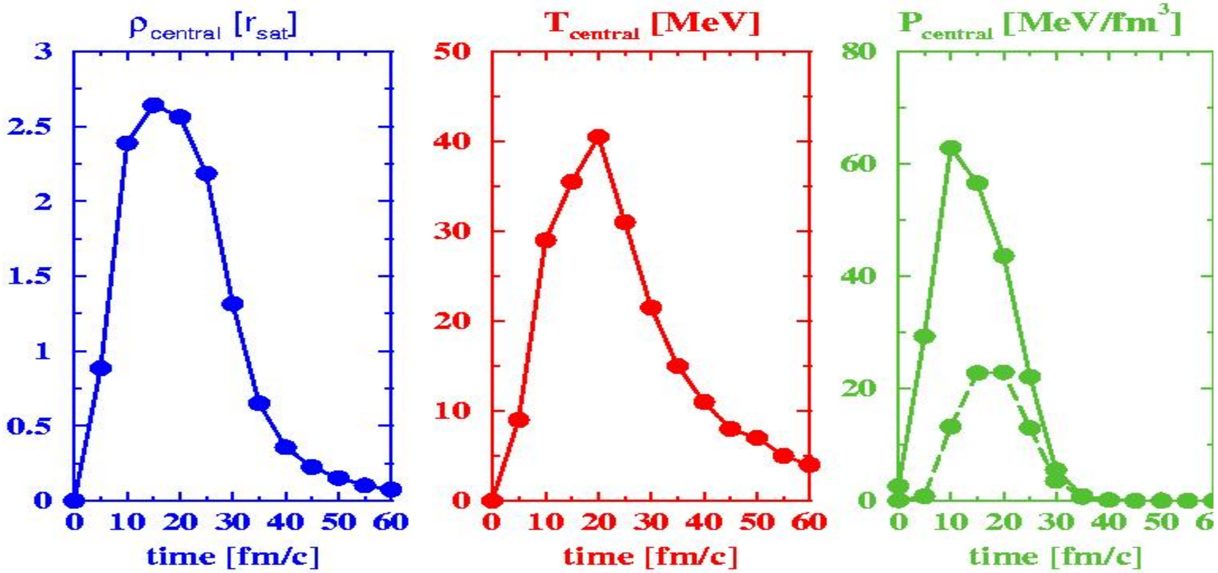
Ground state

T, P=0 MeV

ρ_{sat} = 0.16 fm^{-3}

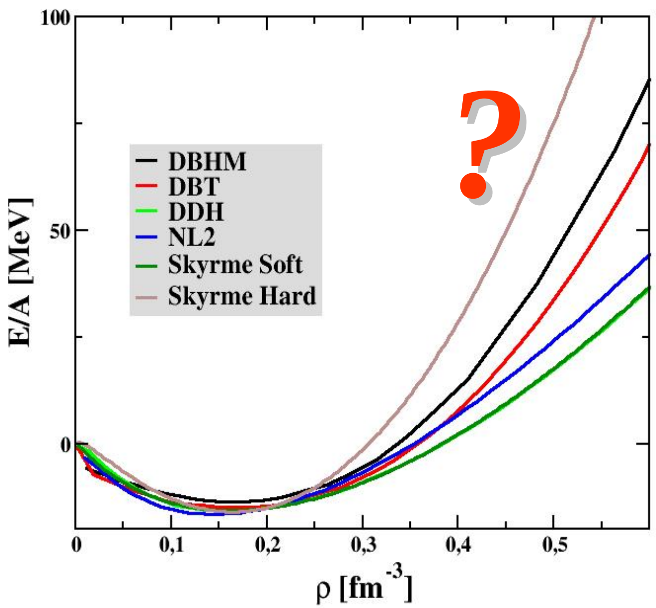
E/A = -16 MeV

Au (E_{beam}=0.6 GeV/nucleon) + Au – central



The nuclear EoS-Uncertainties

...iso-scalar sector



Binding energy/nucleon

E/A = T^{00} (from 00-component of energy-momentum tensor)

hard EoS

κ≈380 MeV

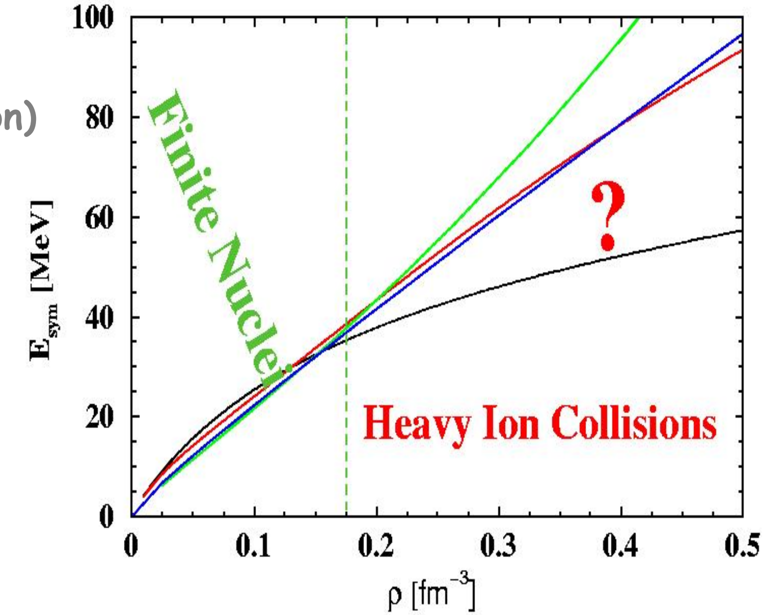
(less compression)

soft EoS

κ≈200 MeV

(more compression)

...iso-vector sector



Symmetry energy

E_{sym} from second derivative of E with respect to asymmetry (N-Z)/(N+Z)

- Different predictions for compression modulus κ (200-400 MeV)
- Different predictions for asym. parameter α_4 (28-36 MeV)

Nuclear matter at supra-normal densities not fixed (crucial differences between models)

Astrophysical Implications of Iso-Vector EOS

Neutron Star Structure

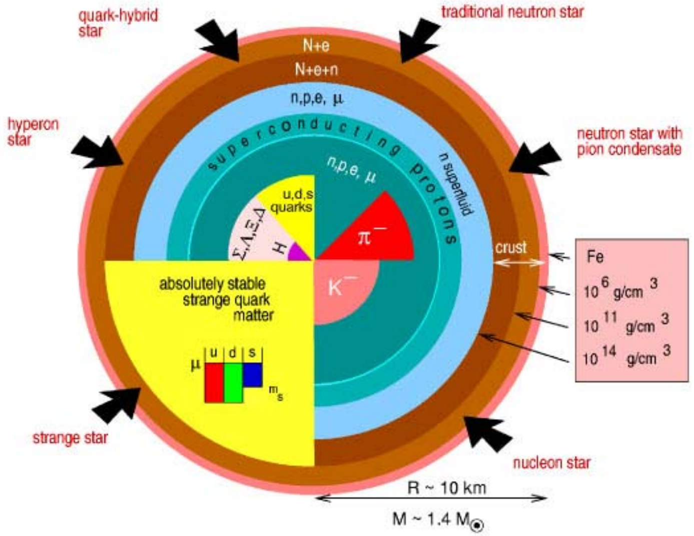
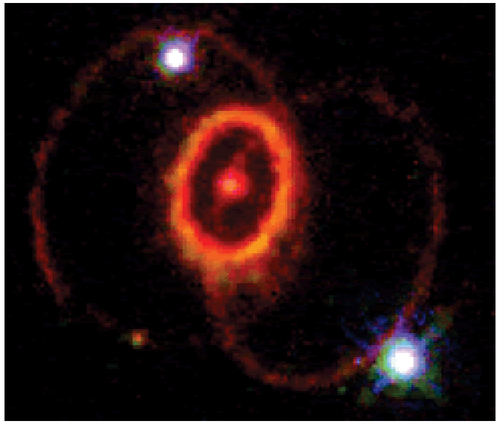


Figure 3.3: Possible novel phases and structures of subatomic matter: (i) a large population of hyperons (Λ,Σ,Ξ), (ii) condensates of negatively charged mesons with and without strange quarks (kaons or pions), (iii) a plasma of up, down, strange quarks and gluons (strange quark matter). Compilation by F. Weber [1].

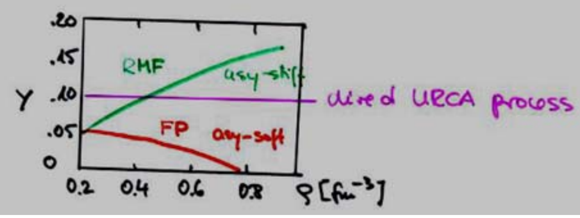


Proton fraction of neutron stars: $Y = \frac{Z}{A} = \frac{1-I}{2}$

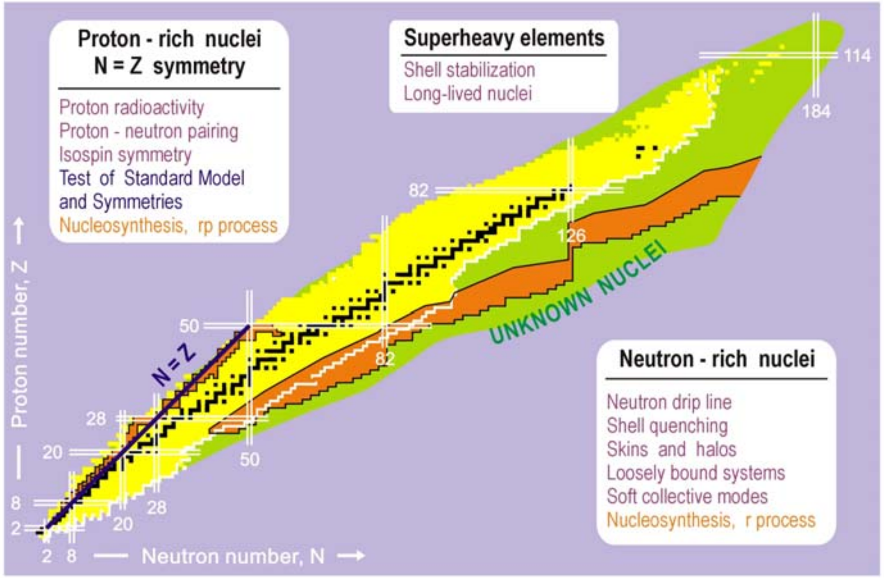
- β-equilibrium $\mu_e = \mu_n - \mu_p = -\frac{\partial \mathcal{E}(\rho, Y)}{\partial Y} = 4 E_{sym}(\rho)(1-2Y)$
- charge neutrality $\rho_e = \rho_p = Y\rho$

$\mu_e \approx p_{F_e} = (3\pi^2 Y \rho)^{1/3}$

→ $Y(\rho)$ determined by $E_{sym}(\rho)$ at high densities

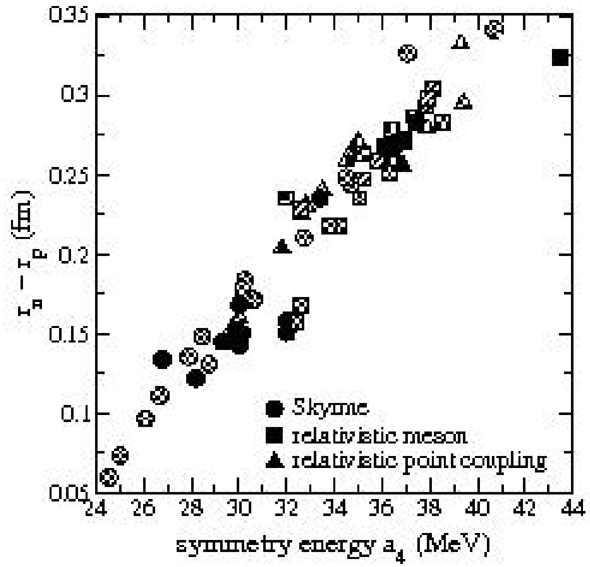


Implications for Nuclear Structure of the Iso-Vector EOS

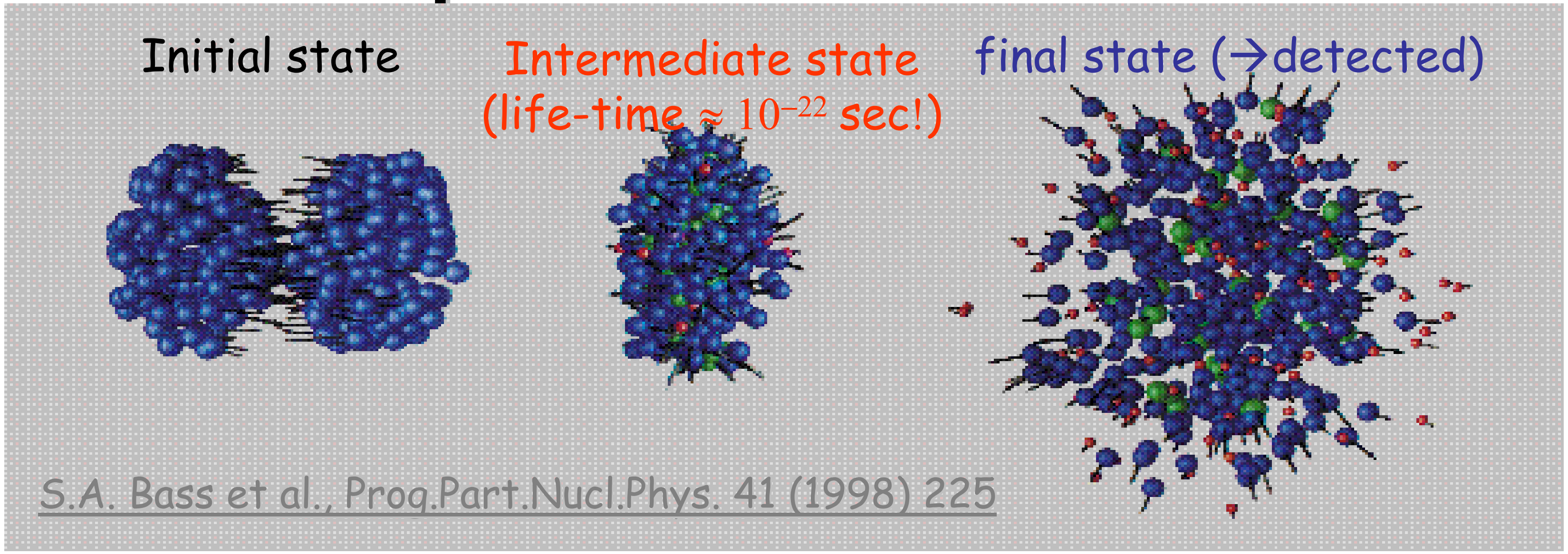


Structure of neutron rich nuclei

Correlation between Neutron Skin of 206Pb and symmetry energy coefficient



Explore EoS in HIC

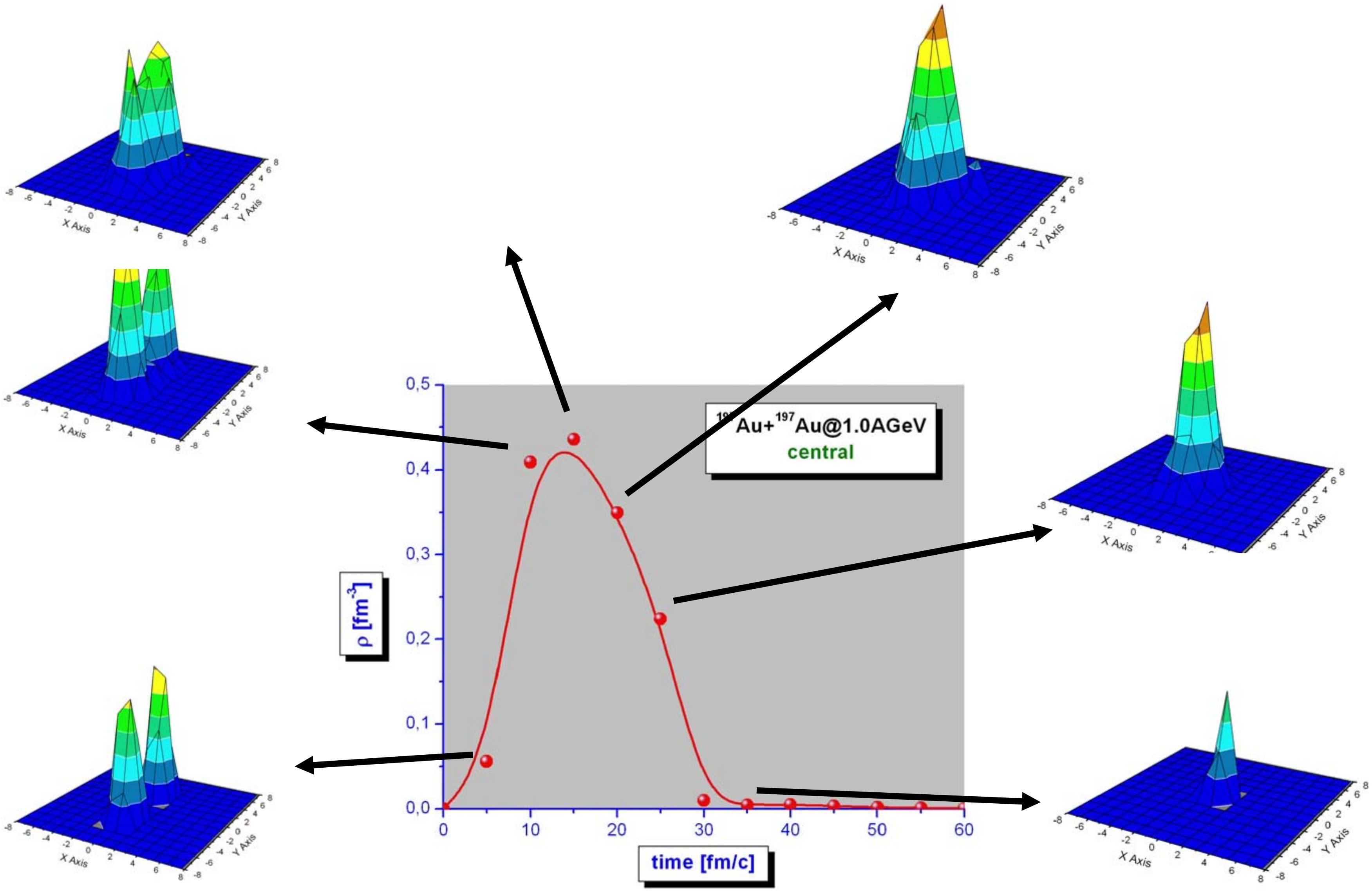


Aim: determine properties of fireball from final state detected in exp.

