Parton Reggeization Approach: Jet and Dijet Jet production at the LHC

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1. **Saleev V.:** *Jet and Dijet Jet production at the LHC*

   B. A. Kniehl, V. A. Saleev, A. V. Shipilova, E. V. Yatsenko. Single jet and prompt-photon inclusive production with multi-Regge kinematics: From Tevatron to LHC. Phys. Rev. D **84**, 074017 (2011);
   M. A. Nefedov, N. N. Nikolaev, V. A. Saleev. Drell-Yan lepton pair production at high energies in the parton Reggeization approach. Phys. Rev. D **87**, 014022 (2013).
   M.A. Nefedov, V.A. Saleev, A. V Shipilova.Dijet azimuthal decorrelations at the LHC in the parton Reggeization approach. Phys. Rev. D**87** (2013) 094030.

2. **Shipilova A.:** *b−jet production at the LHC*

   B. A. Kniehl, A. V. Shipilova and V. A. Saleev. Inclusive b and b anti-b production with quasi-multi-Regge kinematics at the Tevatron. Phys. Rev. D **81**, 094010 (2010);
   V. A. Saleev and A. V. Shipilova. Inclusive b-jet and bb-dijet production at the LHC via Reggeized gluons. Phys. Rev. D **86**, 034032 (2012).

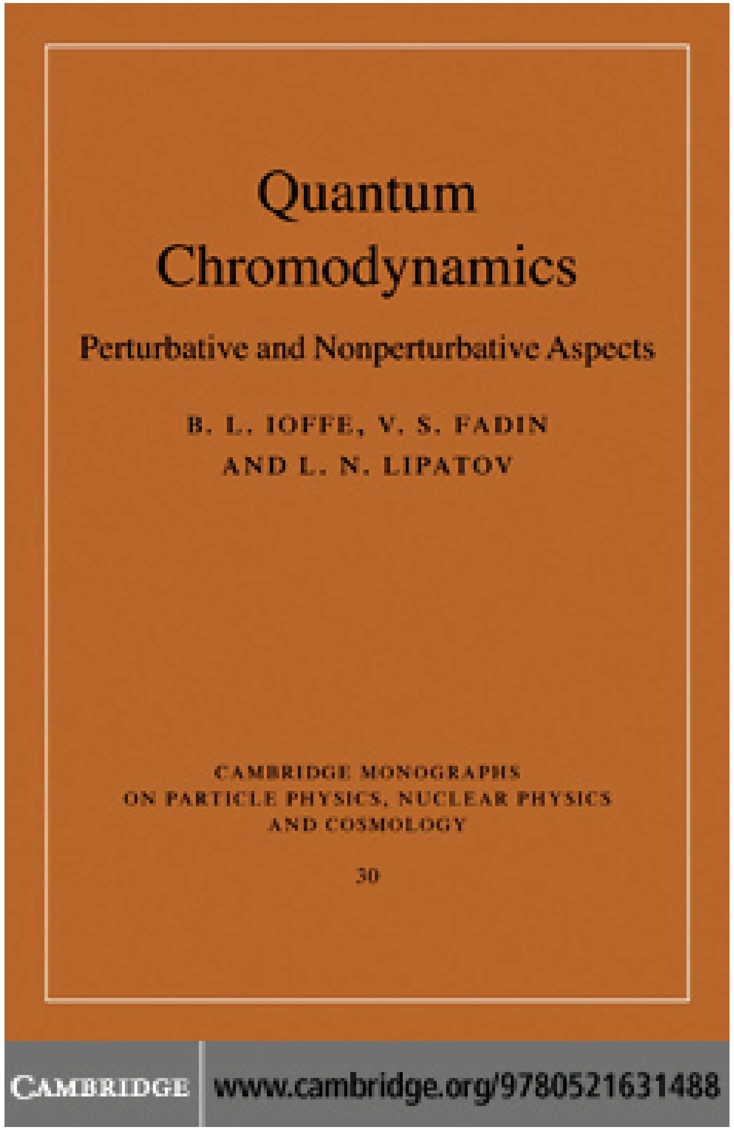
3. **Nefedov M.:** *Heavy quarkonium production at the LHC*

   M.A. Nefedov, V.A. Saleev, A. V. Shipilova.Prompt Upsilon(nS) production at the LHC in the Regge limit of QCD. Phys. Rev. D**87** (2013)
   M.A. Nefedov, V.A. Saleev, A. V. Shipilova.Prompt J/psi production in the Regge limit of QCD: From Tevatron to LHC Phys. Rev. D**85** (2012) 074013.

