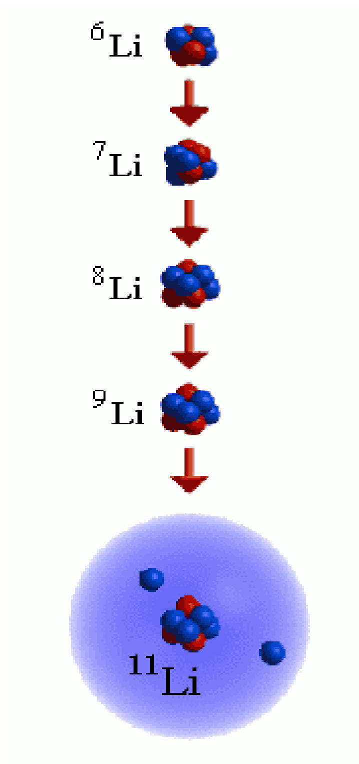
Dubna International Advanced School of Theoretical Physics



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Joint Institute for Nuclear Research

Halo Nuclei: Structure and Reactions

Frontiers of Nuclear Physics

nucleonic matter under *extreme* conditions

(temperature, angular momentum, *very proton / neutron reach nuclei*, …)

Physics of Radioactive Ion Beams

Nuclei ⇒ { alley of *β-stability* to the limits of stability; ~ zero energy to more than 1 GeV/u }

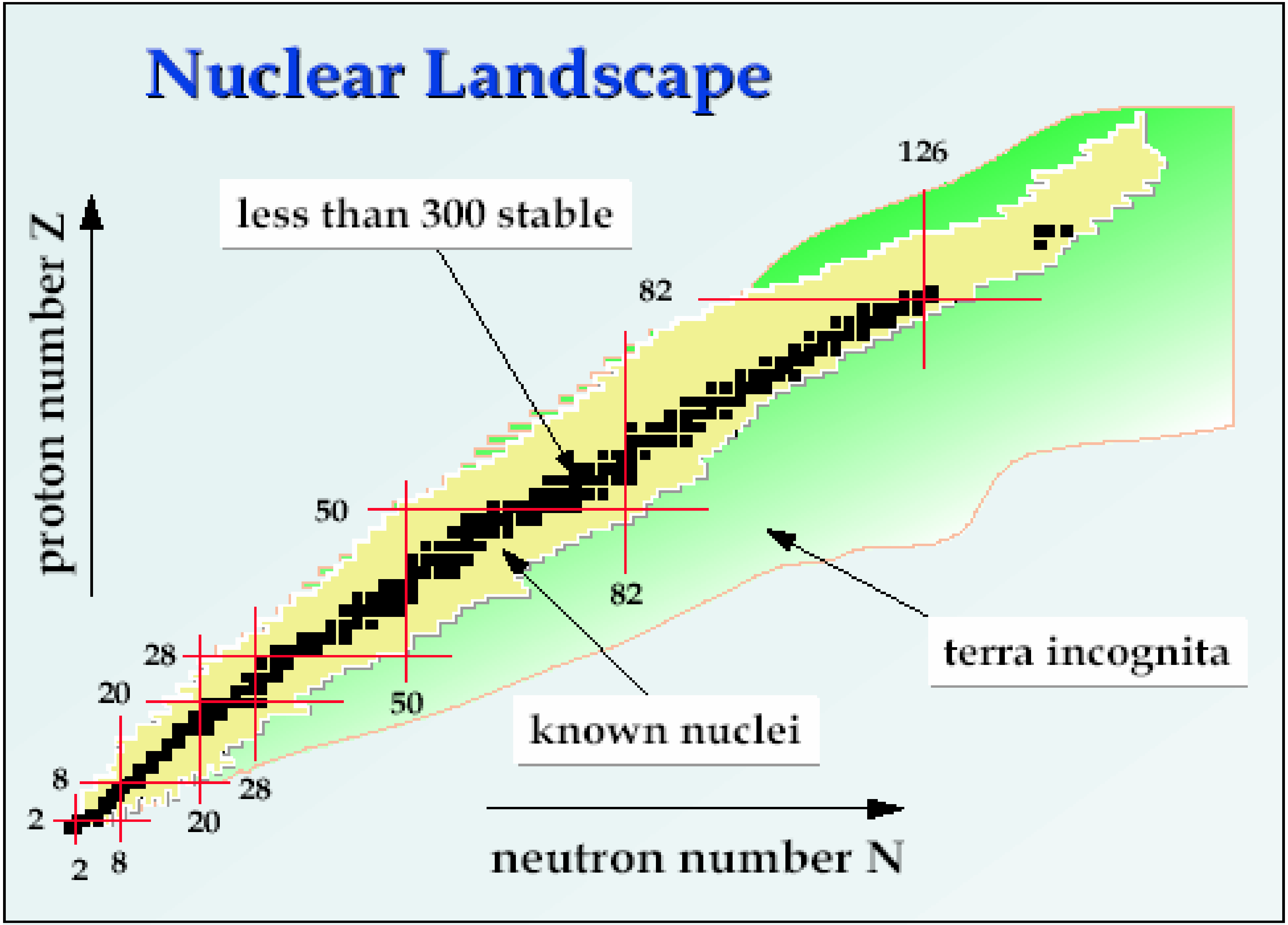
- exact ***locations*** of the neutron and proton driplines
- producing the ***heaviest bound*** nuclei
- learning about the ***astrophysical*** r- and rp- processes
- exploring the ***evolution*** of shell structure
  (*vanishing of magic numbers, new magic numbers, …*)
- resonances (nuclei) ***beyond*** the driplines

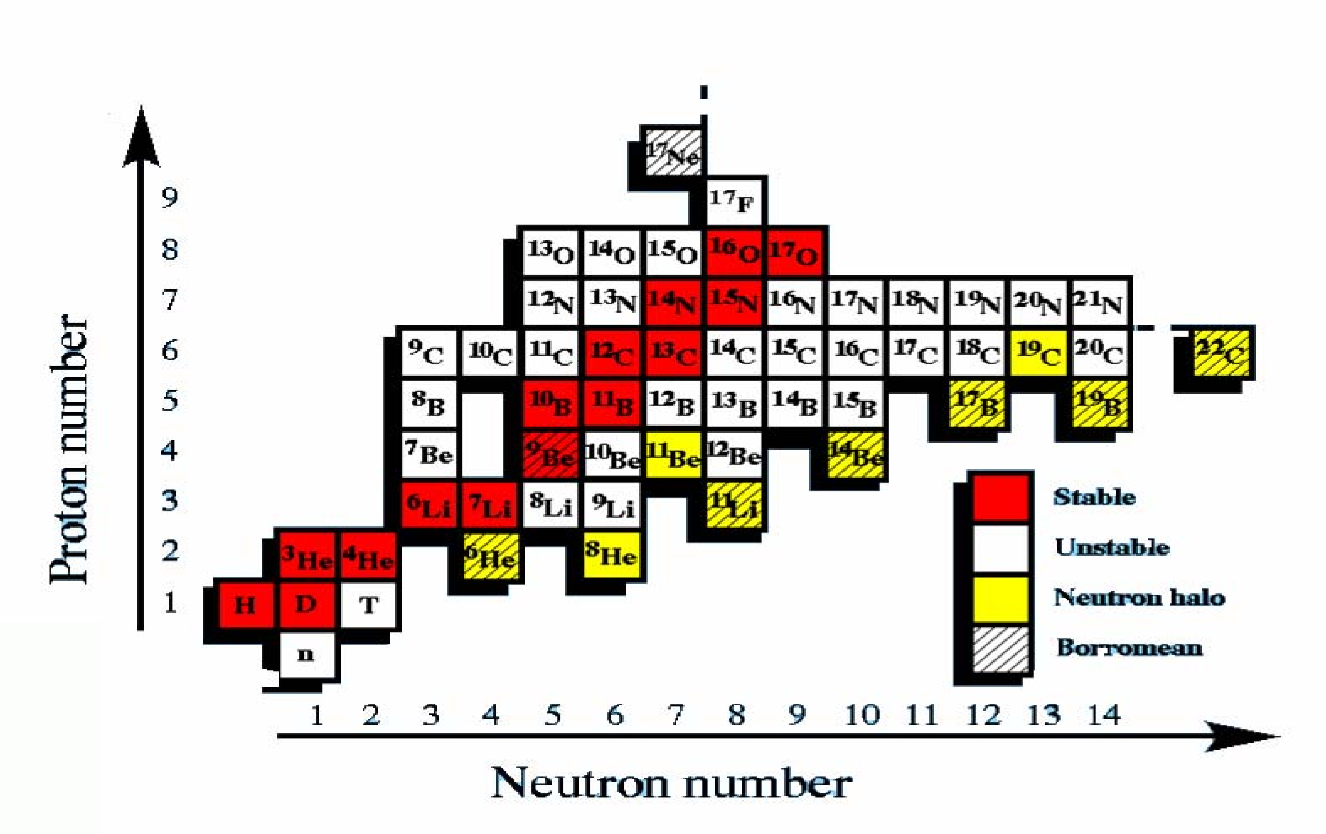
………………..

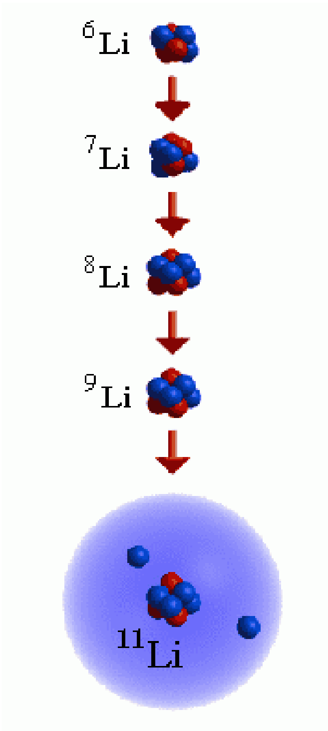
Remarkable ***discoveries*** have already been made with RIBs

HALO :

new structural dripline **phenomenon with clusterization into an ordinary core nucleus and a veil of halo nucleons – forming very dilute neutron matter**


Nuclear Landscape
less than 300 stable
known nuclei
terra incognita
proton number Z
neutron number N
126
82
82
50
50
28
28
20
20
8
8
2
2





Chains of the lightest isotopes (He, Li, Be, B, …) end up with two neutron halo nuclei

Two neutron halo nuclei (^{6}He , ^{11}Li , ^{14}Be , …) break into three fragments and are all Borromean nuclei

One neutron halo nuclei (^{11}Be , ^{19}C , …) break into two fragments

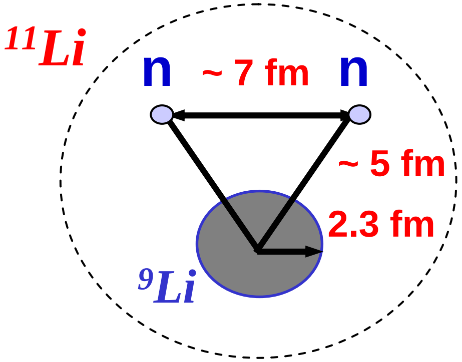
Neutron halo nuclei

Halo

(^{6}He, ^{11}Li, ^{11}Be, ^{14}Be, ^{17}B, …) → weakly bound systems with large extension and space granularity

"Residence in *forbidden* regions"

Appreciable probability for dilute nuclear matter extending far out into *classically forbidden* region



Separation energies of last neutron (s) :

halo	*stable*
< 1	6 - 8 MeV

$\varepsilon(^{11}Li) = 0.3$ MeV

$\varepsilon(^{11}Be) = 0.5$ MeV

$\varepsilon(^{6}He) = 0.97$ MeV

Large size **of halo nuclei**

$\langle r^2(^{11}Li)\rangle^{1/2} \sim 3.5$ fm

(r.m.s. for $A \sim 48$)

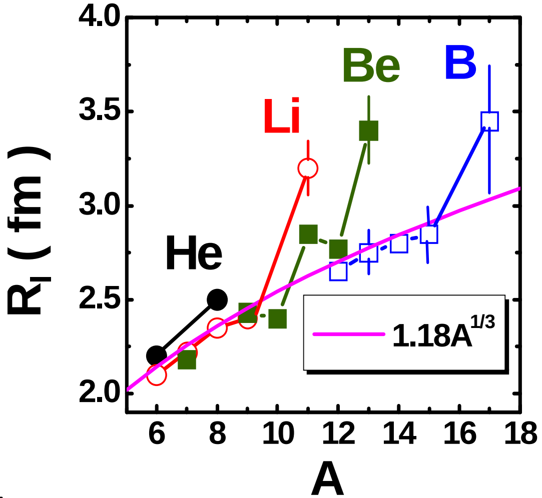
Two-neutron halo nuclei (^{11}Li, ^{6}He, ^{14}Be, ^{17}B, . . .) ⇒ *Borromean* systems

Borromean system is *bound* ⇒ none of the constituent *two-body* subsystems *are* bound

Peculiarities of halo nuclei: the example of ^{11}Li

(i) weakly bound: the two-neutron separation energy (~300 KeV) is about 10 times *less* than the energy of the first excited state in ^{9}Li .