- **Theory**: Put different models of nuclear structure into dynamics & determine EoS dependence on many observables
- **Experiment**: Measure in such a way that your observables are accessible for theory
- **Comparison** between exp. and theory (**could**) provide us the desired EoS
- **Problem**: HIC strongly affected by (local) **non-equilibrium**!
  → relation between dynamics & EoS **not trivial**

consider NE-effects on EoS before explore dynamics C.Fuchs&T.G., NPA714(2003)643

Phase space evolution in a heavy ion Collision



T. Gaitanos

Models for the Equation-of-State

Two (relativistic) approaches:

1. Dirac-Brueckner HF (DB)

Density dependent coupling

2. Quantenhadrodynamics

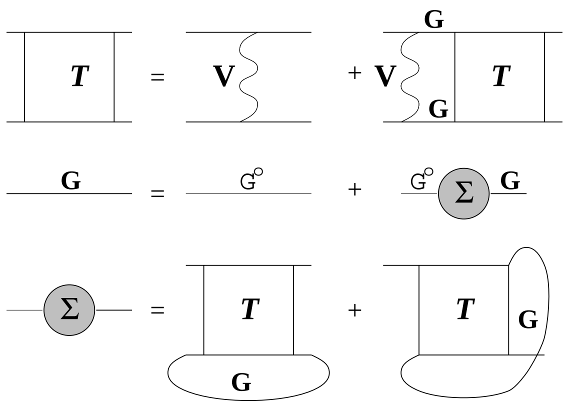


Abbildung 1.2: *Diagrammatische Darstellung der DB-Methode. Die oberste Reihe stellt die Bethe-Salpeter-Gleichung (1.23), die mittlere die Dyson-Gleichung (1.25) und die untere die Bestimmungsgleichung für die Selbstenergie (1.24) dar.*

Die DB Methode besteht nun darin, das Gleichungssystem für die T–Matrix (1.23), die Selbstenergie (1.24) und die Dyson–Gleichung

$$G(1,2) = G^0(1,2) + \int G^0(1,1')\Sigma(1',2')G(2',2) \qquad (1.25)$$

für den Baryonpropagator selbstkonsistent zu lösen. Eine diagrammatische Darstellung der DB–Methode ist in Abbildung 1.2 wiedergegeben. Eine ausführliche Darstellung über die verschiedenen Lösungsverfahren der DB–Methode zu geben, ist nicht das Anliegen dieser Arbeit. Wir wollen stattdessen die für diese Arbeit wesentlichen Eigenschaften der DB–Methode diskutieren, und für Details verweisen wir auf Refs. [20, 21, 22].

$$\mathcal{L}_{QHD} = \mathcal{L}_B + \mathcal{L}_M + \mathcal{L}_{int}$$
$$\mathcal{L}_B = \overline{\Psi}(i\gamma_\mu\partial^\mu - M)\Psi$$
$$\mathcal{L}_M = \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) + U(\sigma) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - m_\omega^2\omega_\alpha\omega^\alpha$$
$$\mathcal{L}_{int} = g_\sigma\overline{\Psi}\Psi\sigma - g_\omega\overline{\Psi}\gamma_\alpha\Psi\omega^\alpha \quad .$$

Die Wahl der 2–Teilchen NN–Wechselwirkung $< 12|V|1'2' >$ geschieht im Rahmen einer relativistischen Quantenfeldtheorie durch das 1–Boson–Austauschmodell. In der Impulsraumdarstellung lautet es [20]

$$V_{\alpha\beta;\gamma\delta}(k) \Rightarrow V^{OBE}_{\alpha\beta;\gamma\delta}(k) = -\sum_i (\mathcal{O})_{\alpha\beta}(\mathcal{O})_{\gamma\delta}\, D_i^o(k) \quad ,$$

wobei sich die Summe über verschiedene Mesonen mit den entsprechenden ungestörten Mesonenpropagatoren D_i^o erstreckt. Die Lorentz–Struktur der OBE–Potentiale wird durch die Lorentzstruktur der Mesonen, charakterisiert durch deren

Analysis of DB self energies

1.4. Das Dirac–Brueckner Modell 19

Decomposition of DB self energy

$$\Sigma(p) = \Sigma^s(p) - \gamma^0\Sigma^0(p) + \bar{\gamma}\cdot\bar{p}\Sigma^v.$$

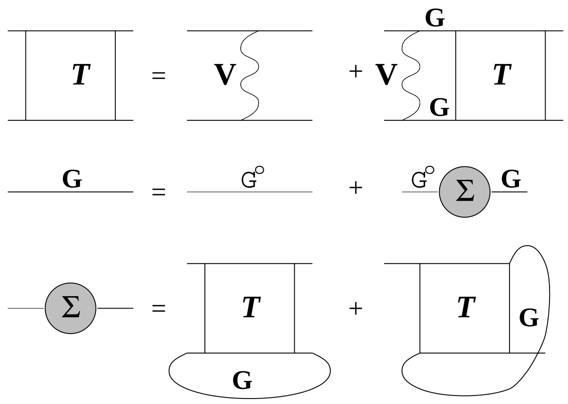
Density (and momentum) dependent coupling coeff.

$$\left(\frac{g_\sigma^*}{m_\sigma}\right)^2 = -\frac{1}{2}\frac{\Sigma_n^s(\bar{p}_f)+\Sigma_p^s(\bar{p}_f)}{\rho_n^s+\rho_p^s}$$

$$\left(\frac{g_\omega^*}{m_\omega}\right)^2 = -\frac{1}{2}\frac{\Sigma_n^0(\bar{p}_f)+\Sigma_p^0(\bar{p}_f)}{\rho_n^v+\rho_p^v}$$

$$\left(\frac{g_\delta^*}{m_\delta}\right)^2 = -\frac{1}{2}\frac{\Sigma_n^s(\bar{p}_f)-\Sigma_p^s(\bar{p}_f)}{\rho_n^s-\rho_p^s}$$

$$\left(\frac{g_\rho^*}{m_\rho}\right)^2 = -\frac{1}{2}\frac{\Sigma_n^0(\bar{p}_f)-\Sigma_p^0(\bar{p}_f)}{\rho_n^v-\rho_p^v},$$



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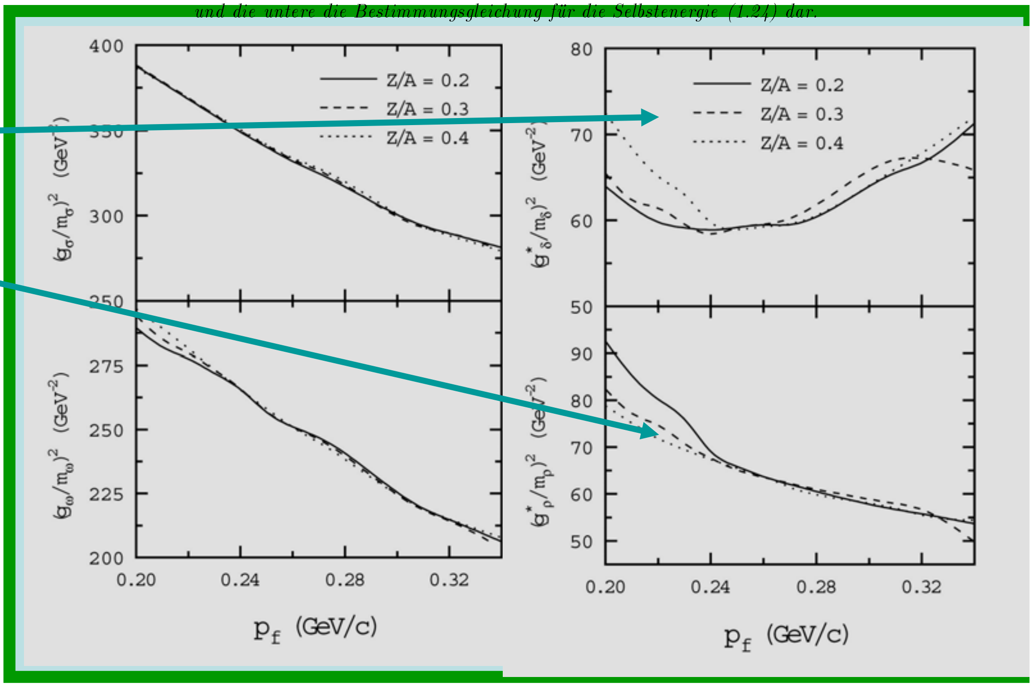
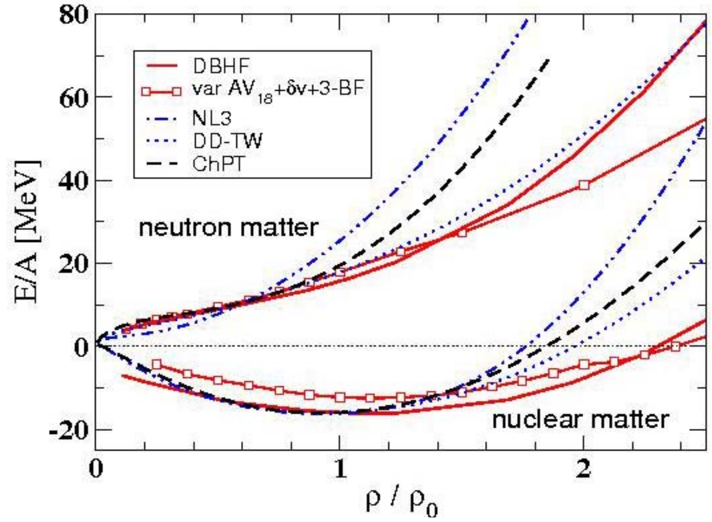
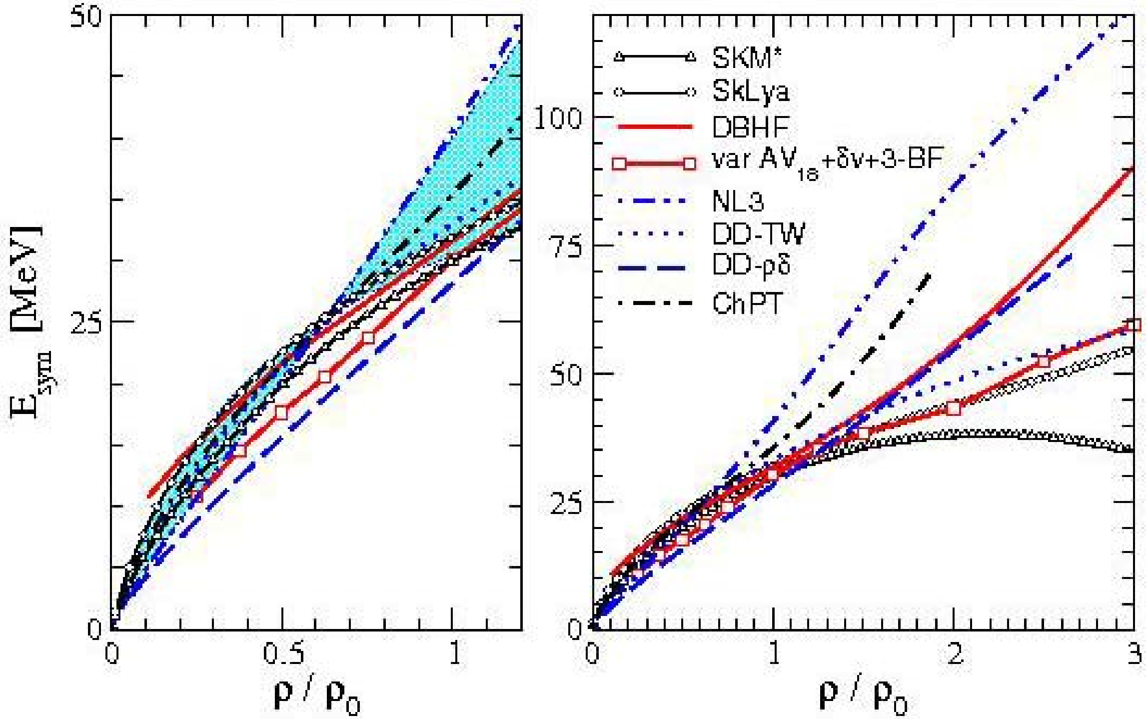


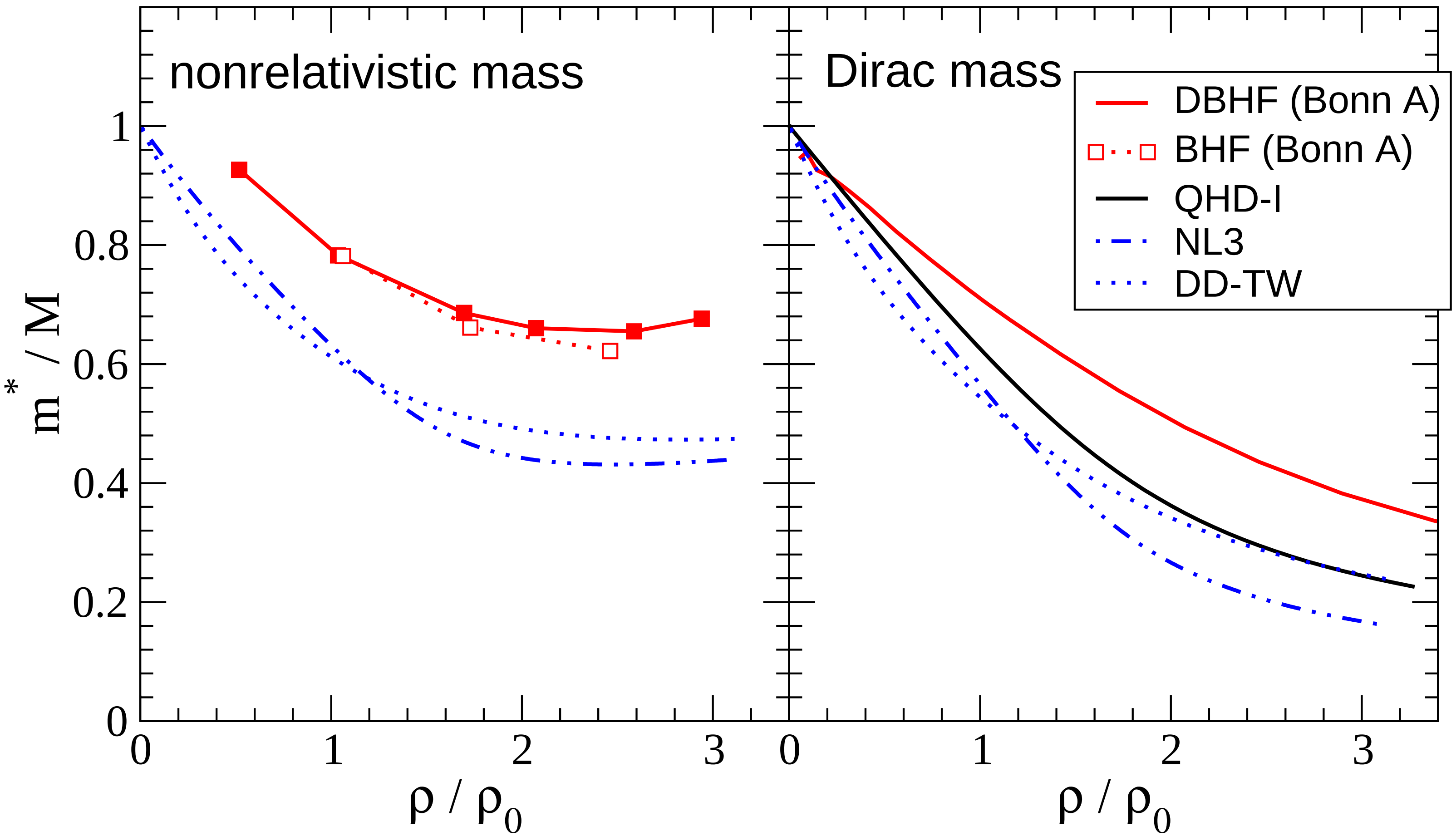
Comparisons of EOS's



Symmetry energy

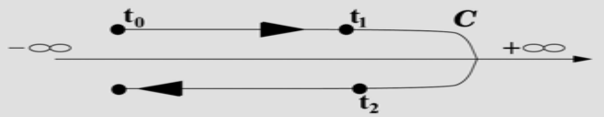


Effective masses



Derivation of a transport equation for heavy ion collisions:

Schwinger Keldysh real time formalism:



$$G(1,1') = (-i) < T_{sk}(\Psi(1)\overline{\Psi}(1')) > \quad .$$

$$\underline{G}(1,1') = \begin{pmatrix} G_{++}(1,1') & G_{+-}(1,1') \\ G_{-+}(1,1') & G_{--}(1,1') \end{pmatrix} = \begin{pmatrix} G^{c}(1,1') & G^{<}(1,1') \\ G^{>}(1,1') & G^{a}(1,1') \end{pmatrix} \quad .$$

Wigner transform to cm-coordinate and relative momentum

$$f(x,k) = \int d^4r \, e^{ik\cdot r} \, f(x+\frac{r}{2}, x-\frac{r}{2}) \quad .$$

Kadanoff-Baym equations

$$\frac{i}{2}\{\partial_k^\mu(\not k^* - m^*), \partial_x^\mu G^<\} - \frac{i}{2}\{\partial_x^\mu(\not k^* - m^*), \partial_k^\mu G^<\} + [(\not k^* - m^*), G^<] = \frac{1}{2}(\Sigma^> G^< + G^< \Sigma^> - \Sigma^< G^> - G^> \Sigma^<) \quad . \quad (3.32)$$

$$\begin{aligned} k^*_\mu &= k_\mu - Re\Sigma^+_\mu \\ m^* &= M - Re\Sigma^+_s \end{aligned}$$

$$\begin{aligned} G^<_{\alpha\beta}(x,k) &= iA_{\alpha\beta}(x,k)F(x,k) \\ G^>_{\alpha\beta}(x,k) &= -iA_{\alpha\beta}(x,k)[1-F(x,k)] \end{aligned}$$

T-Matrix-Näherung

$$\begin{aligned} &\left[(m^*\partial^\mu_x m^* - k^{*\nu}\partial^\mu_x k^*_\nu)\,\partial^k_\mu - (m^*\partial^\mu_k m^* - k^{*\nu}\partial^\mu_k k^*_\nu)\,\partial^x_\mu\right] a(x,k)F(x,k) \\ &= \frac{1}{2}\int \frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\frac{d^4k_4}{(2\pi)^4} a(x,k)a(x,k_2)a(x,k_3)a(x,k_4)W(kk_2|k_3k_4) \\ &\times (2\pi)^4\delta^4(k+k_2-k_3-k_4) \\ &\times \Big[F(x,k_3)F(x,k_4)(1-F(x,k))(1-F(x,k_2)) - \\ &\quad F(x,k)F(x,k_2)(1-F(x,k_3))(1-F(x,k_4)) \Big] \quad . \qquad (3.40 \end{aligned}$$

$$\begin{aligned} W(kk_2|k_3k_4) &= m^*(x,k)m^*(x,k_2)m^*(x,k_3)m^*(x,k_4) \\ &\times < kk_2|T^+|k_3k_4 >< k_3k_4|T^-|kk_2 > \end{aligned}$$

Transport theory in Non-Equilibrium (cont'd)

Quasi-particle Approximation

$$G^{\pm}(x,k) = \frac{1}{k^{*2} - m^{*2} - \Sigma^{\pm}(x,k) \pm i\epsilon} \quad .$$

$$\begin{aligned} a(x,k) &= \frac{2\Gamma(x,k)}{(k^{*2}-m^{*2})^2 + \Gamma^2(x,k)} 2\Theta(k^{*0}) \\ \Gamma(x,k) &= m^* Im\Sigma^+ - k^*_\mu Im\Sigma^{+\mu} \quad . \end{aligned}$$

$$a(x,k) = 2\pi\delta(k^{*2} - m^{*2})2\Theta(k^{*0}) \quad .$$

Boltzmann equation like transport equation

$$\begin{aligned} &\left[\left(m^* \partial_x^\mu m^* - k^{*\nu} \partial_x^\mu k^*_\nu \right) \partial_\mu^k - \left(m^* \partial_k^\mu m^* - k^{*\nu} \partial_k^\mu k^*_\nu \right) \partial_\mu^x \right] f(x,\mathbf{k}) \\ &= \frac{1}{2} \int \frac{d^4k_2}{E^*_{k_2}(2\pi)^3} \frac{d^4k_3}{E^*_{k_3}(2\pi)^3} \frac{d^4k_4}{E^*_{k_4}(2\pi)^3} W(kk_2|k_3k_4)(2\pi)^4\delta^4\left(k + k_2 - k_3 \cdot\right. \\ &\times \Big[f(x,\mathbf{k}_3) f(x,\mathbf{k}_4) \left(1 - f(x,\mathbf{k})\right) \left(1 - f(x,\mathbf{k}_2)\right) - \\ &\quad f(x,\mathbf{k}) f(x,\mathbf{k}_2) \left(1 - f(x,\mathbf{k}_3)\right) \left(1 - f(x,\mathbf{k}_4)\right) \Big] \quad . \end{aligned}$$

How to extract the EOS from HIC ?

