Chiral Condensate in a Hadron Resonance Gas & A Model for Chemical Freeze-Out [1]

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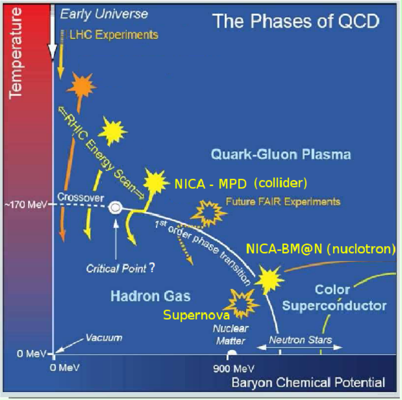
Bilateral FAIR-NICA/JINR Workshop @ FIAS

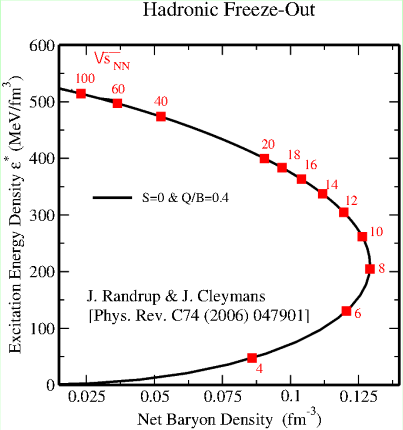
Frankfurt, April 03, 2012

[1] Collaboration: J. Berdermann, J. Cleymans, D. Prorok, K. Redlich, L.Turko ...

QCD Phase Diagram & Heavy-Ion Collisions



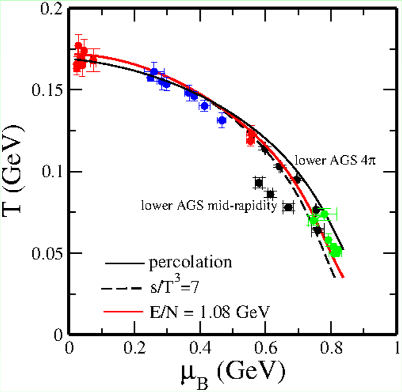
Beam energy scan (BES) programs in the QCD phase diagram



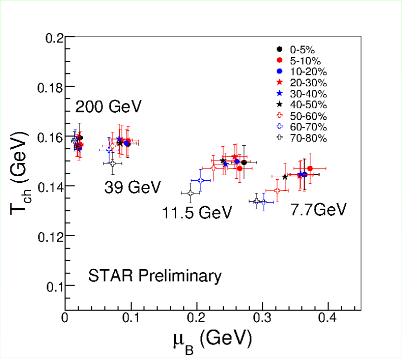
Energy density vs. baryon density at freeze-out for different $\sqrt{s_{NN}}$(GeV)

Highest baryon densities at freeze-out shall be reached for $\sqrt{s_{NN}} \sim 8$ GeV $\longrightarrow$ QGP phase transition ?

Chemical Freeze-out in the QCD Phase Diagram



"Old" freeze-out data from RHIC (red), SPS (blue), AGS (black) and SIS (green).



"New" freeze-out data from STAR BES @ RHIC. Centrality dependence!

F. Becattini, J. Manninen, M. Gazdzicki, Phys. Rev. C73 (2006) 044905
Lokesh Kumar (STAR Collab.), arxiv:1201.4203 [nucl-ex]

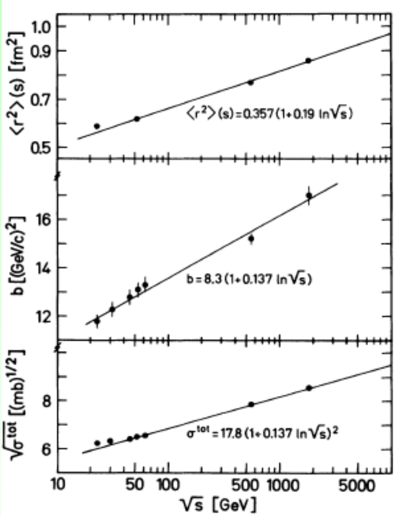
Chemical freeze-out condition

$$\tau_{\rm exp}(T,\mu) = \tau_{\rm coll}(T,\mu)$$

$$\tau_{\rm coll}^{-1}(T,\mu) = \sum_{i,j} \sigma_{ij} n_j$$

$$\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle$$

D.B. et al., Few Body Systems (2011) arxiv:1109.5391 [hep-ph]