(ii) large size: interaction cross section of ^{11}Li is about 30% *larger* than for ^{9}Li



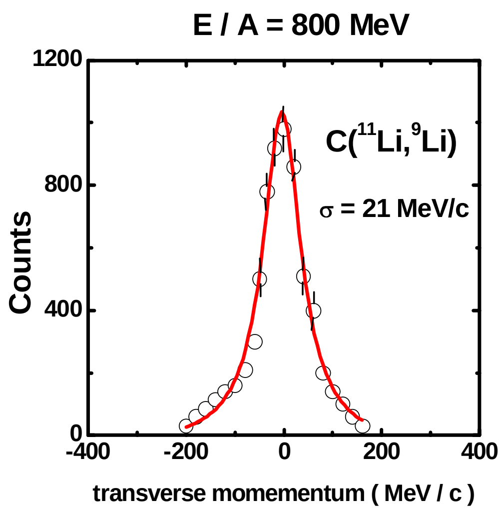
This is very unusual for *strongly interacting* systems held together by *short-range interactions*

Interaction radii : $\sigma_I = \pi \left(R_I(proj) + R_I(targ) \right)^2$

E / A = 790 MeV, *light targets*

I. Tanihata et al.,
Phys. Rev. Lett., 55 (1985) 2676

(iii) very narrow momentum distributions, compared to stable nuclei, of *both neutrons and* ^{9}Li measured in high energy fragmentation reactions of ^{11}Li .



No narrow fragment distributions in breakup on *other fragments*, say ^{8}Li or ^{8}He

(naive picture)
narrow momentum distributions
↓
large spatial extensions

(iv) Relations between interaction and neutron removal cross sections (mb) at 790 MeV/A

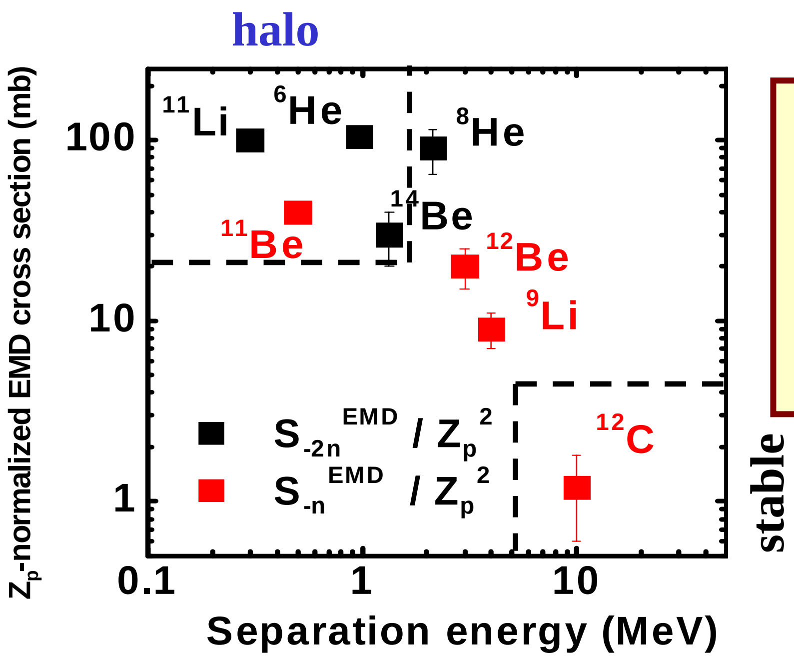
$A + {}^{12}C$	σ_I	σ_{-2n}	σ_{-4n}
^{9}Li	796 ∓ 6		
^{11}Li	1060∓ 10	220∓ 40	
^{4}He	503∓ 5		
^{6}He	722∓ 5	189∓14	
^{8}He	817∓ 6	202∓17	95∓ 5

$$\sigma_I(A=C+xn) = \sigma_I(C) + \sigma_{-xn}$$

Strong *evidence* for the well defined clusterization into the core and two neutrons

Tanihata I. et al.
PRL, 55 (1987) 2670;
PL, B289 (1992) 263

(v) Electromagnetic dissociation cross sections per unit charge are *orders* of magnitude *larger* than for stable nuclei



Evidence for a rather large difference between *charge* and *mass* centers in a body fixed frame

⇓

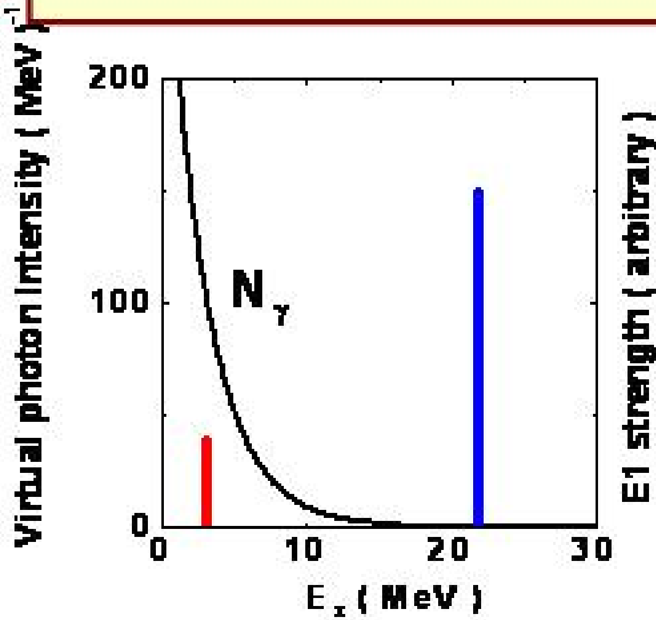
concentration of the dipole strength at *low excitation* energies

T. Kobayashi, Proc. 1st Int. Conf. On Radiactive Nuclear Beams, 1990.

Soft Excitation Modes

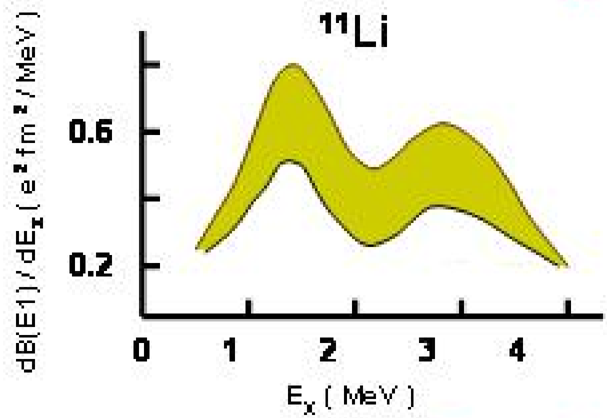
(peculiarities of low energy halo continuum)

***Large* EMD cross sections** ⇨ **specific nuclear property of *extremely* neutron-rich nuclei**



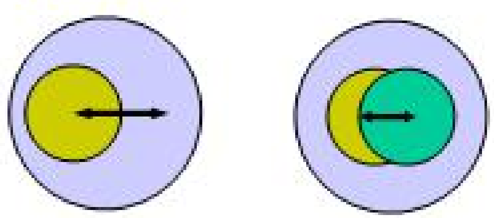
$$\sigma_{EMD} = \int N(E_X)\,\sigma_\gamma(E_X)\,dE_X$$

$$\sigma_\gamma(E_X) = \frac{16\pi^3}{9\hbar c} E_X \frac{dB(E1)}{dE_X}$$



M. Zinser et al., Nucl. Phys. A619 (1997) 151

soft DR **normal GDR**



E_x ~ 1 MeV ~ 20 MeV

excitations of soft modes with

- **different multipolarity**
- **collective excitations *versus* direct transition from weakly bound to continuum states**

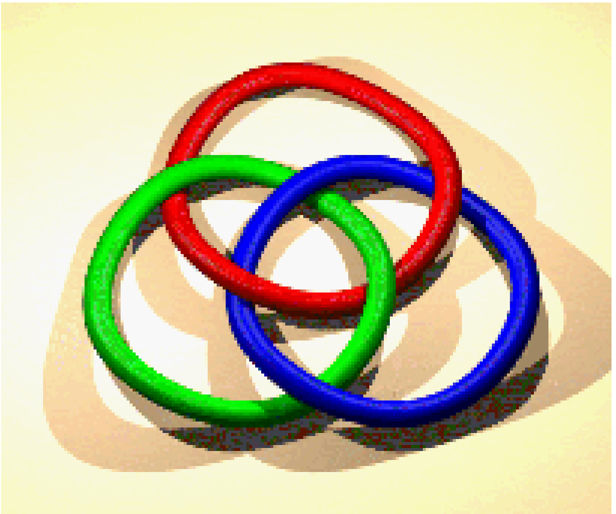
(vi) Ground state properties of ^{11}Li and ^{9}Li :

	^{9}Li	^{11}Li
Spin J^{π} :	$3/2^{-}$	$3/2^{-}$
quadrupole moments :	-27.4 ∓ 1.0 mb	-31.2 ∓ 4.5 m
magnetic moments :	3.4391 ∓ 0.0006 n.m.	3.6678 ∓ 0.0025 n.m.

Schmidt limit : 3.71 n.m.

Previous peculiarities cannot arise from large deformations
core is not *significantly* perturbed by the two valence neutrons

(vii) The three-body system ^{11}Li (^{9}Li + n + n) is Borromean :neither the two neutron nor the core-neutron subsytems *are bound*



Three-body correlations are the most important: due to them the system becomes bound.

The Borromean rings, the heraldic symbol of the Princes of Borromeo, are carved in the stone of their castle in Lake Maggiore in northern Italy.

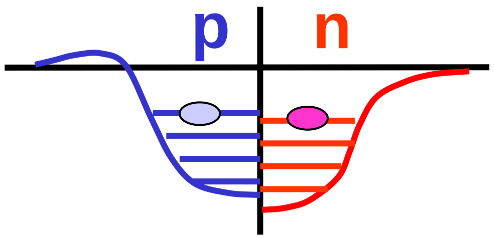
Stable nuclei

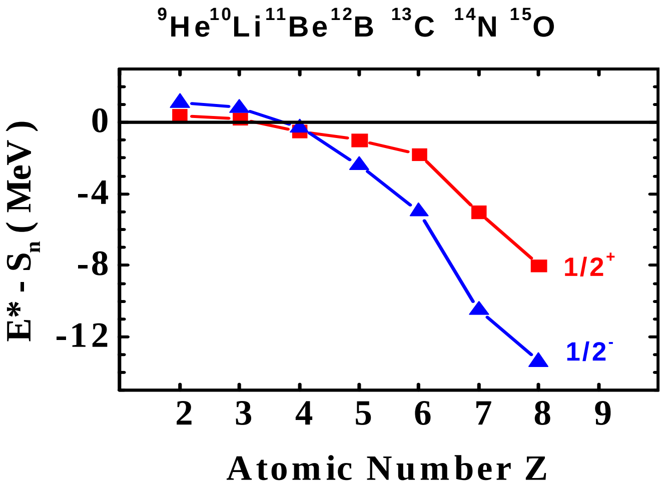
N / Z ~ 1 - 1.5

ε_S **~ 6 - 8 MeV**

ρ_0 **~ 0.16 fm $^{-3}$**

proton and neutrons homogeneously mixed, *no decoupling* of proton and neutron distributions





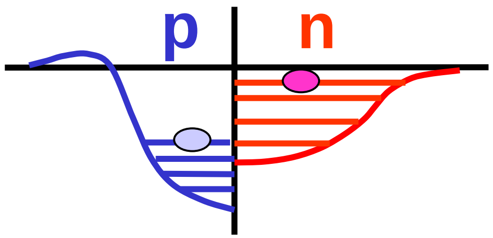
Unstable nuclei

N / Z ~ 0.6 - 4

ε_S **~ 0 - 40 MeV**

***decoupling* of proton and neutron distributions**

neutron halos and neutron skins



Prerequisite of the *halo* formation :

***low* angular momentum motion for halo particles and few-body dynamics**

1s - intruder level

^{11}Be parity inversion of g.s.