EOS ↑ static concept — HIC ↑ dynamical process

Transport description for 1-body phase space distr. $f(x,p)$

e.g. (relativistic) RBUU

$$\left[p^{*}_{\mu}\partial^{\mu} + \left(p^{*}_{\nu}F^{\nu\mu} + m^{*}\partial^{\mu}m^{*}\right)\partial^{(p)}_{\mu}\right] f(x,p) = I_{coll}[f, \sigma_{med}]$$

$m^{*} = m - \Sigma_s$ scalar

$p^{*}_{\mu} = p_{\mu} - \Sigma_{\mu}$ vector

$F^{\nu\mu} = \partial^{\nu}\Sigma^{\mu} - \partial^{\mu}\Sigma^{\nu}$

} mean fields (self energies) (EOS) ← relation? → σ_{med}

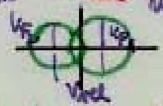
$$\Sigma = i\,\mathrm{Tr}\left[G(f)\,f\right] \quad \longleftrightarrow \quad \sigma_{med} \approx |G(f)|^{2}$$

↑ Dirac-Brueckner G-Matrix

Strategies:

(A) microscopic

$$G^{DB} \longrightarrow \left\{ \begin{array}{l} \Sigma^{DB}(k,\rho) \\ \sigma^{DB}_{med} \end{array} \right|_{equil} \longrightarrow \left\{ \begin{array}{l} \Sigma^{eff} \\ \sigma^{eff}_{med} \end{array} \right|_{non\text{-}eq} \longrightarrow \text{observables}$$



parameter free, consistent Σ, ρ ; test of DB out of equil.!

(B) phenomenological

$\Sigma(k,\rho)$ e.g. Skyrme hard/soft + MD

mom. dep. ↗ ↖ dens. dep

fit to data → determine EOS! σ_{med}?

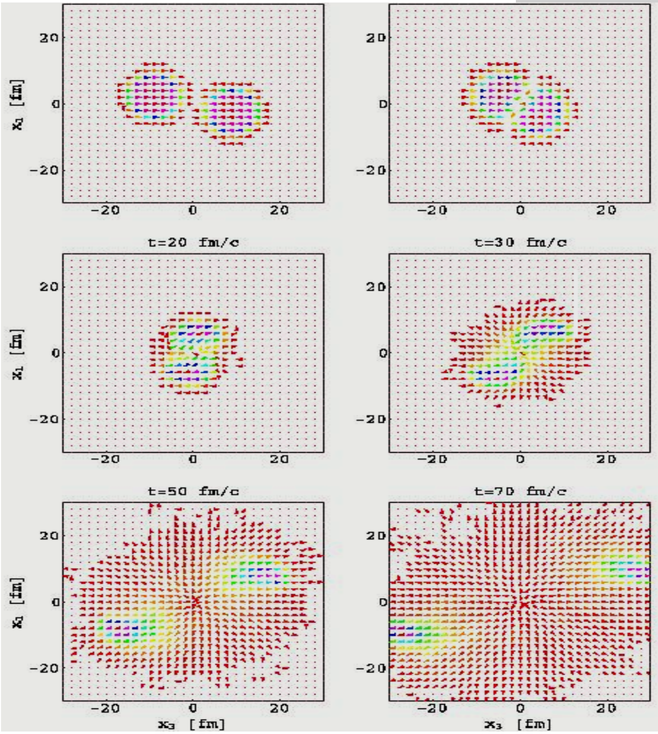
Method of solution of Transport Equation: Testparticles

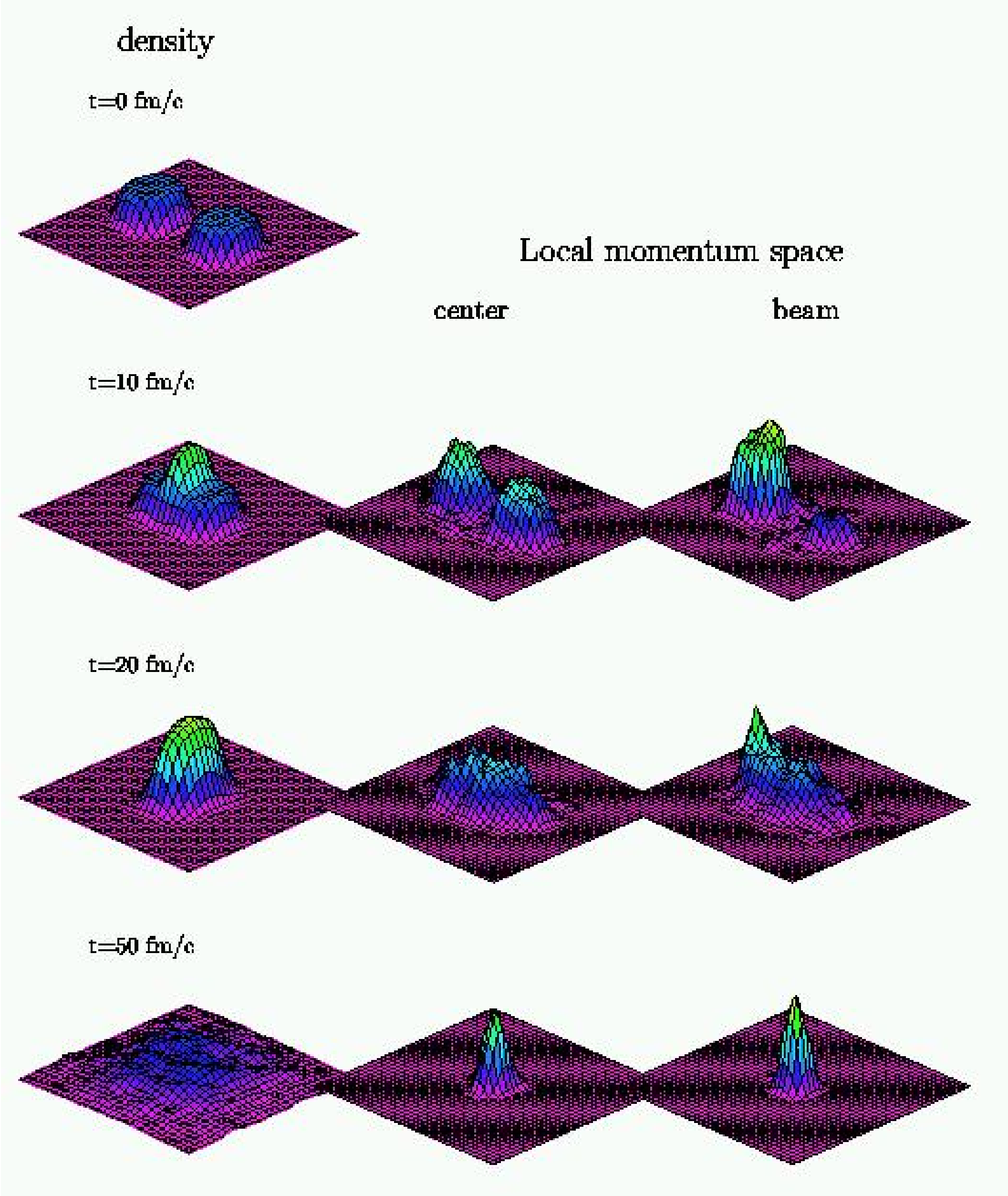
Relativistic Gaussians:

$$
\begin{aligned}
(aF)(x,k^*) &= \frac{C}{N}\sum_{i=1}^{A\cdot N}\int_{-\infty}^{\infty} d\tau\, g\left(x - x_i(\tau)\right)\tilde{g}\left(k^* - k_i^*(\tau)\right) \\
&= \frac{C}{N\left(\pi\sigma\sigma_k\right)^3}\sum_{i=1}^{A\cdot N}\int_{-\infty}^{\infty} d\tau\, e^{(x-x_i(\tau))^2/\sigma^2} e^{\left(k^*-k_i^*(\tau)\right)^2/\sigma_k^2} \\
&\quad \times\delta\left[\left(x_\mu - x_{i\mu}(\tau)\right)u_i^\mu(\tau)\right]\delta\left[k_\mu^* k_i^{*\mu}(\tau) - m_i^{*2}\right] \quad ,
\end{aligned}
$$

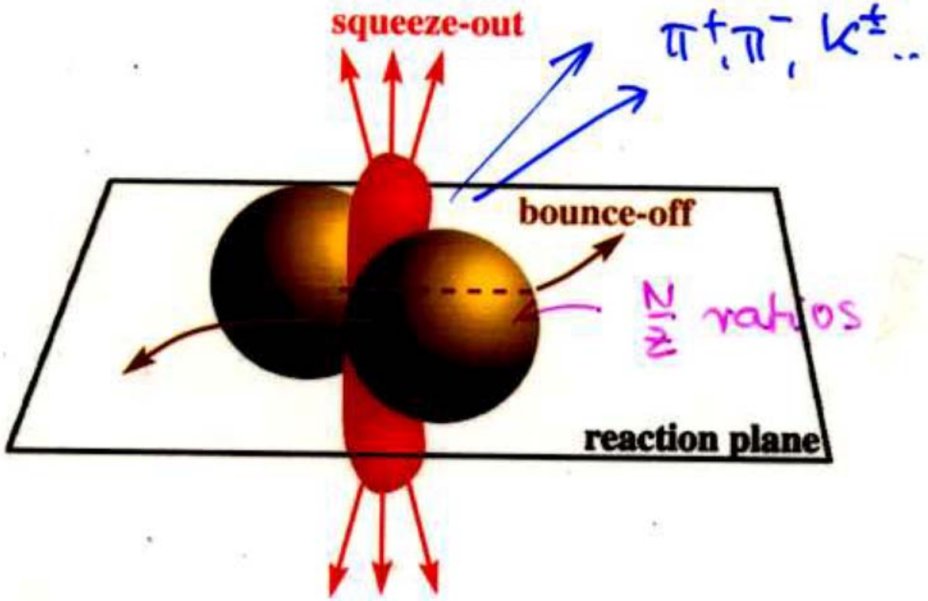
Hamiltonian equations of motion:

$$
\begin{aligned}
\frac{d}{d\tau}x_i^\mu &= u_i^\mu(\tau) \\
\frac{d}{d\tau}u_i^\mu &= \frac{1}{m_i^*}\left(u_{i\nu}F^{\mu\nu} + \partial^\mu m_i^* - (\partial^\nu m_i^*)u_{i\nu}u_i^\mu\right)
\end{aligned}
$$




density
t=0 fm/c
Local momentum space
center
beam
t=10 fm/c
t=20 fm/c
t=50 fm/c

Observables:



- Flow: $N(\theta; y, p_t, b) = N_0(1 + v_1(y, p_t)\cos\theta + v_2(y, p_t)\cos 2\theta + \ldots)$

  direct. flow (v_1), ellipt. flow (v_2)

  $v_{1,2}^{pn} = v_{1,2}^{p} - v_{1,2}^{n}$ proton – neutron diff. flow

- π^+/π^- ratios: $n/p \searrow \rightarrow \begin{cases} Y(\Delta^0, \Delta^-) \searrow \\ Y(\Delta^+, \Delta^{++}) \nearrow \end{cases} \rightarrow \pi^-/\pi^+ \searrow$

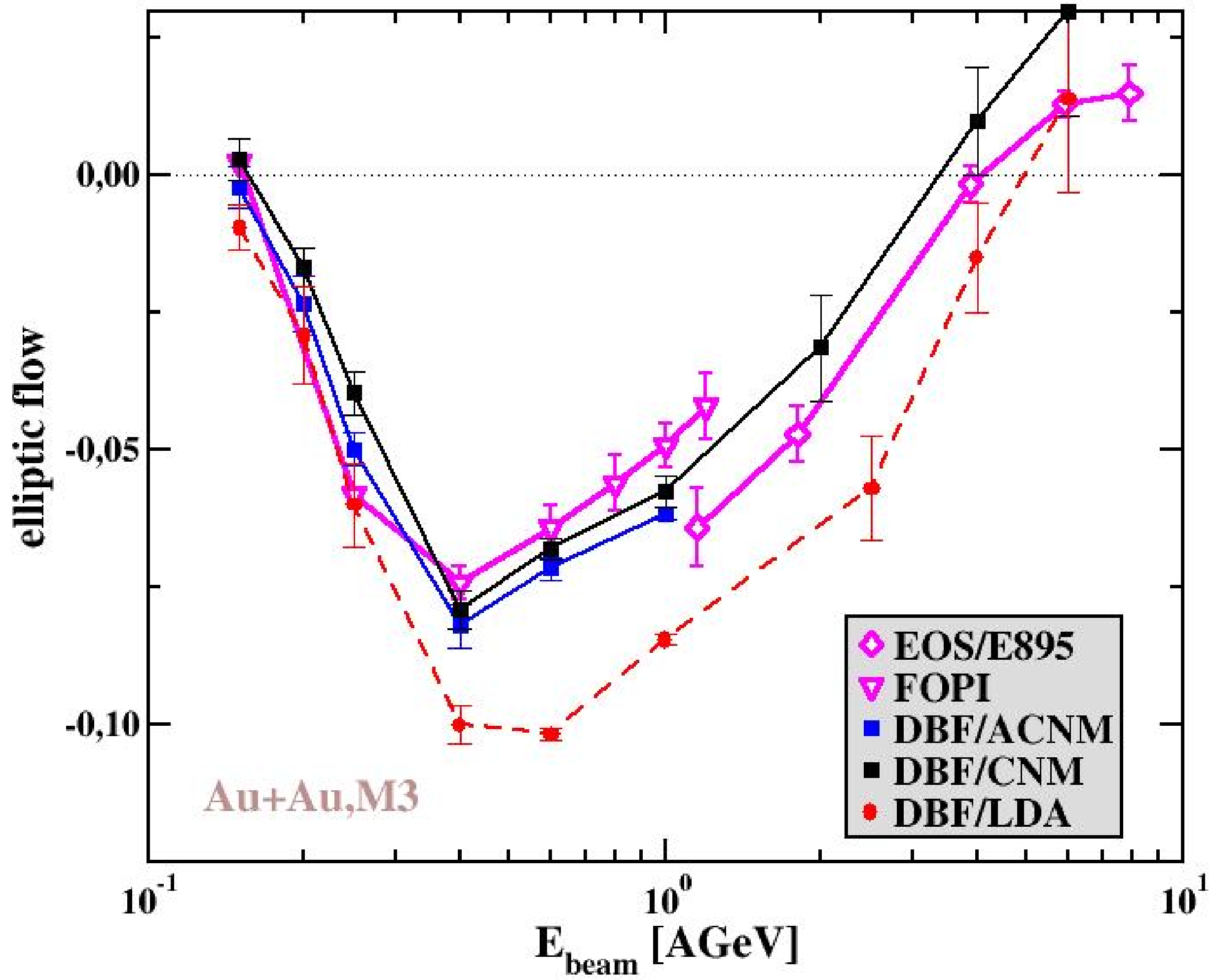
  plus threshold eff: $\frac{m_n^*}{m_p^*} \searrow \rightarrow \pi^-/\pi^+ \searrow$

- Isospin tracing

  isospin as indicator of stopping

Some results for symmetric nuclear matter

Elliptic flow

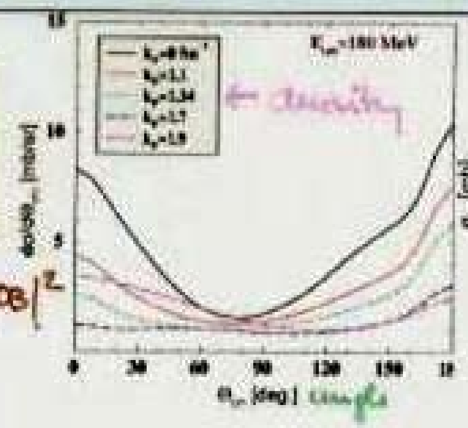


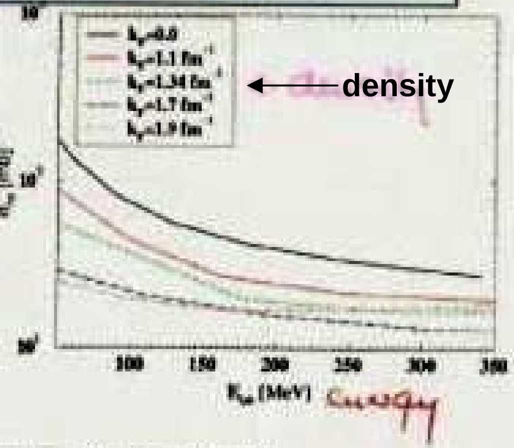
Effect of in-medium cross sections on stopping and flow: Au+Au