B. Povh, J. Hüfner, PRD 46 (1992) 990

Hadronic radii and chiral condensate

$$r_\pi^2(T,\mu) = \frac{3}{4\pi^2} F_\pi^{-2}(T,\mu) \ .$$

$$F_\pi^2(T,\mu) = -m_0 \langle \bar{q}q \rangle_{T,\mu} / m_\pi^2 .$$

$$r_\pi^2(T,\mu) = \frac{3 m_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_{T,\mu}|^{-1} \ .$$

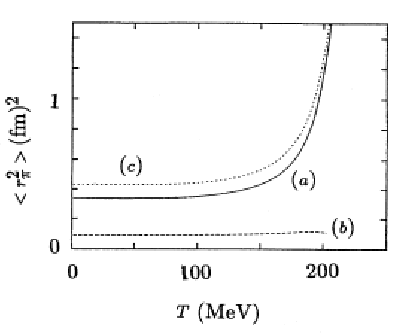
$$r_N^2(T,\mu) = r_0^2 + r_\pi^2(T,\mu) \ ,$$

Expansion time from entropy conservation

$S = s(T,\mu)\ V(\tau_{\rm exp}) = {\rm const}$

$$\tau_{\rm exp}(T,\mu) = a\ s^{-1/3}(T,\mu) \ ,$$

D.B. et al., Few Body Systems (2011) arxiv:1109.5391



H.-J. Hippe and S. Klevansky, PRC 52 (1995) 2172

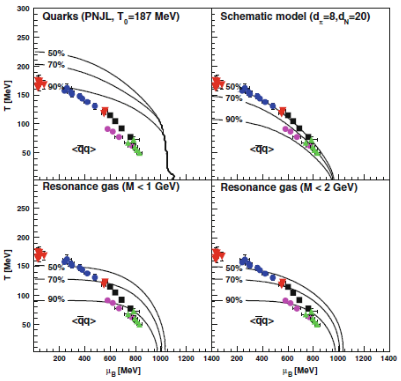
Chiral Condensate in a Hadron Resonance Gas

$$
\begin{aligned}
\frac{\langle\bar{q}q\rangle}{\langle\bar{q}q\rangle_{\rm vac}} &= 1-\frac{m_0}{F_\pi^2 m_\pi^2}\Bigg\{4N_c\int\frac{dp\,p^2}{2\pi^2}\frac{m}{\varepsilon_p}\left[f_\Phi^+ + f_\Phi^-\right] \\
&+ \sum_{M=f_0,\omega,\dots} d_M(2-N_s)\int\frac{dp\,p^2}{2\pi^2}\frac{m_M}{E_M(p)}f_M(E_M(p)) \\
&+ \sum_{B=N,\Lambda,\dots} d_B(3-N_s)\int\frac{dp\,p^2}{2\pi^2}\frac{m_B}{E_B(p)}\left[f_B^+(E_B(p))+f_B^-(E_B(p))\right]\Bigg\} \\
&- \sum_{G=\pi,K,\eta,\eta'}\frac{d_G r_G}{4\pi^2F_G^2}\int dp\,\frac{p^2}{E_G(p)}f_G(E_G(p))
\end{aligned}
$$

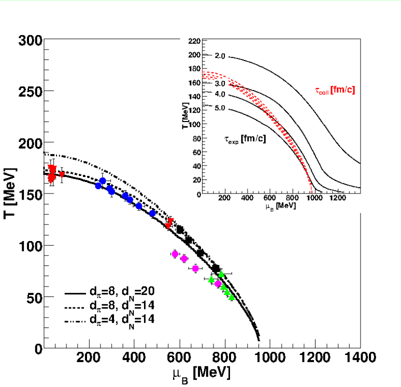
S. Leupold, J. Phys. G (2006)