PLAN

1. Introduction to BFKL or small$-x$ physics
2. Parton Reggeization Approach (PRA)
3. Single jet production at the LHC
4. Dijet production at the LHC
5. Conclusions

Introduction to BFKL or small$-x$ physics



BFKL = **B**alitsky, **F**adin, **K**uraev, **L**ipatov

List of pioneer papers on BFKL approach, it was 40 years ago ...

- Lipatov, L. N., Sov. J. Nucl. Phys. 23 (1976) 338;
- Fadin, V. S., Kuraev, E.A. and Lipatov, L.N., Phys. Lett. B60 (1975) 50;
- Kuraev, E. A., Lipatov, L. N. and Fadin, V. S., Sov. Phys. JETP 44 (1976) 443; 45 (1977) 199;
- Balitsky I. I. and Lipatov L. N., Sov. J. Nucl. Phys. 28 (1978) 822.

MAIN AIM OF OUR TALKS TO SHOW MODERN PHENOMENOLOGICAL APPLICATIONS OF BFKL APPROACH

Hard processes:

$$\sqrt{S}, \sqrt{|t|}, p_T, M \gg \Lambda_{QCD},$$

$\mu_R \sim p_T, M, \sqrt{|t|}, ...$ is the renormalization scale parameter

$$\alpha_s \ll 1$$

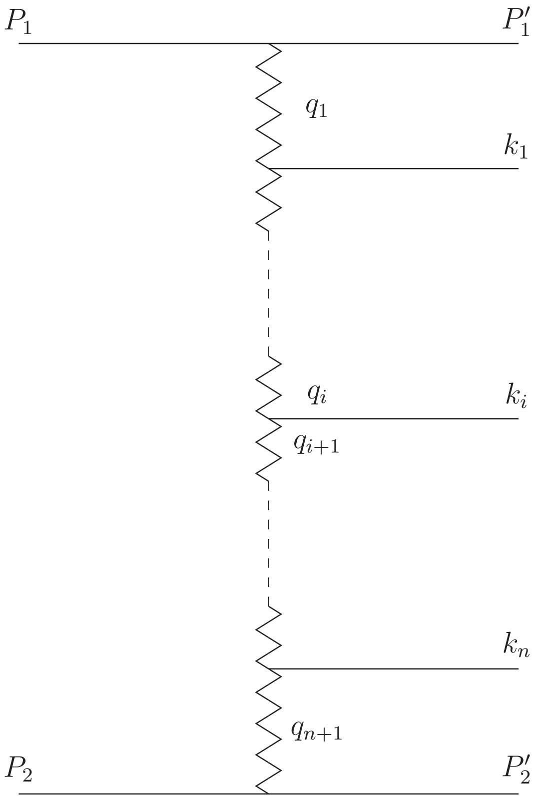
Region of Regge kinematics:

$$\sqrt{S} >> \sqrt{|t|}, p_T, M$$

$\mu_F \sim p_T, M, \sqrt{|t|}, ...$ is the factorization scale parameter, which controls initial state radiation and hard scattering process.

$$x = \frac{\mu}{\sqrt{S}} \ll 1$$

The subprocesses in the Regge kinematics with $t-$channel exchange dominate at the high energy !



$$k_i = \beta_i P_1 + \alpha_i P_2 + k_{iT}$$
$$S = (P_1 + P_2)^2$$
$$S\alpha_i\beta_i = k_i^2 - k_{iT}^2$$
$$1/S \sim \beta_{n+1} \ll \beta_n \ll ... \ll \beta_0 \sim 1$$
$$1/S \sim \alpha_0 \ll \alpha_1 \ll ... \ll \alpha_{n+1} \sim 1$$
$$S_i = (k_{i-1} + k_i)^2 = S\beta_{i-1}\alpha_i$$
$$S_i \gg |k_{iT}^2| \sim |t_i| = |q_i^2|$$