^{10}Li g.s. : $\left[\pi 0p_{\frac{3}{2}} \otimes \nu 1s_{\frac{1}{2}}\right]_{2^-}$

Peculiarities of halo

<u>in *ground* state</u>

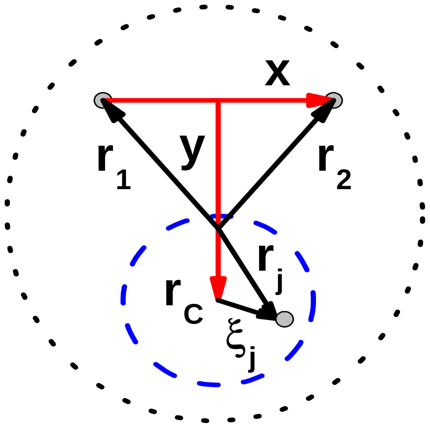
weakly bound,
with large extension
and space granularity

elastic scattering
some inclusive observables
(reaction cross sections, …)

<u>in *low-energy* continuum</u>

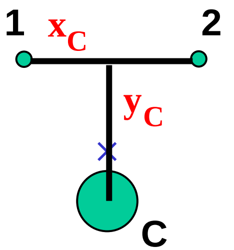
concentration of the transition strength near break up threshold - *soft modes*

nuclear reactions
(transition properties)

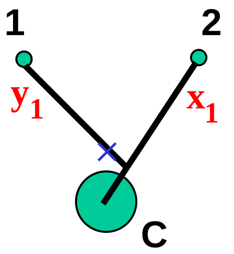


BASIC dynamics of halo nuclei ⇒ *Decoupling* of *halo* and nuclear *core* degrees of freedom

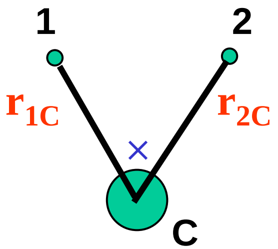
$$\Phi\left(\overline{r_1},\ldots,\overline{r_A}\right) = \phi_C\left(\overline{\xi_1},\ldots,\overline{\xi_{A_C}}\right)\psi\left(\overline{x},\overline{y}\right)$$



T - basis



Y - basis



V - basis

The *T*-set of Jacobin coordinates ($A_i = m_i / m, \quad A = A_1 + A_2 + A_C$)

$$\bar{x}_C = \bar{r}_1 - \bar{r}_2, \qquad \bar{y}_C = \bar{r}_C - \frac{A_1 \bar{r}_1 + A_2 \bar{r}_2}{A_1 + A_2}, \qquad \bar{R} = \frac{1}{A}\left(A_1 \bar{r}_1 + A_2 \bar{r}_2 + A_C \bar{r}_C\right)$$

The hyperspherical coordnates : $\rho, \ \alpha_C, \theta_{x_C}, \varphi_{x_C}, \theta_{y_C}, \varphi_{y_C}$

$$\rho^2 = \mu_{x_i} x_C^2 + \mu_{y_i} y_C^2 = \sum_{i=1}^{3} A_i \left(\bar{r}_i - \bar{R}\right)^2 = \frac{1}{A} \sum_{i>j=1}^{3} A_i A_j \left(\bar{r}_i - \bar{r}_j\right)^2$$

$$\alpha_C = \arctan\left(\frac{|x_C|}{|y_C|}\right), \quad 0 \le \alpha_C \le \frac{\pi}{2}$$

ρ is the *rotation, translation and permutation* invariant variable

$$\sqrt{\mu_{x_i}}\, x_i = \rho \sin \alpha_i, \quad \sqrt{\mu_{y_i}}\, y_i = \rho \cos \alpha_i$$

Volume element in the 6-dimensional space

$$d\bar{x}_i\, d\bar{y}_i = x_i^2\, dx_i\ \ y_i^2\, dy_i\ d\Omega_{x_i}\ d\Omega_{y_i} = \frac{1}{\left(\mu_x \mu_y\right)^{3/2}} \rho^5\, d\rho\, d\Omega_5^i$$

$$= \frac{1}{\left(\mu_x \mu_y\right)^{3/2}} \rho^5\, d\rho\ \sin^2\alpha_i \cos^2\alpha_i\, d\alpha_i\ d\Omega_{x_i}\ d\Omega_{y_i}$$

The kinetic energy operator T has the separable form

$$T = -\frac{\hbar^2}{2m}\left(\frac{1}{\mu_x}\Delta_x + \frac{1}{\mu_y}\Delta_y\right) = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial\rho^2} + \frac{5}{\rho}\frac{\partial}{\partial\rho} - \frac{1}{\rho^2}\hat{K}^2\left(\Omega_5^i\right)\right) = -\frac{\hbar^2}{2m}\Delta_6$$

$\hat{K}^2\left(\Omega_5^i\right)$ is a square of the 6-dimensional hyperorbital momentum

$$\hat{K}^2\left(\Omega_5^i\right) = -\frac{\partial^2}{\partial\alpha^2} - 4\cot(2\alpha)\frac{\partial}{\partial\alpha} + \frac{1}{\sin^2\alpha}\hat{l}^2\left(\hat{x}\right) + \frac{1}{\cos^2\alpha}\hat{l}^2\left(\hat{y}\right)$$

Eigenfunctions of Δ_6 are the homogeneous harmonic polynomials

$$\Delta_6 P_K\left(\bar{x},\bar{y}\right) = \Delta_6\, \rho^K\, \Phi_K^{l_x m_x,\, l_y m_y}\left(\Omega_5^i\right) = 0$$

$$\left\{\hat{K}^2\left(\Omega_5^i\right) - K(K+4)\right\}\Phi_K^{l_x m_x,\, l_y m_y}\left(\Omega_5^i\right) = 0$$

$\Phi_K^{l_x m_x,\, l_y m_y}\left(\Omega_5^i\right)$ are hyperspherical harmonics or K-harmonics.

They give a *complete set of orthogonal functions* in the 6-dimensional space on unit hypersphere $\left(K = l_x + l_y + 2n\right)$

$$\Phi_K^{l_x m_x,\, l_y m_y}\left(\Omega_5^i\right) = N_K^{l_x l_y}\, Y_{l_x m_x}\left(\hat{x}\right) Y_{l_y m_y}\left(\hat{y}\right) \left(\sin\alpha\right)^{l_x} \left(\cos\alpha\right)^{l_y} P_{\frac{1}{2}(K-l_x-l_y)}^{\left(l_x+\frac{1}{2},\, l_y+\frac{1}{2}\right)}\left(\cos 2\alpha\right)$$

$P_n^{(\alpha,\beta)}(z)$ are the Jacobi polynomials, $Y_{lm}\left(\hat{x}\right)$ are the spherical harmonics

The functions with fixed total orbital moment $\overline{L} = \overline{l}_x + \overline{l}_y$

$$\Phi_{KLM}^{l_x, l_y}\left(\Omega_5^i\right) = \sum_{m_x, m_y} \left(l_x m_x l_y m_y \middle| L M\right) \Phi_K^{l_x m_x, l_y m_y}\left(\Omega_5^i\right)$$

a normalizing coefficient $N_K^{l_x l_y}$ is defined by the relation

$$\int d\Omega_5^i \, \Phi_{K'L'M'}^{l_x', l_y' *}\left(\Omega_5^i\right) \Phi_{KLM}^{l_x, l_y}\left(\Omega_5^i\right) = \delta_{KK'} \delta_{LL'} \delta_{MM'} \delta_{l_x l_x'} \delta_{l_y l_y'}$$

The parity of HH depends only on $K = l_x + l_y + 2n$ ⇒ + (positive), if K – even; - (negative), if K - odd

The three equivalent sets of Jacobi coordinates are connected by transformation (kinematic rotation)

$$\begin{cases} \sqrt{\mu_{x_j}}\,\overline{x}_j = -\cos\varphi_{ji}\sqrt{\mu_{x_i}}\,\overline{x}_i - \sin\varphi_{ji}\sqrt{\mu_{y_i}}\,\overline{y}_i \\ \sqrt{\mu_{y_j}}\,\overline{y}_j = \sin\varphi_{ji}\sqrt{\mu_{x_i}}\,\overline{x}_i - \cos\varphi_{ji}\sqrt{\mu_{y_i}}\,\overline{y}_i \end{cases}$$

$$\varphi_{ji} = \varphi_{ji}\left(A_1, A_2, A_C\right)$$

Quantum numbers *K, L, M* don't change under a kinematic rotation. HH are transformed in a simple way and the parity is also conserved.

$$\Phi_{KLM}^{l_{x_i}, l_{y_i}}\left(\Omega_5^i\right) = \sum_{l_{x_k}, l_{y_k}} \underbrace{< l_{x_k}, l_{y_k} \middle| l_{x_i}, l_{y_i} >_{KL}}_{\Downarrow} \Phi_{KLM}^{l_{x_k}, l_{y_k}}\left(\Omega_5^k\right)$$

Reynal-Revai coefficients

The three-body bound-state and continuum wave functions
(within cluster representation)

$$\Psi_{JM} = \phi_C(\xi_C)\,\Phi_{JM}(\bar{\mathbf{x}},\bar{\mathbf{y}})\,\exp\{i(\bar{P}\circ\bar{R})\}/(2\pi)^{3/2}$$

The Schrodinger 3-body equation : $(T + V - E)\,\Phi_{JM}(\bar{\mathbf{x}},\bar{\mathbf{y}}) = 0$

where the kinetic energy operator : $\mathbf{T} = -\frac{\hbar^2}{2m}\left(\frac{1}{\mu_\mathrm{x}}\Delta_\mathrm{x} + \frac{1}{\mu_\mathrm{y}}\Delta_\mathrm{y}\right)$

and the interaction : $V = V_{12}(\bar{r}_{12}) + V_{1C}(\bar{r}_{1C}) + V_{2C}(\bar{r}_{2C})$

The bound state wave function (E < 0)

$$\Phi_{JM}(\bar{\mathbf{x}},\bar{\mathbf{y}}) = \rho^{-5/2}\sum_{LSKl_Xl_Y}\chi^{LS}_{Kl_Xl_Y}(\rho)\left[\Phi^{l_\mathrm{x},l_\mathrm{y}}_{KL}(\Omega^{\mathrm{i}}_5)\otimes\chi_S\right]_{JM}$$

$\chi_{SM_S} = \left[\,|1/2\rangle_1 \otimes |1/2\rangle_2\right]_{SM_S}$ - spin function of two nucleons

The continuum wave function (E > 0)

$$\Phi_{S'M'_S}(\bar{k}_\mathrm{x},\bar{k}_\mathrm{y},\bar{\mathbf{x}},\bar{\mathbf{y}}) = (\kappa\rho)^{-5/2}\sum_{\gamma,\gamma'}\chi^{LS,L'S'}_{Kl_Xl_Y,K'l'_Xl'_Y}(\kappa,\rho)\left[\Phi^{l_\mathrm{x},l_\mathrm{y}}_{KL}(\Omega^{\mathrm{i}}_5)\otimes\chi_S\right]_{JM}$$

$$*\; i^{K'}\left(L'M'_L\,S'M'_S\,|\,JM\right)\Phi^{l_\mathrm{x},l_\mathrm{y}}_{KL}(\Omega^{\mathrm{i}}_5)$$

$\kappa = \sqrt{k_x^2 + k_y^2} = \frac{1}{\hbar}\sqrt{2m|E|}$ is the hypermomentum conjugated to ρ

The HH expansion of the 6-dimensional plane wave

$$\exp\left\{i\left(\bar{k}_x \circ \bar{\mathbf{x}} + \bar{k}_y \circ \bar{\mathbf{y}}\right)\right\} = \frac{(2\pi)^3}{(\kappa\rho)^2}\sum_{\gamma} i^K J_{K+2}(\kappa\rho)\, \Phi_{KL}^{l_x,l_y}\left(\Omega_5^{i}\right)\Phi_{KL}^{l_x,l_y\,*}\left(\Omega_5^{\kappa}\right)$$

