

Event structure of multiparticle production in nucleus-nucleus collisions

Rogachevsky Oleg

LHEP

NICA/JINR-FAIR Bilateral Workshop

3.04.2012

Multiparticle production at HIC

The statistical and hydrodynamical models predict an approximately exponential form of particle transverse momentum spectra

Pioneering ideas/models:

-1950: E. Fermi

statistical hadron production: $T = T_i \sim s_{NN}^{1/4}$

-1951: I. Pomeranchuk

freeze-out at $T = T_{FO} \approx m_\pi$

-1953: L. D. Landau

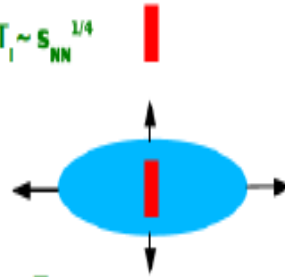
hydrodynamical expansion from T_i to T_{FO}

$T = f(m, v_T, T_{FO})$

-1965: R. Hagedorn

statistical hadron production at $T = T_H \approx 150 \text{ MeV}$

$$f(m_T) \sim e^{-m_T/T}$$



A fireball is

**... a statistical equilibrium of an undetermined number of all kinds of fireballs, each of which in turn is considered to be...*

R.Hagedorn

(selfsimilarity)

The Fractal geometry of nature

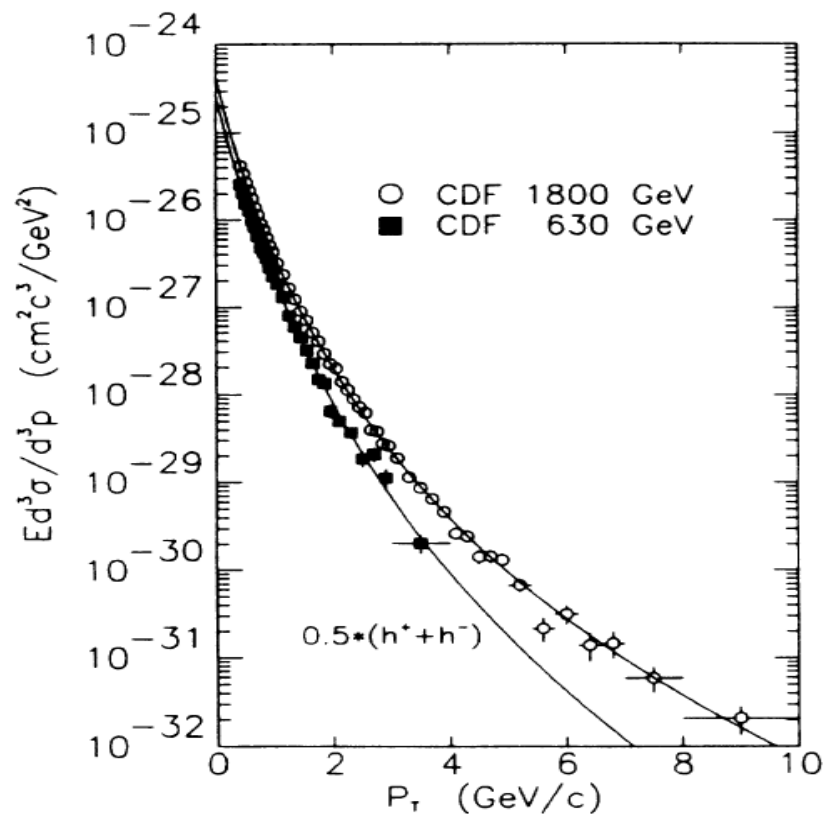
B.B. Mandelbrot Science 155, 636 (1967)

The Non-extensive statistical mechanics

C. Tsallis, J. Stat. Phys. 52, 479 (1988)

(Thanks to M,Gazdzicki)

PPbar collisions



Abe F. et al 1988 Phys. Rev. Lett. 61 1819

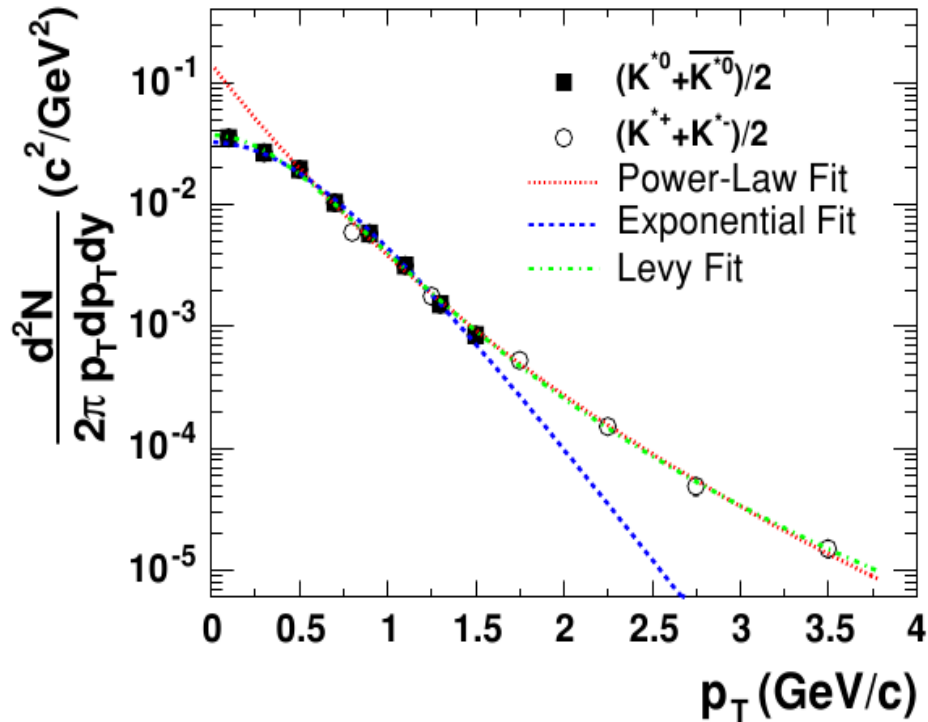
ppbar \rightarrow h + X

$$E \frac{d^3 \sigma}{d^3 p} = \frac{A p_0^n}{(p_T + p_0)^n}$$

Inclusive cross sections for rapidity $|y| < 1.0$
and fitted curves with p_0 fixed at 1.3 GeV/c.

Levy function

STAR collaboration
Phys. Rev. C 71, 064902 (2005)



exp

$$\frac{1}{2\pi m_T} \frac{d^2N}{dy dm_T} = \frac{dN}{dy} \frac{1}{2\pi T(m_0 + T)} \exp\left(\frac{-(m_T - m_0)}{T}\right)$$

Power law

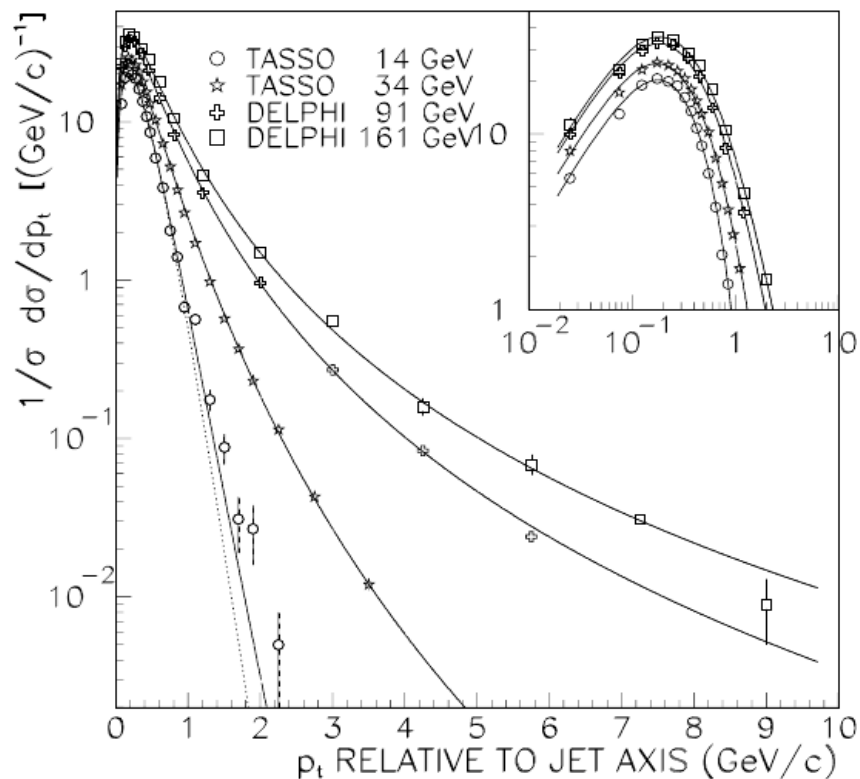
$$\frac{1}{2\pi p_T} \frac{d^2N}{dy dp_T} = \frac{dN}{dy} \frac{2(n-1)(n-2)}{\pi(n-3)^2 \langle p_T \rangle^2} \times \left(1 + \frac{p_T}{\langle p_T \rangle (n-3)/2}\right)^{-n}$$

Levy

$$\frac{1}{2\pi p_T} \frac{d^2N}{dy dp_T} = \frac{dN}{dy} \frac{(n-1)(n-2)}{2\pi nT[nT + m_0(n-2)]} \times \left(1 + \frac{\sqrt{p_T^2 + m_0^2} - m_0}{nT}\right)^{-n}$$

The invariant yields for both $(K^0 + \bar{K}^0)/2$ and $(K^+ + K^-)/2$ as a function of p_T for $|y| < 0.5$ in minimum bias $p + p$ interactions. The dotted curve is the fit to the power-law function for $p_T > 0.5$ GeV/c and extended to lower values of p_T . The dashed curve is the K^0 spectrum fit to the exponential function and extended to higher values of p_T . The dashed-dotted curve is the fit to the Levy function for $p_T < 4$ GeV/c. Errors are statistical only.

e^-e^+ collisions



Bediaga I, Curado EMF and Miranda J.

A non-extensive thermodynamical equilibrium approach in $e(+) e(-) \rightarrow$ hadrons

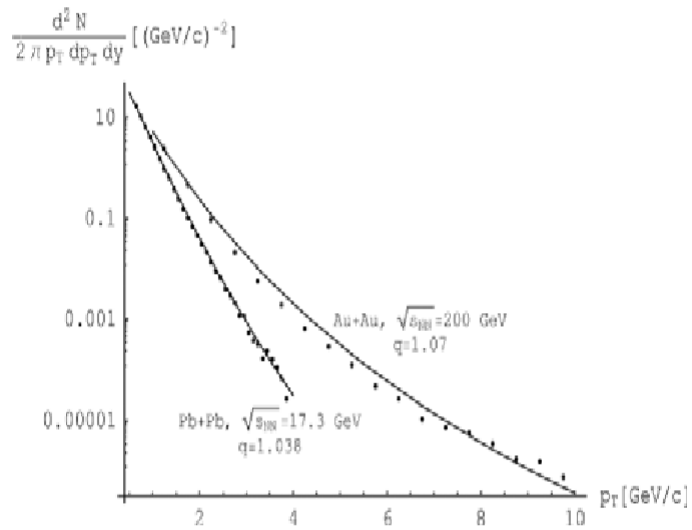
Physica A 286: 156-163 2000.

The transverse momentum p_t of charged hadrons with respect to jet axis is sketched for four different experiments, whose center-of-mass energies vary from 14 and 34 GeV (TASSO) up to 91 and 161 GeV (DELPHI).

$$\frac{1}{\sigma} \frac{d\sigma}{dp_t} = c p_t \int_0^\infty dp_l \left[1 - \frac{1-q}{T_0} \sqrt{p_l^2 + \mu^2} \right]^{q/(1-q)}$$

PbPb & AuAu collisions

Eur. Phys. J. A 40, 313 (2009)



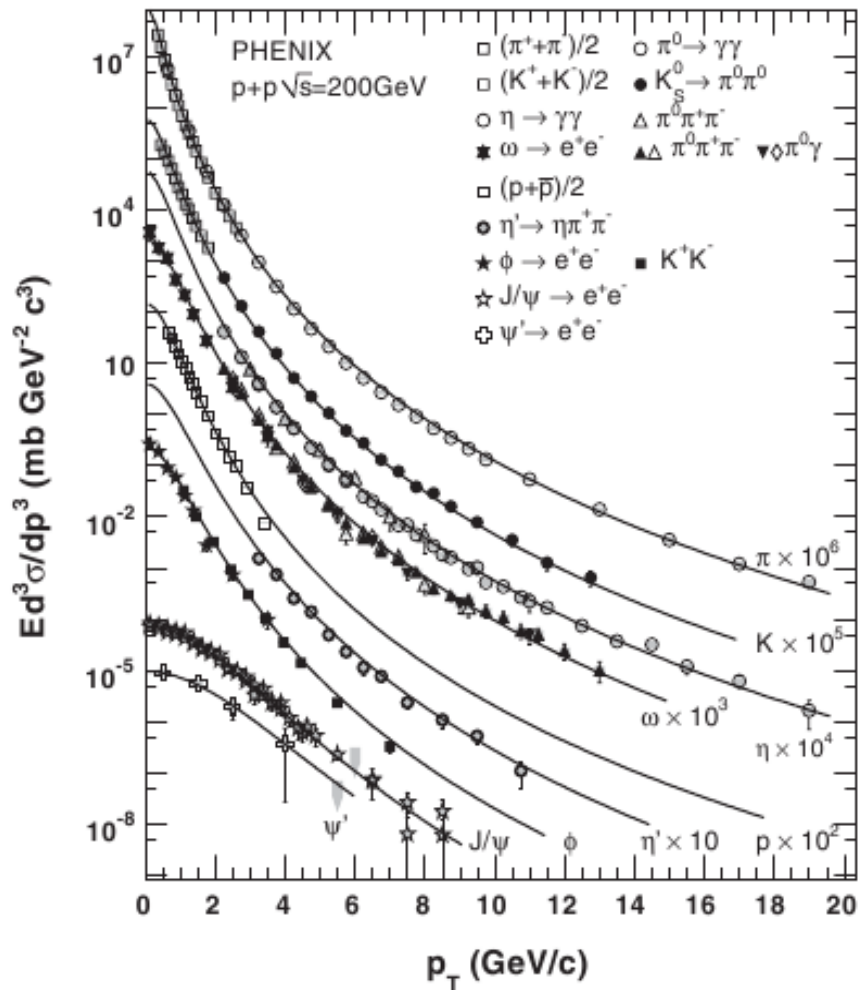
Experimental neutral pion invariant yields in central Pb+Pb collisions at $\sqrt{s_{NN}} = 17.3$ GeV and in central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV compared with the modified thermal distribution shape by using non-extensive statistics

($q = 1.038$ for Pb+Pb and $q = 1.07$ for Au+Au collisions.)

$$\frac{d^2 N}{2\pi p_{\perp} dp_{\perp} dy} = C m_{\perp} \left[1 - (1 - q) \frac{m_{\perp}}{T} \right]^{1/(1-q)}$$

PP collisions

PHENIX collaboration
PRD 83, 052004 (2011)



Tsallis distribution

$$G_q(E) = C_q \left(1 - (1 - q) \frac{E}{T} \right)^{1/(1-q)},$$

$$n = - \frac{1}{1 - q}.$$

$$E \frac{d^3 \sigma}{dp^3} = \frac{1}{2\pi} \frac{d\sigma}{dy} \frac{(n-1)(n-2)}{(nT + m_0(n-1))(nT + m_0)} \left(\frac{nT + m_T}{nT + m_0} \right)^{-n}$$

Invariant differential cross sections of different particles measured in p + p collisions at $\sqrt{s} = 200$ GeV in various decay modes.