The schematic presentation of the parton ladder in Multi-Regge kinematics

BFKL evolution equation

$$\frac{\partial \Phi(x, \vec{q}_T^{\,2})}{\partial \ln(1/x)} = K_{BFKL} \bigotimes_{\vec{q}_T} \Phi(x, \vec{q}_T^{\,2})$$

$$\ln(1/x) \gg 1$$

DGLAP evolution equation

$$\frac{\partial G(x, Q^2)}{\partial \ln(Q^2/\Lambda_{QCD}^2)} = K_{DGLAP} \bigotimes_{x} G(x, Q^2)$$

$$\ln(Q^2/\Lambda_{QCD}^2) \gg 1$$

At the energy range of Tevatron and LHC Colliders the both large logarithms, $\ln(1/x)$ and $\ln(Q^2/\Lambda_{QCD}^2)$, are important !

There are more complicated evolution equation, which take into account DGLAP and BFKL evolution:

KMR by Kimber, Martin and Ryskin

CCFM by Ciafaloni, Catani, Fiorani and Marchesini

Electron Reggeization in QED:

M. Gell-Mann, M. L. Goldberger, F. E. Low, E. Marx, and F. Zachariasen, **1964**.

Quark Reggeization in QCD:

V. S. Fadin and V. E. Sherman, **1976**

Qluon Reggeization in QCD:

E. A. Kuraev, L. N. Lipatov, and V. S. Fadin, **1974**
I. I. Balitsky and L. N. Lipatov, **1976**

Reggeization is an effective way to take into account amount part of radiative corrections at high energy Regge kinematics.

In the Multi-Regge kinematics the radiative corrections to the Born amplitude do not destroy form of these amplitude but give only simple factorized Regge factors.

Reggeized amplitude can be presented formally like Born amplitude with change of initial $t-$channel partons to the Reggeized ones and change of QCD-vertices to the effective Reggeon-parton vertices.

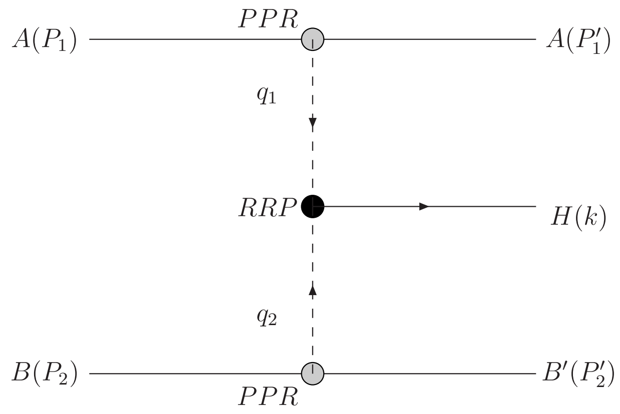


Figure 1: Multi-Regge kinematics: $y_A >> y_H >> y_B$

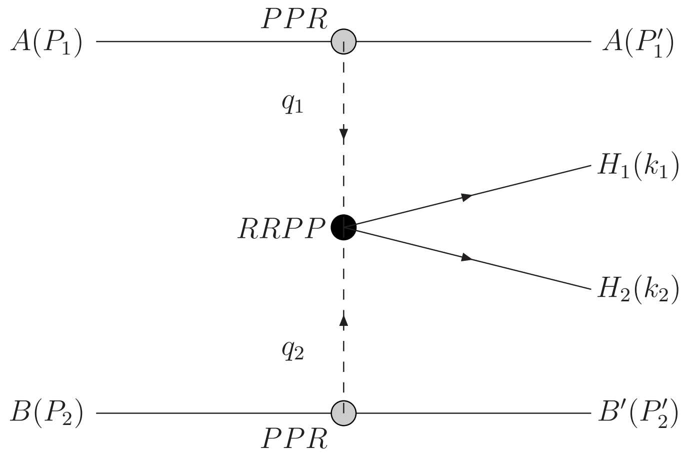


Figure 2: Quasi-multi-Regge kinematics: $y_A >> y_{H1} \approx y_{H2} >> y_B$

Parton Reggeization Approach

At present time we have theoretical background and tools:

- The Effective Field Theory contains field of Reggeized gluons [*Lipatov, 1995.*]
- Effective action with Reggeized quarks [*Lipatov, Vyazovky, 2001.*]
- The Feynman rules for Reggeized gluons [*Antonov, Kuraev, Lipatov, Cherednikov, 2005.*]

PRA consists two main parts:

1. Lipatov's effective theory for Reggeized gluons and quarks,
2. High energy factorization hypothesis with unintegrated PDFs $\Phi(x, \vec{q}_T, \mu)$

Some definitions:

$$P_1 = E_1(1,0,0,1), \qquad P_2 = E_2(1,0,0,-1), \qquad S = 4E_1E_2$$

$$(n^+)^\mu = P_2^\mu/E_2, \qquad (n^-)^\mu = P_1^\mu/E_1, \qquad k^\pm = k \cdot n^\pm = k^\mu n_\mu^\pm$$

Reggeized parton 4-momenta:

$$q_1 = x_1P_1 + q_{1T}, \qquad q_2 = x_2P_2 + q_{2T}, \qquad t_1 = -q_1^2 = -q_{1T}^2, \qquad t_2 = -q_2^2 = -q_{2T}^2$$

$$q_T = (0, \vec{\mathbf{q}}_T, 0)$$

MRK and QMRK:

$$x_1, x_2 \ll 1$$

Reggeon-Reggeon-Particle

1. *Fadin-Kuraev-Lipatov (1974)* effective vertex $RR \to g$.

$$C^{g,\mu}_{\mathcal{R}\mathcal{R}}(q_1,q_2) = -\sqrt{4\pi\alpha_s} f^{abc} \frac{q_1^+ q_2^-}{2\sqrt{t_1 t_2}} \left[(q_1 - q_2)^\mu + \frac{(n^+)^\mu}{q_1^+} \left(q_2^2 + q_1^+ q_2^- \right) - \frac{(n^-)^\mu}{q_2^-} \left(q_1^2 + q_1^+ q_2^- \right) \right]$$

2. *Fadin-Sherman (1976)* effective vertex $Q\bar{Q} \to g$.

$$C^{g,\mu}_{\mathcal{Q}\overline{\mathcal{Q}}}(q_1,q_2) = -i\sqrt{4\pi\alpha_s} T^a \left[\gamma^\mu - \hat{q}_1 \frac{(n^-)^\mu}{q_1^- + q_2^-} - \hat{q}_2 \frac{(n^+)^\mu}{q_1^+ + q_2^+} \right]$$

3. $RQ \to q$ effective vertex

$$C^{q}_{\mathcal{Q}\mathcal{R}}(q_1,q_2) = -i\sqrt{4\pi\alpha_s} T^a \left[\gamma^\mu + \hat{q}_1 \frac{n^\pm_\mu}{(q_1 + q_2)^\pm} \right] \frac{q_1^\mp n^\pm_\mu}{2\sqrt{t_1}}$$

4. $Q\bar{Q} \to \gamma, \gamma^*$ effective vertex

Reggeon-Reggeon-Particle-Particle

1. $RR \to gg$ effective vertex [Fadin, Kuraev,Lipatov]
2. $RR \to q\bar{q}$ effective vertex [Fadin, Kuraev,Lipatov]
3. $RQ \to gq$ effective vertex [Lipatov, Vyazovsky]
4. $QQ \to qq$, $Q\bar{Q} \to q\bar{q}$, $Q\bar{Q} \to gg$ effective vertices [Lipatov, Vyazovsky]
5. $Q\bar{Q} \to g\gamma$, $QR \to q\gamma$

COLLINEAR FACTORIZATION

$$d\sigma^{\rm PM}(p+p\to\mathcal{H}+X,S)=\int dx_1\,G(x_1,\mu^2)\int dx_2\,G(x_2,\mu^2)d\hat\sigma(g+g\to\mathcal{H}+X,\hat s),\quad (1)$$