Normalization condition for bound state wave function

$$\int d\bar{\mathbf{x}}\, d\bar{\mathbf{y}}\; \Phi^{*}_{J'M'}\left(\bar{\mathbf{x}},\bar{\mathbf{y}}\right)\Phi_{JM}\left(\bar{\mathbf{x}},\bar{\mathbf{y}}\right) = \delta_{JJ'}\delta_{MM'}$$

Normalization condition for continuum wave function

$$\int d\bar{\mathbf{x}}\, d\bar{\mathbf{y}}\; \Phi^{*}_{S'M'_S}\left(\bar{k}'_x, \bar{k}'_y, \bar{\mathbf{x}}, \bar{\mathbf{y}}\right)\Phi_{SM_S}\left(\bar{k}_x, \bar{k}_y, \bar{\mathbf{x}}, \bar{\mathbf{y}}\right) =$$

$$\delta_{SS'}\delta_{M_S M'_S}\delta\left(\bar{k}'_x - \bar{k}_x\right)\delta\left(\bar{k}'_y - \bar{k}_y\right) = \delta_{SS'}\delta_{M_S M'_S}\frac{1}{\kappa^5}\delta\left(\kappa' - \kappa\right)\delta\left(\Omega'^{\kappa}_5 - \Omega^{\kappa}_5\right)$$

After projecting onto the hyperangular part of the wave function the Schrodinger equation is reduced to a set of coupled equations

$$\left\{-\frac{\hbar^2}{2m}\left[\frac{d^2}{d\rho^2} + \frac{\Lambda(\Lambda+1)}{\rho^2}\right] + V_{K\gamma,K\gamma}(\rho) - E\right\}\chi_{K\gamma}(\rho) = -\sum_{K'\gamma'\neq K\gamma} V_{K\gamma,K'\gamma'}(\rho)\,\chi_{K'\gamma'}(\rho)$$

where $\Lambda = K + 3/2$ and partial-wave coupling interactions

$$V_{K\gamma,K'\gamma'}(\rho) = \left\langle \Phi_{K\gamma}\left(\Omega_5^{i}\right)\middle| V_{12}\left(\bar{r}_{12}\right) + V_{1C}\left(\bar{r}_{1C}\right) + V_{2C}\left(\bar{r}_{2C}\right)\middle| \Phi_{K'\gamma'}\left(\Omega_5^{i}\right)\right\rangle$$

the boundary conditions: $\chi_{K\gamma}\left(\rho \Rightarrow 0\right) \sim \rho^{\Lambda+1} = \rho^{K+5/2}$

The asymptotic hyperradial behaviour of $V_{K\gamma,K'\gamma'}(\rho)$

The simplest case : $K=K',\ \gamma=\gamma',\ K=0,\ l_x=0,\ l_y=0$

two-body potentials : $\mathrm{V_{ij}}=\mathrm{V_{jk}}=\mathrm{V_{ki}} \Rightarrow$ a square well, radius *R*

$$\mathrm{V_{00}}(\rho)=3\int d\Omega_5^{\mathrm{i}}\,\Phi_{000}^{00}(\Omega_5^{\mathrm{i}})\,\mathrm{V_{jk}}(\bar{\mathrm{x}}_{\mathrm{i}})\,\Phi_{000}^{00}(\Omega_5^{\mathrm{i}})$$

$$=3\int_0^{\pi/2} d\alpha\,\sin^2\alpha\,\cos^2\alpha\,\mathrm{V_{jk}}(\rho\sin\alpha)\xrightarrow[\rho\to\infty]{}\int_0^{R/\rho} d\alpha\,\alpha^2\sim\frac{1}{\rho^3}$$

$\dfrac{1}{\rho^3}\Rightarrow$ a general behaviour of three-body effective potential if the two-body potentials are short-range potentials

At $\rho\to\infty$ the system of differential equations is decoupled since effective potentials can be neglected

$$\left\{-\frac{\hbar^2}{2m}\left[\frac{d^2}{d\rho^2}+\frac{\Lambda(\Lambda+1)}{\rho^2}\right]-E\right\}\chi_{K\gamma}(\rho)=0$$

if E < 0

$$\chi_{K\gamma}(\rho\to\infty)\sim\exp(-\kappa\rho)\Rightarrow\Phi_{JM}(\bar{\mathrm{x}},\bar{\mathrm{y}})\sim\frac{1}{\rho^{5/2}}\exp(-\kappa\rho)$$

if E > 0

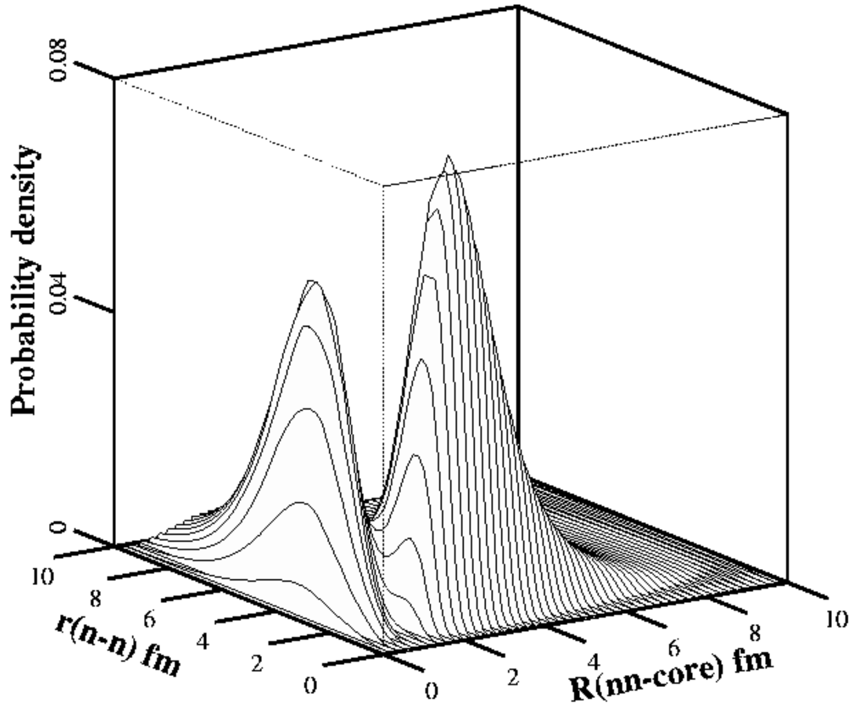
$$\chi_{K\gamma,K'\gamma'}(\rho\to\infty)\sim\sqrt{\kappa\rho}\left[H_{K+2}^{(-)}(\kappa\rho)\,\delta_{K\gamma,K'\gamma'}-S_{K\gamma,K'\gamma'}H_{K+2}^{(+)}(\kappa\rho)\right]$$

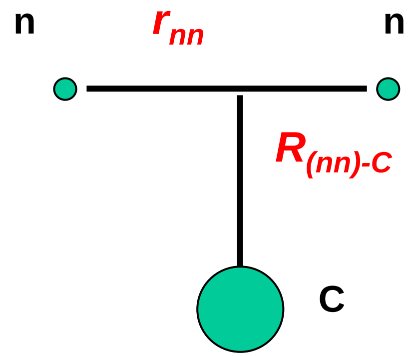
$$\Phi_{SM_S}(\bar{k}_{\mathrm{x}},\bar{k}_{\mathrm{y}},\bar{\mathrm{x}},\bar{\mathrm{y}})\sim\frac{1}{\rho^{5/2}}\left(\mathrm{A}\sin(\kappa\rho)+\mathrm{B}\cos(\kappa\rho)\right)$$

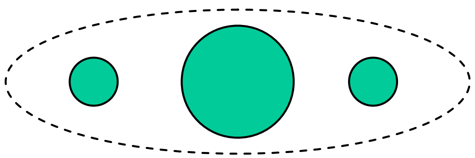
$$H_{K+2}^{(\pm)}(\kappa\rho)\sim\frac{1}{\sqrt{\kappa\rho}}\exp(\pm i\kappa\rho)$$

Correlation density for the ground state of ^{6}He

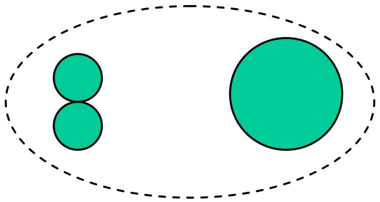
$$P\left(r_{nn}, R_{nn\text{-}C}\right) = r_{nn}^2\, R_{nn\text{-}C}^2\, \frac{1}{2J+1} \sum_M \int d\Omega_{nn}\, d\Omega_{nn\text{-}C} \left|\Phi_{JM}\left(\bar{r}_{nn}, \bar{R}_{nn\text{-}C}\right)\right|^2$$





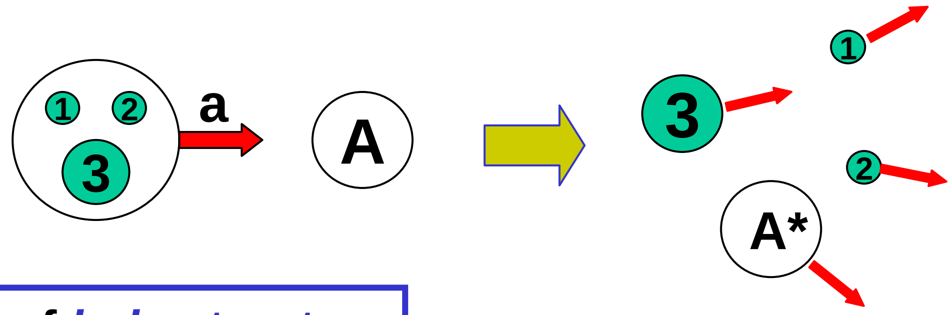


cigar-like configuration



dineutron configuration

Three-body halo fragmentation reactions



Study of *halo* structure

events with *undestroyed core* → ***peripheral* reactions**

***complex* constituents**

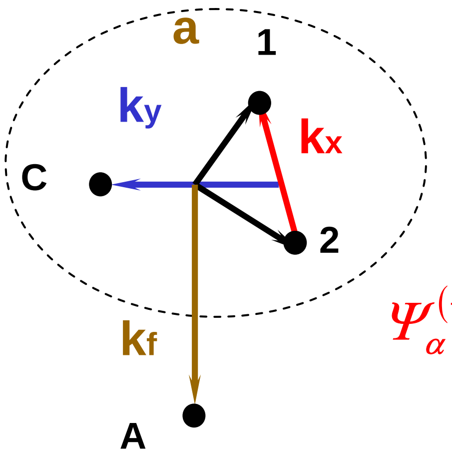
a + A → **1 + 2 + 3 + A_{gr} , elastic (4-body)**
→ **1 + 2 + 3 + A* , inelastic (≥ 4-bodies)**

Cross section

$$\sigma = \frac{(2\pi)}{\hbar \mathbf{v}_i} \sum_{\alpha} \int d\bar{k}_1\, d\bar{k}_2\, d\bar{k}_C\, d\bar{k}_{A^*}\, \delta\left(E_i - E_f\right) \delta\left(\bar{P}_i - \bar{P}_f\right) \left|T_{fi}\right|^2$$

Reaction amplitude T_{fi} (*prior* representation)

$$T_{fi} = \left\langle \Psi_\alpha^{(-)}\left(\bar{k}_f, \bar{k}_x, \bar{k}_y\right) \middle| \sum_{p,t} V_{p,t} - U_{aA} \middle| \Phi_0, \Psi_{A_{gr}}, \chi_i^{(+)}\left(\bar{k}_i\right) \right\rangle$$



$$E_a^* = \frac{k_x^2}{2\mu_x} + \frac{k_y^2}{2\mu_y}$$

$\Phi_0 \Rightarrow$ halo *ground* state wave function

$\Psi_{A_{gr}} \Rightarrow$ target *ground* state wave function

$\chi_i^{(+)}\left(\bar{k}_i\right) \Rightarrow$ distorted wave for relative projectile-target motion

$\Psi_\alpha^{(-)}\left(\bar{k}_f, \bar{k}_x, \bar{k}_y\right) \Rightarrow$ exact scattering wave function

$V_{p,t} \Rightarrow$ *NN* - interaction between *projectile* and *target* nucleons