Charged particle transverse momentum spectra in pp

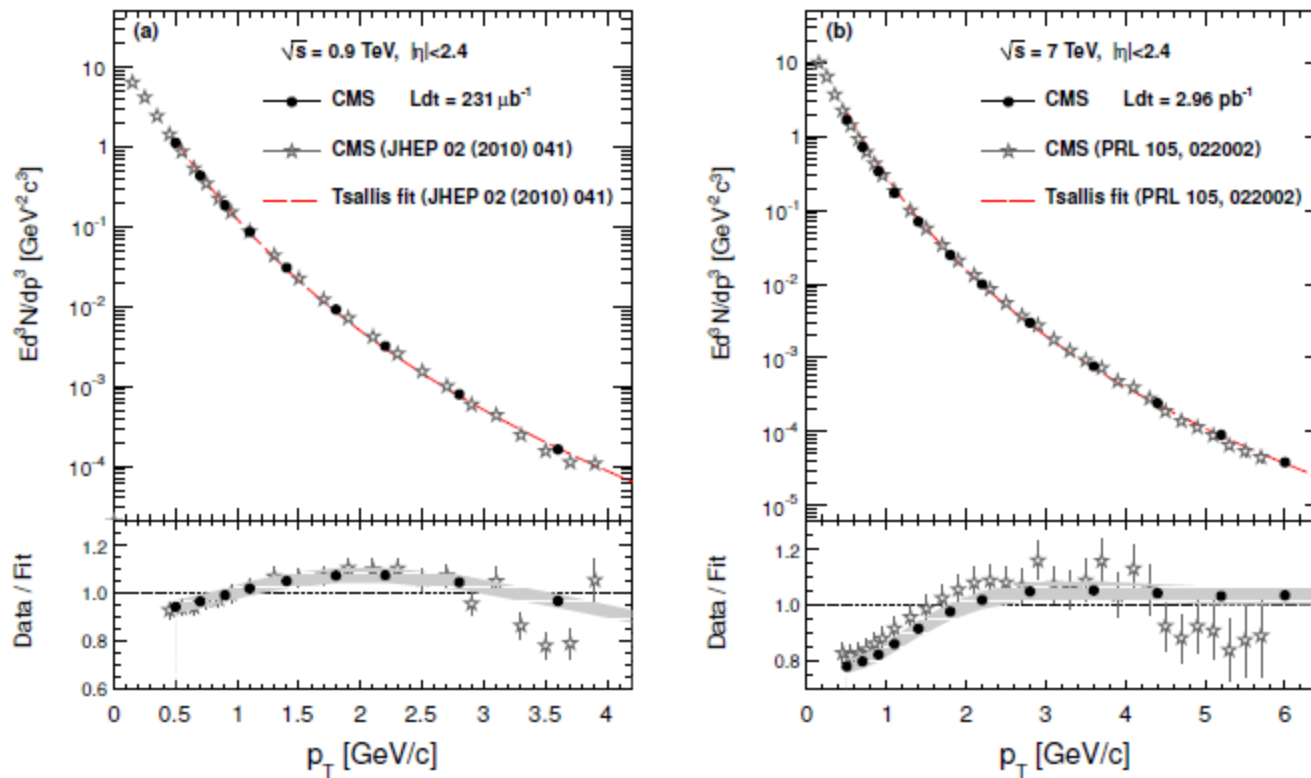
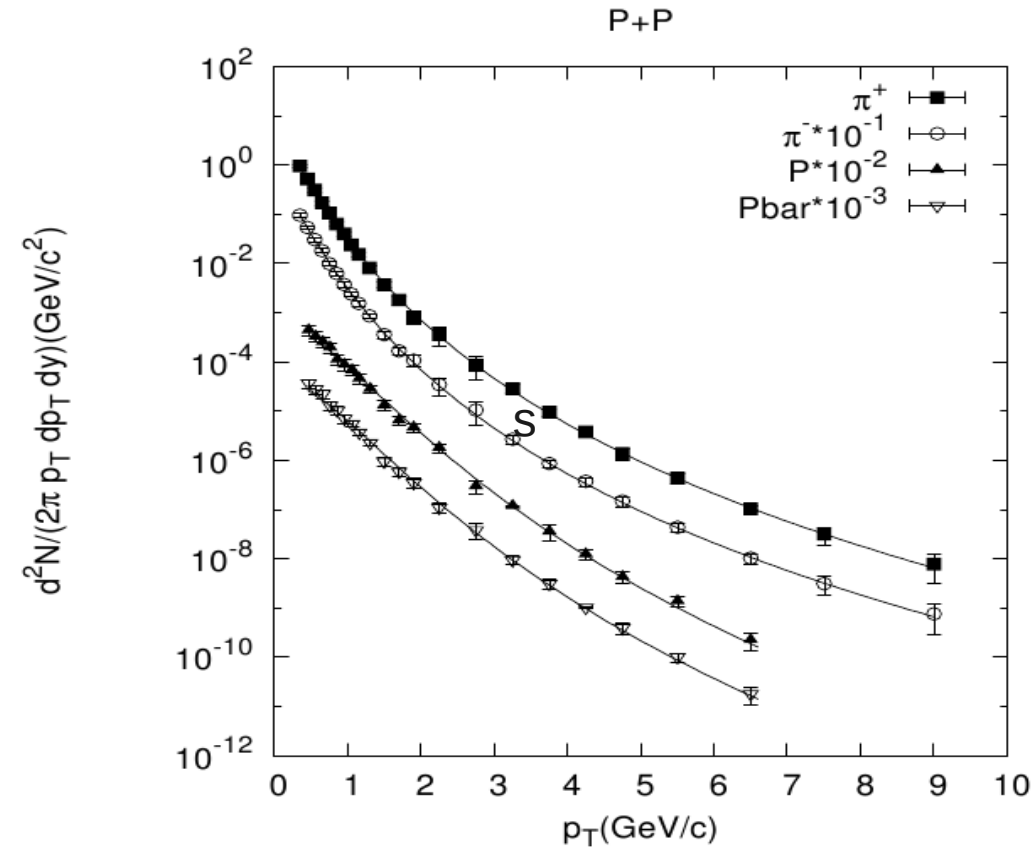


Figure 4. (a) Upper panel: the invariant charged particle differential yield from the present analysis (solid circles) and the previous CMS measurements at $\sqrt{s} = 0.9$ TeV (stars) over the limited p_T range of the earlier result. Lower panel: the ratio of the new (solid circles) and previous (stars) CMS results to a Tsallis fit of the earlier measurement. Error bars on the earlier measurement are the statistical plus systematic uncertainties added in quadrature. The systematic uncertainty band around the new measurement consists of all contributions, except for the common event selection uncertainty. (b) The same for $\sqrt{s} = 7$ TeV.

Tsallis distribution

- ◆ Transverse-Momentum and Pseudorapidity Distributions of Charged Hadrons in Collisions at $\sqrt{s}=7$ TeV
Physical Review Letters 105, 022002 (2010)
- ◆ Transverse momentum spectra of charged particles in proton–proton collisions at $\sqrt{s} = 900$ GeV with ALICE at the LHC
Physics Letters B 693 53–68 (2010)
- ◆ Measurement of neutral mesons in p+p collisions at $\sqrt{s} = 200$ GeV and scaling properties of hadron production
Physical Review D 83, 052004 (2011)
- ◆ Nuclear modification factors of ϕ mesons in d + Au, Cu + Cu, and Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV
Physical Review C 83, 024909 (2011)
- ◆ Strange particle production in proton–proton collisions at $\sqrt{s} = 0.9$ TeV with ALICE at the LHC
The European Physical Journal C 71, 1594(2011)
- ◆ Production of pions, kaons and protons in pp collisions at $\sqrt{s} = 900$ GeV with ALICE at the LHC
The European Physical Journal C 71, 1655(2011)
- ◆ Charged-particle multiplicities in pp interactions measured with the ATLAS detector at the LHC
New Journal of Physics 13, 053033 (2011)

Tsallis statistics (1)



Bhaskar De
 S. Bhattacharyya
 Goutam Sau
 S. K. Biswas

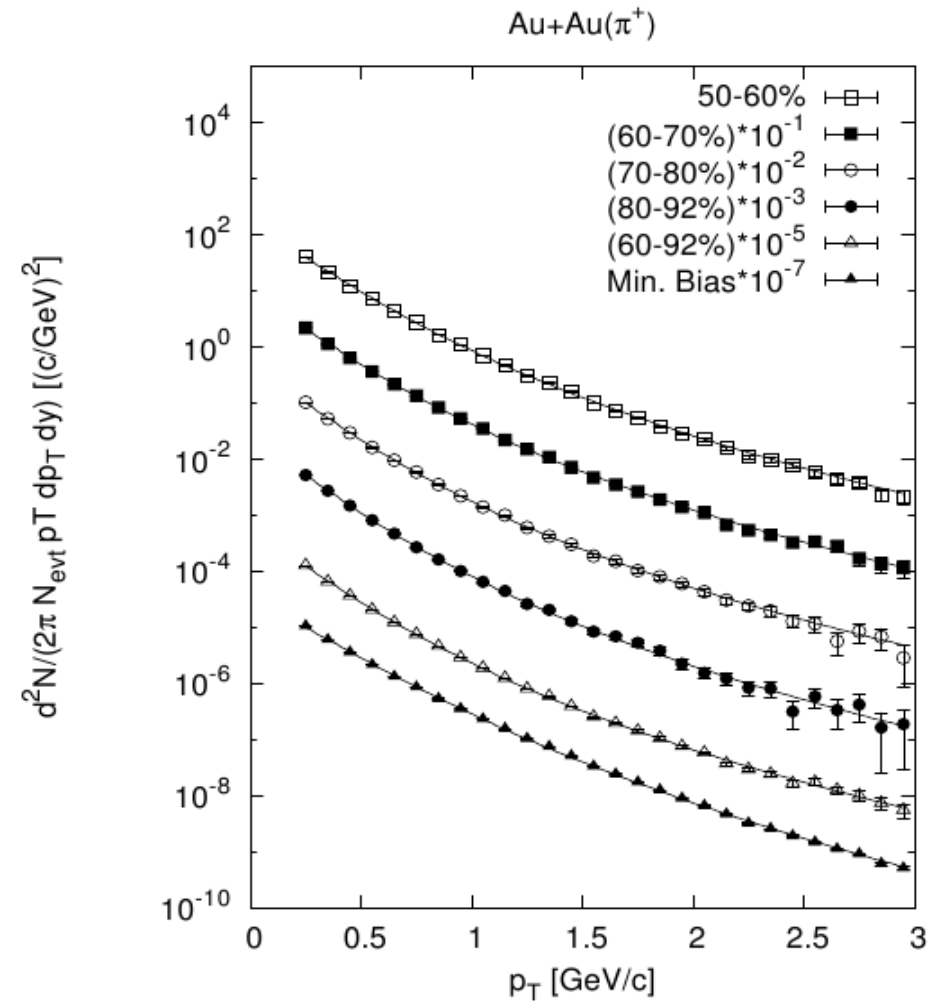
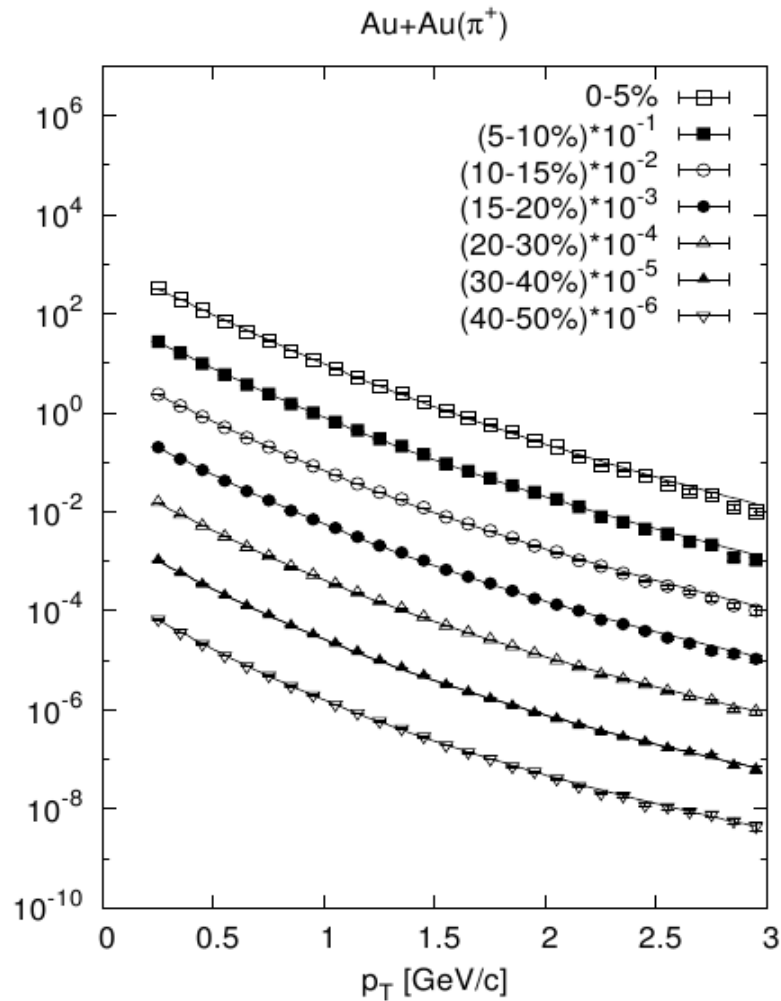
IJMP E Vol. 16, No. 6 (2007) 1687

STAR & PHENIX
 $\sqrt{s} = 200 \text{ GeV}$

$$\frac{1}{\sigma} \frac{d\sigma}{dp_T} \approx c p_T \int_0^\infty dp_L (1 + (q-1)\beta \sqrt{p_T^2 + p_L^2 + m_0^2})^{-q/(q-1)}$$

Tsallis statistics (2)

Ibid.