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations for

$$G(x,\mu_0^2)\to G(x,\mu^2)$$

HIGH-ENERGY FACTORIZATION

$$d\sigma(p+p\to\mathcal{H}+X,S)=\int\frac{dx_1}{x_1}\int d|\mathbf{q}_{1T}|^2\int\frac{d\varphi_1}{2\pi}\Phi(x_1,|\mathbf{q}_{1T}|^2,\mu^2)$$
$$\times\quad\int\frac{dx_2}{x_2}\int d|\mathbf{q}_{2T}|^2\int\frac{d\varphi_2}{2\pi}\Phi(x_2,|\mathbf{q}_{2T}|^2,\mu^2)d\hat\sigma(R+R\to\mathcal{H}+X,\mathbf{q}_{1T},\mathbf{q}_{2T},\hat s),\quad (2)$$

Balitsky-Fadin-Kuraev-Lipatov (BFKL) evolution equations for

$$\Phi(x,|\mathbf{q}_{2T}|^2,\mu_0^2)\to\Phi(x,|\mathbf{q}_{2T}|^2,\mu^2)$$

NORMALIZATION CONDITIONS

$$xG(x,\mu^2) = \int^{\mu^2} d|\mathbf{q}_T|^2\, \Phi(x, |\mathbf{q}_T|^2, \mu^2), \tag{3}$$

$$\hat\sigma(g+g\to\mathcal{H}+X,\hat s) = \int \frac{d\varphi_1}{2\pi}\int\frac{d\varphi_2}{2\pi} d\hat\sigma(R+R\to\mathcal{H}+X, t_1=0, t_2=0, \hat s) \tag{4}$$

Unintegrated PFDs

In our numerical analysis, we adopt the prescriptions proposed by *Blümlein (1994)* and by *Kimber, Martin, and Ryskin (2001)* to obtain unintegrated gluon and quark distribution functions for the proton from the conventional integrated ones.

$$\Phi(x,t,\mu^2) = K(x,t,\mu^2) \otimes G(x,\mu^2)$$

Blumlein gluon PDF bases on BFKL equation with contribution of the large logarithms $\log(\frac{S}{\mu^2}) \simeq \log(\frac{1}{x})$.

KMR PDFs base on DGLAṔ equation with contribution of the large logarithms $\log(\frac{\mu^2}{\Lambda^2_{QCD}})$ and include additionally (model dependent) large logarithms $\log(\frac{S}{\mu^2}) \simeq \log(\frac{1}{x})$ and $\vec{\mathbf{q}}_T$.

As input for this procedure, we use the *Martin-Roberts-Stirling-Thorne (MRST)* proton PDFs.

Single jet hadroproduction

$$\frac{d\sigma}{dp_T\,dy}(pp \to j + X) = \frac{1}{p_T^3}\int d\phi_1 \int dt_1 \Phi_g^p(x_1, t_1, \mu^2)\Phi_g^p(x_2, t_2, \mu^2)\overline{|\mathcal{M}\,(\mathcal{R}\mathcal{R} \to g)|^2},$$

$$\overline{|\mathcal{M}(\mathcal{R} + \mathcal{R} \to g)|^2} = \frac{3}{2}\pi\alpha_s \mathbf{p}_T^2.$$