$U_{aA} \Rightarrow$ optical potential in *initial* channel

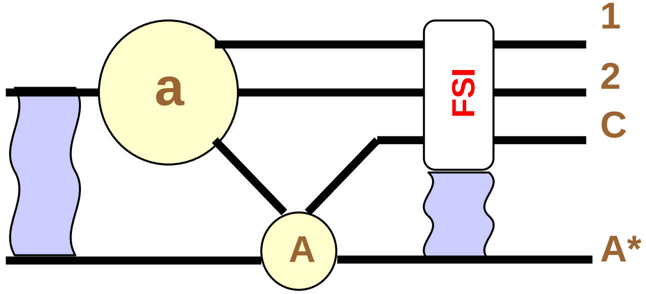
Reaction amplitude T_{fi} (prior representation)

$$T_{\mathrm{fi}} = \left\langle \Psi_\alpha^{(-)}\left(\bar{k}_{\mathrm{f}}, \bar{k}_{\mathrm{x}}, \bar{k}_{\mathrm{y}}\right) \left| \sum_{\mathrm{p,t}} \mathrm{V}_{\mathrm{p,t}} - \mathrm{U}_{\mathrm{aA}} \right| \Phi_0, \Psi_{A_{\mathrm{gr}}}, \chi_{\mathrm{i}}^{(+)}\left(\bar{k}_{\mathrm{i}}\right) \right\rangle$$

Approximations:

DW: *low-energy* halo excitations $\Rightarrow$ *small* k_x & k_y
(no spectators, three-body continuum, full scale FSI)

$$T_{\mathrm{fi}} = \left\langle \chi_{\mathrm{f}}^{(-)}\left(\bar{k}_{\mathrm{f}}\right), \Psi_{A_{\mathrm{gr}}}, \Phi^{(-)}\left(\bar{k}_{\mathrm{x}}, \bar{k}_{\mathrm{y}}\right) \left| \sum_{\mathrm{p,t}} \mathrm{V}_{\mathrm{p,t}} \right| \Phi_0, \Psi_{A_{\mathrm{gr}}}, \chi_{\mathrm{i}}^{(+)}\left(\bar{k}_{\mathrm{i}}\right) \right\rangle$$



Kinematically complete experiments
- sensitivity to 3-body correlations (halo)
- selection of halo excitation energy
- variety of observables
- elastic & inelastic breakup

$$\bar{k}_{\mathrm{x}} = \mu_{\mathrm{x}}\left(\frac{\bar{k}_1}{m_1} - \frac{\bar{k}_2}{m_2}\right) \qquad \bar{k}_{\mathrm{y}} = \mu_{\mathrm{y}}\left(\frac{\bar{k}_{\mathrm{C}}}{m_{\mathrm{C}}} - \frac{\bar{k}_1 + \bar{k}_2}{m_1 + m_2}\right) \quad \Rightarrow \quad \bar{k}_{\mathrm{C}}$$

(in the halo rest frame)

Model assumptions

$$T_{\rm fi} = \left\langle \chi_{\rm f}^{(-)}\left(\bar{k}_{\rm f}\right), \Psi_{A_{\rm gr}}, \Phi^{(-)}\left(\bar{k}_{\rm x}, \bar{k}_{\rm y}\right) \left| \sum_{\rm p,t} \mathrm{V}_{\rm pt} \right| \Phi_0, \Psi_{A_{\rm gr}}, \chi_{\rm i}^{(+)}\left(\bar{k}_{\rm i}\right) \right\rangle$$

Nuclear structure

Transition densities ⇒

$$\left\langle \Phi^{(-)}\left(\bar{k}_{\rm x}, \bar{k}_{\rm y}\right) \left\| \sum_p \frac{\delta\left(r - r_p\right)}{r\, r_p} \left[Y_L\left(\hat{r}_p\right) \otimes \sigma_p^{\rm S} \right]_J \right\| \Phi_0 \right\rangle$$

Three-body models

$$\Phi\left(\bar{r}_1, \ldots, \bar{r}_A\right) = \phi_C\left(\bar{\xi}_1, \ldots, \bar{\xi}_{A_C}\right) \psi\left(\bar{x}, \bar{y}\right)$$

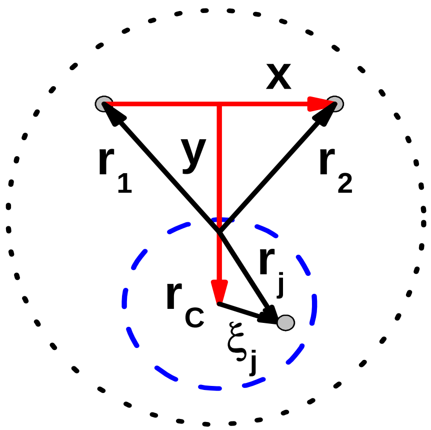
⇒ **effective interactions (NN & N-core)**

⇓

Method of hyperspherical harmonics:
3-body *bound* and *continuum* states

⇓

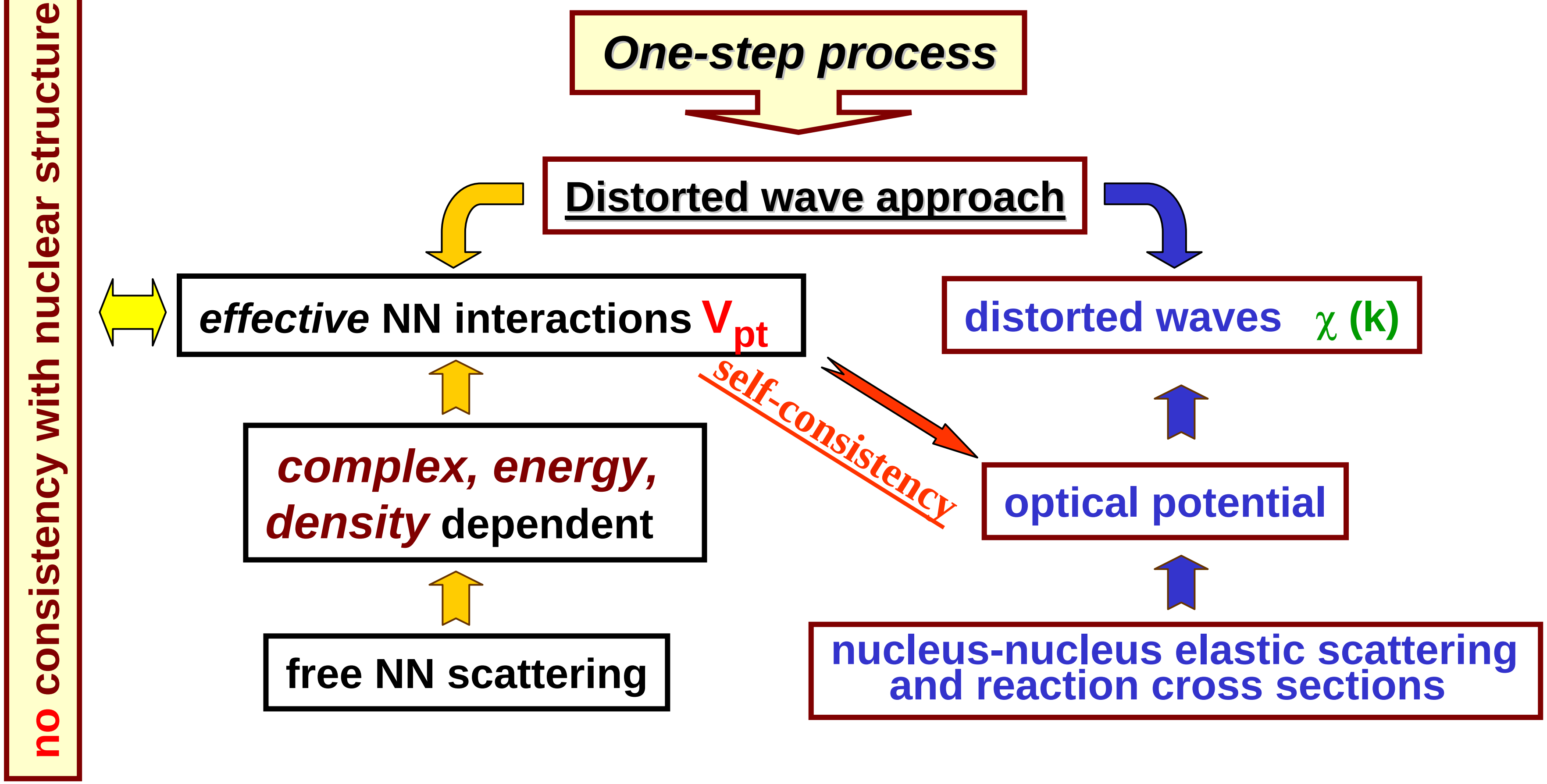
binding energy
electromagnetic moments
electromagnetic formfactors
geometrical properties
density distributions
. . . .



Model assumptions

$$T_{\mathrm{fi}} = \left\langle \chi_{\mathrm{f}}^{(-)}\left(\bar{k}_{\mathrm{f}}\right), \Psi_{A_{\mathrm{gr}}}, \Phi^{(-)}\left(\bar{k}_{\mathrm{x}}, \bar{k}_{\mathrm{y}}\right) \middle| \sum_{\mathrm{p,t}} \mathrm{V}_{\mathrm{pt}} \middle| \Phi_0, \Psi_{A_{\mathrm{gr}}}, \chi_{\mathrm{i}}^{(+)}\left(\bar{k}_{\mathrm{i}}\right) \right\rangle$$

Reaction mechanism



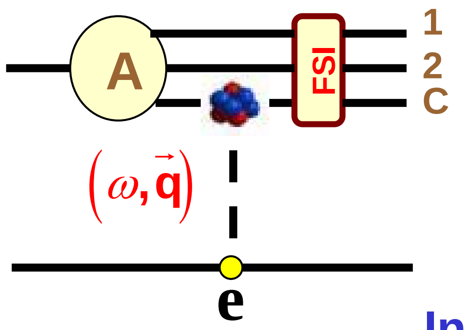
ELECTRON SCATTERING

Electromagnetic forces are **well known** and **weak** → Reaction mechanism can **be disentangled** from nuclear structure

Maxwell equation & ***Continuity equation***

$$\Box A_\mu(x) = 4\pi e \langle f | J_\mu(x) | i \rangle$$

$$\partial_\mu J^\mu(x) = 0$$



Approximations:

- one photon exchange
- ultrarelativistic electrons
- small *energy* and *momentum* transfer

Inelastic cross-section

$$d\sigma = d\bar{k}_f\, d\bar{k}_1\, d\bar{k}_2\, d\bar{k}_C\, \delta^4\left(k_i + P_i - k_f - P_f\right) \frac{(\hbar c)^2}{\varepsilon_f^2} \sigma_M \sum V_{\alpha\beta} W_{\alpha\beta}$$

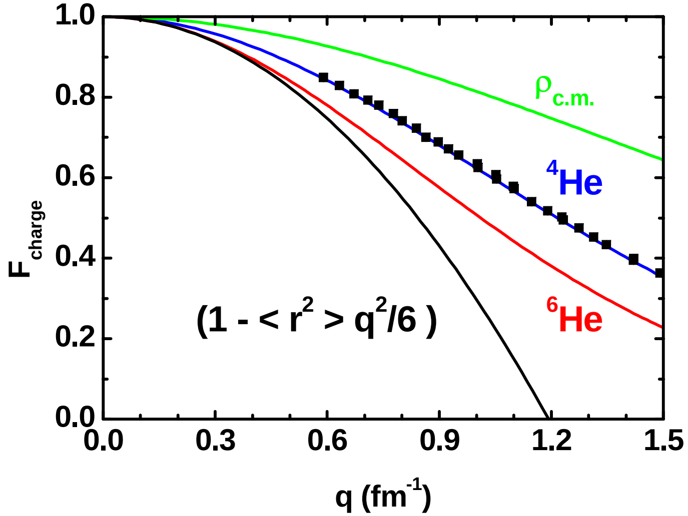
Coulomb contribution:

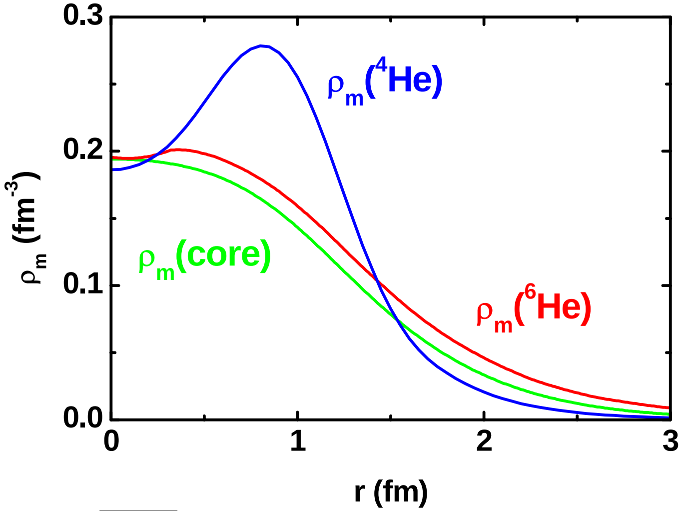
$$W_{00} = \frac{1}{2J_A + 1} \sum \left| \left\langle \Phi^{(-)}_{m_i}\left(\bar{k}_x, \bar{k}_y\right) \middle| \hat{\rho}\left(\bar{q}\right) \middle| \Phi_{J_A M_A} \right\rangle \right|^2, \quad V_{00} = \frac{Q^4}{\left|\bar{q}\right|^4}$$

Inclusive cross section:

$$\frac{d^3\sigma}{d\hat{k}_f dE_x} = \frac{4\varepsilon_f^2 \alpha^2}{(\hbar c)^2} \frac{2E_x^2 \cos^2\frac{\theta}{2}}{1 + \frac{\varepsilon_f}{M_A c^2}\left(1 + \frac{\left|\bar{k}_i\right| \cos\theta}{\left|\bar{k}_f\right|}\right)} \frac{1}{\left|\bar{q}\right|^4}\, 2\left(\frac{\mu_x \mu_y}{\hbar^4}\right)^{\frac{3}{2}} \frac{4\pi}{\hat{J}_A^2} \sum \left| \rho^{101}_{\gamma J_f J_A}\left(\bar{q}\right) \rho_C\left(\bar{q}\right) \right|^2$$

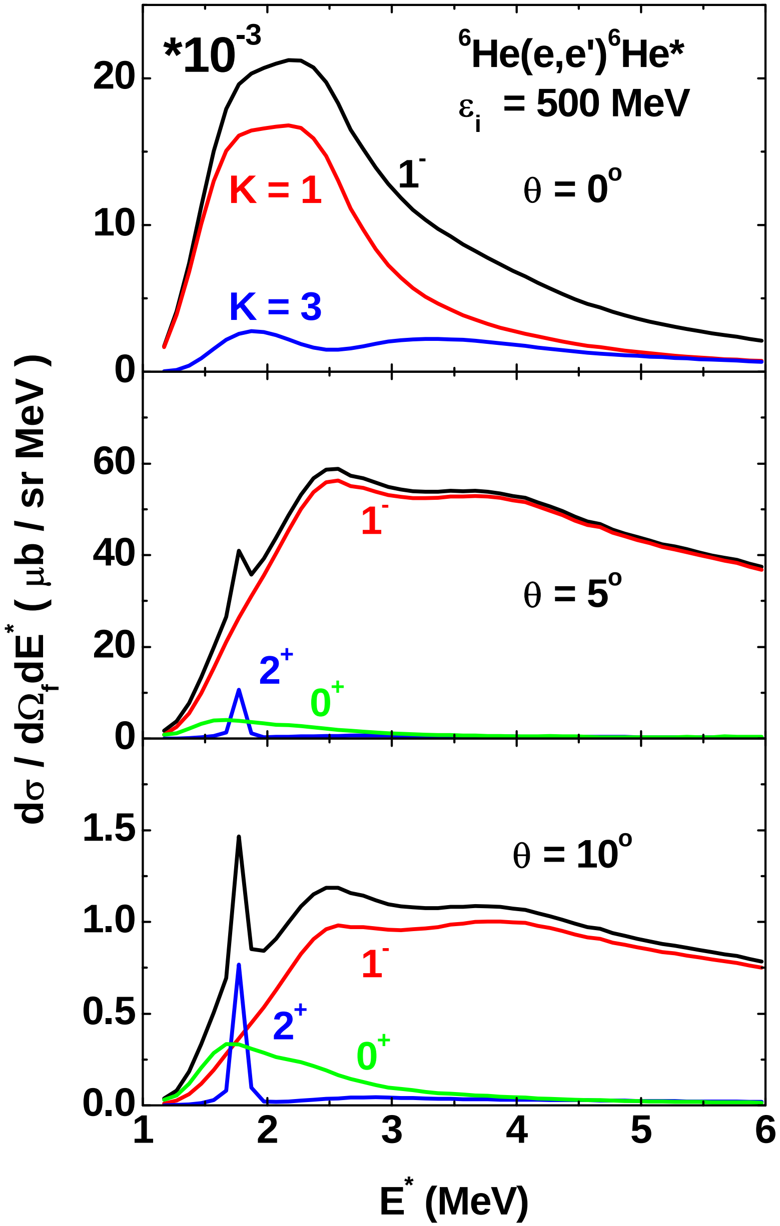
$$F_{ch}^{^6He}(q) = F_{ch}^{^4He}(q)\,\rho_{cm}^{C}(q)$$



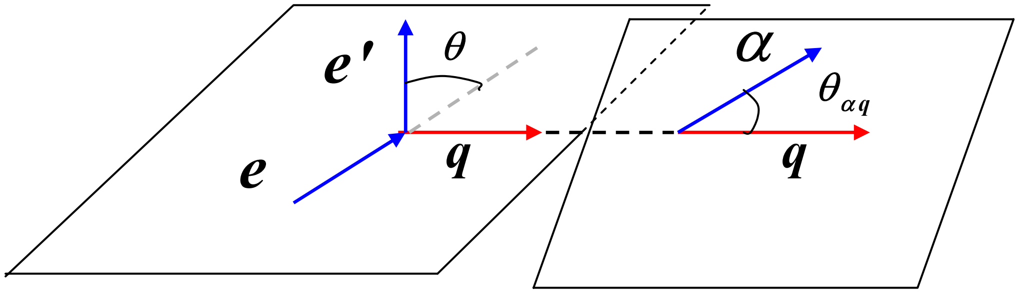


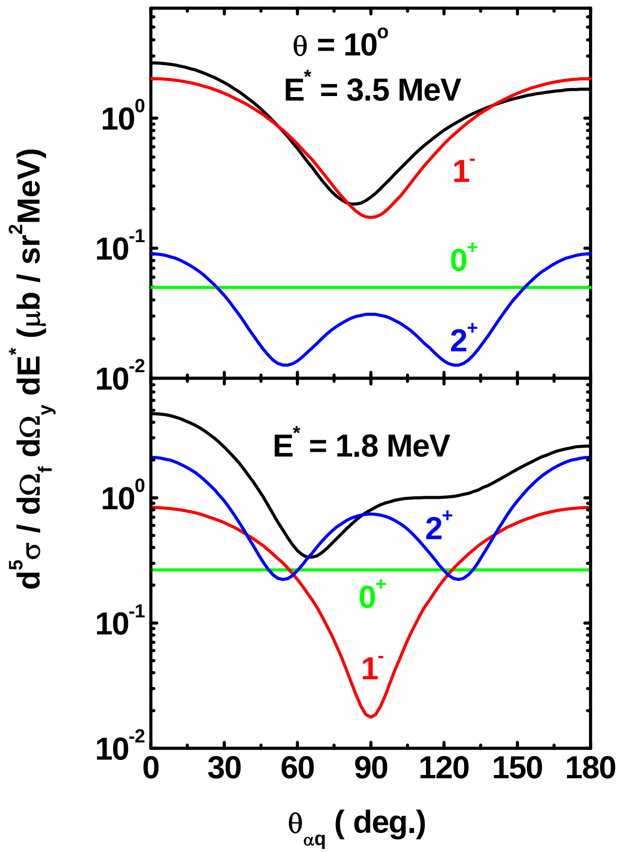
$$\sqrt{\langle r_{ch}^2 \rangle} = 2.054 \pm 0.014 \ fm$$

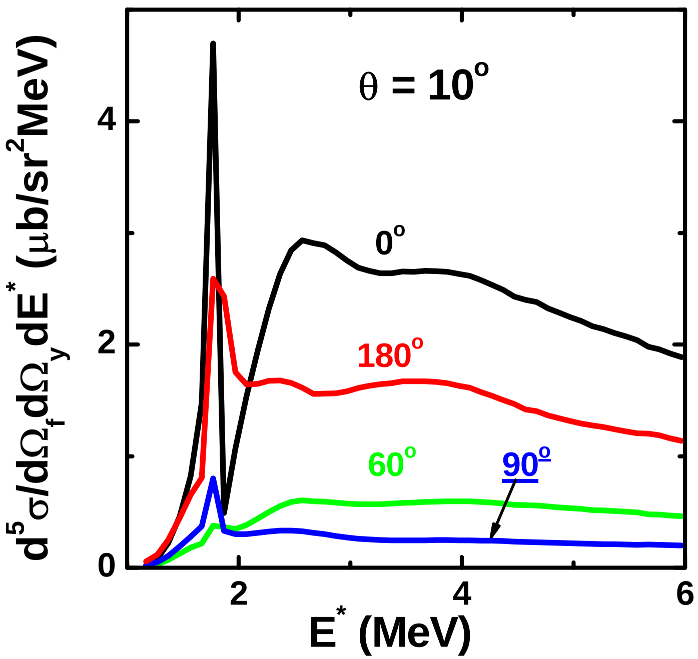
L.-B. Wang et al., Phys. Rev. Lett. 93(2004) 142501



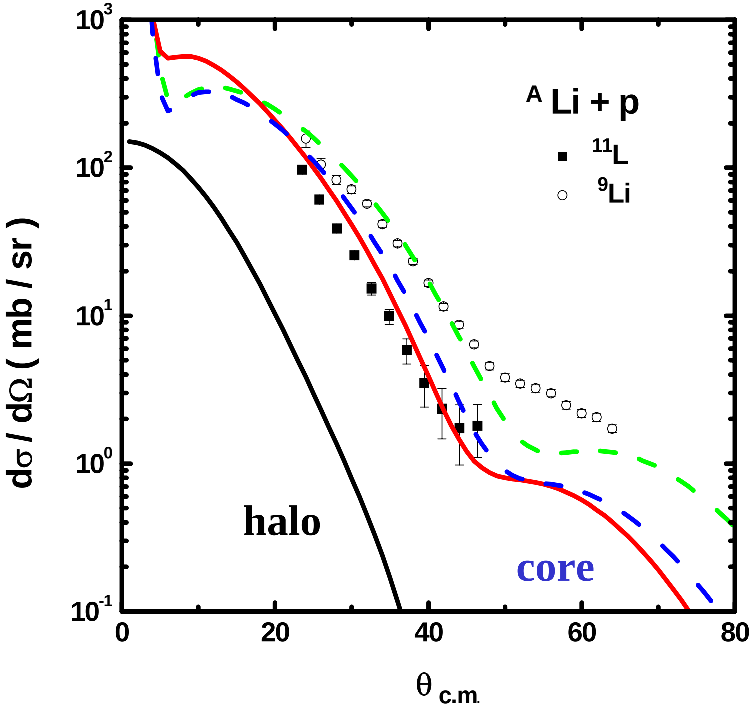
$$^{6}He(e,e'\alpha)2n \quad E_e = 500\,MeV$$