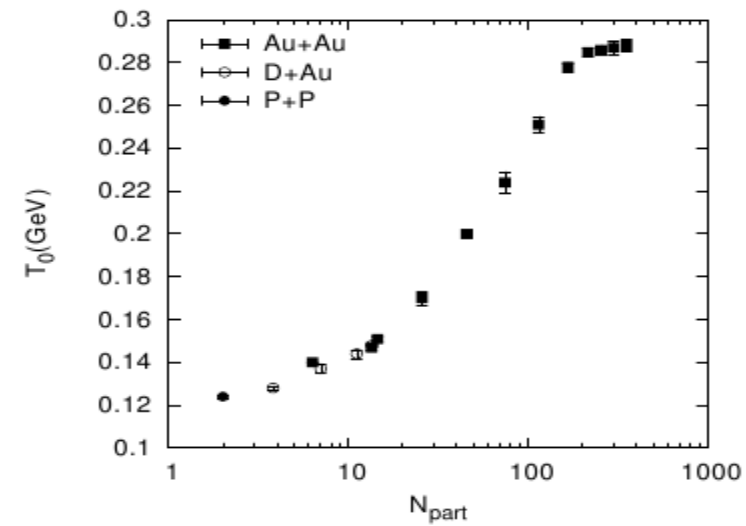
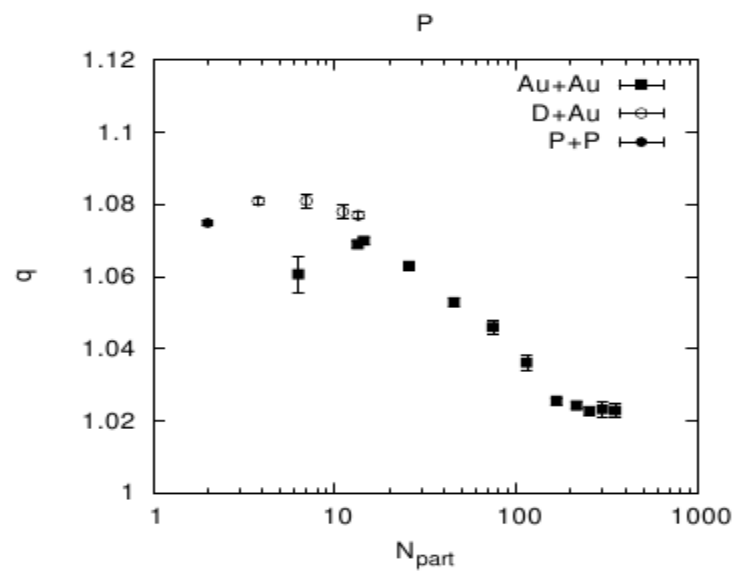
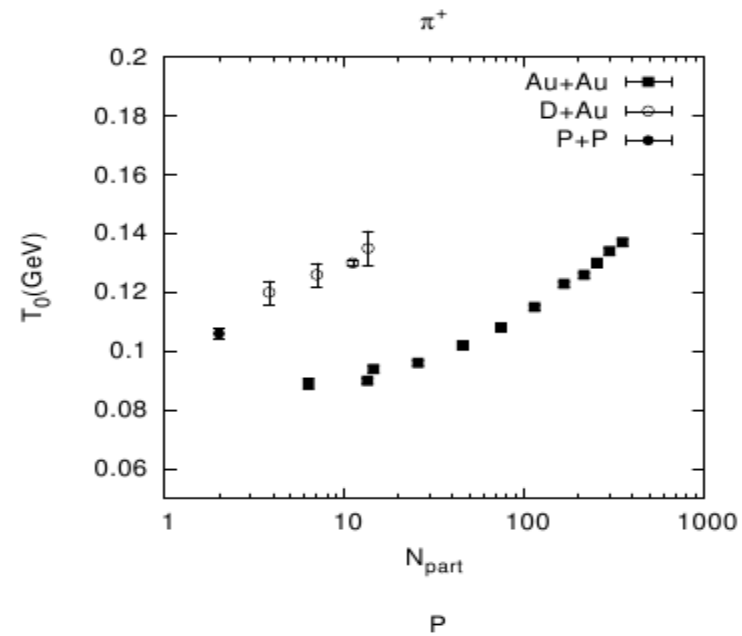
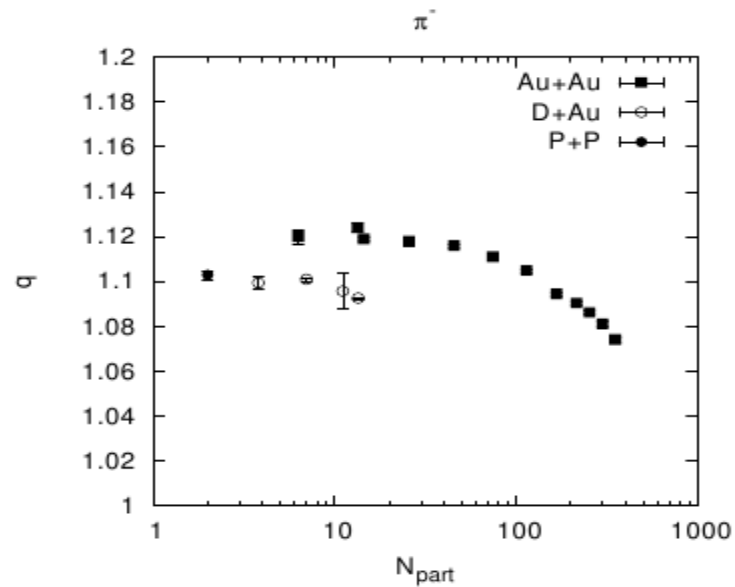


Tsallis statistics (3)

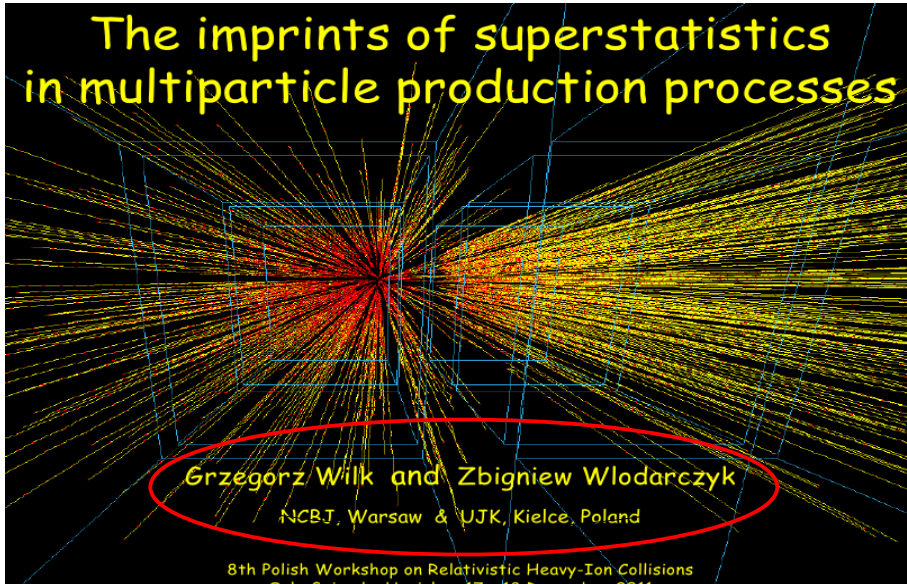
Table 1. Values of fitted parameters with respect to experimental data on π^+ -spectra at different centralities of $Au + Au$, $D + Au$ and $P + P$ collisions at RHIC.

	Centrality	C	q	$T_0(\text{GeV})$	χ^2/ndf
$Au + Au$	0–5	7963 ± 198	1.080 ± 0.002	0.137 ± 0.001	35.180/25
	5–10	7000 ± 180	1.084 ± 0.002	0.134 ± 0.001	33.758/25
	10–15	6500 ± 170	1.090 ± 0.002	0.130 ± 0.001	28.543/25
	15–20	6010 ± 160	1.095 ± 0.002	0.126 ± 0.001	27.543/25
	20–30	4915 ± 123	1.099 ± 0.002	0.123 ± 0.001	28.704/25
	30–40	3873 ± 114	1.107 ± 0.002	0.115 ± 0.001	31.913/25
	40–50	2918 ± 53	1.114 ± 0.001	0.108 ± 0.001	21.123/25
	50–60	2089 ± 45	1.118 ± 0.001	0.102 ± 0.001	22.208/25
	60–70	1324 ± 40	1.122 ± 0.002	0.096 ± 0.001	24.902/25
	70–80	769 ± 32	1.125 ± 0.003	0.090 ± 0.001	19.718/25
	80–92	408 ± 23	1.119 ± 0.003	0.089 ± 0.002	14.443/25
	60–92	815 ± 24	1.121 ± 0.001	0.094 ± 0.001	12.717/25
	Min. Bias	3003 ± 36	1.091 ± 0.001	0.128 ± 0.001	24.140/25
$D + Au$	0–20	259 ± 51	1.092 ± 0.003	0.135 ± 0.006	5.682/21
	20–40	196 ± 4	1.096 ± 0.001	0.130 ± 0.001	7.505/21
	40–100	107 ± 13	1.099 ± 0.002	0.120 ± 0.004	5.022/21
	Min. Bias	145 ± 21	1.099 ± 0.002	0.126 ± 0.004	5.077/21
$P + P$	—	81 ± 6	1.102 ± 0.002	0.106 ± 0.002	2.813/20

Tsallis statistics parameters



q interpretations



Tsallis statistics as Superstatistics

C. Beck et al., Physica A322 (2003) 267

Superstatistics is a superposition of two different statistics relevant to system under consideration with a **stationary state** and **intensive parameter fluctuations**

$$h(E/T) = \int_0^\infty f(E/T) g(1/T) d(1/T)$$

G. Wilk, Z. Włodarczyk, Phys. Rev. Lett. 84, 2770 (2000); Physica A376(2007)279 PRC79(2009)054903; EPJA40(2009)299; JPG38(2011)065101; Physica A390(2011)3566 G. Wilk, Z. Włodarczyk, W. Wolak, APPB(2011)1277

M. Biyajima et al., EPJC40(2005)243 and C48(2006)593 (p_T fits).
T. Osada et al., PRC77(2008)044903; PTPSuppl.174(2008)168 (2008); CEJP7(2009)432; IJP85(2011)825 (q-hydrodynamics)

Tsallis distribution

C. Tsallis, J.Stat.Phys. 52 (1988) 479

$$f(E) = \frac{2-q}{T} \left[1 - (1-q) \frac{E}{T} \right]^{1-q}$$

$q \rightarrow 1$ meaning of q ?