$$\frac{p_T^2}{2} < \mu < 2p_T^2$$

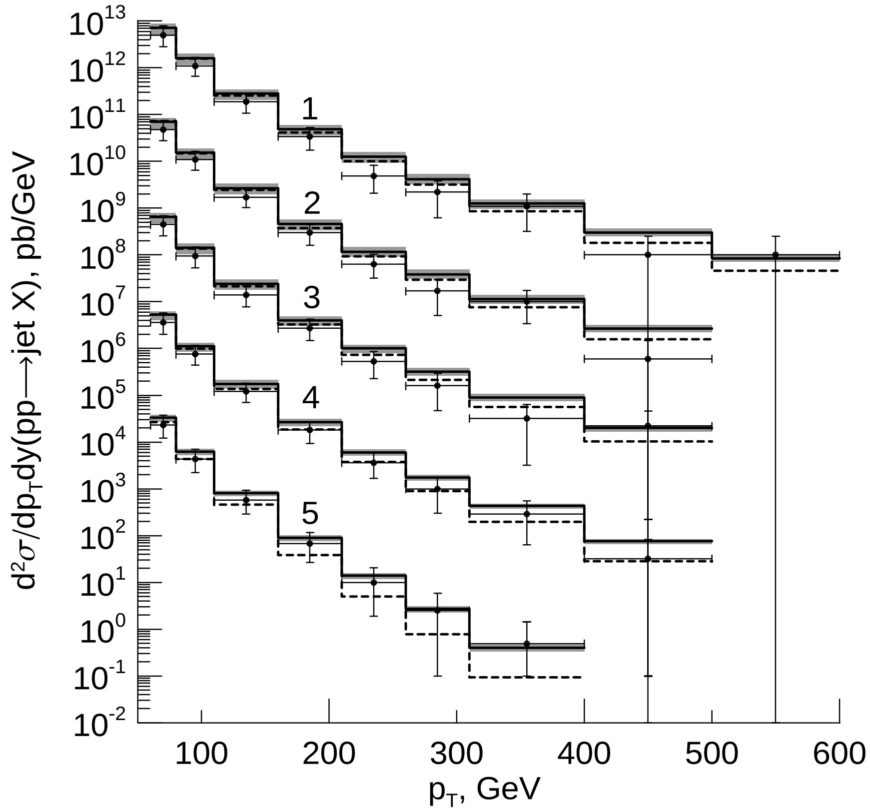


Figure 3: LHC, $\sqrt{S} = 7$ TeV, ATLAS. Solid lines - KMR unPDF, dashed lines - Blumlein unPDF. (1) $|y| < 0.3$ ($\times 10^8$), (2) $0.3 < |y| < 0.8$ ($\times 10^6$), (3) $0.8 < |y| < 1.2$ ($\times 10^4$), (4) $1.2 < |y| < 2.1$ ($\times 10^2$), and (5) $2.1 < |y| < 2.6$

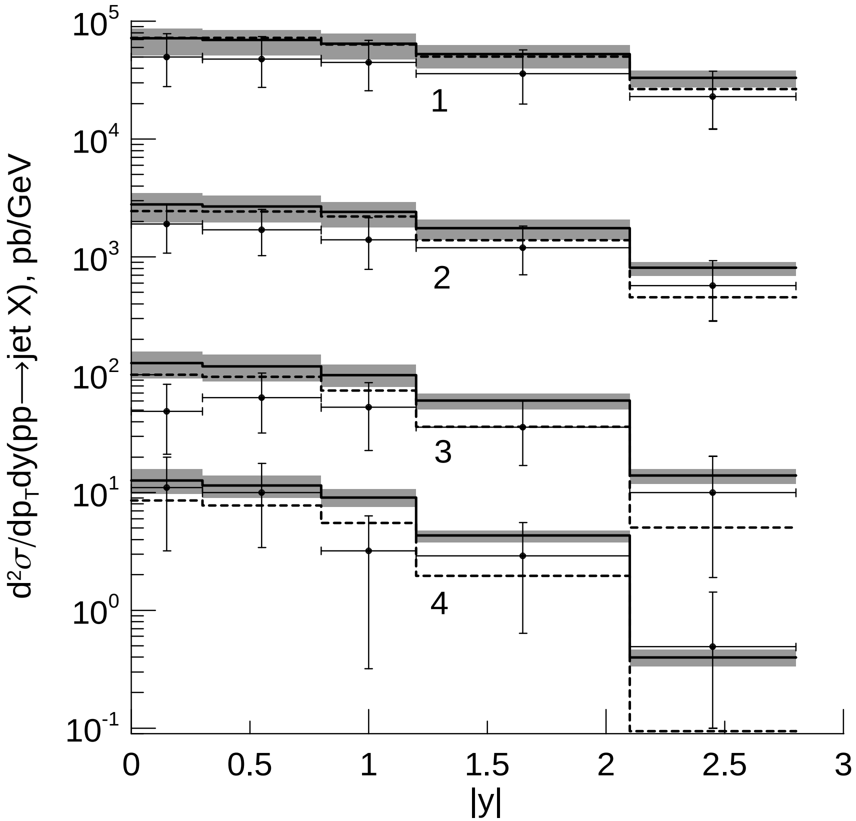


Figure 4: LHC, $\sqrt{S} = 7$ TeV, ATLAS. Solid lines - KMR unPDF, dashed lines - Blumlein unPDF. (1) 60 GeV $< p_T <$ 80 GeV, (2) 110 GeV $< p_T <$ 160 GeV, (3) 210 GeV $< p_T <$ 250 GeV, and (4) 310 GeV $< p_T <$ 400 GeV