Elastic Scattering of *Halo Nucleus* on Proton



$^{11}Li + p$, E/A = 68 MeV

A.A. Korsheninnikov et al., PRL, 78 (1997) 2317

$^{9}Li + p$, E/A = 60 MeV

C.B. Moon et al., PL, B297 (1992) 39

single folded optical potential :

$$U_{^{11}Li} = U_{core} + U_{halo}$$

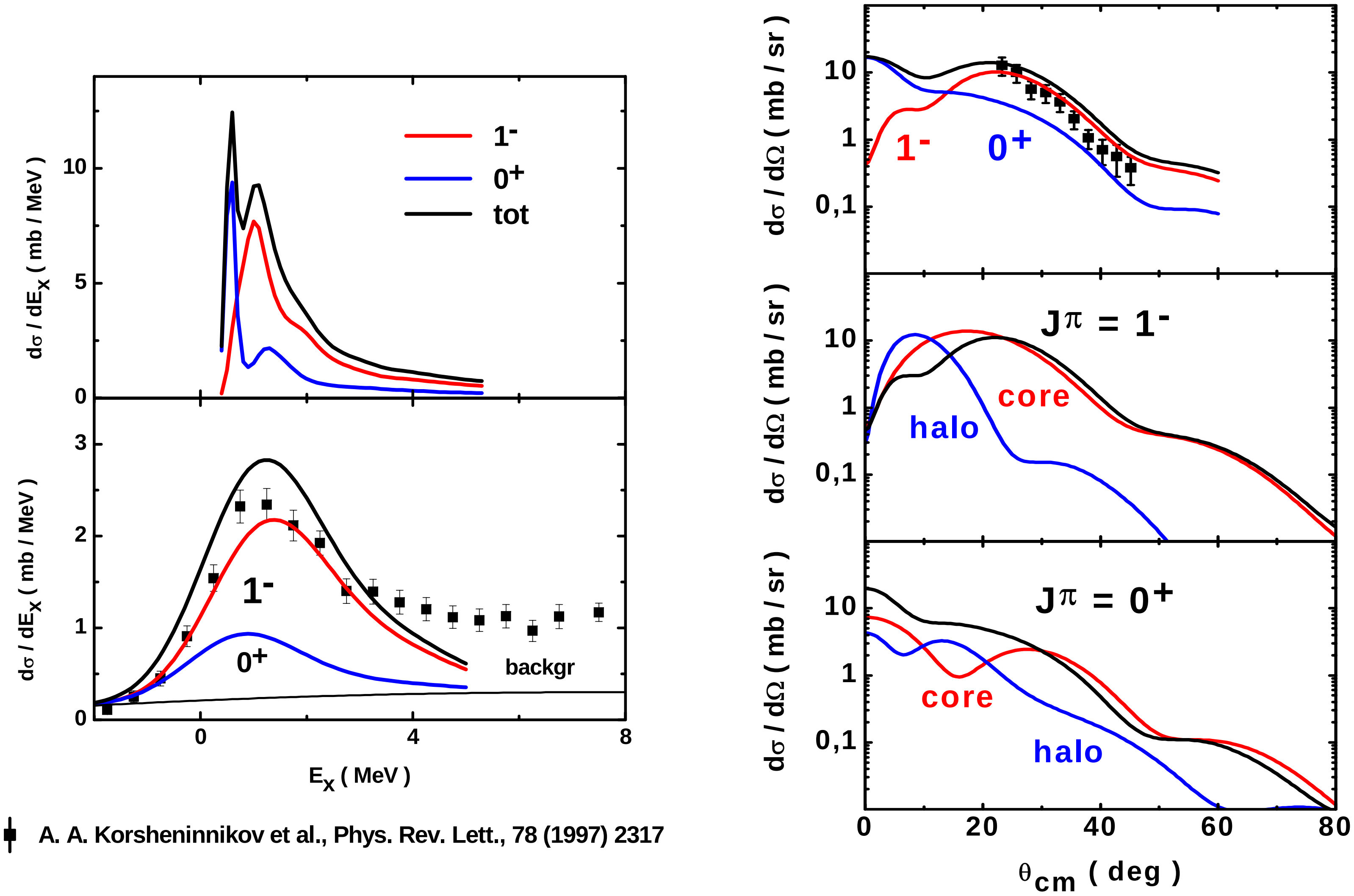
halo nucleons	core nucleons
$U_{halo} = \int t_{NN}\, \rho_{2n}$	$U_{core} = \int V_{NN}\, \rho_{core}$
free *NN t*-matrix interaction	density dependent *JLM* interaction

$$\rho_{core}(\bar{q}) = \rho_{cm}(\bar{q})\, \rho_{^{9}Li}(\bar{q})$$

Reaction cross sections :

$U_{^{11}Li}$	U_{core}	U_{halo}	$U_{^{9}Li}$
387 mb	214 mb	231 mb	219 mb

^{11}Li + p, E/A = 68 MeV



A. A. Korsheninnikov et al., Phys. Rev. Lett., 78 (1997) 2317

Identification of a *'true'* three-body resonance

The resonant 3-body wave function for given J^{π}

(in the interior region)

$$\Psi\left(\rho\Omega_5^{\rho}, E_{\kappa}\Omega_5^{\kappa}\right) \sim \frac{1}{(\kappa\rho)^{5/2}} \sum_{K\gamma} C_{K\gamma}\left(E_{\kappa}\right) \Psi_{K\gamma}^{R}(\rho) Y_{K\gamma}\left(\Omega_5^{\rho}\right) Y_{K\gamma}\left(\Omega_5^{\kappa}\right)$$

with $$\left|C_{K\gamma}\left(E_{\kappa}\right)\right|^2 = \frac{\Gamma_{K\gamma}}{\left(E_{\kappa}-E_0\right)^2 + \Gamma^2/4}$$

$$E_{\kappa} = \varepsilon_x + \varepsilon_y = \frac{k_x^2}{2\mu_x} + \frac{k_y^2}{2\mu_y}$$

$$\sin^2\theta_{\kappa} = \varepsilon_x / E_{\kappa}$$

Double differential cross section

$$\frac{d^2\sigma}{d\varepsilon_x\, d\varepsilon_y} \sim \left(E_{\kappa}\right)^{-5/2} \sqrt{\varepsilon_x\varepsilon_y} \sum_{\gamma} \left|C_{K\gamma}\left(E_{\kappa}\right)\right|^2 \left|\psi_{K_0}^{l_x l_y}\left(\theta_{\kappa}\right)\right|^2 = \frac{\left(E_{\kappa}\right)^{-5/2} \sqrt{\varepsilon_x\varepsilon_y} \sum_{\gamma} \Gamma_{K\gamma} \left|\psi_{K_0}^{l_x l_y}\left(\theta_{\kappa}\right)\right|^2}{\left(E_{\kappa}-E_0\right)^2 + \Gamma^2/4}$$

$\psi_{K_0}^{l_x l_y}\left(\theta_{\kappa}\right)$ **: the hyperangular part of hyperharmonics** $Y_{K\gamma}\left(\Omega_5^{\kappa}\right)$

In the **simplest** approximation:

width (*near threshold*)

$\Gamma \sim E_\kappa^2$ (~ 3-body phase volume)

parametrization: $\Gamma = \Gamma_0 \left(E_\kappa / E_0\right)^2$

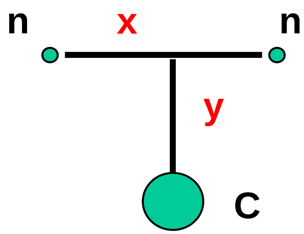
$$\frac{d^2\sigma}{d\varepsilon_x\, d\varepsilon_y} \sim \frac{\sqrt{\varepsilon_x \varepsilon_y}\left(\varepsilon_x+\varepsilon_y\right)^{-1/2}}{\left(\varepsilon_x+\varepsilon_y-E_0\right)^2+\Gamma_0\left(E_\kappa/E_0\right)^2/4}\sum_{l_x l_y}\left(\frac{\varepsilon_x}{E_\kappa}\right)^{l_x}\left(\frac{\varepsilon_y}{E_\kappa}\right)^{l_y}$$

noninvariant for different sets of Jacobi coordinates

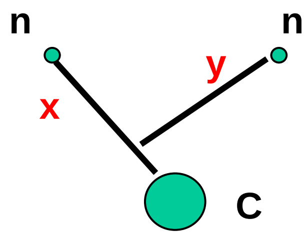
the asymmetric resonance shape for the 3-body decayng state

$$\frac{d\sigma}{dE_\kappa} \sim \frac{\left(E_\kappa\right)^{3/2}}{\left(E_\kappa-E_0\right)^2+\Gamma_0\left(E_\kappa/E_0\right)^2/4}$$

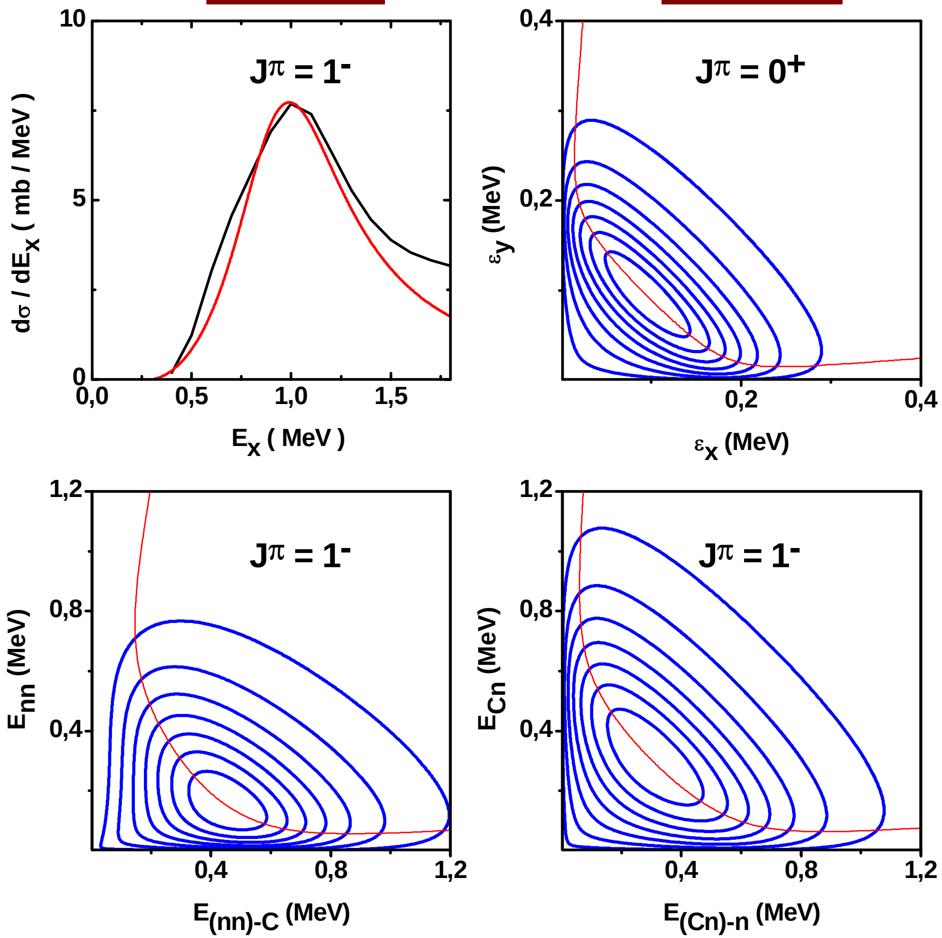
Correlation energy plots for a 'true' 3-body resonance