DB in-medium cross sections

$$\frac{d\sigma}{d\Omega}(E,\Omega,k_F) = \frac{M^{*2}}{c^4 4\pi^2}|T^{DB}|^2$$

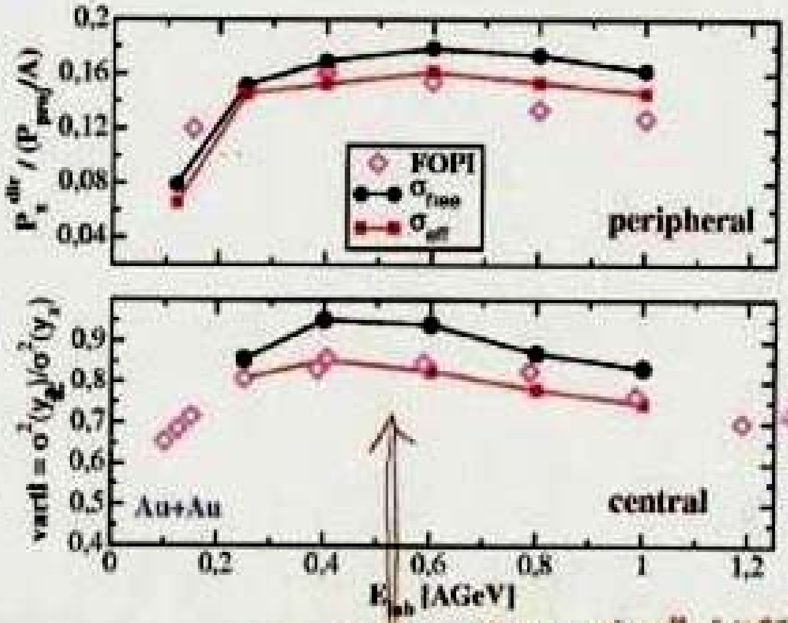
CMM {k_F1, k_F2, v_rel}

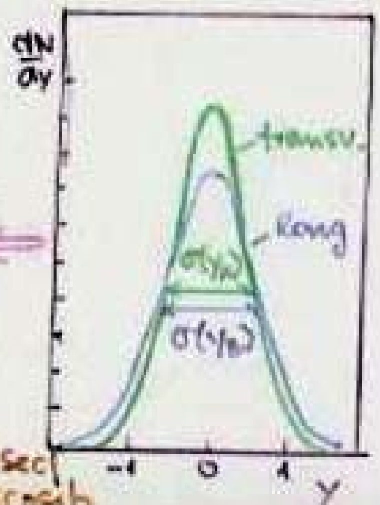




Directed flow

Ratio of rapidity Distribution in beam and transverse directions

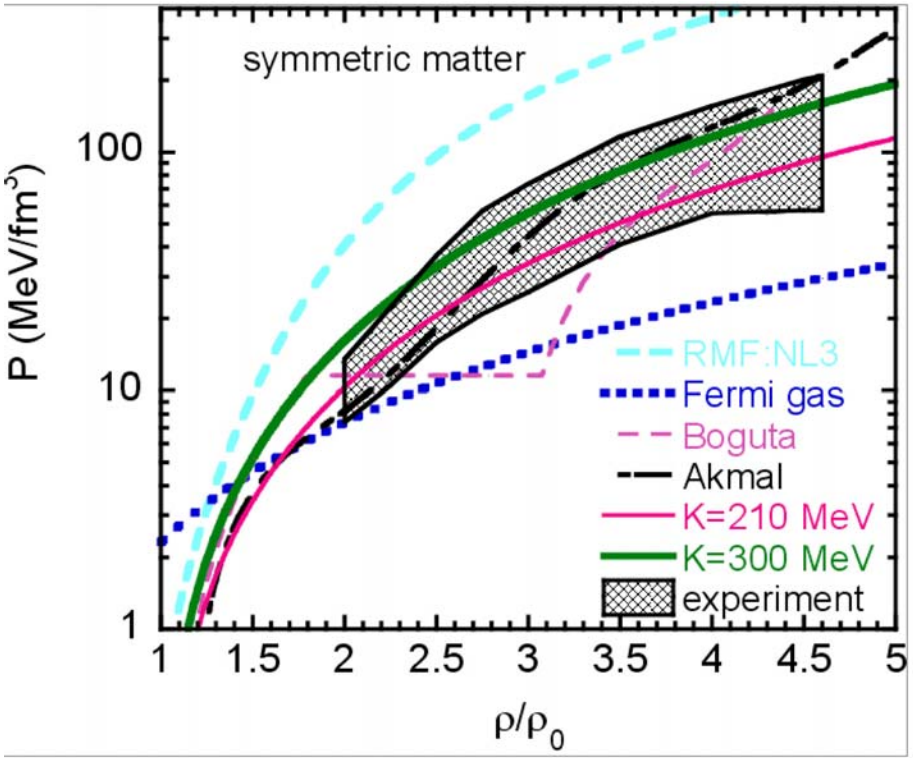


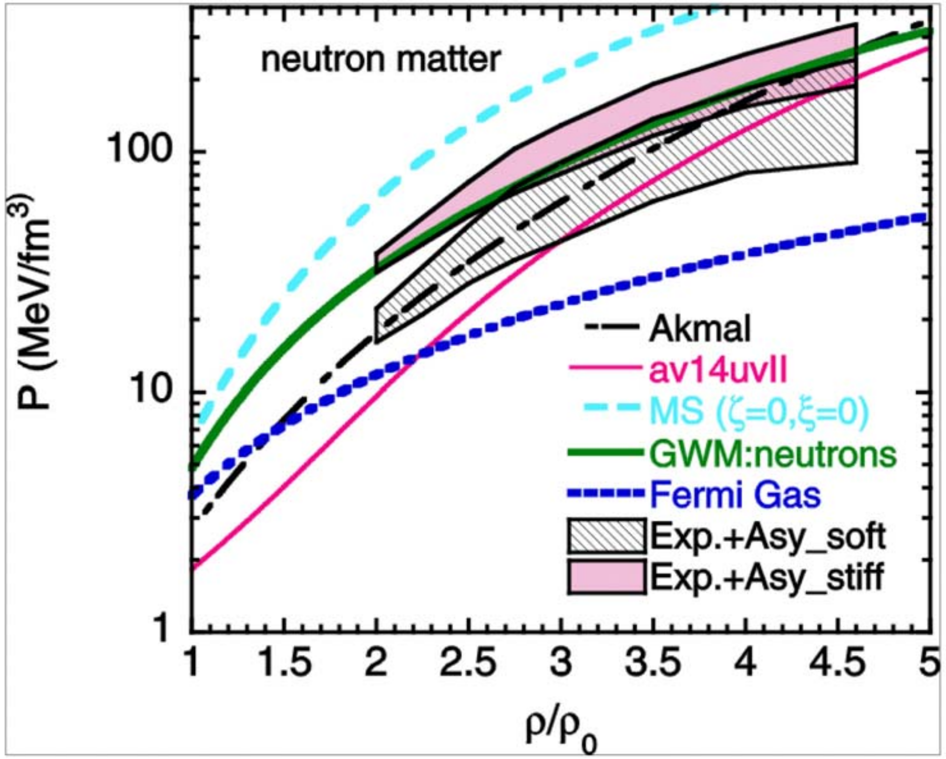


correlation { max of eff. cross sec / max of shear viscosity

Results from Flow Analysis

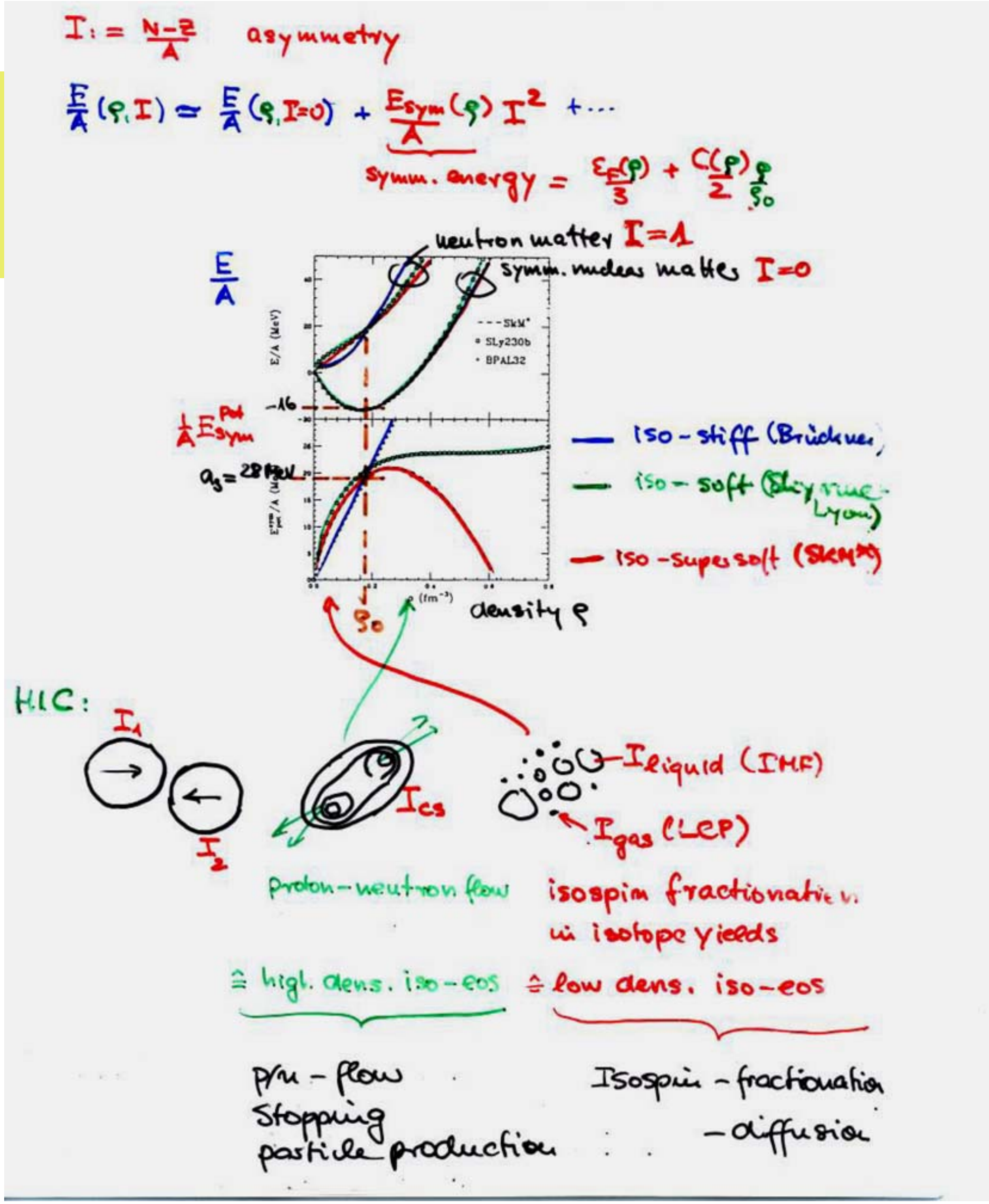
(P. Danielewicz, R.Lacey)



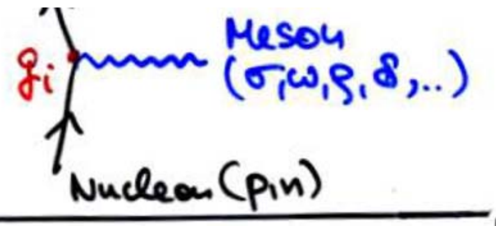


Isovector EOS

and heavy ion collisions



RELATIVISTIC MEAN FIELD



QHD:

$$\mathcal{L} = \mathcal{L}_{Nucl} + \mathcal{L}_{Meson}$$

$$\mathcal{L}_{Nucl} = \bar{\psi}\left[i\gamma^{\mu}\left(\partial_{\mu} - g_{\omega}\omega_{\mu} - g_{\rho}\vec{\tau}\cdot\vec{\rho}\right) - \left(m - g_{\sigma}\sigma - g_{\delta}\vec{\tau}\cdot\vec{\delta}\right)\right]\psi$$

$$\mathcal{L}_{Meson} = \mathcal{L}^{0}_{Meson} + U^{NL}(\sigma^3, \sigma^4, ..)$$

Self interactions, non-linearity
(also density dependence $g_i(\rho)$)

	Isoscalar	Isovector
Scalar	σ	δ
Vector	ω	ρ

↑ cancellation! ↑

"meson-like" - fields
(Lorentz structure)

$$U_0 \simeq \left(g_{\sigma}\left(\frac{m^*}{E_F^*}\right)^2 - \frac{E}{m} g_{\omega}\right)\rho_B \qquad E^{pot}_{sym} = \frac{1}{2}\left(g_{\rho} - g_{\delta}\left(\frac{m^*}{E_F^*}\right)^2\right)\rho_B$$

$$m^*_{p/n} = m - (g_{\sigma} \mp g_{\delta})\,\sigma$$

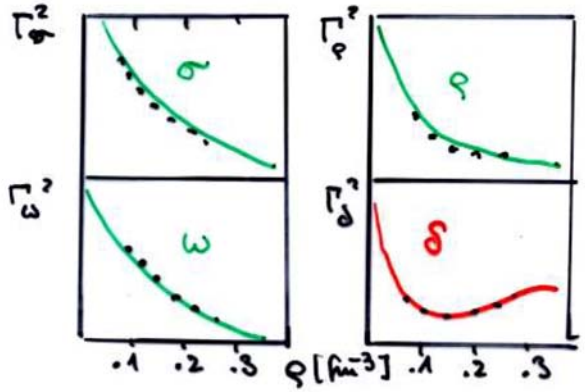
Relativistic language

	isoscalar	isovector	
scalar	σ	δ	meson-exch (-like fields)
vector	ω	ρ	

2 Strategies again:

(A) microscopic (DB) $\Sigma_i = \Gamma_i(k,\rho)\,\rho_i \; ; \; i=\sigma,\omega,\rho,\delta$

vertex fct.



(B) phenomenological NLρ NL$\rho\delta$

Consequences

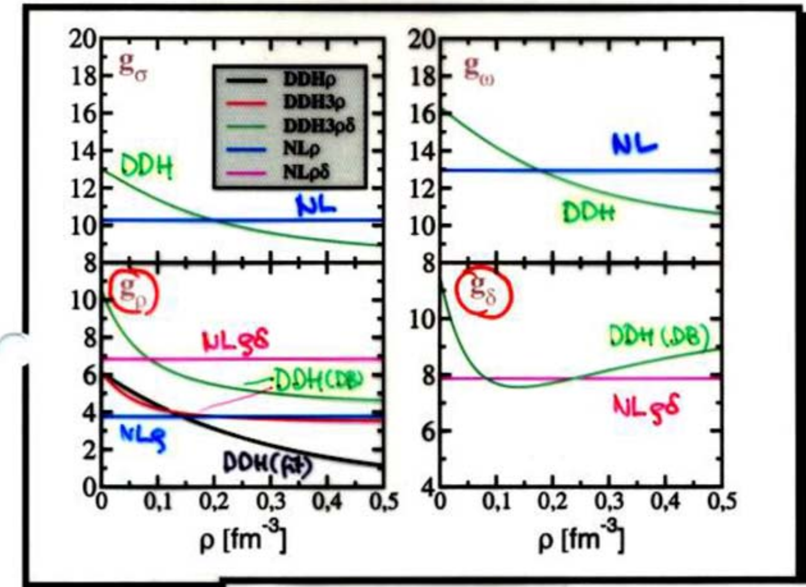
$$E_{sym} = \frac{1}{6}\frac{k_F^2}{E_F} + \frac{1}{2}\left(\Gamma_\rho - \Gamma_\delta\left(\frac{m^*}{E_F}\right)^2\right)\rho_B$$

dens. dep. cancellation

$$m^*_{p/n} = m - (\Gamma_\sigma \mp \Gamma_\delta)\,\sigma$$

effective mass splitt.

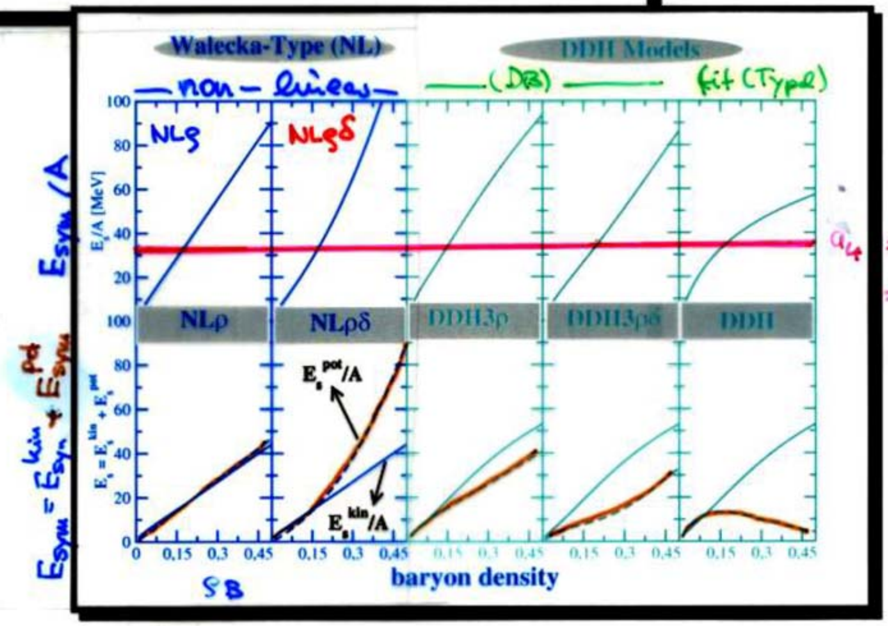
Iso-vector EOS in RMF



Coupling vertices:
g_i ($i = \sigma, \omega, \rho, \delta$)

Isoscalar

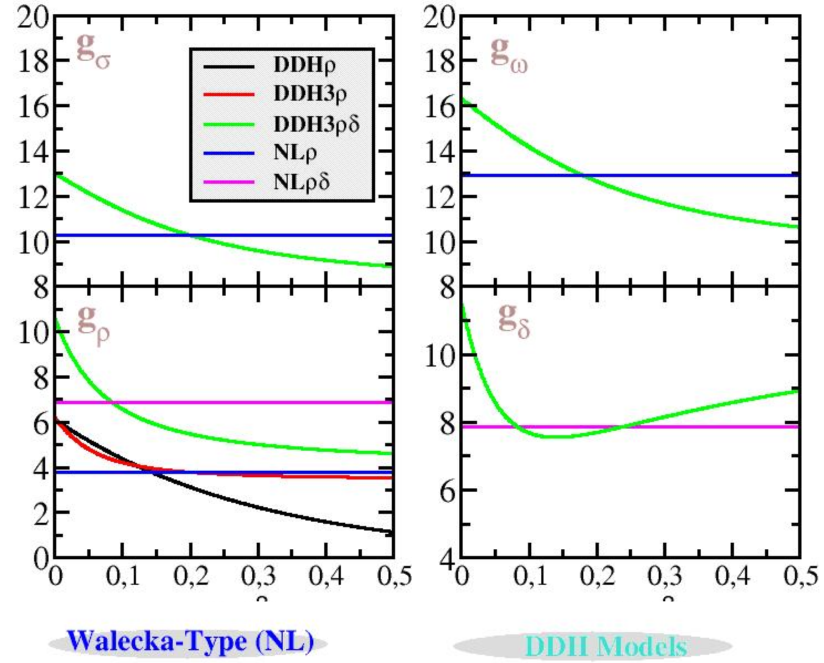
Isovector

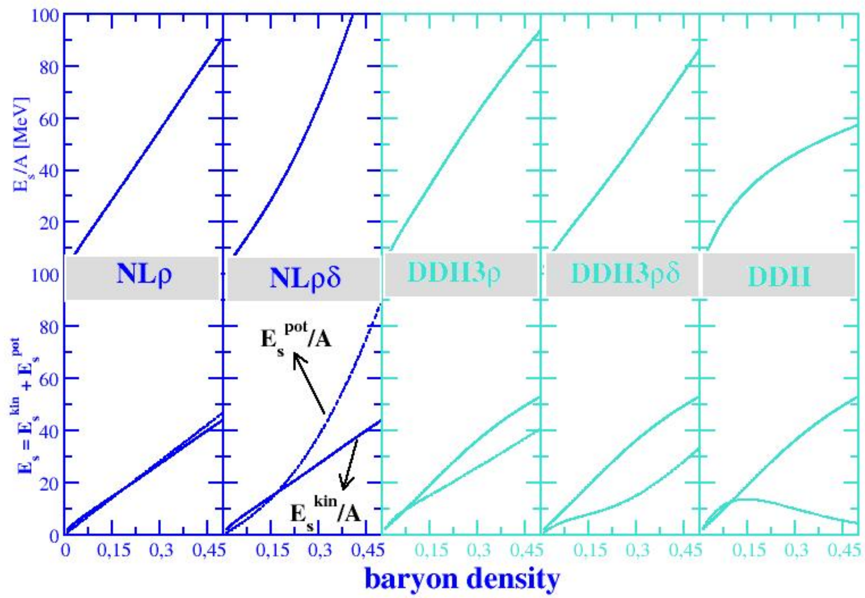


$a_4 = 33.4$ MeV $= E_{sym}(\rho_0)$

T. Gaitanos, M. Di Toro, S. Typel, V. Baran, C. Fuchs, V. Greco, H. H. W, Nucl. Phys A732 (04) 24

A $\rho\delta$ parametrization of the isovector dpendence

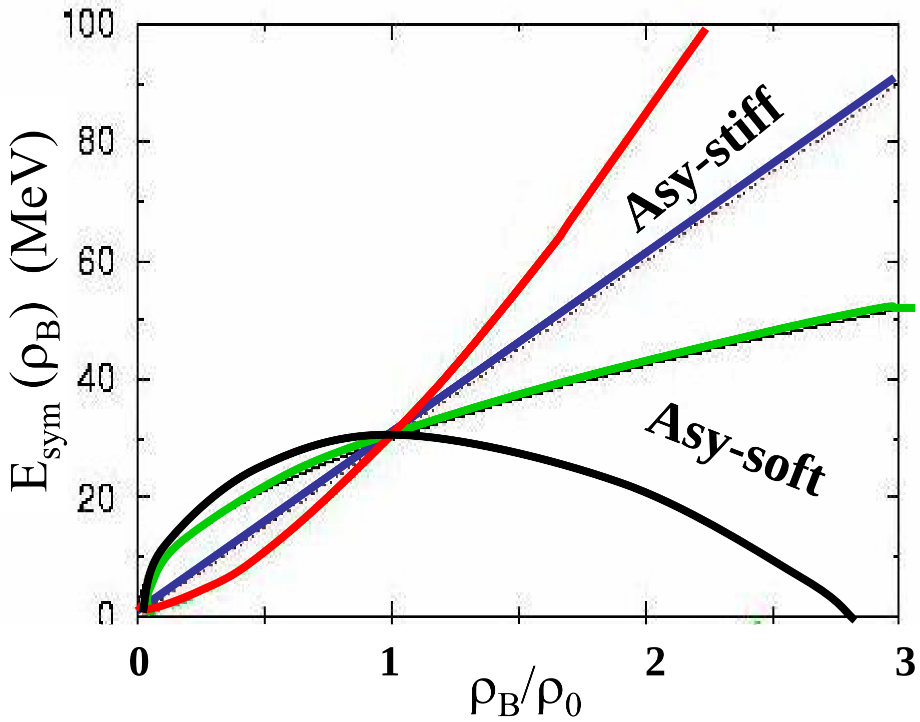




Symmetry Energy

$$E(\rho_B,\alpha) = E(\rho_B) + E_{sym}(\rho_B) I^2 + O(I^4) + \ldots$$

$$I = \frac{\rho_n - \rho_p}{\rho_n + \rho_p} \qquad E_{sym} = \frac{1}{2} \frac{\partial^2 E}{\partial I^2}\bigg|_{I=0}$$