BG

$$f(E) = \frac{1}{T} \exp\left(-\frac{E}{T}\right)$$

R. Hagedorn (1965)

3

In full phase space q measures dynamical fluctuations in $P(N)$

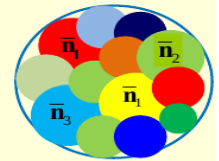
(*) Experiment: $P(N)$ is adequately described by NBD depending on $\langle N \rangle$ and k ($k \geq 1$) affecting its width:

$$\frac{1}{k} = \frac{\sigma^2(N)}{\langle N \rangle^2} - \frac{1}{\langle N \rangle}$$

(*) If $1/k$ is understood as a measure of fluctuations of $\langle N \rangle$, then

$$P(N) = \int_0^\infty d\bar{n} \frac{\bar{n}^N \exp(-\bar{n})}{N!} \frac{\gamma^k \bar{n}^{k-1} \exp(-\gamma \bar{n})}{\Gamma(k)}$$

$$= \frac{\Gamma(k+N)}{\Gamma(1+N)\Gamma(k)} \frac{\gamma^k}{(\gamma+1)^{k+N}} \quad \text{with} \quad \gamma = \frac{k}{\langle \bar{n} \rangle}$$



(P. Carruthers, C.C. Shih, Int.J.Phys. A4 (1989)5587)

$$\frac{1}{k} = D(\bar{n}) = \frac{\sigma^2(\bar{n})}{\langle \bar{n} \rangle^2} = q-1$$

(*) \rightarrow one expects: $q=1+1/k$ what indeed is observed

G. Wilk, Z. Włodarczyk, EPJA40(2009)299; F. Navarra, O. Utyuzh, WW, PRD67(2003)114002

8

q interpretations

Tsallis Distribution in High-Energy Collisions

(arXiv:1101.3023, accepted in EPL)

Gergely Gábor Barnaföldi

KFKI RMKI of the HAS

in collaboration

T.S. Biró, G. Kalmár, K. Ürmössy, P. Ván

High-pT Physics for the LHC 2011, Utrecht 4-7 April 2011

MOTIVATION

- New LHC pp data (CMS)

CMS: JHEP 1002:041(2010)

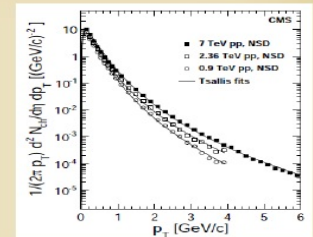
fitted Tsallis distribution for p_T spectra:

$$E \frac{d^3 N_{ch}}{dp^3} = \frac{1}{2\pi p_T} \frac{E}{p} \frac{d^2 N_{ch}}{d\eta dp_T} = C(n, T, m) \frac{dN_{ch}}{dy} \left(1 + \frac{E_T}{nT}\right)^{-n}$$

Parameters:

0.9 TeV T = 130 MeV, q = 1.13

2.36 TeV T = 140 MeV, q = 1.15



$n := (q-1)^{-1}$

- RHIC analysis on AuAu data ($y=0$)

Cooper-Frye model: K. Ürmössy, T.S. Biró: PL B689 14 (2010)

Parameters: $f(E) = A[1 + (q-1)E/T]^{-1/(q-1)}$

200 GeV T = 51 MeV, q = 1.062 (fit for $p_T < 6$ GeV/c)

G.G. Barnaföldi: Tsallis Distribution in High-Energy Collisions

3

Basics of non-extensive thermodynamics

Non-extensive thermodynamics (Based on: T.S. Biró: EPL84, 56003,2008) associative composition rule, (non-additive):

$$h(h(x, y), z) = h(x, h(y, z))$$

Then should exist a strict monotonic function, $X(x)$ 'generalised logarithm' (an entropy-like quantity), for which:

$$h(x, y) = X^{-1}(X(x) + X(y))$$

$$X(h(x, y)) = X(x) + X(y).$$

Examples: (i) Classical Boltzmann-Gibbs thermodynamics:

$$f(E) = e^{-\beta E} / Z$$

$$h(x, y) = x + y.$$

(ii) Tsallis-Pareto-like distribution with $a = q - 1$:

$$f(E) = \frac{1}{Z} e^{-\frac{E}{a} \ln(1+aE)} = \frac{1}{Z} (1+aE)^{-\beta/a}$$

$$h(x, y) = x + y + axy$$

$$S = \int f \frac{e^{-a \ln(f)} - 1}{a} = \frac{1}{a} \int (f^{1-a} - f).$$

G.G. Barnaföldi: Tsallis Distribution in High-Energy Collisions

7

Hadronization via non-extensive way

Our program:

- Search and fit Tsallis distribution to data from AA, pp, ee.

- Test: can a BFKL / DGLAP-like evolution equation be obtained?

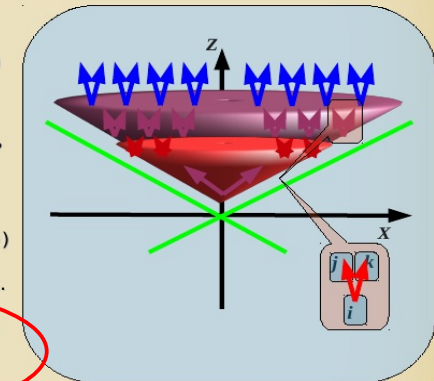
$$D(x, Q^2) \sim f(E, T, q) * f(\ln(Q^2))$$

$$D(x, Q^2) \sim f(E, T(\ln(Q^2)), q(\ln(Q^2)))$$

- Build up a simple theory to test.

- Search for physical meaning of T and q parameters.

→ This is a hard thing...

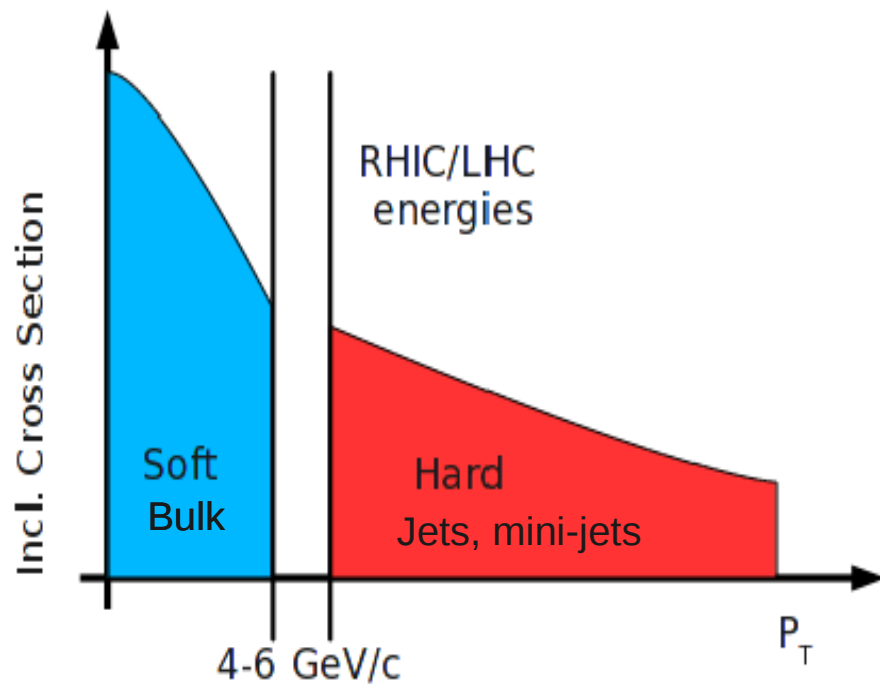


G.G. Barnaföldi: Tsallis Distribution in High-Energy Collisions

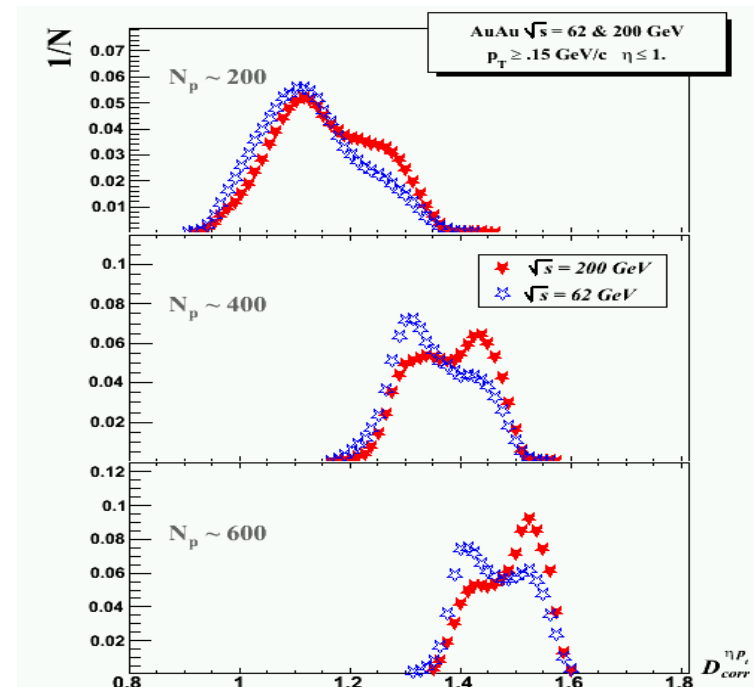
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High & low p_T hadron spectra

Interpretation ?



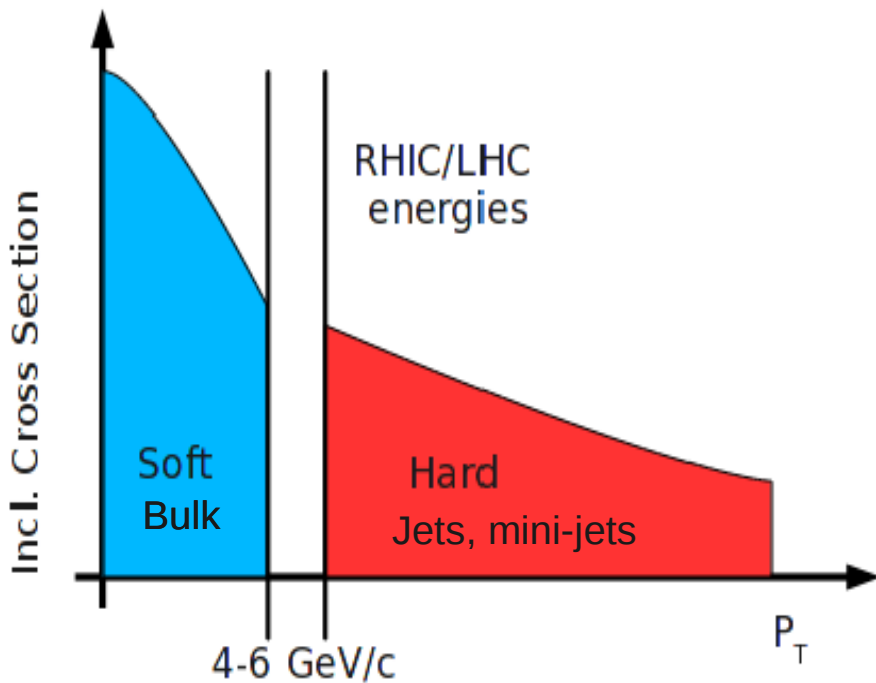
R.O. ICHEP 2006



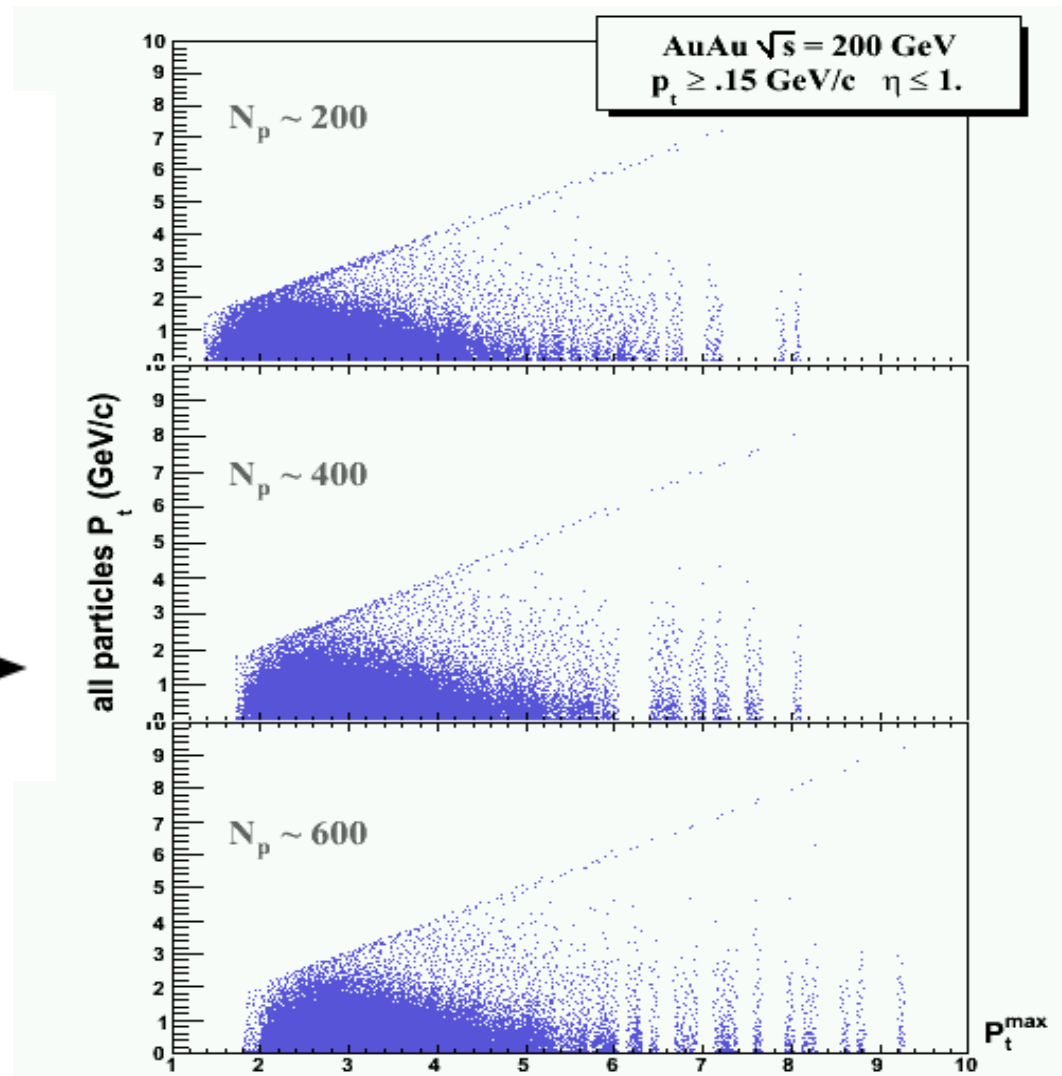
Fractal dimensions of events in rapidity-transverse momentum space