T basis



Y basis



V_{nn} ⇒ s – wave: *attractive*
p – wave: *repulsive*

$J^{\pi} = 0^{+}$: $K_0 = 0$

$l_x = l_y = 0$ ⇐ in *both* T and Y bases

$J^{\pi} = 1^{-}$: $K_0 = 1$

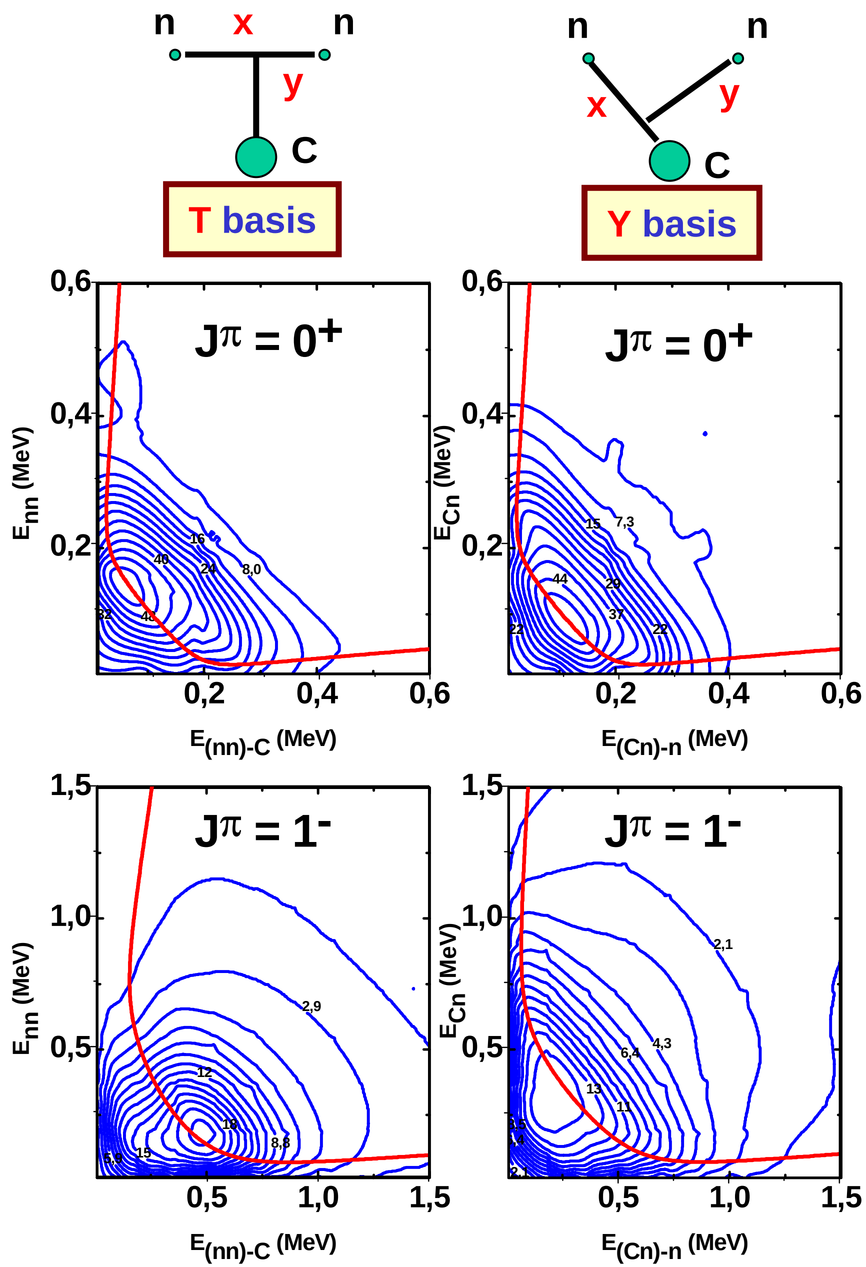
$$\begin{cases} l_x = 1, \; l_y = 0 \\ l_x = 0, \; l_y = 1 \end{cases}$$

$l_x = 0, \; l_y = 1$ ⇐ dominates in T basis

in Y basis reduces to

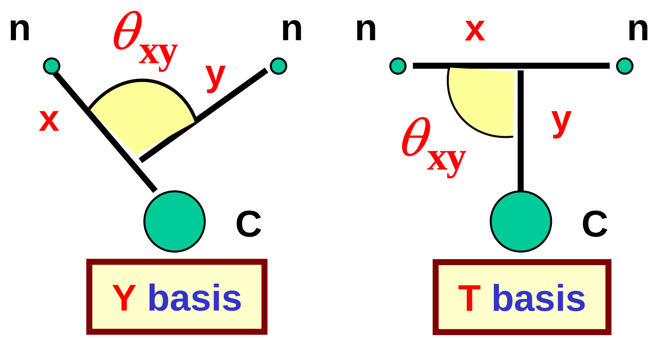
$$\begin{cases} l_x = 1, \; l_y = 0, \sim 50\% \\ l_x = 0, \; l_y = 1, \sim 50\% \end{cases}$$

Fragment energy correlations, $^{11}Li + p$, E/A = 68 MeV



Fragment *angular* correlations

$$\frac{d\sigma}{d\cos\theta_{xy}} \sim \sum_{n\gamma} P_n\left(\cos\theta_{xy}\right)\left(l_x\,0\,l_x'\,0\middle|n\,0\right)\left(l_y\,0\,l_y'\,0\middle|n\,0\right)B_\gamma$$



Angular asymmetry : *n* is odd

(*n* = 1 is the main term)

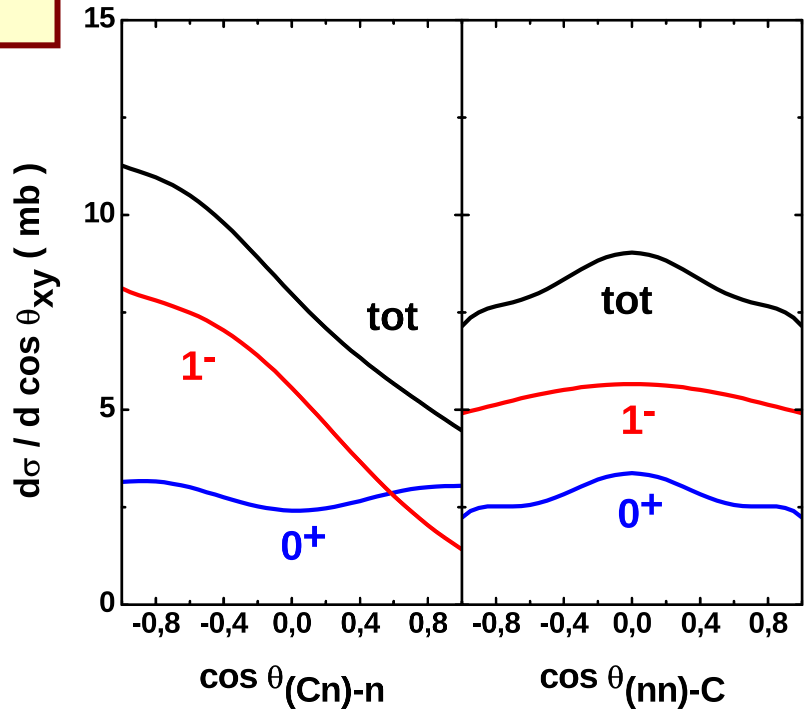
Evidence about mixing of angular momenta with different parities

$^{11}Li + p$, E / A = 68 MeV

$J^\pi = 0^+, K_0 = 0$		
	l_x	l_y
T :	0	0 : Y

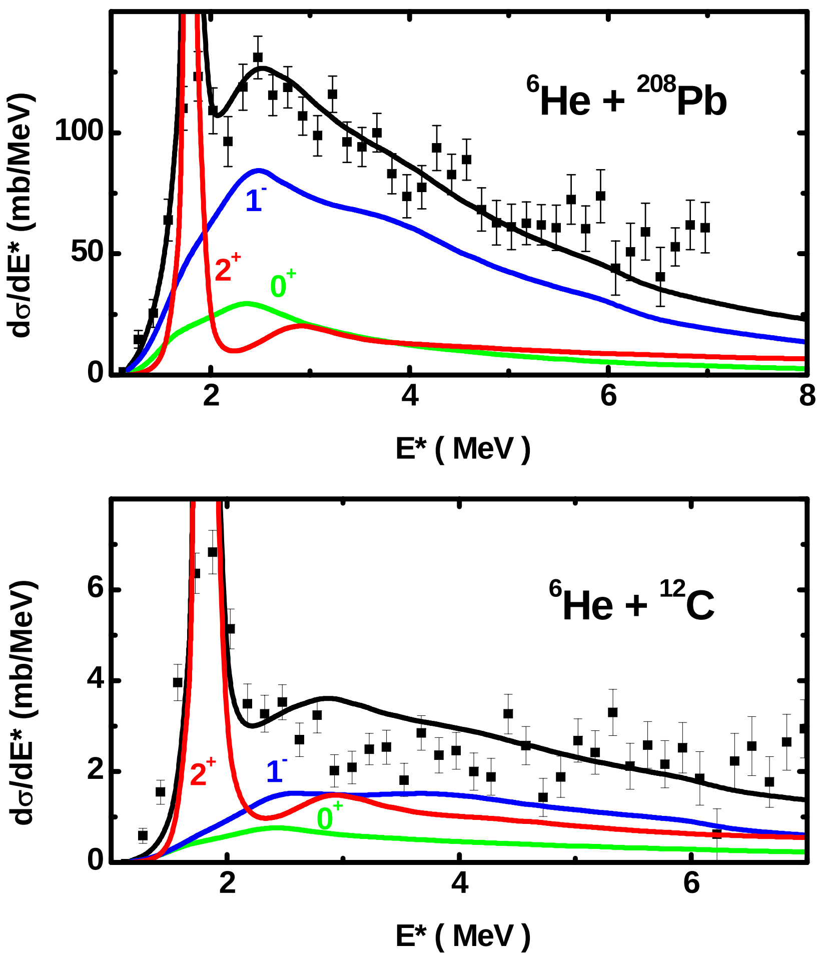
n = 1 →

$J^\pi = 1^-, K_0 = 1$		
	l_x	l_y
T :	0	1
Y :	0	1, ~ 50%
	1	0, ~ 50%



Halo scattering on nuclei

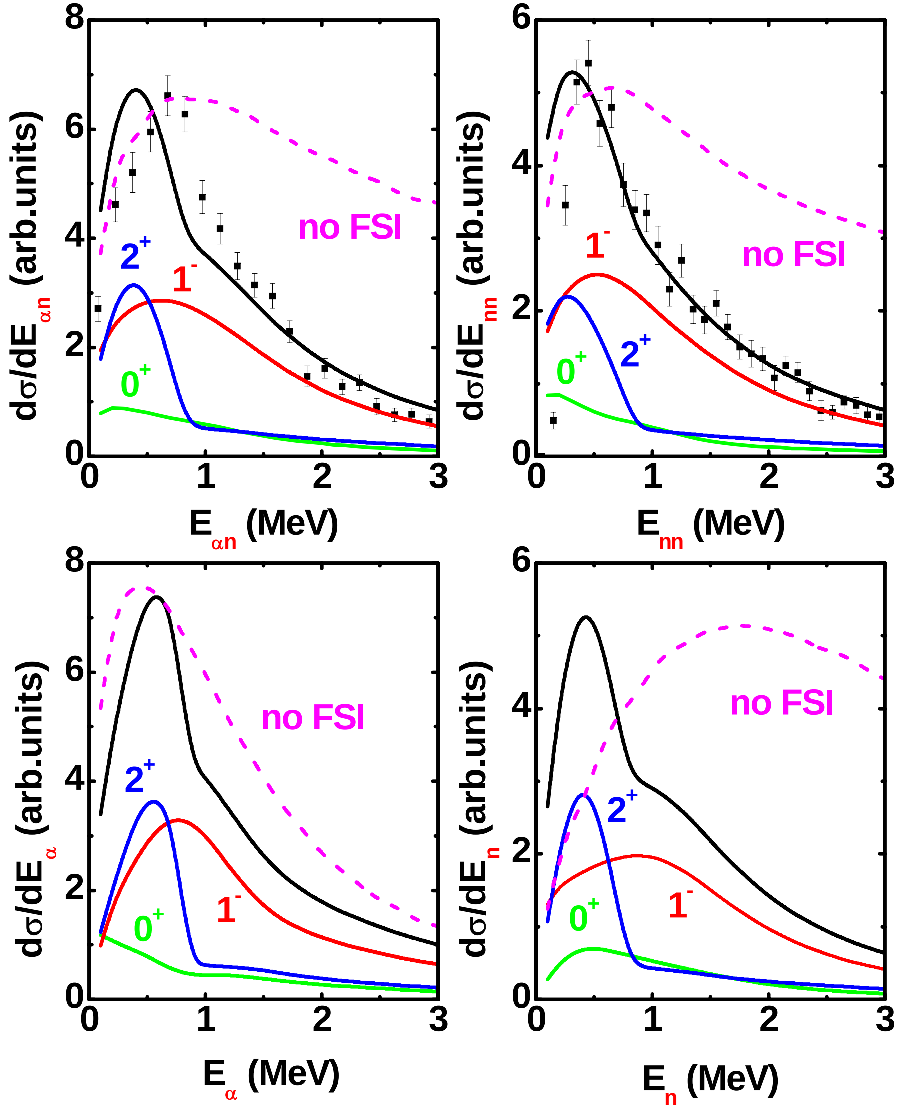
E / A = 240 MeV



T. Aumann et al., Phys. Rev., C59 (1999) 1252.

^{6}He + ^{208}Pb

E / A = 240 MeV/A



CONCLUSIONS

- **The remarkable discovery of new type of nuclear structure at driplines, *HALO,* have been made with radioactive nuclear beams.**

- **The theoretical description of dripline nuclei is an exciting challenge. The coupling between *bound* states and the *continuum* asks for a strong interplay between various aspects of nuclear structure and reaction theory.**

- **Development of new experimental techniques for production and /or detection of radioactive beams is the way to unexplored**

“ TERRA INCOGNITA “