Expansion around ρ_0

$$E_{sym} = a_4 + \frac{L}{3}\left(\frac{\rho_B - \rho_0}{\rho_0}\right) + \frac{K_{sym}}{18}\left(\frac{\rho_B - \rho_0}{\rho_0}\right)^2$$

Pressure & compressibility

$$L = 3\rho_0 \frac{\partial E_{sym}}{\partial \rho_B}\bigg|_{\rho_B=\rho_0} = \frac{3}{\rho_0} P_{sym}$$

$$K_{sym} = 9\rho_0^2 \frac{\partial^2 E_{sym}}{\partial \rho_B^2}\bigg|_{\rho_B=\rho_0}$$

Pressure gradient $\quad \dfrac{dP_{sym}}{d\rho} = \dfrac{2}{3} L + \dfrac{1}{9} K_{sym}$

RMF Symmetry Energy: $\delta - contrib.$

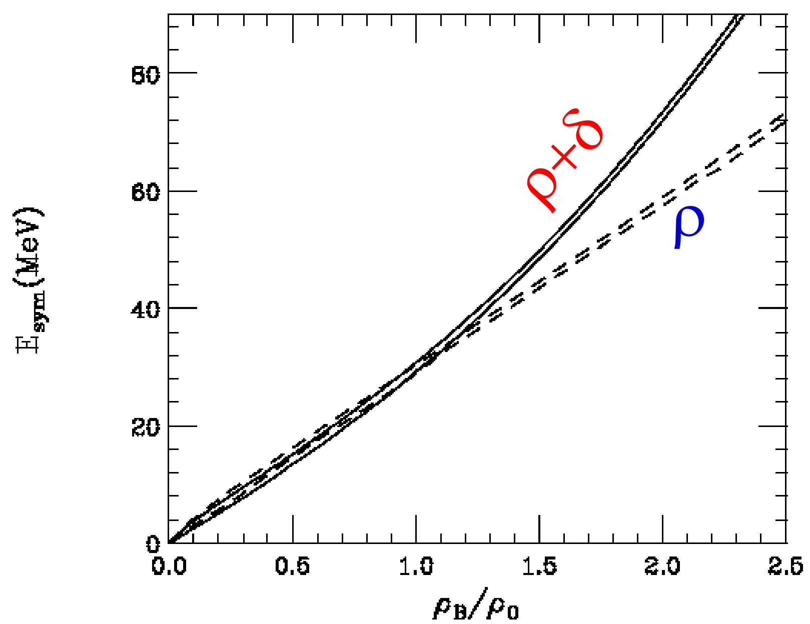
$$E_{sym} = \frac{1}{6}\frac{k_F^2}{E_F^{*2}} + \frac{1}{2}\left[f_\rho - f_\delta\left(\frac{M^*}{E^*}\right)^2\right]\rho_B$$

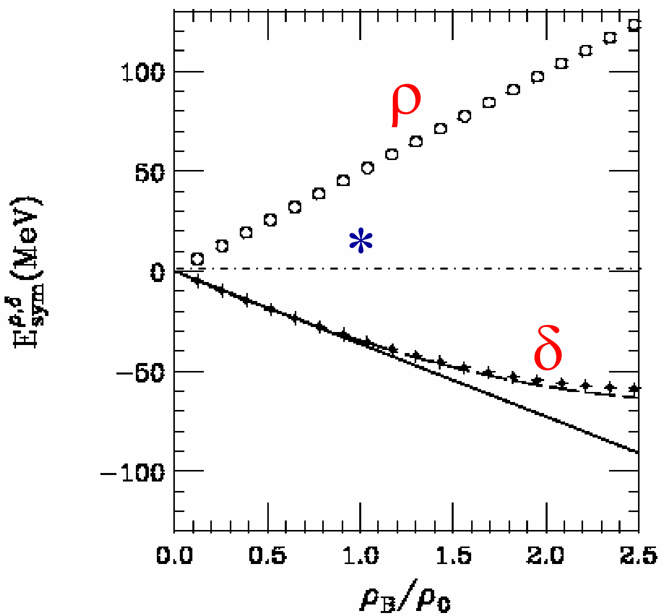
→ No δ ➡ $f_\rho \cong 1.5 f_\rho^{\mathrm{FREE}}$

→ $f_\delta = 2.5\ fm^2$ ➡ $f_\rho \cong 5 f_\rho^{\mathrm{FREE}}$

$a_4 = E_{sym}(\rho_0)$ fixes (f_ρ, f_δ)

DBHF, DHF } $f_\delta \approx 2.0 \div 2.5\ fm^2$

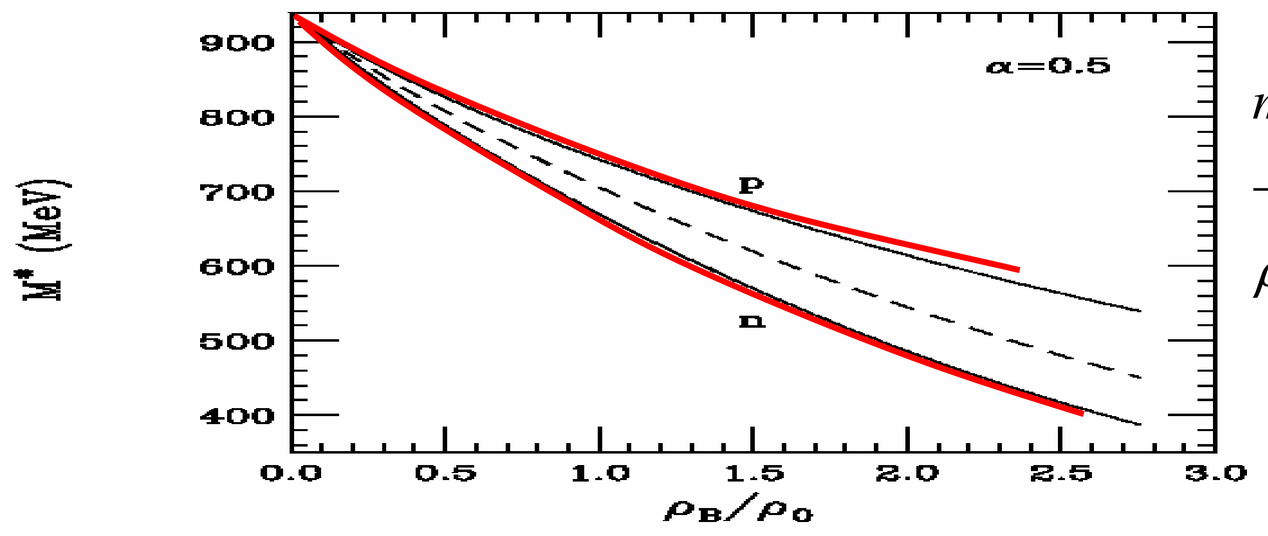




PRC65(2002)045201

Effective Mass Splitting: Dirac Masses

Minimal Effective Field Approach: $(\sigma, \omega, \delta, \rho)$



$$m_D^*(q) = m + \Sigma_s(\sigma) \pm f_\delta \rho_{S3}$$

$$\rightarrow +n, -p$$

$$\rho_3 \equiv \rho_p - \rho_n, < 0 \Rightarrow n - rich$$

RMF−(ρ+δ)

RMF-ρ

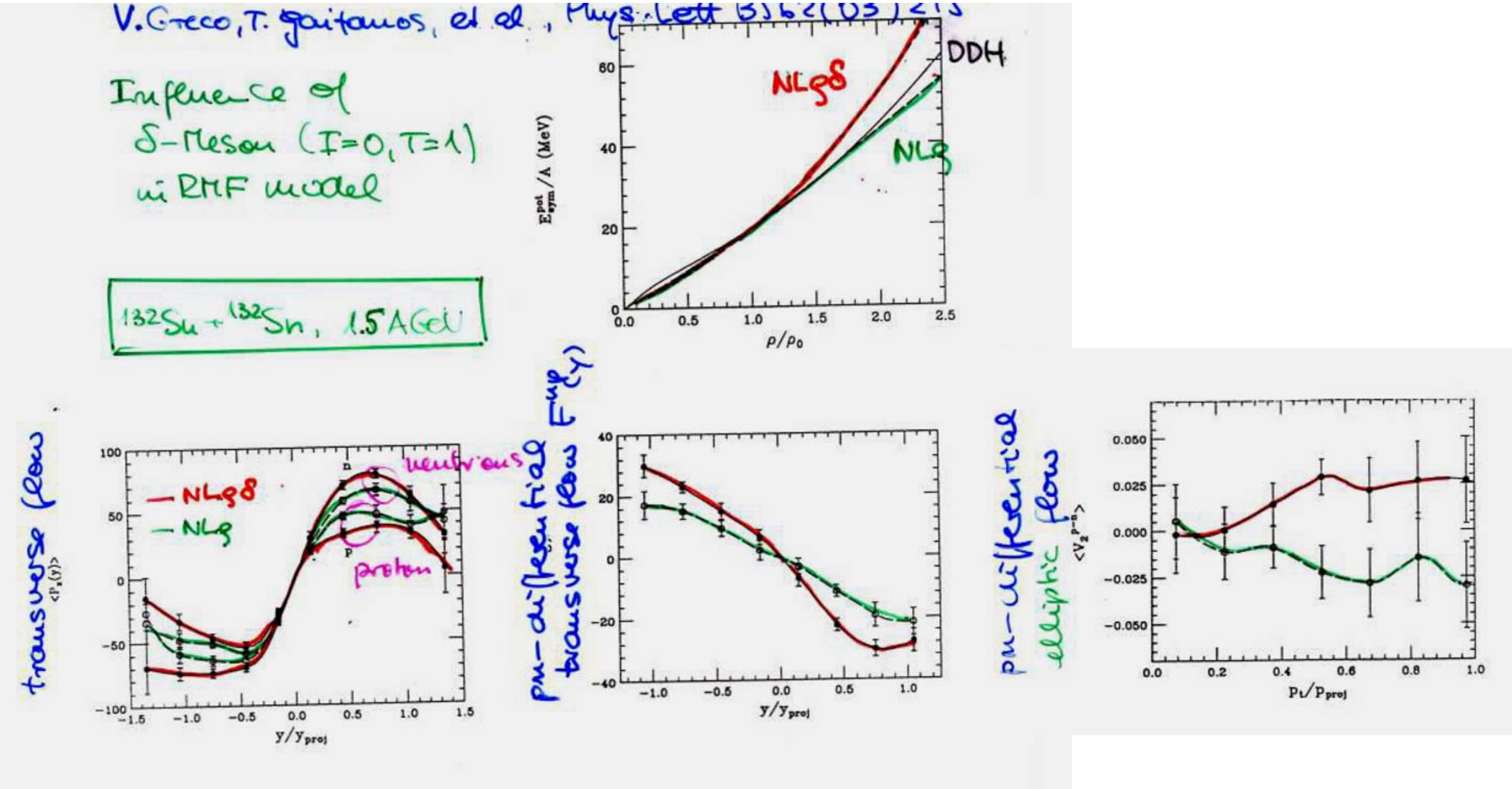
Splitting sign $(m_D^*(n) - m_D^*(p))$

Agree {
- RMFT, DHF(V. Greco et al., PRC63, PRC64 (2001))
- DBHF (F. Hofmann et al., PRC64 (2001))
- SLy (E. Chabanat et al., NPA 627 (1997)) non rel.

Disagree {
- BHF (W.Zuo, PRC60 (1999) 24605) non rel.
- "Old" Skyrme

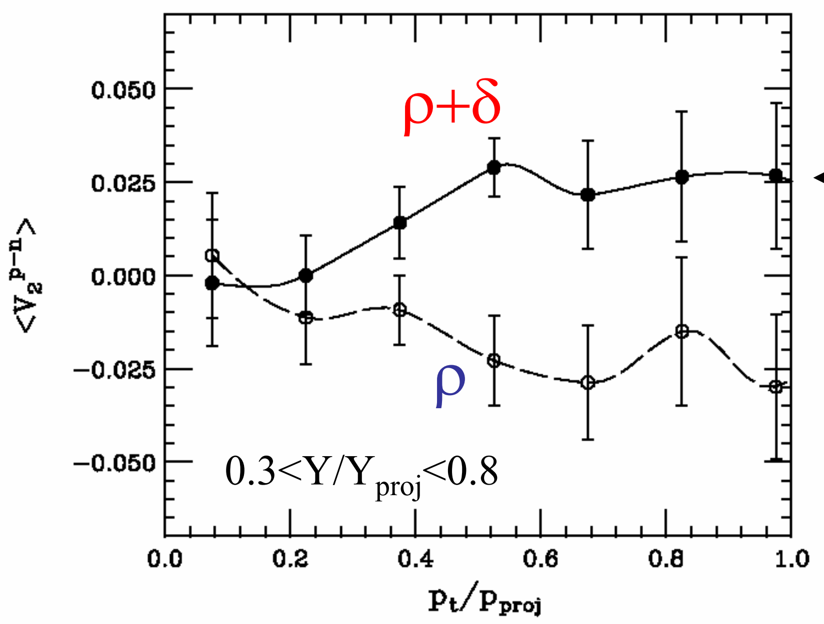
PRC65(2002)045201

Dynamical Isospin flow effects



Elliptic flow

- Difference at high p_t ⇔ first stage



High p_t neutrons are emitted "earlier"

Equilibrium (ρ,δ) dynamically broken
Importance relativistic structure

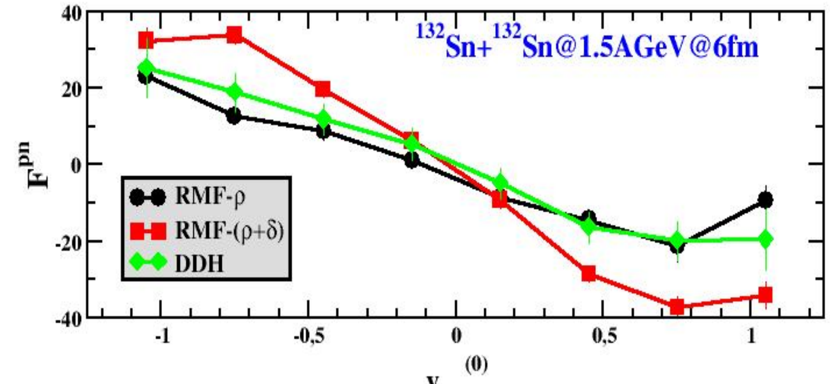
approximations

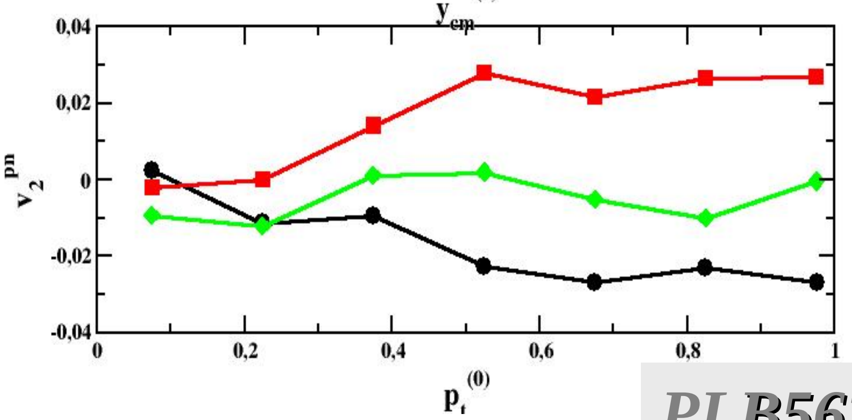
$$\frac{d\vec{p}_p^{\,*}}{d\tau} - \frac{d\vec{p}_n^{\,*}}{d\tau} \simeq 2\left[\gamma f_\rho - \frac{f_\delta}{\gamma}\right]\vec{\nabla}\rho_3 = \frac{4}{\rho_B}E^*_{sym}\vec{\nabla}\rho_3$$

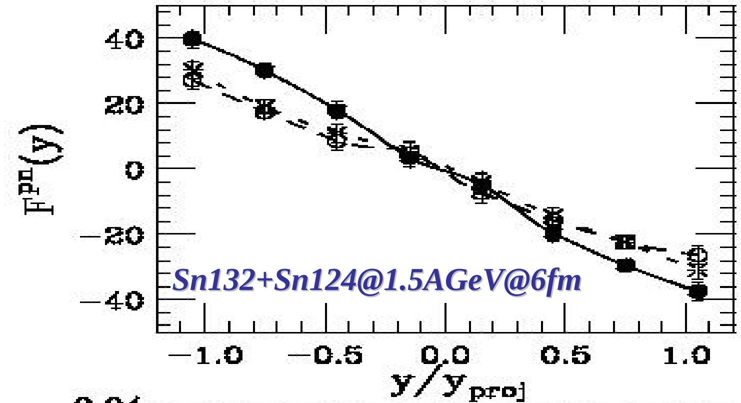
$$2\left[f_\rho - f_\delta \frac{M^*}{E_F^*}\right] = \frac{4}{\rho_B}E_{sym}^{pot}$$