D.B., J. Berdermann, J. Cleymans, K. Redlich, Few Body Systems (2011)

Chemical Freeze-out and Chiral Condensate



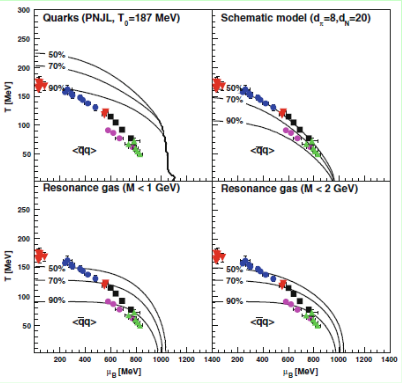
Chemical freeze-out vs. Condensate



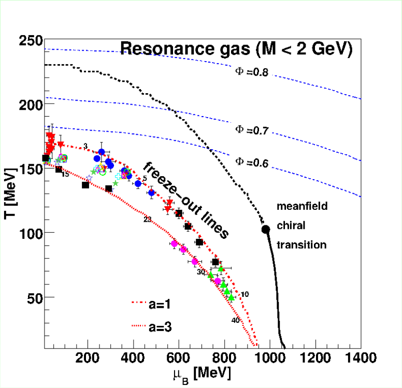
Chemical freeze-out from kinetic condition, schematic model

D.B., J. Berdermann, J. Cleymans, K. Redlich, Few Body Systems (2011)

Chemical Freeze-out and Chiral Condensate



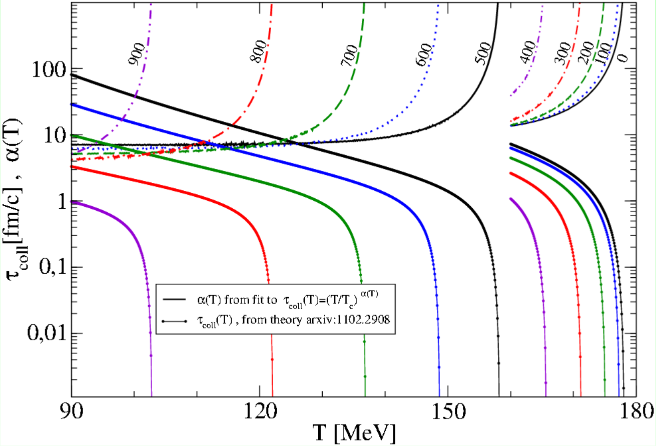
Chemical freeze-out vs. Condensate



Chemical freeze-out from kinetic condition, $a \sim$ inverse system size

D.B., J. Berdermann, J. Cleymans, K. Redlich, in preparation (2012)

Strong T-Dependence of (inelastic) Collision Time



Compare talk C. Blume and

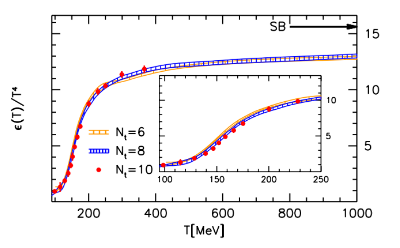
C. Wetterich, P. Braun-Munzinger, J. Stachel, PLB

D.B., J. Berdermann, J. Cleymans, K. Redlich, in preparation (2012)

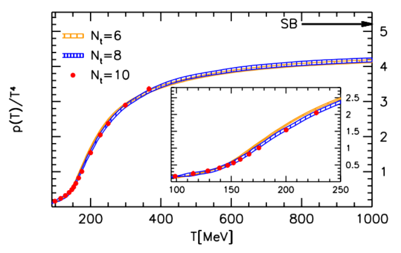
Conclusions

- The model works super!
- Improvements are plenty:
  - Hadron mass formulae from holografic QCD
  - Spectral functions - generalized Beth-Uhlenbeck
  - Thermodynamics ... hydrodynamics .
- Thanks for your attention

Theoretical laboratory of QCD



The energy density normalized by T^4
as a function of the temperature
on N_t =6,8 and 10 lattices.