Event fractal dimensions

R.O. ICHEP 2006

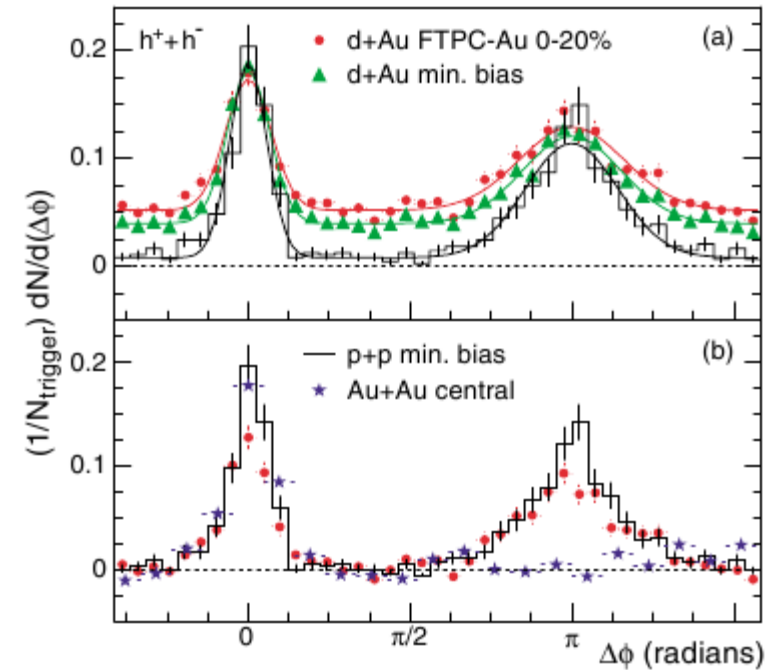
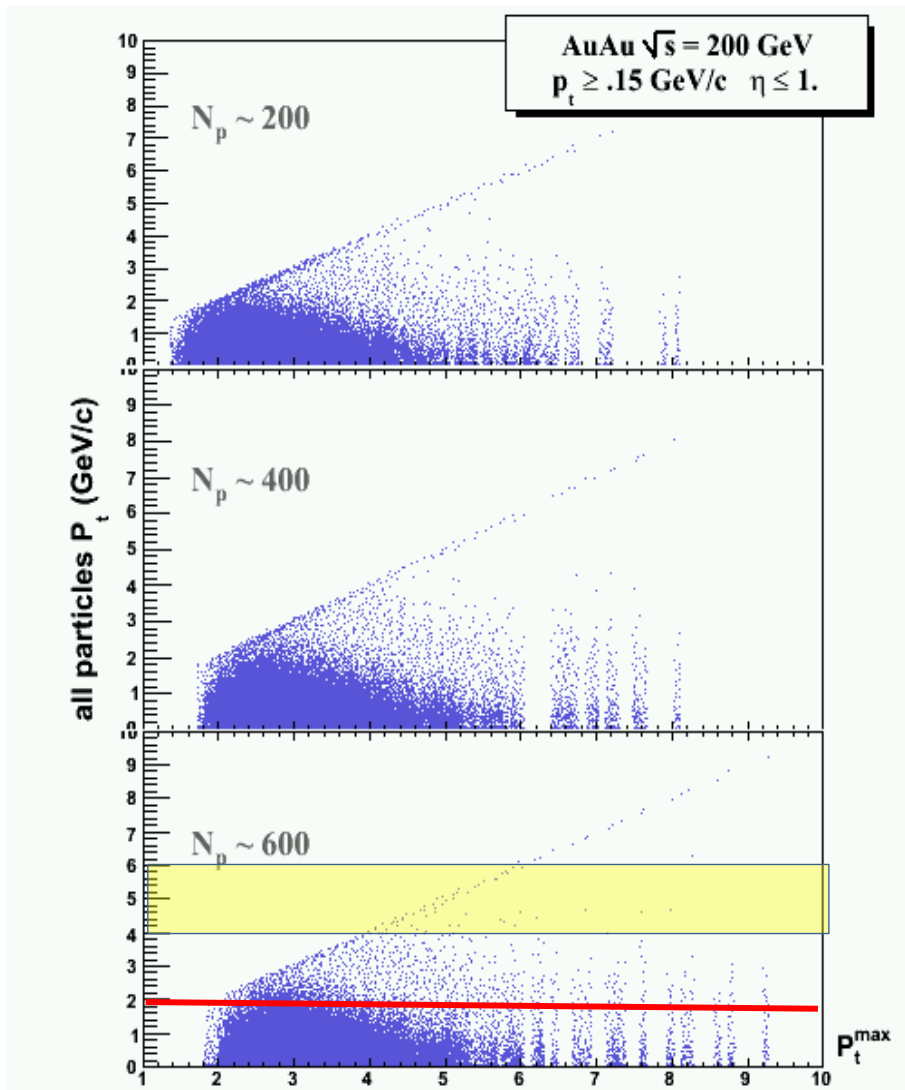


p_T^{\max} – maximum transverse momentum of a particle in the event



Disappearance of away side jet

Phys. Rev. Lett. 91, 072304 (2003).

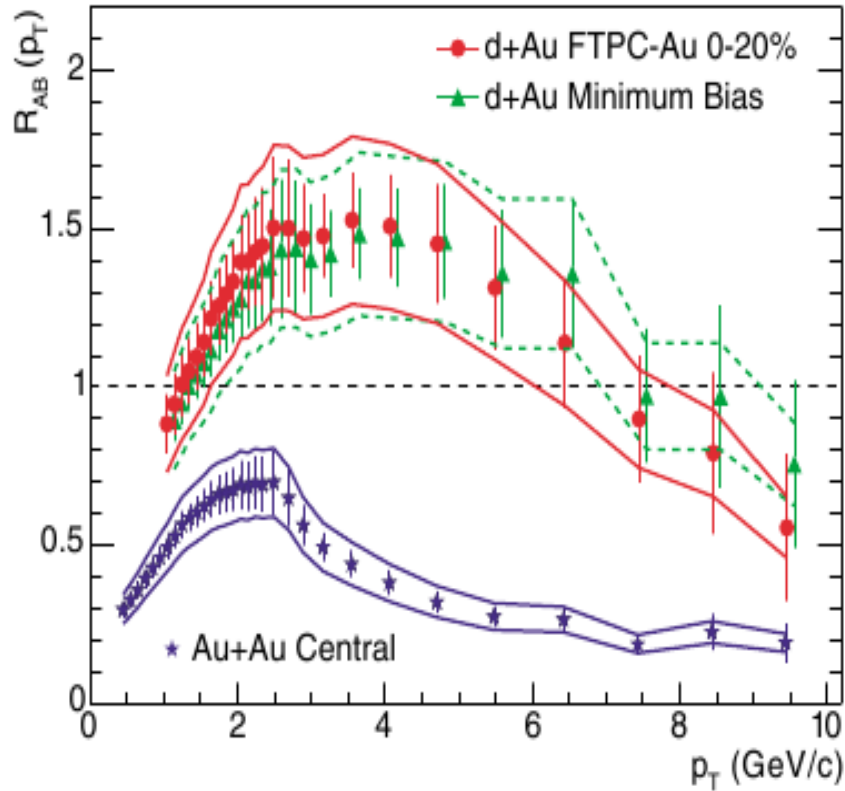


$$D(\Delta\phi) \equiv \frac{1}{N_{\text{trigger}}} \frac{1}{\epsilon} \frac{dN}{d(\Delta\phi)}$$

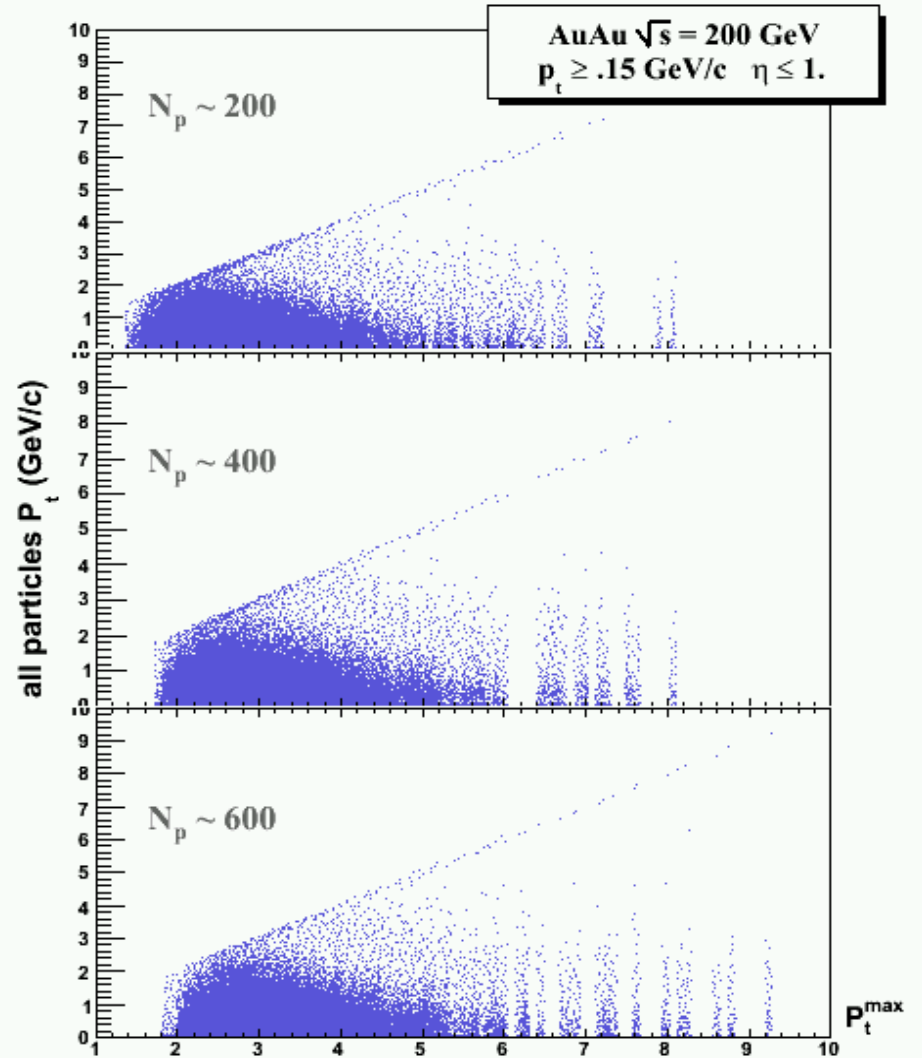
. Only particles within $|\eta| < 0.7$ are included in the analysis. N_{triggers} is the number of particles within $4 < p_T(\text{trig}) < 6$ GeV/c, referred to as trigger particles. The distribution results from the correlation of each trigger particle with all associated particles in the same event having $2 < p_T < p_T(\text{trig})$, where ϵ is the tracking efficiency of the associated particles.

High p_T suppression

Phys. Rev. Lett. 91, 072304 (2003).

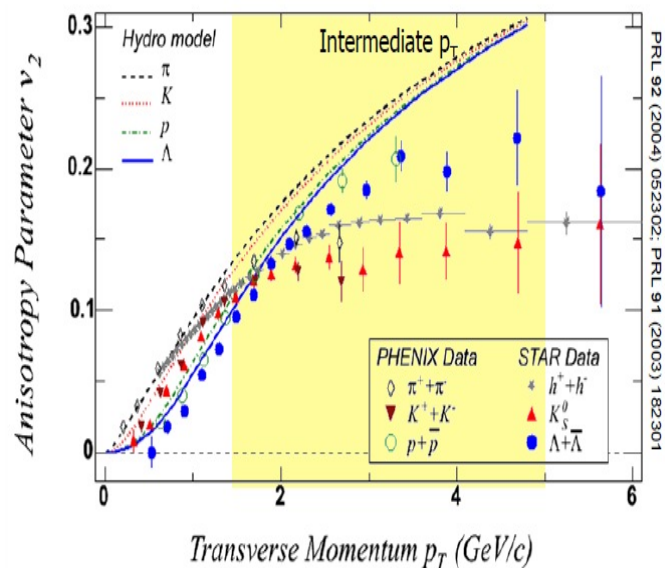


$$R_{AB}(p_T) = \frac{d^2N/dp_T d\eta}{T_{AB} d^2\sigma^{pp}/dp_T d\eta}$$

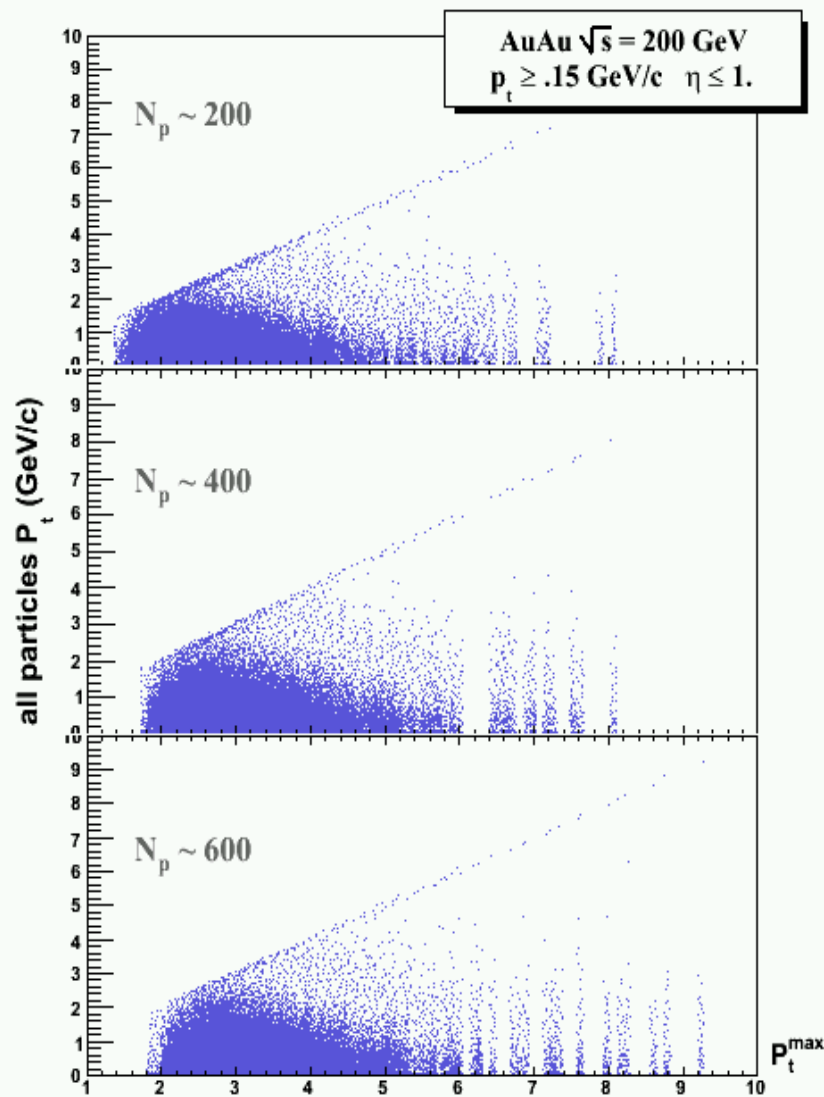
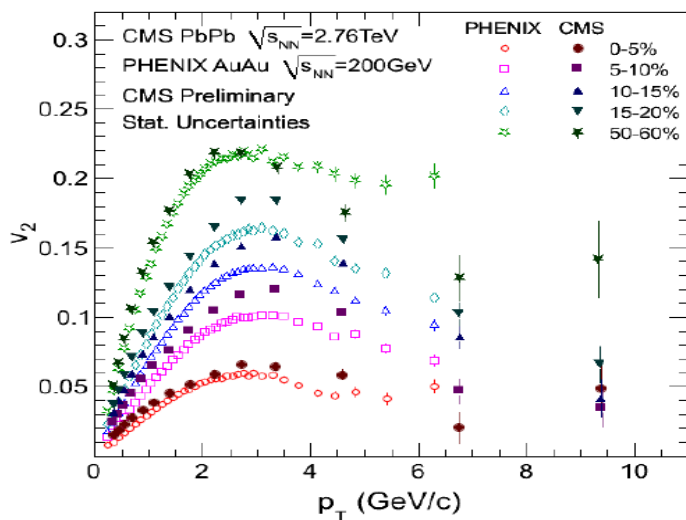