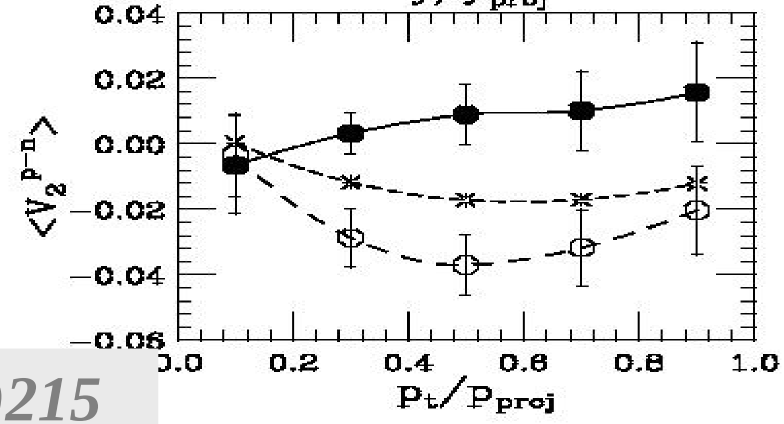
Test with ρ & $E_{sym} \approx$ NL-(ρ+δ)

Collective isospin flows@SIS









PL**B562**(2003)215

Strong isospin dependence of isospin flow

→*Pt-dependence: Chronometer of collision (high pt's reflects earlier high compression)*

→ *NLρδ: more I-Flow due to Lorentz decomposition of iso-vector channel:* $\frac{d\vec{p}_p^{\,*}}{d\tau} - \frac{d\vec{p}_n^{\,*}}{d\tau} \approx 2\left[\gamma f_\rho - \frac{f_\delta}{\gamma}\right]\vec{\nabla}\rho_I$

ρ-meson enhanced by γ *δ-meson suppressed by scalar density*

need neutron (light isobars) detection from experiments

STOPPING and ISOSPIN TRACING METHOD (FOPI)

Ru + Zr System

$^{96}_{40}Zr_{56}$ $N/Z = 1.4$

$^{96}_{44}Ru_{52}$ $N/Z = 1.18$

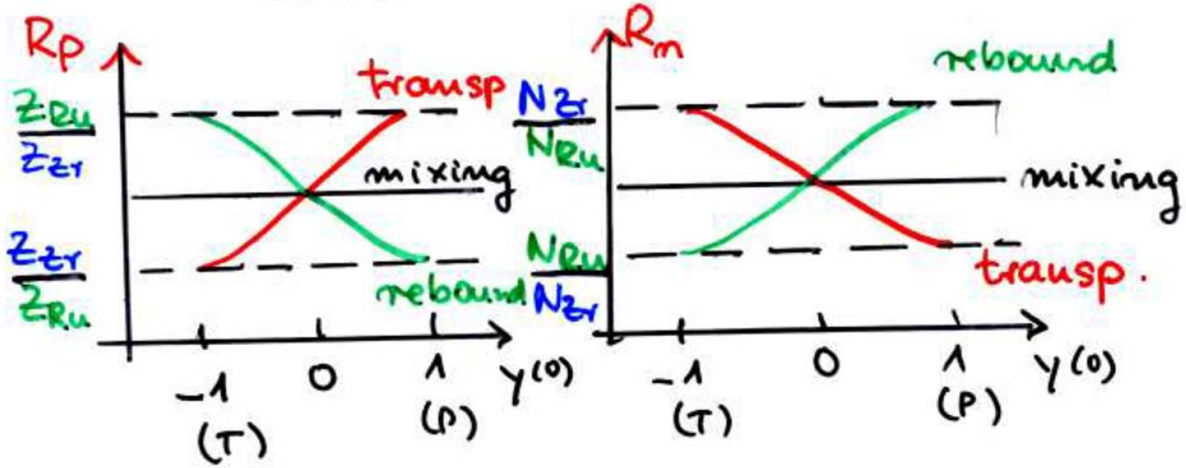
$\frac{Z_{Zr}}{Z_{Ru}} = 0.91$; $\frac{N_{Zr}}{N_{Ru}} = 1.08$

Isospin tracing Ratio

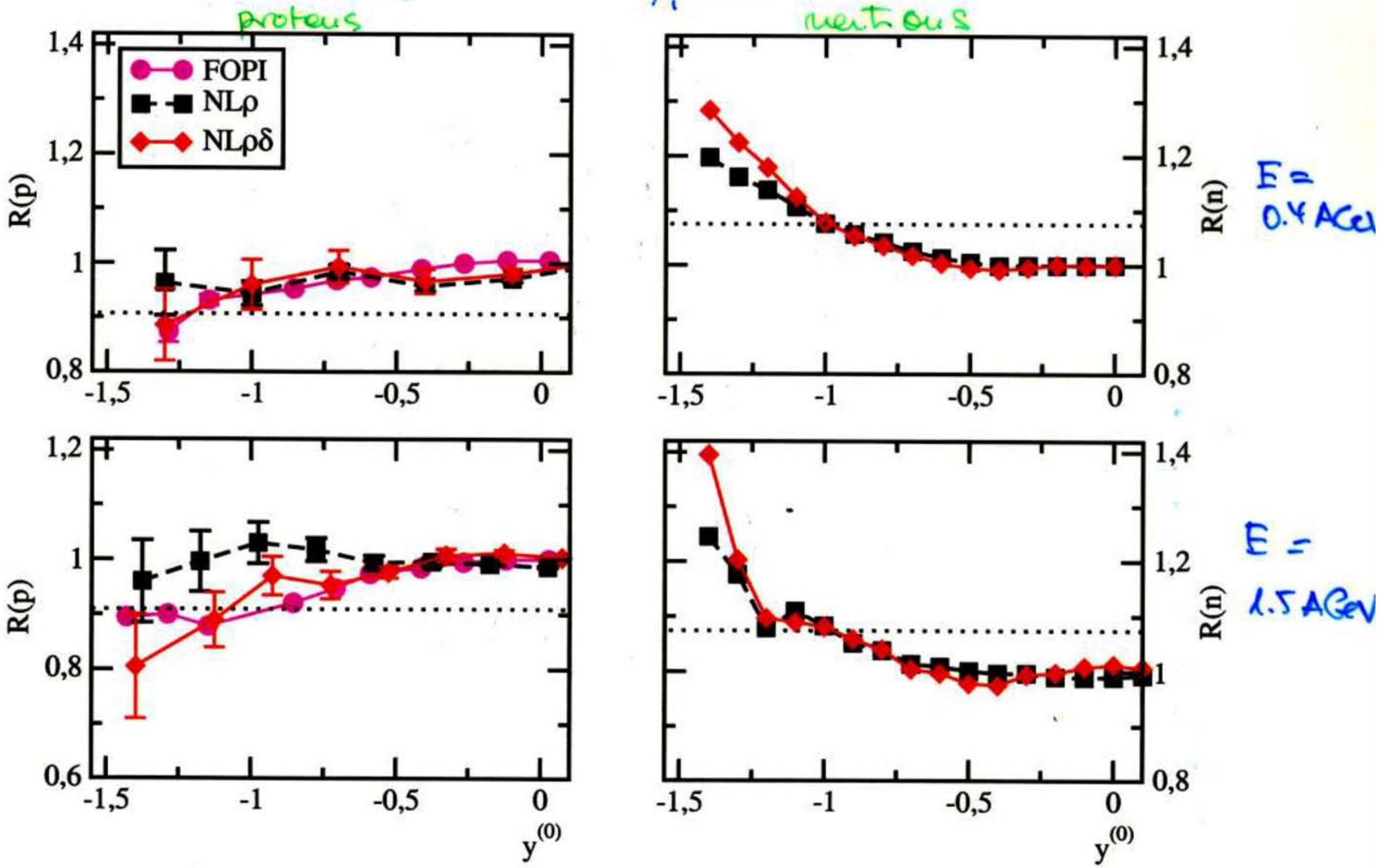
$$R_i = \frac{Y_i^{Ru\,Zr}}{Y_i^{Zr\,Ru}}$$

(P) (T)

; $i = p, n, d, t, {}^3He, \pi^\pm, \ldots$

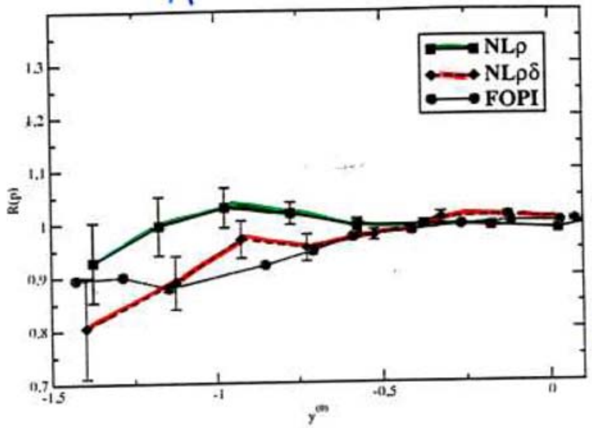


Isospin Tracing Ratio's $\frac{Y_i^{Ru+Zr}}{Y_i^{Zr+Ru}}$; central



Isospin Tracing $\frac{Y_i^{RuZr}}{Y_i^{ZrRu}}$

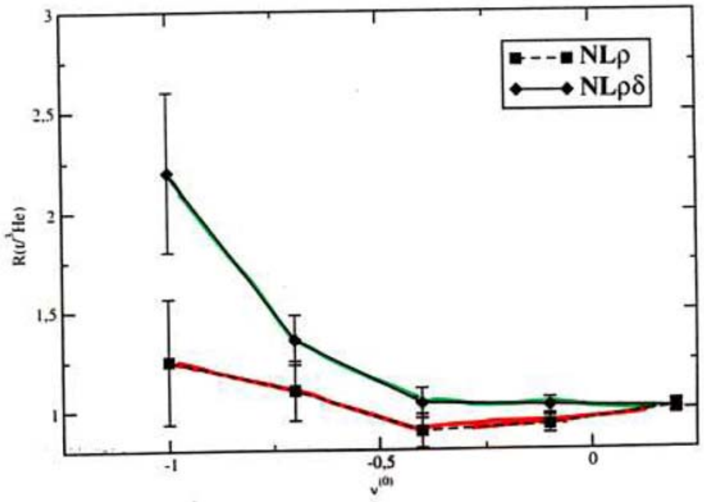
Protons



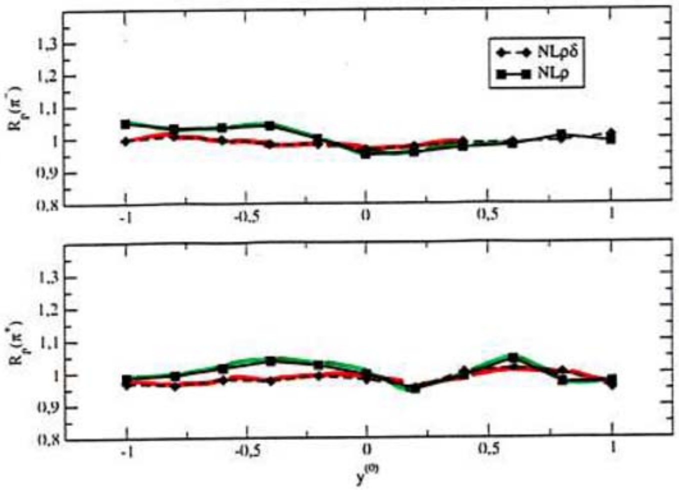
1.5GeV

$\pi^{\pm}$

t/³He



0.4GeV



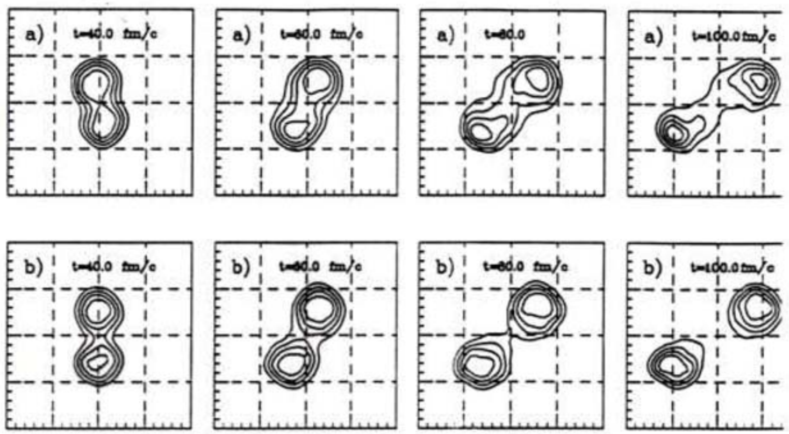
0.4GeV

Isospin Transport through Neck

$^{124,112}_{(H)\ (L)}Sn + {}^{124,112}Sn$, 50 MeV/A

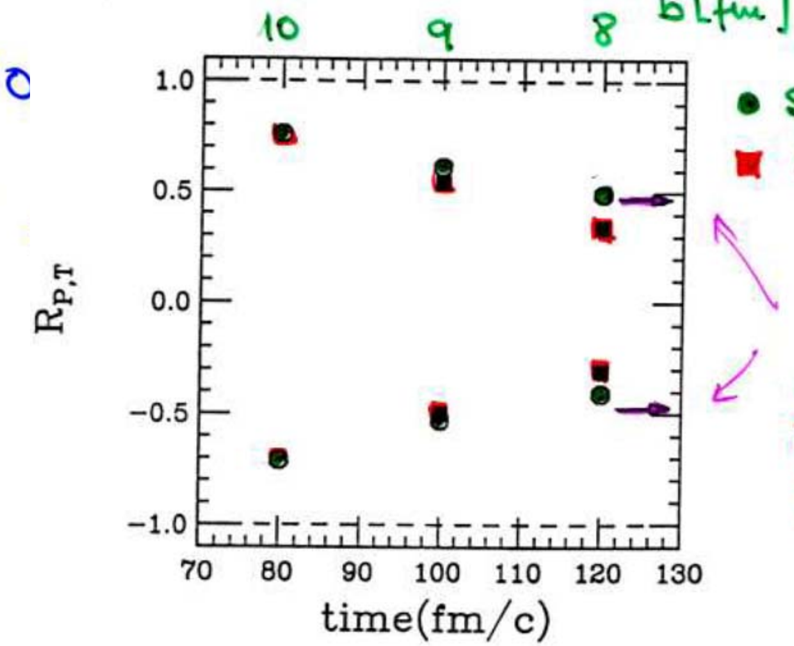
b=8 fm

b=10 fm



Isospin Imbalance Ratio

$$R_i = \frac{2I_i^M - (I_i^{HH} + I_i^{LL})}{I_i^{HH} - I_i^{LL}}$$



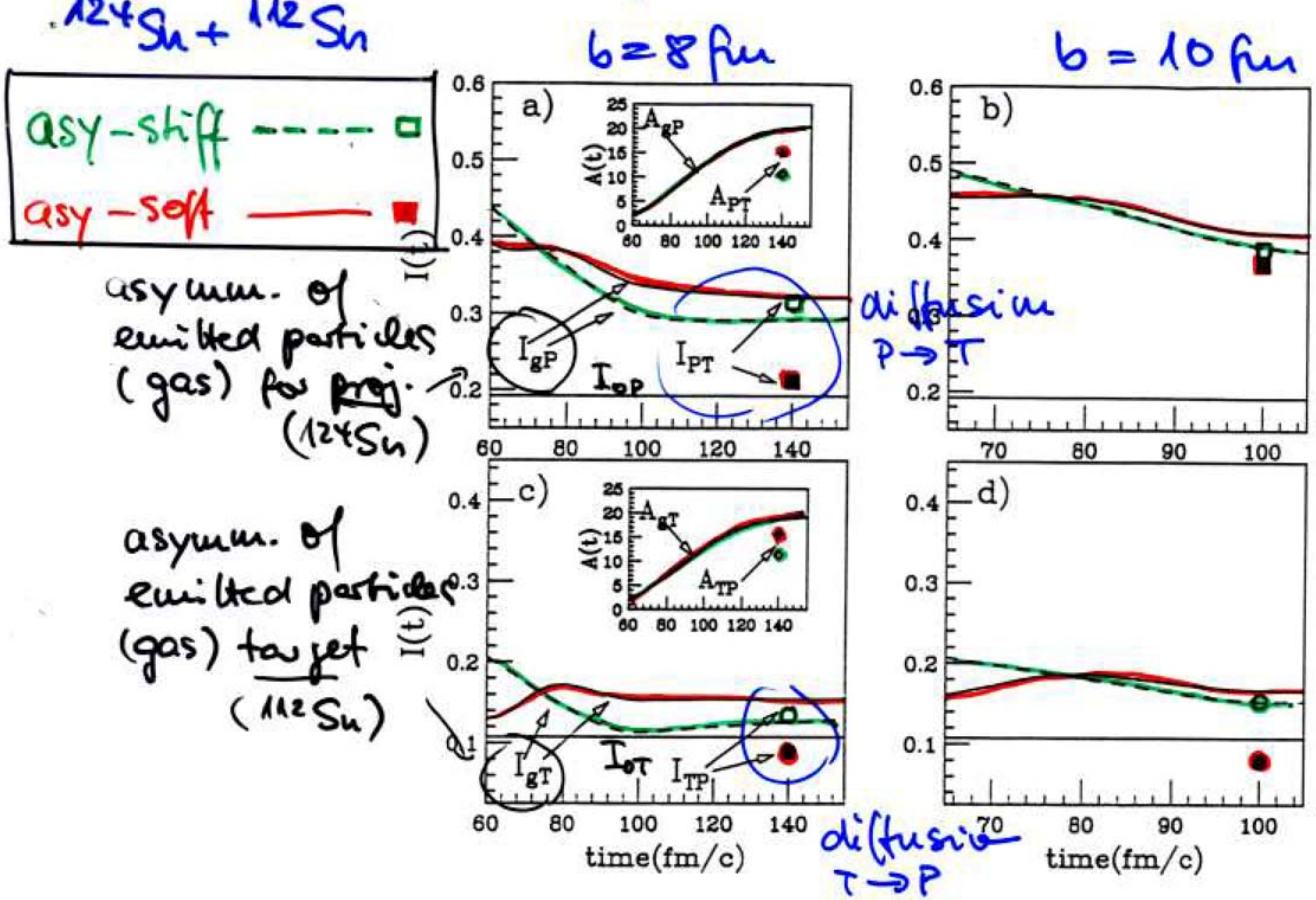
● superstiff

■ soft

MSU Exp. +Calc.