The pressure normalized by T^4
as a function of the temperature
on N_t =6,8 and 10 lattices.

S. Borsanyi *et al.* *"The QCD equation of state with dynamical quarks,"* JHEP **1011**, 077 (2010)

Hagedorn resonance gas: hadrons with finite widths

The energy density per degree of freedom with the mass M

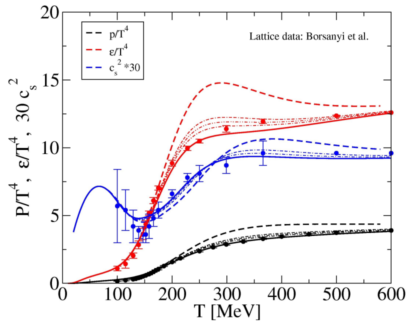
$$\varepsilon(T,\mu_B,\mu_S) = \sum_{i:\ m_i<m_0} g_i\ \varepsilon_i(T,\mu_i;m_i) + \sum_{i:\ m_i\geq m_0} g_i \int_{m_0^2}^{\infty} d(M^2)\ A(M,m_i)\ \varepsilon_i(T,\mu_i;M),$$

Spectral function

$$A(M,m) = N_M \frac{\Gamma\cdot m}{(M^2-m^2)^2+\Gamma^2\cdot m^2}\ ,$$

$$\Gamma(T) = C_\Gamma \left(\frac{m}{T_H}\right)^{N_m} \left(\frac{T}{T_H}\right)^{N_T} \exp\left(\frac{m}{T_H}\right)$$

Hagedorn resonance gas: hadrons with finite widths



$$P(T) = T \int_0^T dT' \, \frac{\varepsilon(T')}{T'^2} \, .$$

N_m in the range from $N_m = 2.5$ (dashed line) to $N_m = 3.0$ (solid line).

$C_\Gamma = 10^{-4}$

$N_T = 6.5$

$T_H = 165$ MeV

$$\Gamma(T) = C_\Gamma \left(\frac{m}{T_H}\right)^{N_m} \left(\frac{T}{T_H}\right)^{N_T} \exp\left(\frac{m}{T_H}\right)$$

D. Blaschke & K.A. Bugaev, Fizika B **13**, 491 (2004); PPNP **53**, 197 (2004)

Mott-Hagedorn resonance gas

State-dependent hadron resonance width

$$A_i(M, m_i) = N_M \frac{\Gamma_i \cdot m_i}{(M^2 - m_i^2)^2 + \Gamma_i^2 \cdot m_i^2} \ ,$$

$$\Gamma_i(T) = \tau^{-1}_{\text{coll,i}}(T) = \sum_j \lambda \langle r_i^2 \rangle_T \langle r_j^2 \rangle_T \ n_j(T)$$

D. B. Blaschke, J. Berdermann, J. Cleymans, K. Redlich: [arXiv:1102.2908]

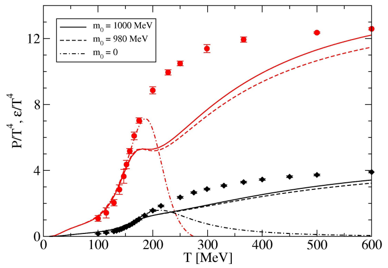
For pions (mesons)

$$r_\pi^2(T, \mu) = \frac{3M_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_T|^{-1} \ ; \qquad \langle \bar{q}q \rangle_T = 304.8 \, [1 - \tanh{(0.002 \, T - 1)}]$$

For nucleons (baryons)

$$r_N^2(T, \mu) = r_0^2 + r_\pi^2(T, \mu); \qquad r_0 = 0.45\text{fm} \ \text{ pion cloud.}$$

Mott-Hagedorn resonance gas



Quarks and gluons are missing!