Azimutal anisotropy

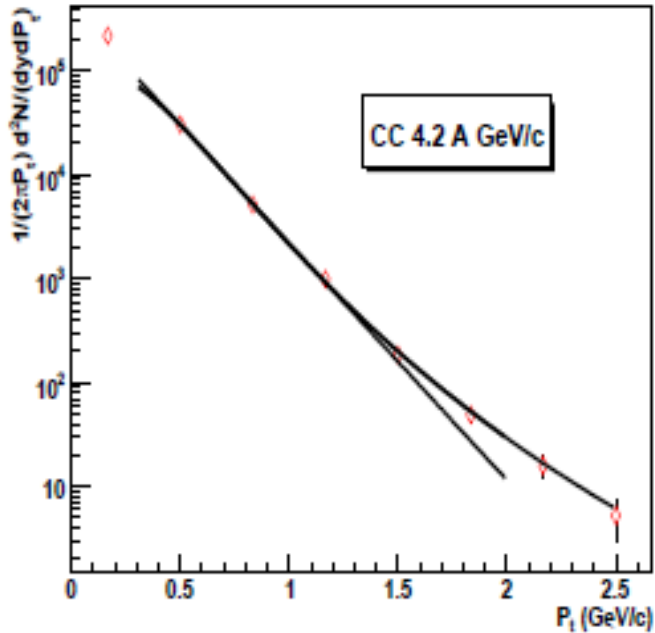
RHIC



LHC

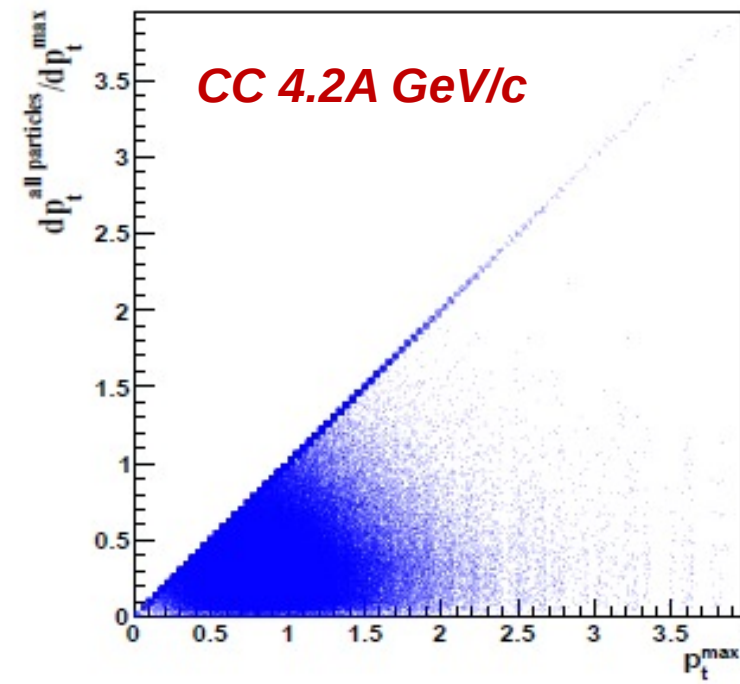


pC dC aC CC @ 4.2A GeV/c



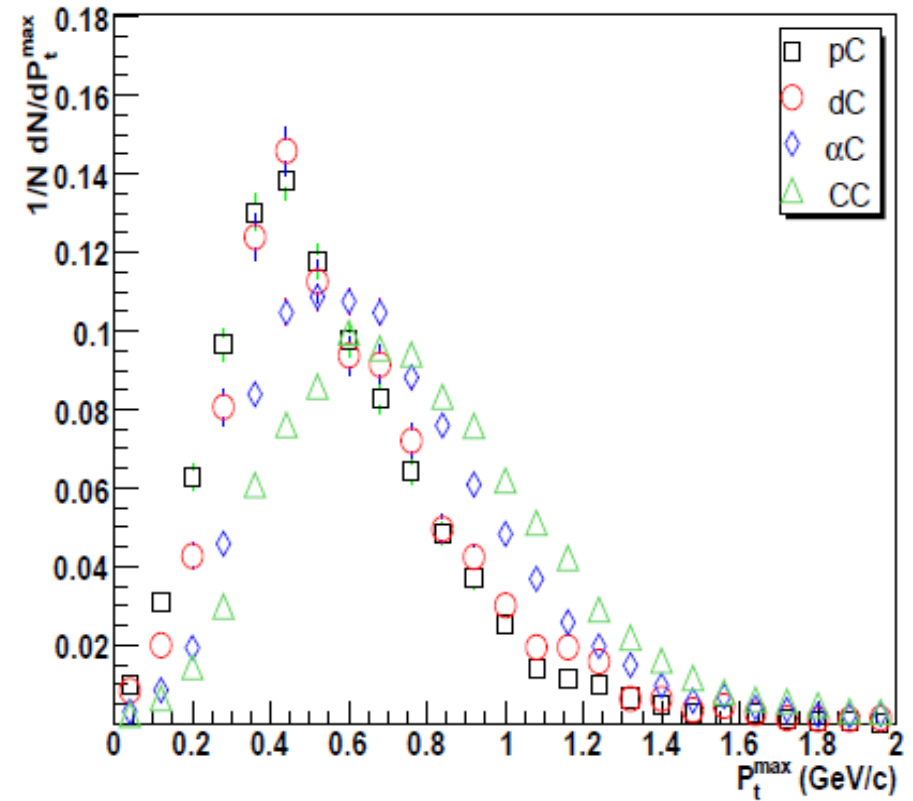
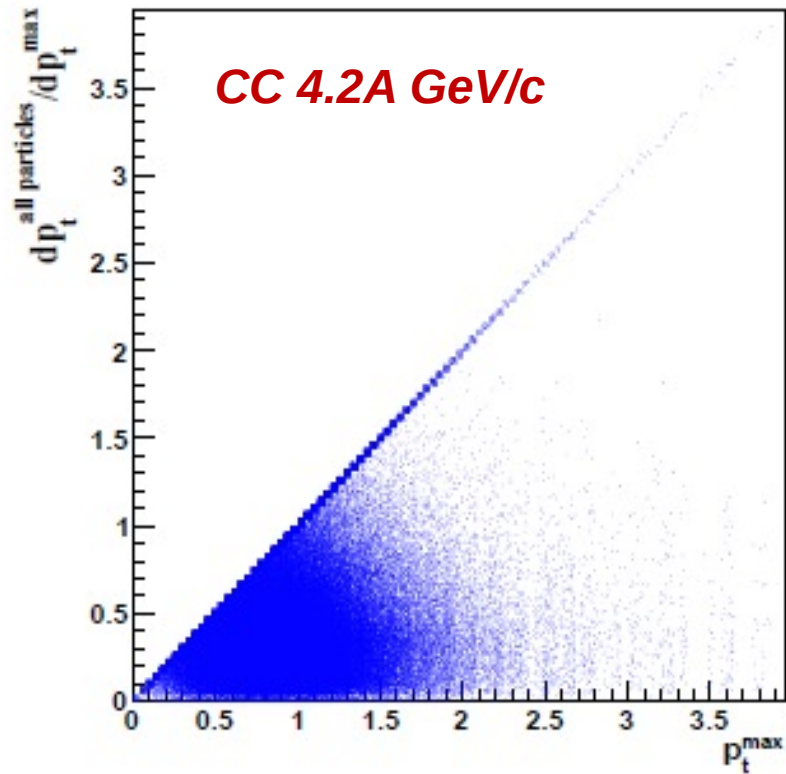
Levy distribution fit

Interaction	Number of events
<i>pC</i>	5722
<i>dC</i>	3826
<i>αC</i>	9643
<i>CC</i>	15842



	parameter value			
	n	T	B	χ^2/ndf
<i>pC</i>	7.45 ± 1.22	$6.86e-02 \pm 6.04e-03$	$2.29e+04 \pm 2.40e+03$	0.19
<i>dC</i>	6.68 ± 1.10	$6.63e-02 \pm 6.78e-03$	$1.85e+04 \pm 2.31e+03$	0.19
<i>αC</i>	6.59 ± 0.57	$7.08e-02 \pm 3.66e-03$	$6.10e+04 \pm 3.70e+03$	1.20
<i>CC</i>	6.84 ± 0.39	$7.69e-02 \pm 2.39e-03$	$1.30e+05 \pm 4.55e+03$	3.20