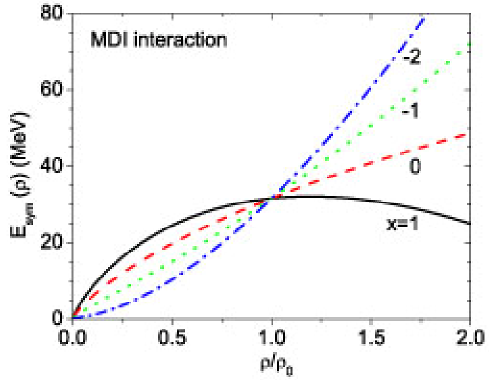
H.B.Tsang, P.Danielewicz et al PRL 92 (04) 062701

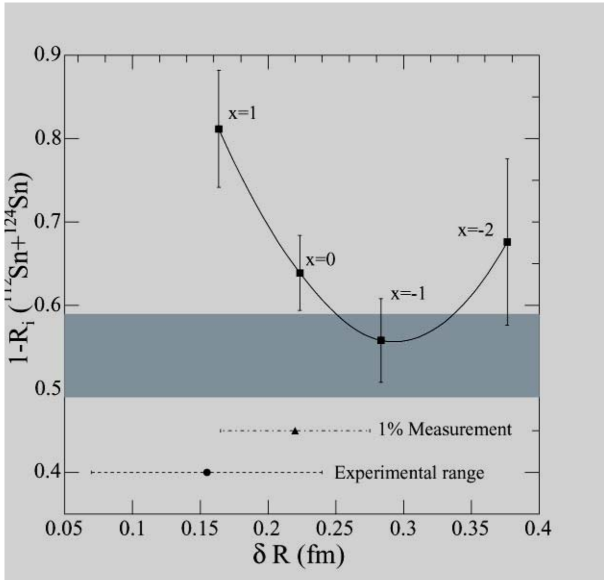
Detailed Analysis of Isospin Transport

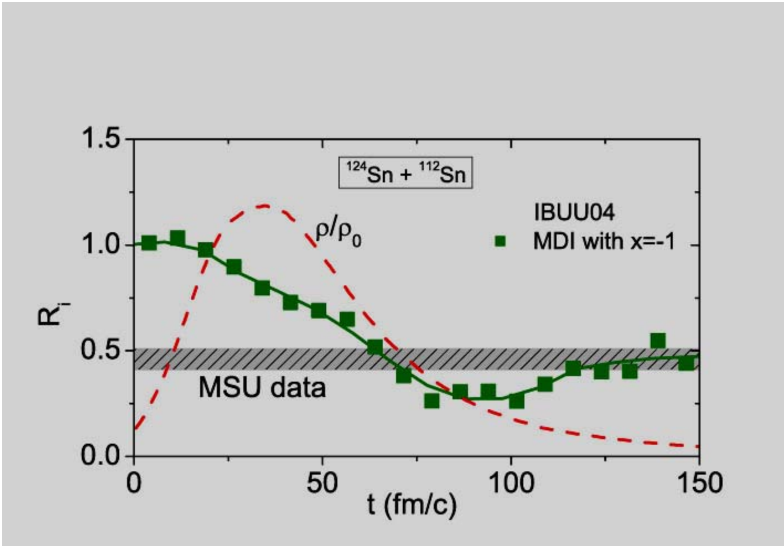


Effect of momentum dependence on

Isospin transport







K ,K -Production (as Test of the Isovector EOS)

Produced at high density in secondary reactions: $NN \to N\Delta \nearrow N\pi$

$\pi N \to Y K$

2 Effects: $\Delta N \to N Y K$

A) Neutron richness of source $I = \frac{N-Z}{N+Z}$

$nn \to N\Delta^{0,-}$

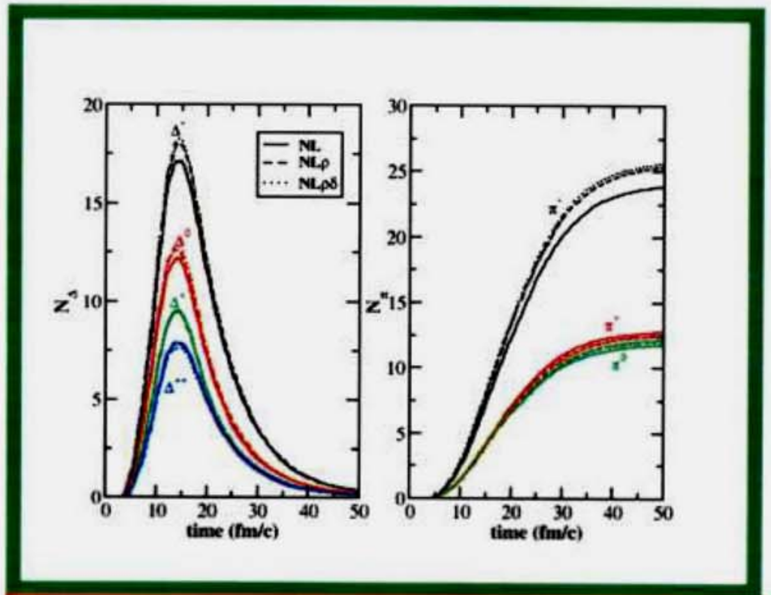
$\hookrightarrow \pi^- N \to Y K^0$

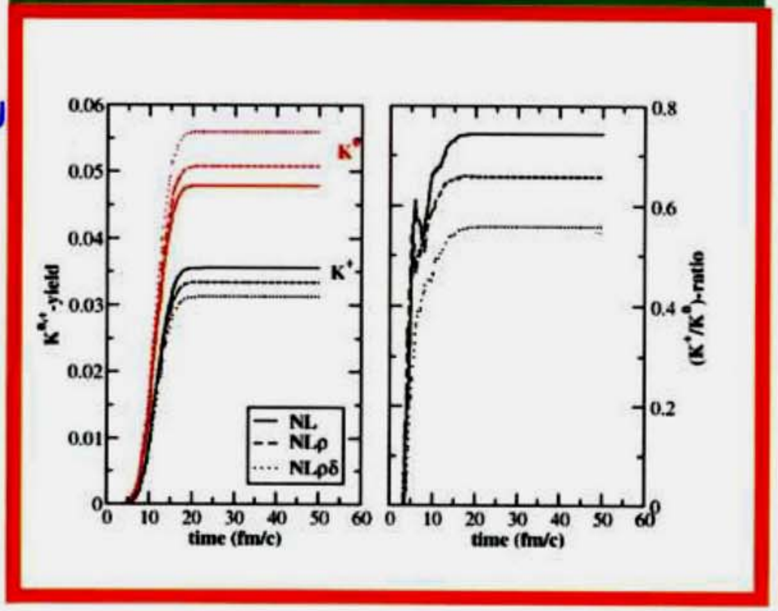
asy-stiff: $n \searrow \to K^0 \searrow$

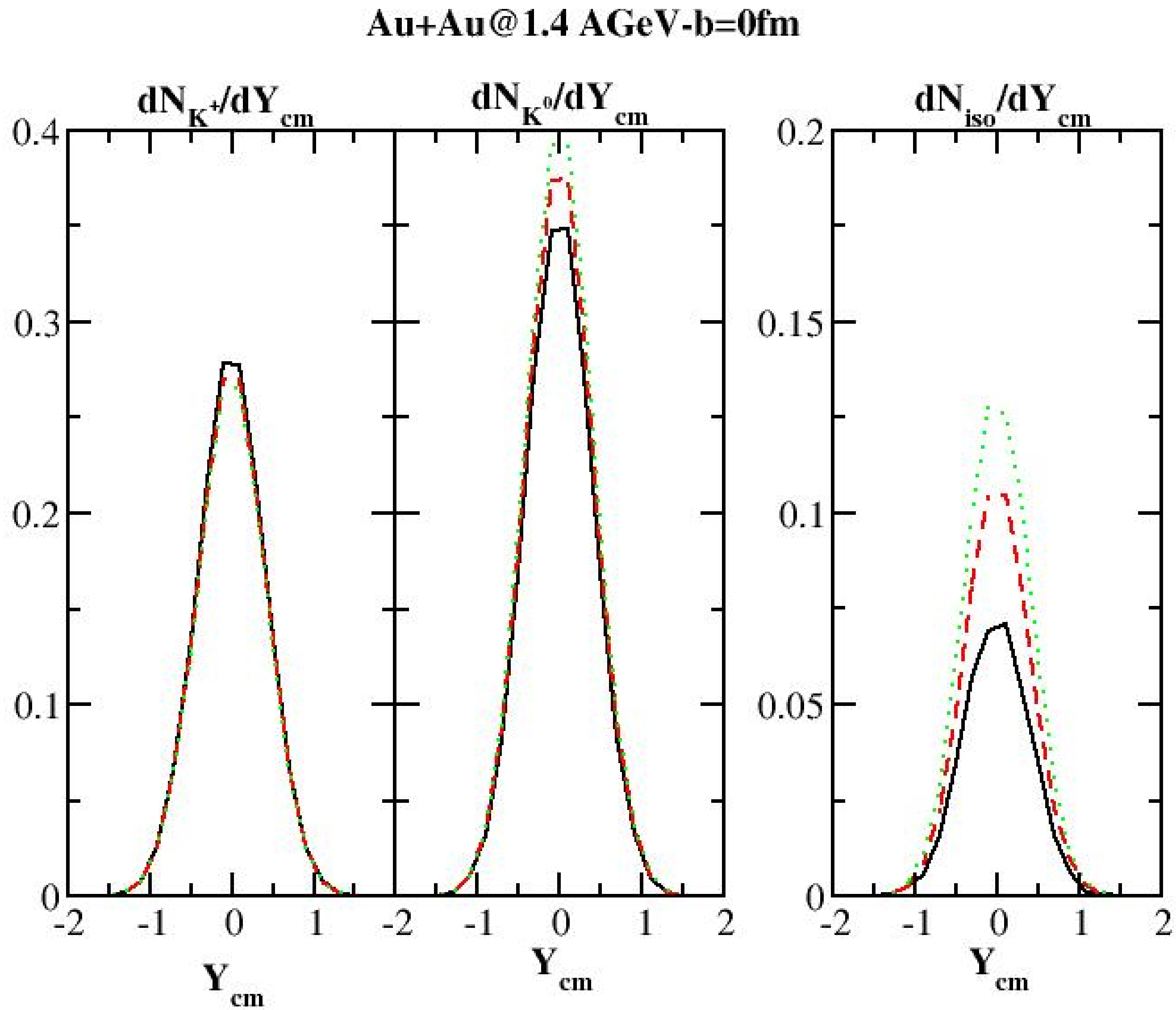
B) Threshold effect, e.g. $p\pi^- \to \Lambda K^0$

$$s_{th} = -\Sigma_p^0 + \sqrt{s_{in} + \vec{\Sigma}_p^{\,2}} + \Sigma_\Lambda^s \geq M_\Lambda + M_{K^0} = 1.6\ \mathrm{GeV}$$

asystiff: $\Sigma_p^0 \searrow \to s_{th} \nearrow \to K^0 \nearrow$







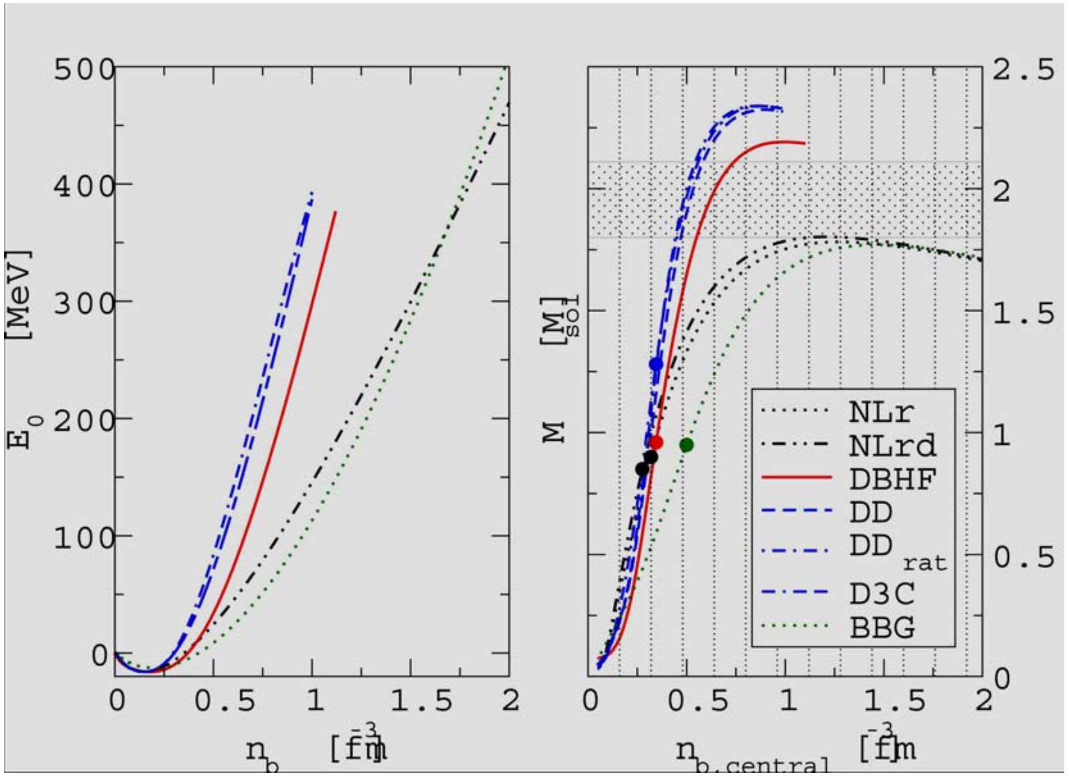
Au+Au@1.4 AGeV-b=0fm
dN_K+/dY_cm
dN_K0/dY_cm
dN_iso/dY_cm
0.4
0.3
0.2
0.1
0
0.2
0.15
0.1
0.05
0
-2
-1
0
1
2
Y_cm

Neutron star cooling and iso-vector EOS

Tolman-Oppenheimer-Volkov equation to determine mass of neutron star

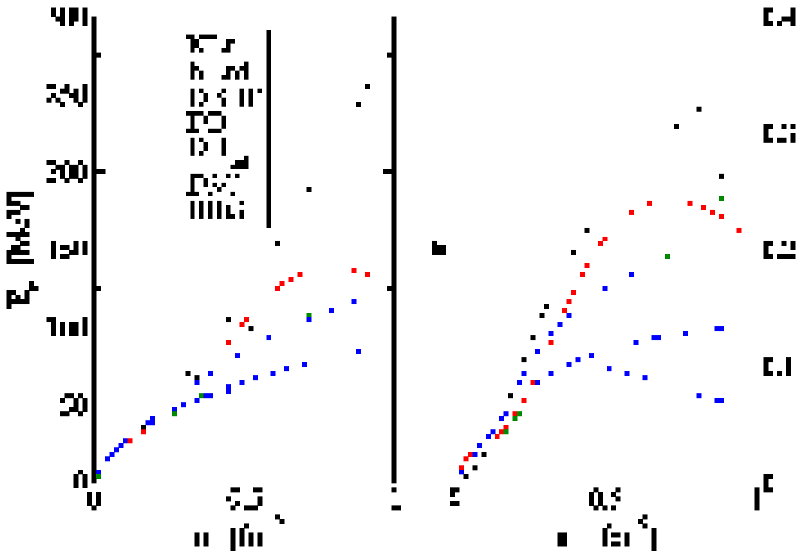
$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\frac{Gm(r)\varepsilon(r)}{r^2}\left(1+\frac{P(r)}{\varepsilon(r)}\right)\left(1+\frac{4\pi r^3 P(r)}{m(r)}\right)\left(1-\frac{2Gm(r)}{r}\right)^{-1}$$

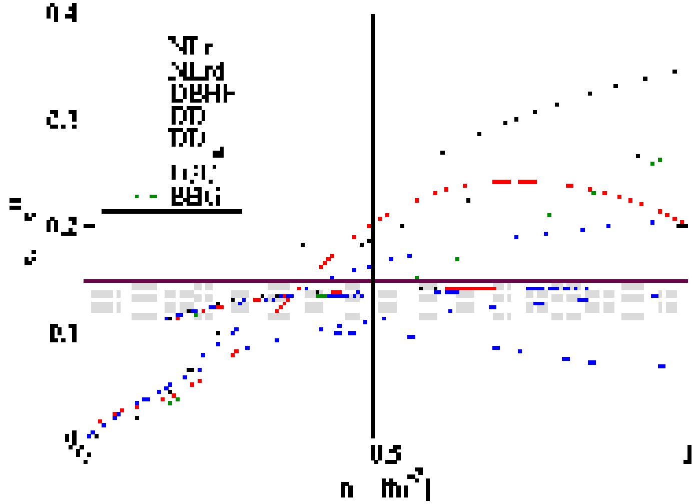
$$m(r) = 4\pi \int_0^r \mathrm{d}r'\, r'^2 \varepsilon(r').$$

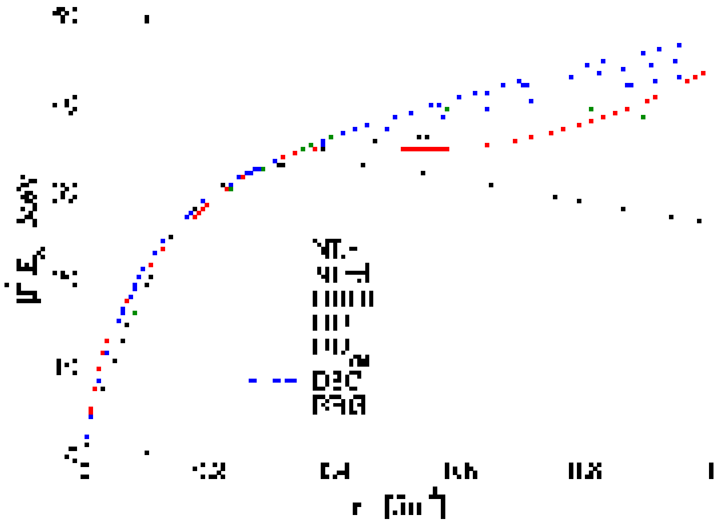


Neutron star cooling (cont'd)

Symmetry energy and protron fraction

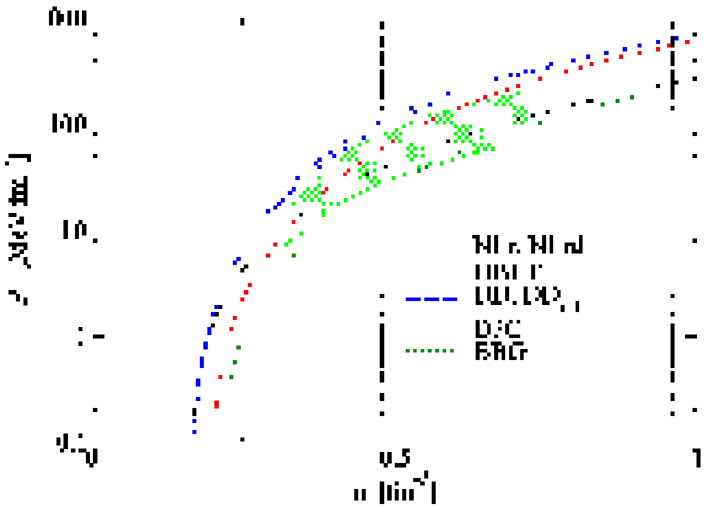


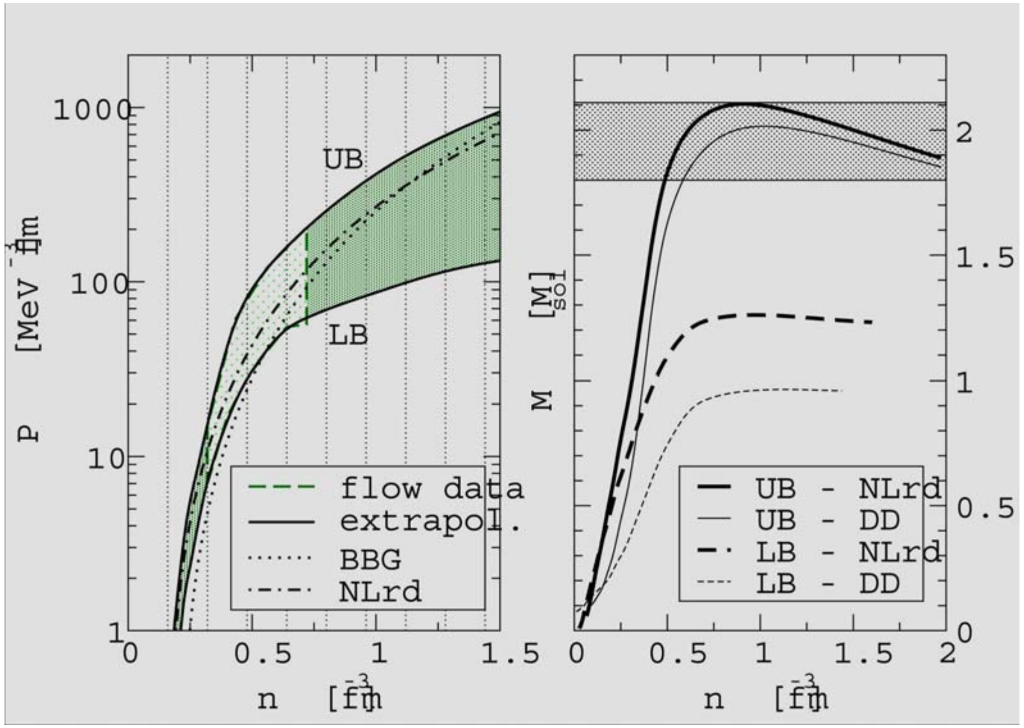




Neutron star cooling (cont'd)