Mott-Hagedorn resonance gas: Pressure and energy density for three values of the mass threshold

$m_0 = 1.0$ GeV (solid lines)

$m_0 = 0.98$ GeV (dashed lines)

and

$m_0 = 0$ (dash-dotted lines)

Quarks and gluons in the PNJL model

Systematic expansion of the pressure as the thermodynamical potential in the grand canonical ensemble for a chiral quark model of the PNJL type beyond its mean field description $P_{\rm PNJL,MF}(T)$ by including perturbative corrections

$$P(T) = P^*_{\rm HRG}(T) + P_{\rm PNJL,MF}(T) + P_2(T) \ ,$$

$$P^*_{\rm HRG}(T) = \frac{P_{\rm HRG}(T)}{1 + (P_{\rm HRG}(T)/(aT^4))^\alpha} \ ,$$

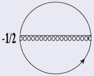
with $a = 2.7$ and $\alpha = 1.8$.

Quark and gluon contributions

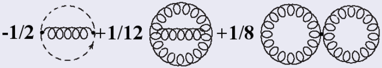
$$P_2(T) = P_2^{\rm quark}(T) + P_2^{\rm gluon}(T)$$

Quark and gluon contributions

$P_2^{\rm quark}(T)$



$P_2^{\rm gluon}(T)$



Total perturbative QCD correction

$$P_2 = -\frac{8}{\pi}\alpha_s T^4 (I_\Lambda^+ +$$

$$\frac{3}{\pi^2}((I_\Lambda^+)^2 + (I_\Lambda^-)^2))$$

$$\underset{\Lambda/T\to 0}{\longrightarrow} -\frac{3\pi}{2}\alpha_s T^4$$

where

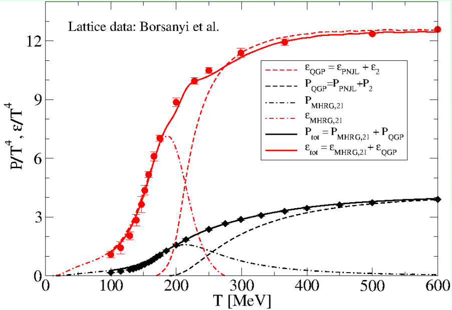
$$I_\Lambda^\pm = \int_{\Lambda/T}^\infty \frac{\mathrm{d}x\ x}{\mathrm{e}^x \pm 1}$$

- Energy corrections

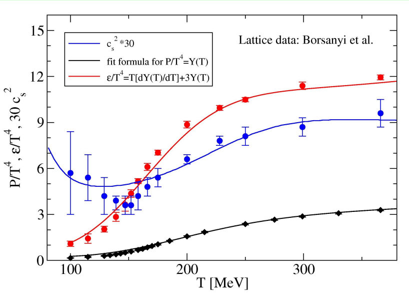
$$\varepsilon_2(T) = T\frac{dP_2(T)}{dT} - P_2(T)\ .$$

Quarks, gluons and hadron resonances

$$P_{\mathrm{MHRG}}(T) = \sum_i \delta_i d_i \int \frac{d^3p}{(2\pi)^3} \int dM\, A_i(M, m_i) T \ln\left\{1 + \delta_i \mathrm{e}^{-[\sqrt{p^2+M^2}-\mu_i]/T}\right\} ,$$



- Quark-gluon plasma contributions are described within the improved PNJL model with α_s corrections .
- Heavy hadrons are described within the resonance gas with finite width exhibiting a Mott effect at the coincident chiral and deconfinement transitions.



- Contribution restricted to the region around the chiral/deconfinement transition 170-250 MeV
- Fit formula for the pressure

$$P = aT^4 + bT^{4.4}\tanh(cT - d),$$

$$a = 1.0724,\ b = 0.2254,$$
$$c = 0.00943,\ d = 1.6287$$

Conclusions

- An effective model description of QCD thermodynamics at finite temperatures which properly accounts for the fact that in the QCD transition region it is dominated by a tower of hadronic resonances.
- A generalization of the Hagedorn resonance gas thermodynamics which includes the finite lifetime of hadronic resonances in a hot and dense medium

To do

- Join hadron resonance gas with quark-gluon model.
- Calculate kurtosis and compare with lattice QCD.
- Spectral function for low-lying hadrons from microphysics (PNJL model ...).