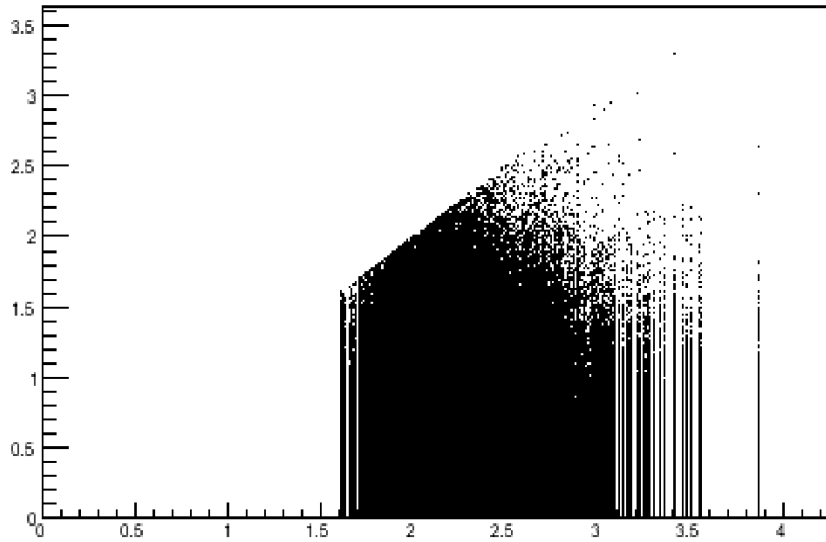
Transition region @4.2 GeV/c



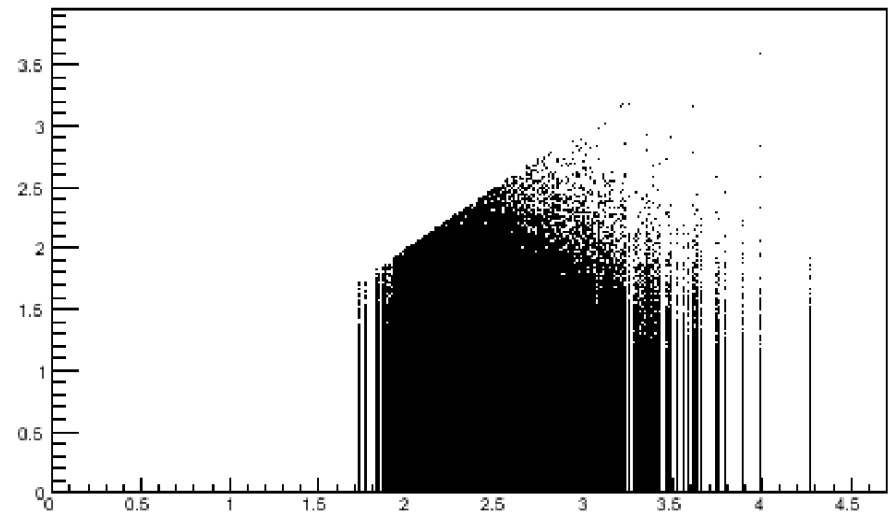
- Shifts with A
- Broadening with A

UrQMD AuAu & NICA energies

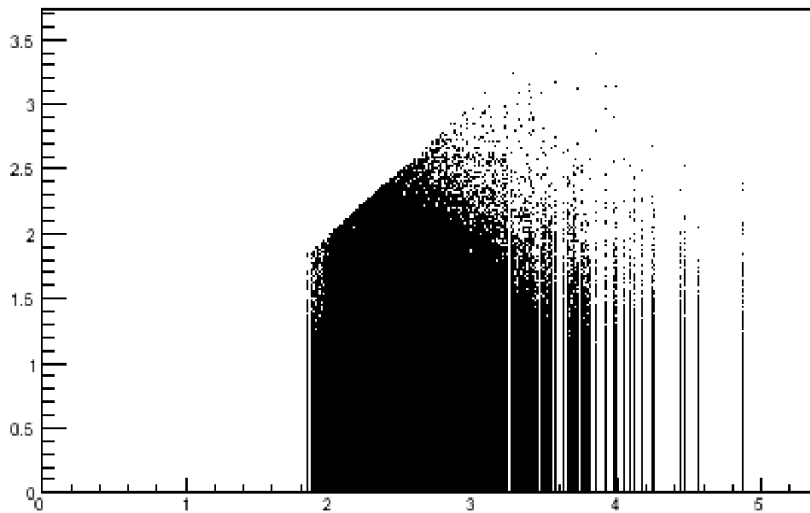
All Pt 4GeV



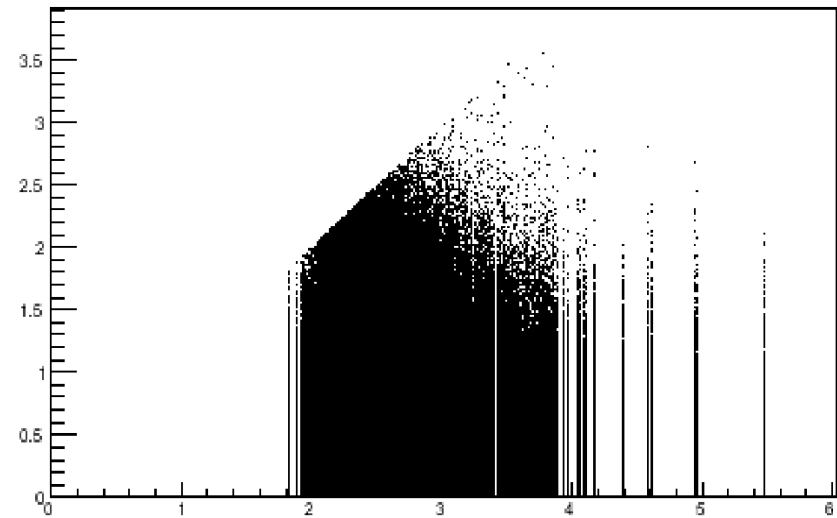
All Pt 7GeV



All Pt 9GeV



All Pt 11GeV

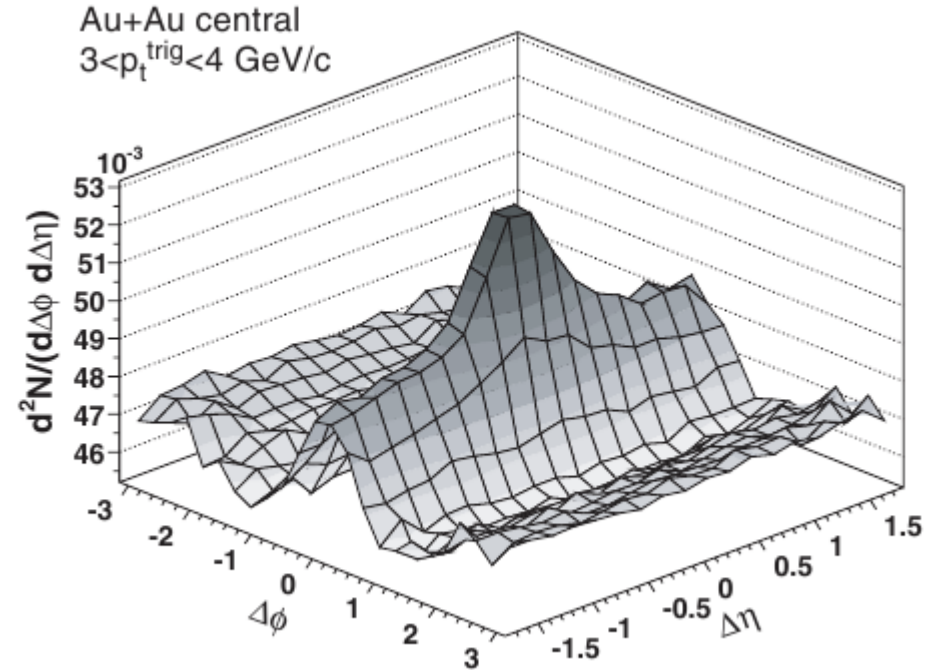
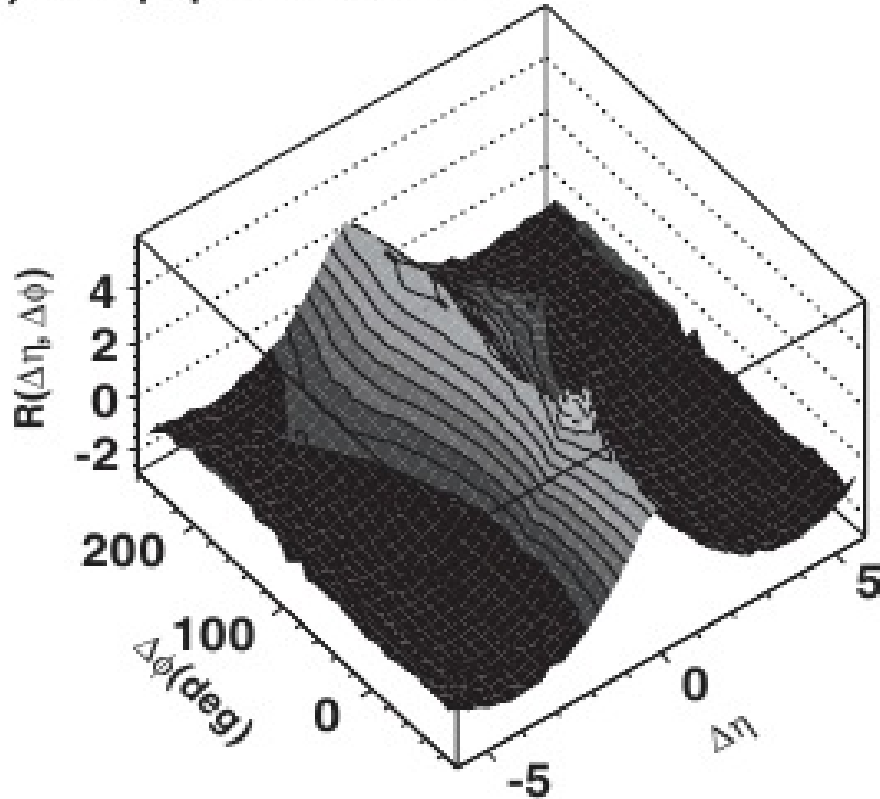


Ridge @ RHIC

PHYSICAL REVIEW C 75, 054913 (2007)

PHYSICAL REVIEW C 80, 064912 (2009)

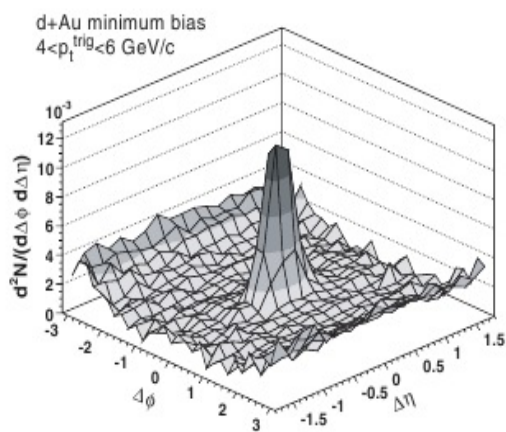
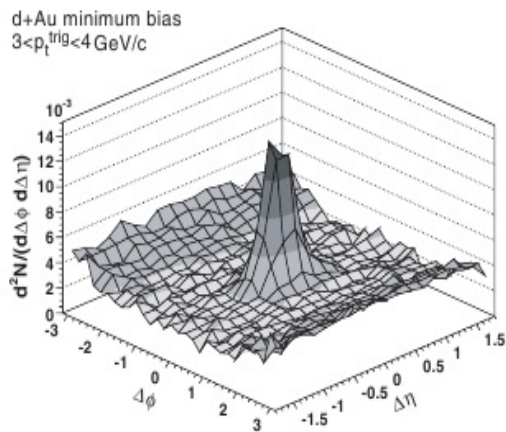
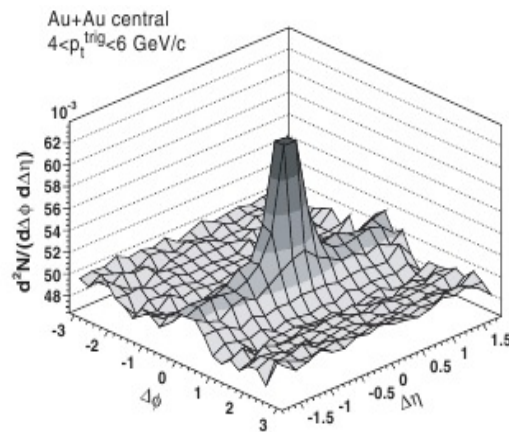
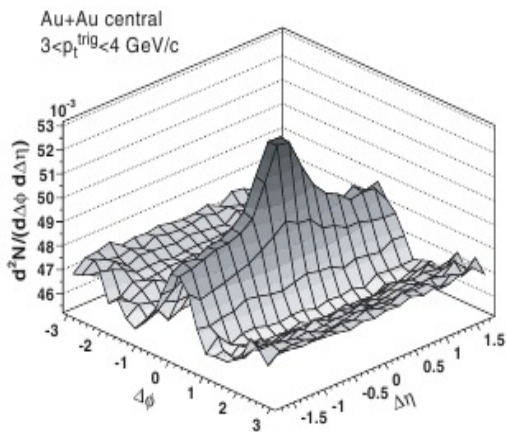
(a) final p+p data 200 GeV



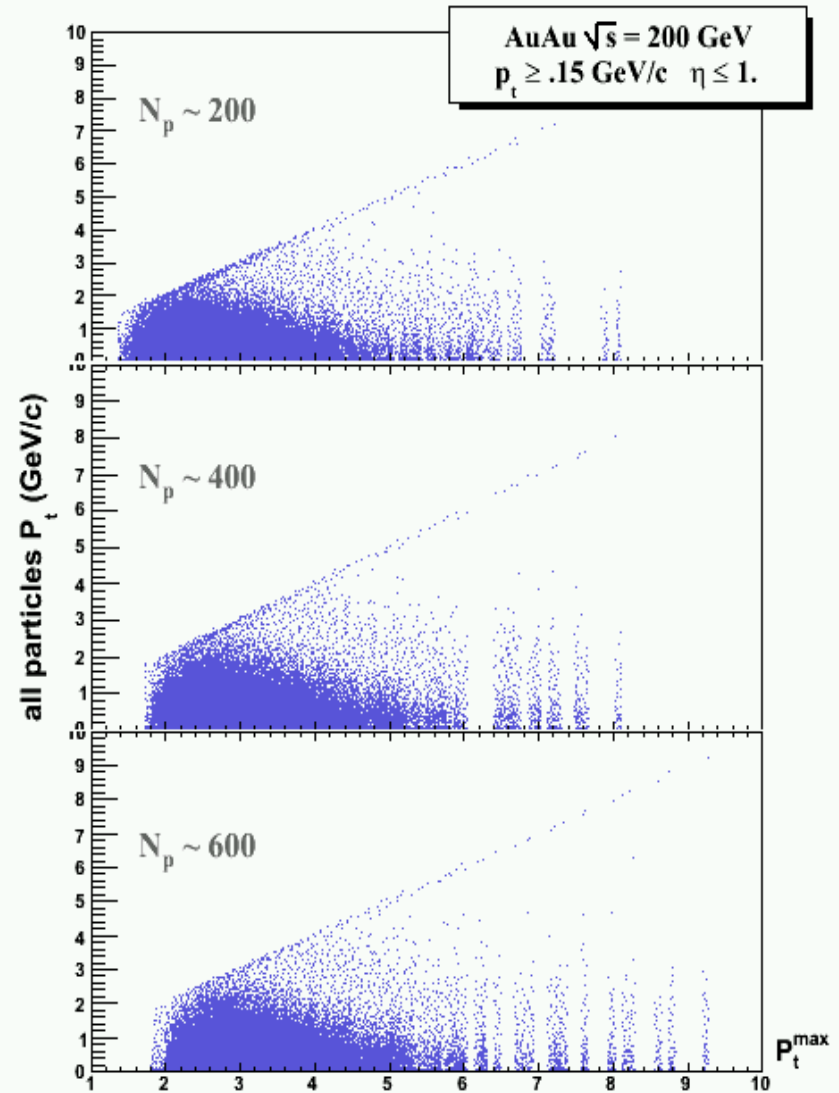
They cover an acceptance of $3 < |\eta| < 4.5$ and $-180^\circ < \phi < 180^\circ$. About 5×10^5 200-GeV and 8×10^5 410-GeV p+p events were selected for further analysis by requiring that the main collision vertex fell within $|z_{vtx}| < 10$ cm along the beam axis.

$$2 \text{ GeV}/c < p_T^{\text{assoc}} < p_T^{\text{trig}}$$

Ridge dAu vs AuAu



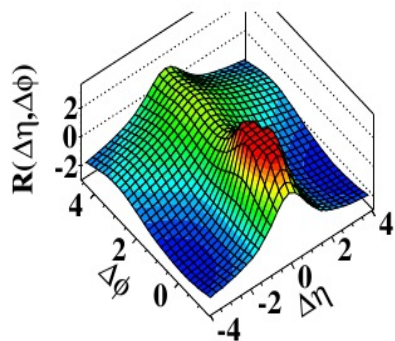
$$2 \text{ GeV}/c < p_T^{\text{assoc}} < p_T^{\text{trig}}$$



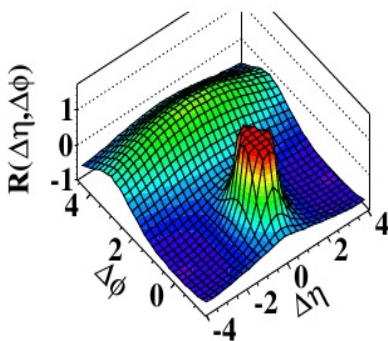
Ridge @ LHC

pp @ 7 TeV

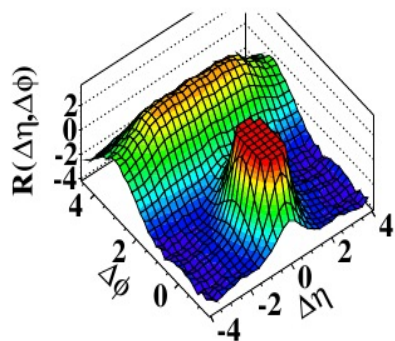
(a) CMS MinBias, $p_T > 0.1 \text{ GeV}/c$