Contraints from heavy ion collisions (flow)





Conclusions

- EOS can be determined from heavy ion collisions,
  in particular also the isovevtor part
  - at low density: fragmentation reactions at low energy
  - at high density: flow and particle production at relativistic energy
- Density dependence is not well determined from theory,
  but is important for nuclear structure and astrophysics
- Investigated in the framework of effective theories
  - DB
  - QHD (rho- and delta-mesons, evidence for delta-field)
- Sensitivity from various variables:
  - proton-neutron differential flow
  - isospin transparency and isospin tracing
  - production of pion and kaons
- data with more asymmetric (exotic) colliding systems helpful

Thanks to Collaborators:

T. Gaitanos (Munich)

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M. Colonna, M. Di Toro, V. Greco (Catania)

V. Baran (Bucharest)

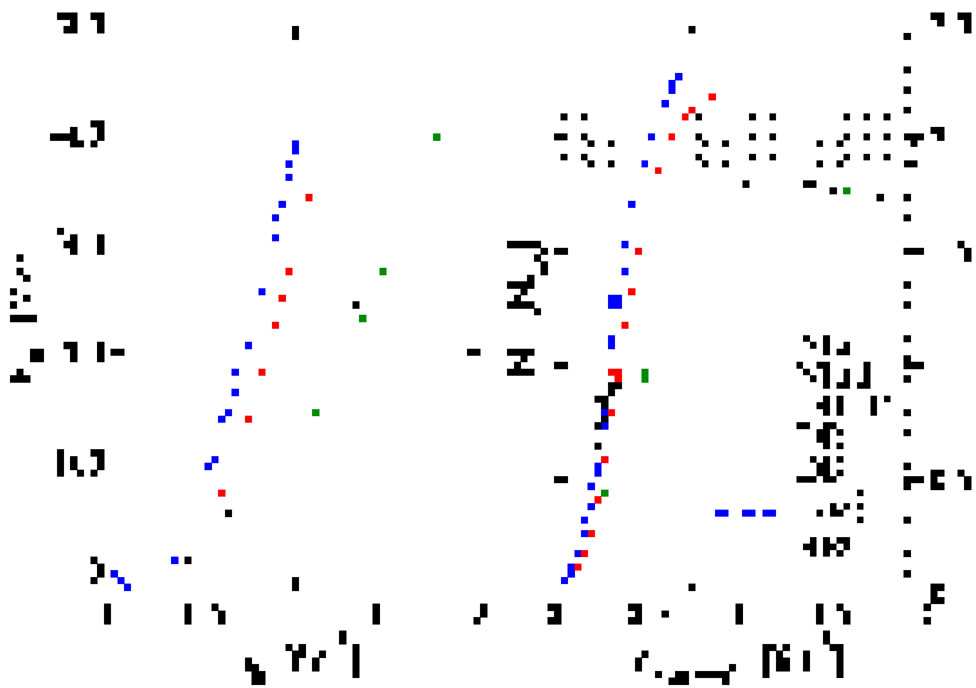
M. Zielinska-Pfabe (Smith College)

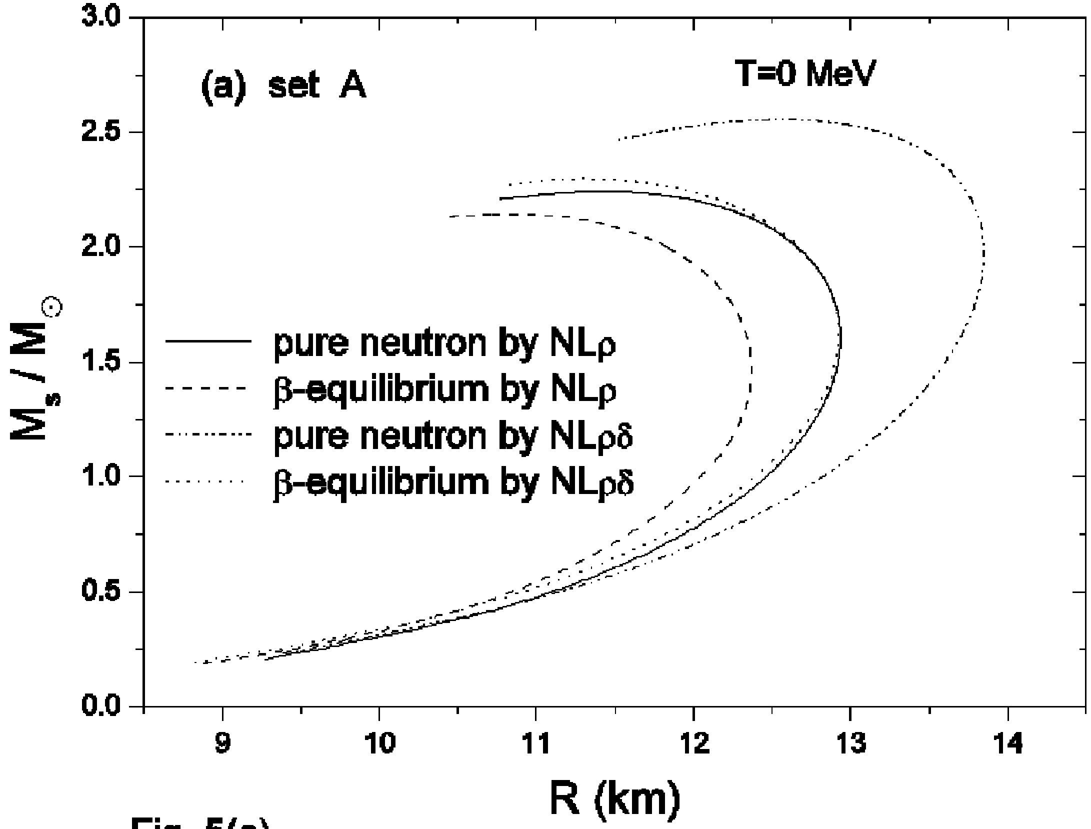
T. Mikhailova (Dubna)

S. Typel (GSI)

T. Klähn, H. Gregorian, D. Blaschke (Rostock and GSI)

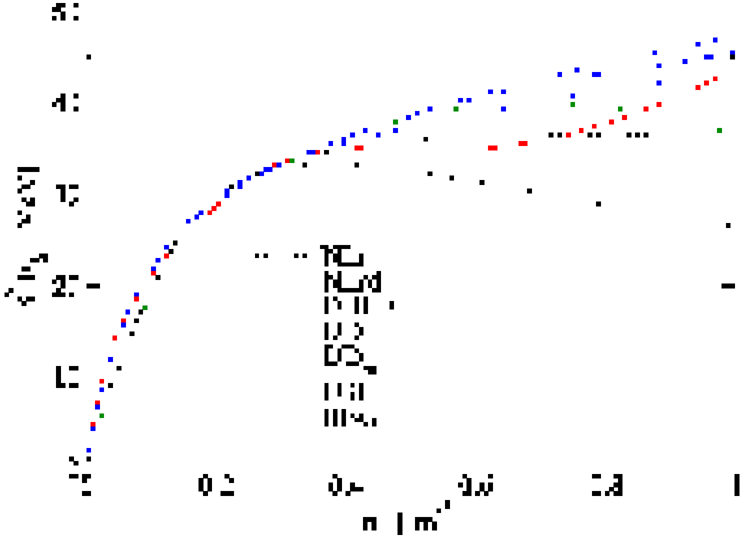
Thank you for attention !

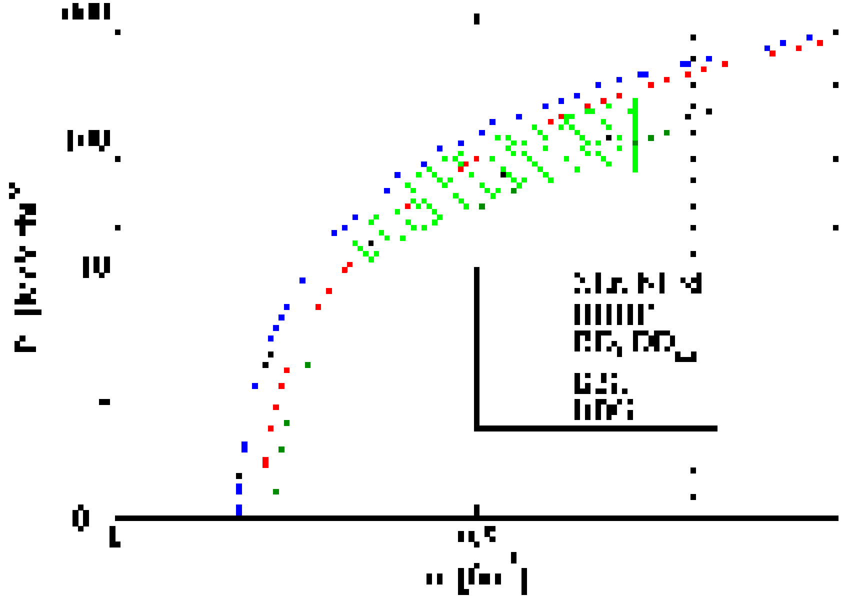




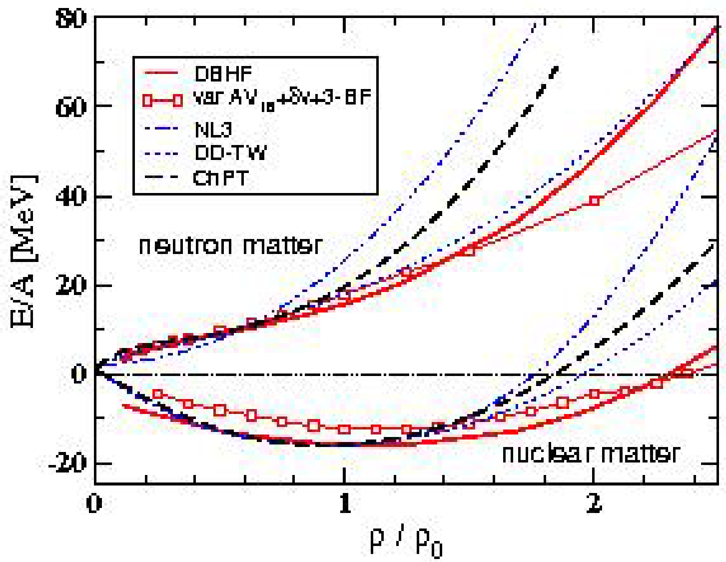
(a) set A
T=0 MeV
pure neutron by NLρ
β-equilibrium by NLρ
pure neutron by NLρδ
β-equilibrium by NLρδ
M_s / M_⊙
R (km)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
9
10
11
12
13
14

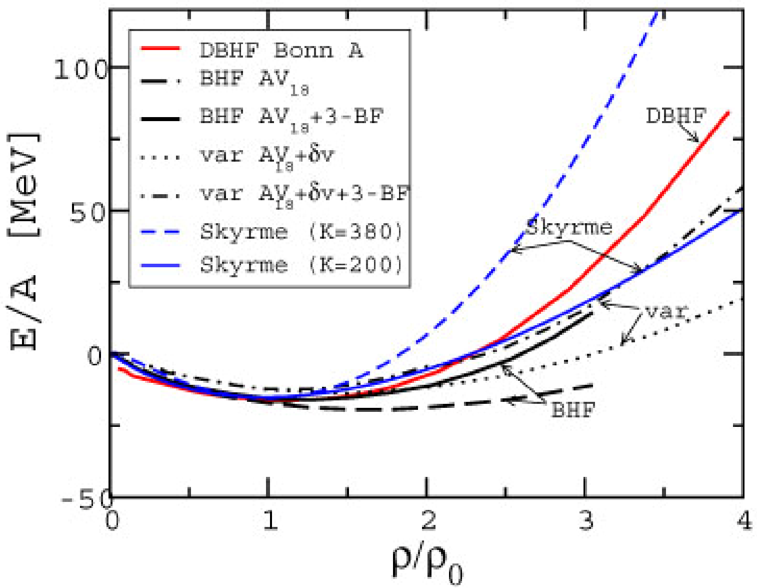
Fig. 5(a)


0.2
0.4
0.6
0.8
1

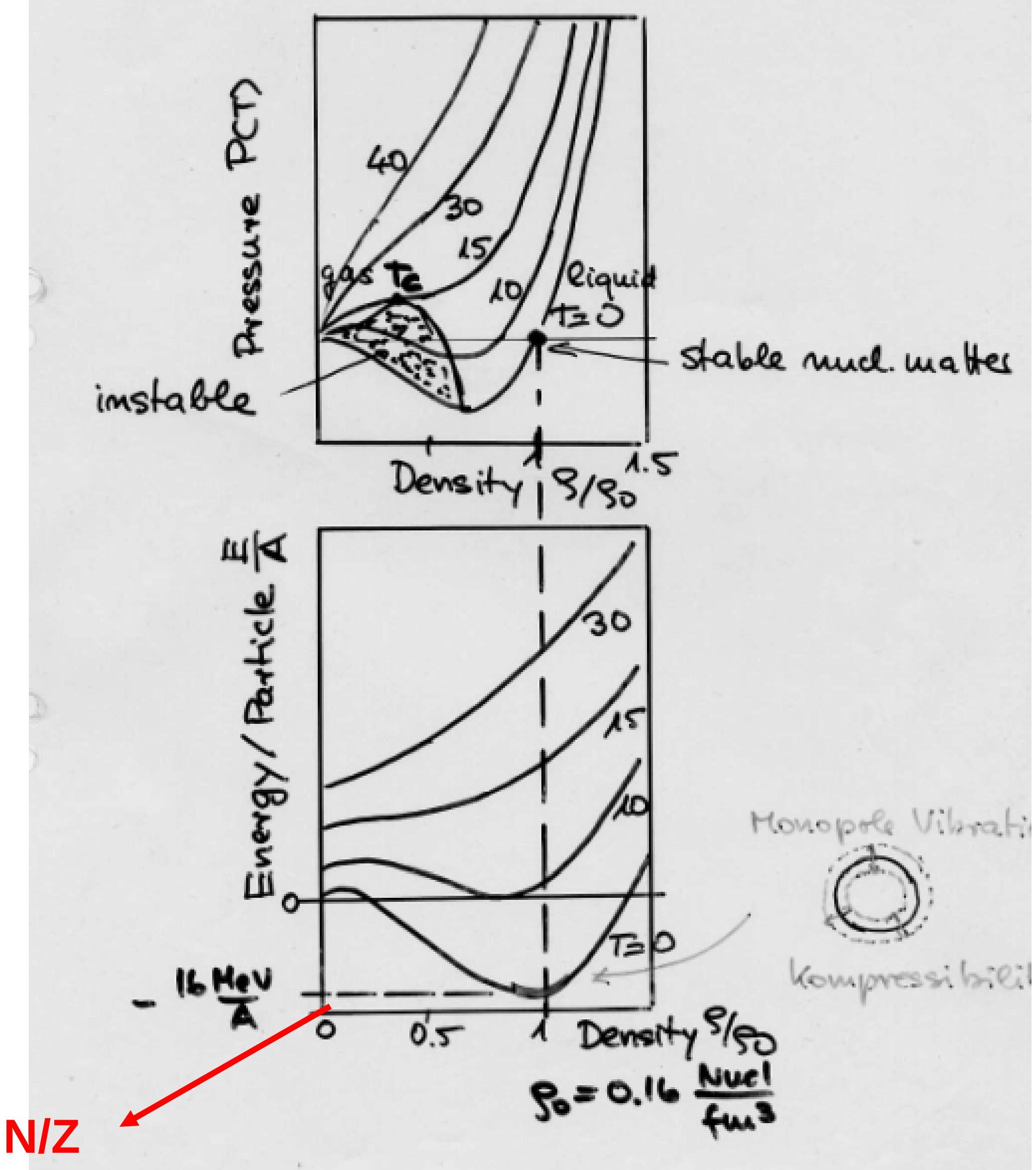

10
0
0.5

$$
\begin{aligned}
U_{\mathrm{MDI}}(\rho,\delta,\mathbf{p},\tau) &= A_u\frac{\rho_{\tau'}}{\rho_0}+A_l\frac{\rho_\tau}{\rho_0}+B\left(\frac{\rho}{\rho_0}\right)^\sigma(1-x\delta^2)\\
&- 8\tau x\frac{B}{\sigma+1}\frac{\rho^{\sigma-1}}{\rho_0^\sigma}\delta\rho_{\tau'}\\
&+ \frac{2C_{\tau,\tau}}{\rho_0}\int d^3\mathbf{p}'\frac{f_\tau(\mathbf{r},\mathbf{p}')}{1+(\mathbf{p}-\mathbf{p}')^2/\Lambda^2}\\
&+ \frac{2C_{\tau,\tau'}}{\rho_0}\int d^3\mathbf{p}'\frac{f_{\tau'}(\mathbf{r},\mathbf{p}')}{1+(\mathbf{p}-\mathbf{p}')^2/\Lambda^2}, \quad (2)
\end{aligned}
$$


DBHF
var AV18+δv+3-BF
NL3
DD-TW
ChPT
neutron matter
nuclear matter
E/A [MeV]
ρ / ρ0


DBHF Bonn A
BHF AV18
BHF AV18+3-BF
var AV18+δv
var AV18+δv+3-BF
Skyrme (K=380)
Skyrme (K=200)
DBHF
Skyrme
var
BHF
E/A [MeV]
ρ/ρ0

Equation-of-State of Nuclear Matter



TRANSPORT CALCULATION with

MICROSCOPIC, NON-EQUILIBRIUM SELF ENERGIES

(C. Fuchs, T. Gaitanos, H.H.W.)

$\frac{df}{dt}[\Sigma] = I_{coll}[f, \sigma_{med}]$ transport — Coupled

$\Sigma = -i\,Tr\,[T[f]f]$ microscopic int.

$T = V + V\,\frac{Q[f]}{e}\,T$ $\quad \sigma_{med} \sim |T|^2$ (e.g. Dirac-Brueck)

local dens. LDA — equilibr. approx.

Coll. nucl. matter CNM — local non-equilibrium

asymm. ACNM — " (with asymm. density)

constructed covar. from DB