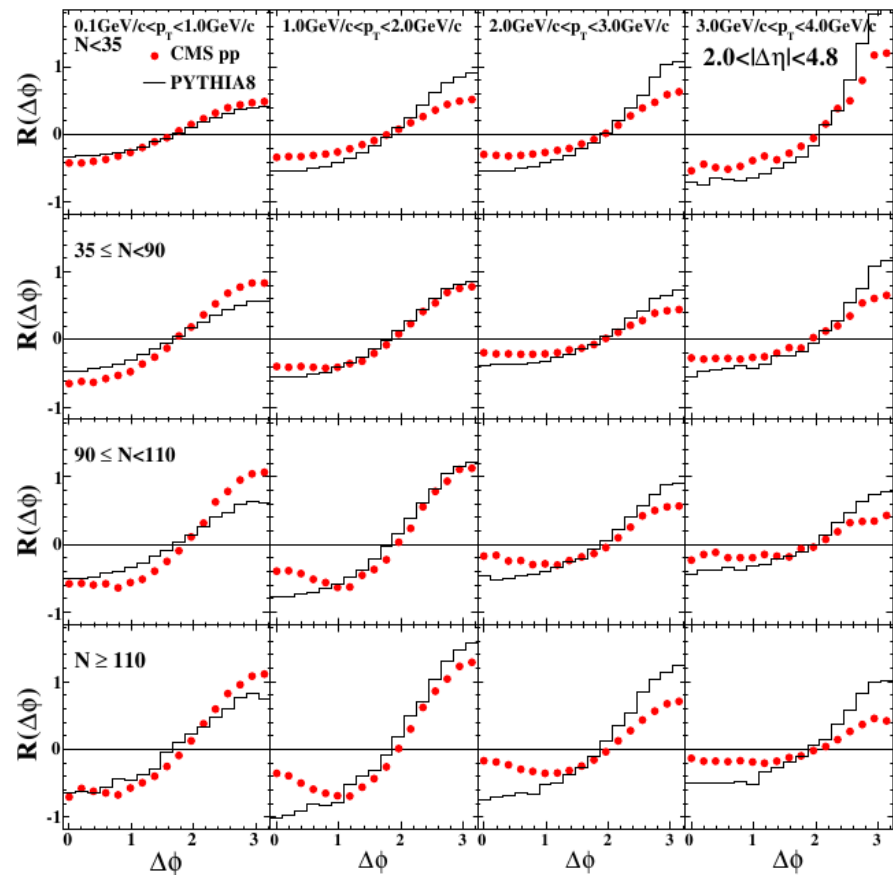
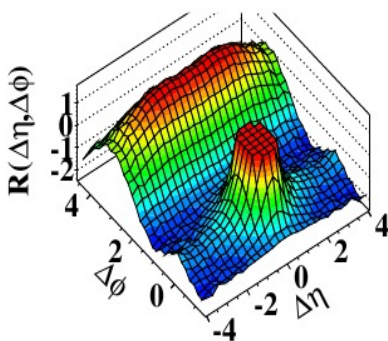
(b) CMS MinBias, $1.0 \text{ GeV}/c < p_T < 3.0 \text{ GeV}/c$



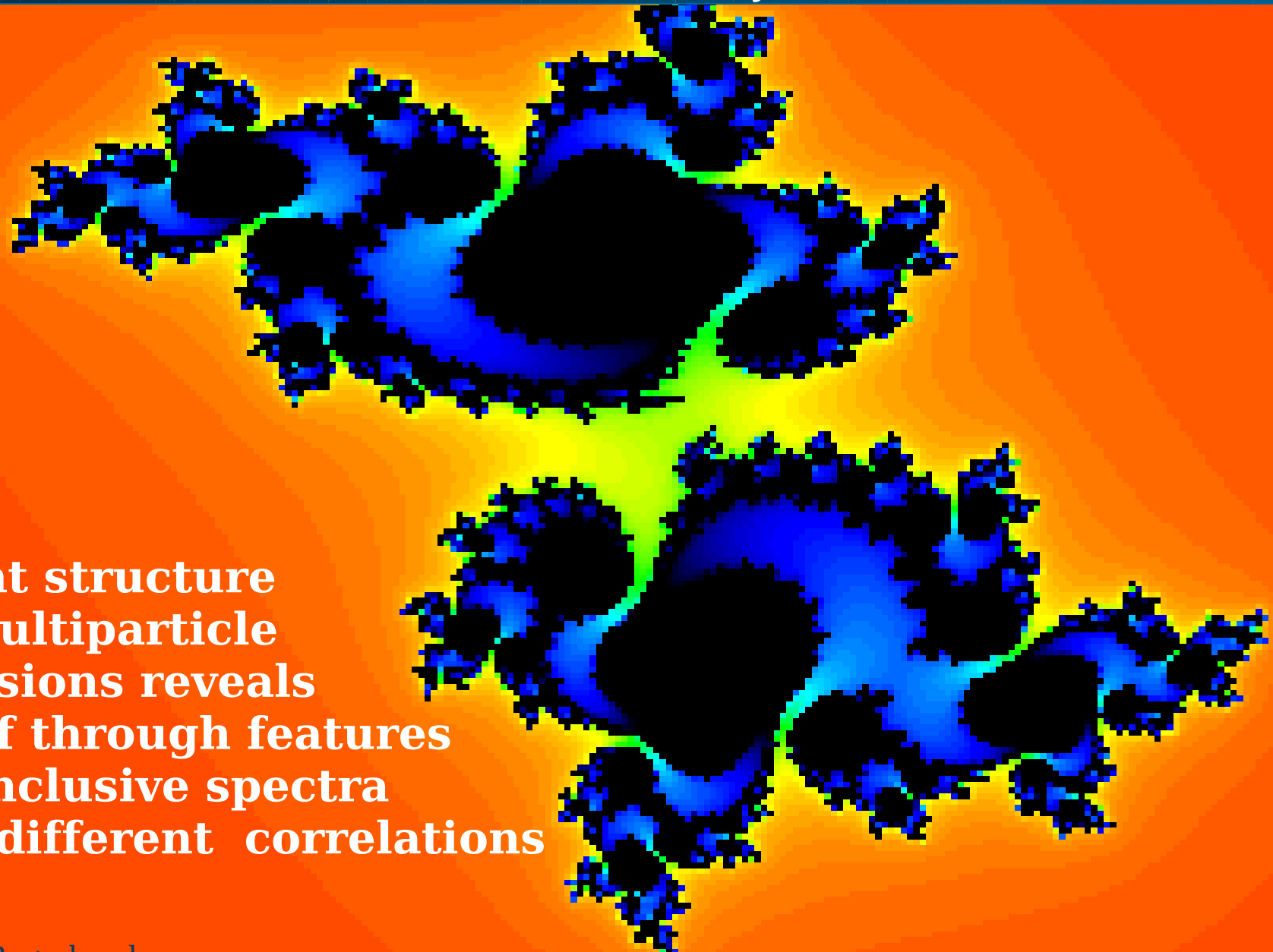
(c) CMS $N \geq 110$, $p_T > 0.1 \text{ GeV}/c$



(d) CMS $N \geq 110$, $1.0 \text{ GeV}/c < p_T < 3.0 \text{ GeV}/c$



Summary

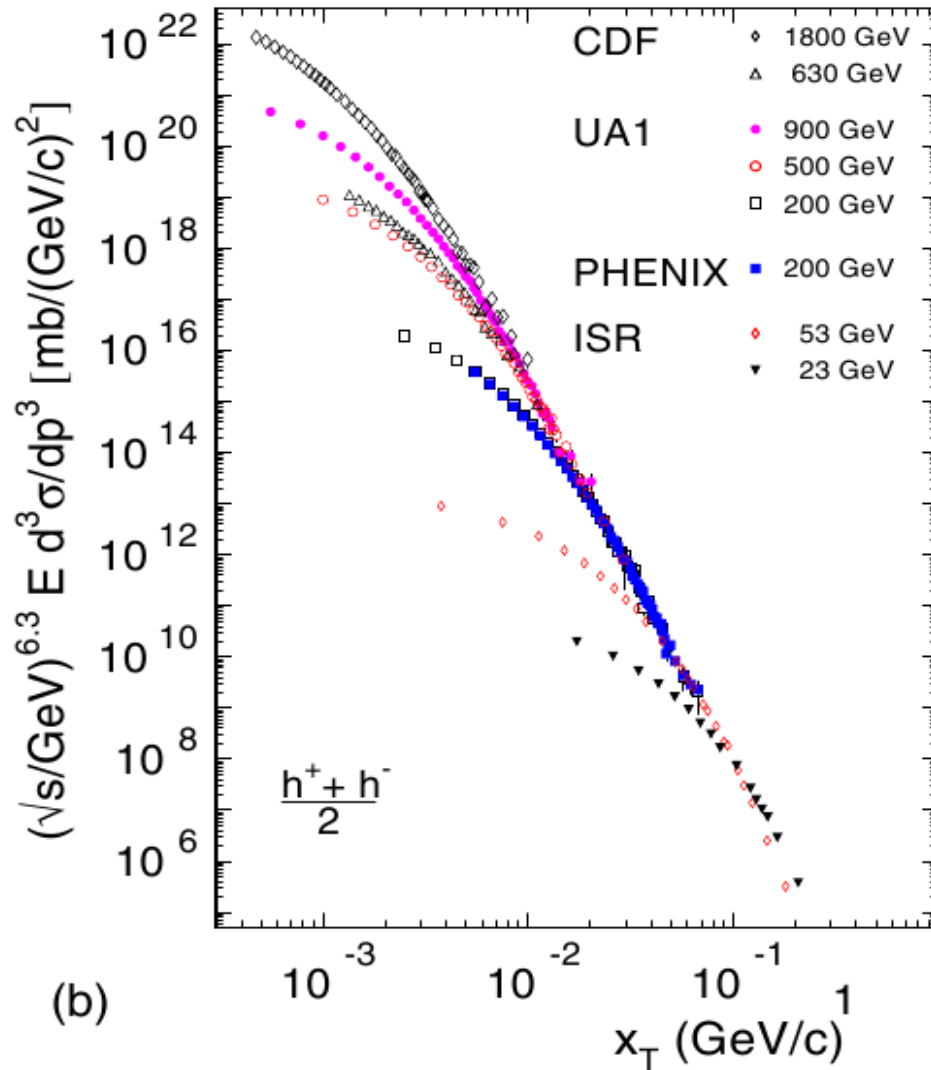


**Event structure
of multiparticle
collisions reveals
itself through features
in inclusive spectra
and different correlations**

**Thanks for your
attention**

X_T scaling

Adcox K et al PHENIX Collaboration 2005 Nucl. Phys. A 757 184–283



$$\sqrt{s}(\text{GeV})^{6.3} \times E d^3\sigma/d^3p \text{ vs. } x_T = 2p_T/\sqrt{s}$$

Z scaling

First LHC data on charged hadron production

pp collisions at low p_T

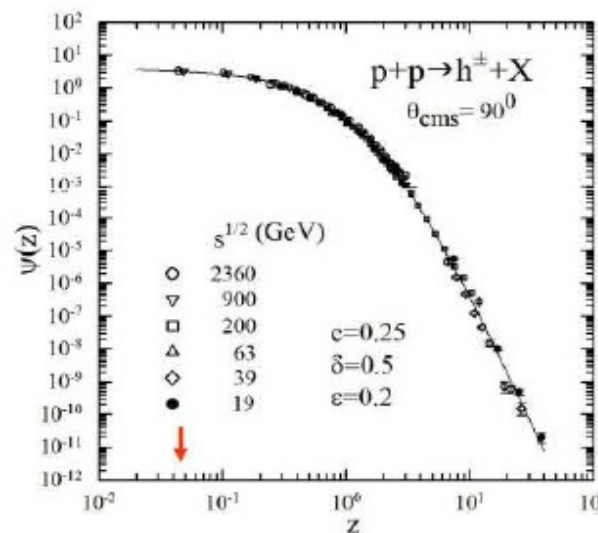
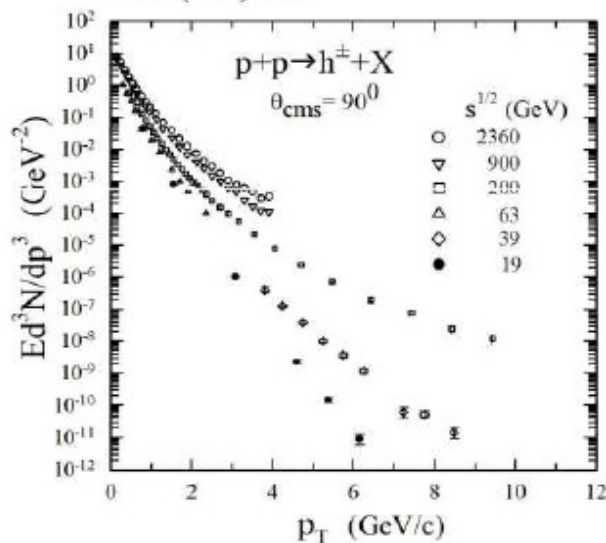
MT & I.Zborovsky
J. Phys.G: Nucl.Part.Phys.
37,085008(2010)

FNAL (fixed target)
PRD 19 (1979) 764
PRD 40 (1989) 2777

ISR: BS Coll.
Nucl.Phys. B 100 (2007) 237

RHIC: STAR Coll.
PRL 91 (2003) 172302

LHC: CMS Coll.
JHEP02(2010) 041



- Energy independence at low p_T
- CMS data confirm onset of saturation for $z < 0.1$



M.Tokarev



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