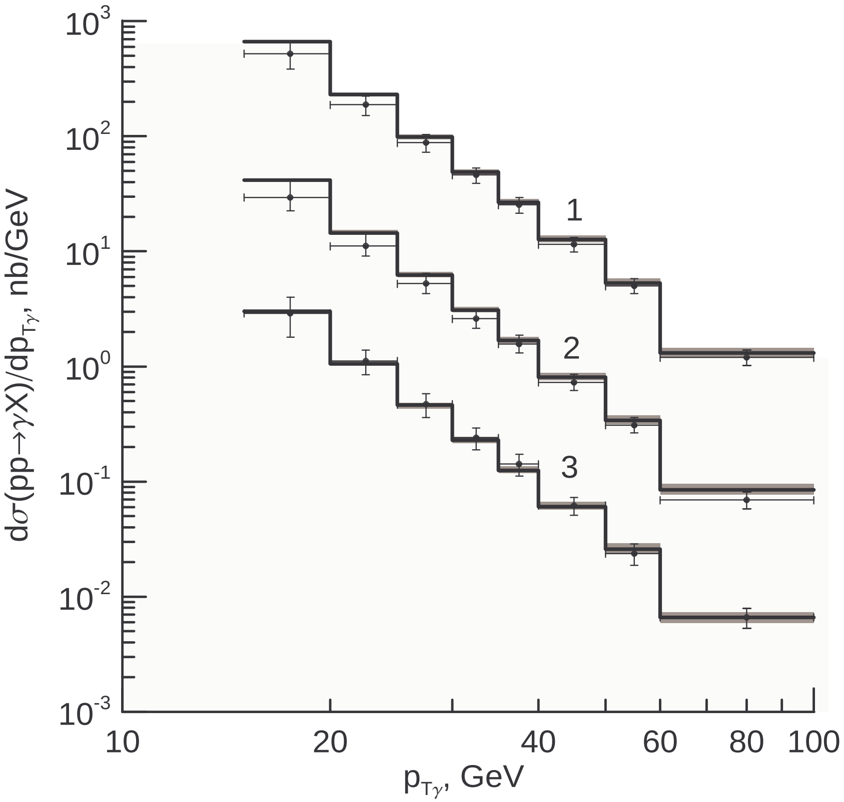


Figure 5: LHC, $\sqrt{S} = 7$ TeV, ATLAS. Solid lines - KMR unPDF. 1 – $|y| < 0.6$ ($\times 100$), 2 – $0.6 < |y| < 1.37$ ($\times 5$), 3 – $1.52 < |y| < 1.81$

Spin effects in Drell-Yan pair production

In PRA we obtain for the squared amplitude of the subprocess $Q\bar{Q} \to \gamma^* \to l^+l^-$:

$$\overline{|M(\mathcal{Q}_i\bar{\mathcal{Q}}_i \to l^+l^-)|^2} = \frac{16\pi^2}{3Q^4}\alpha^2 e_i^2 L^{\mu\nu} w_{\mu\nu}^{PRA},$$

where the tensor of Reggeized quarks reads:

$$\begin{aligned} w_{\mu\nu}^{PRA} &= x_1x_2\Big[-Sg^{\mu\nu} + 2(P_1^\mu P_2^\nu + P_2^\mu P_1^\nu)\frac{(2x_1x_2S - Q^2 - t_1 - t_2)}{x_1x_2S} + \\ &+ \frac{2}{x_2}(q_1^\mu P_1^\nu + q_1^\nu P_1^\mu) + \frac{2}{x_1}(q_2^\mu P_2^\nu + q_2^\nu P_2^\mu) + \\ &+ \frac{4(t_1 - x_1x_2S)}{Sx_2^2}P_1^\mu P_1^\nu + \frac{4(t_2 - x_1x_2S)}{Sx_1^2}P_2^\mu P_2^\nu\Big]. \end{aligned} \quad (5)$$

$$\frac{dN}{d\Omega} = \frac{4}{\lambda+3}\left(1 + \lambda\cos^2\theta + \mu\sin 2\theta\cos\phi + \frac{\nu}{2}\sin^2\theta\cos 2\phi\right)$$

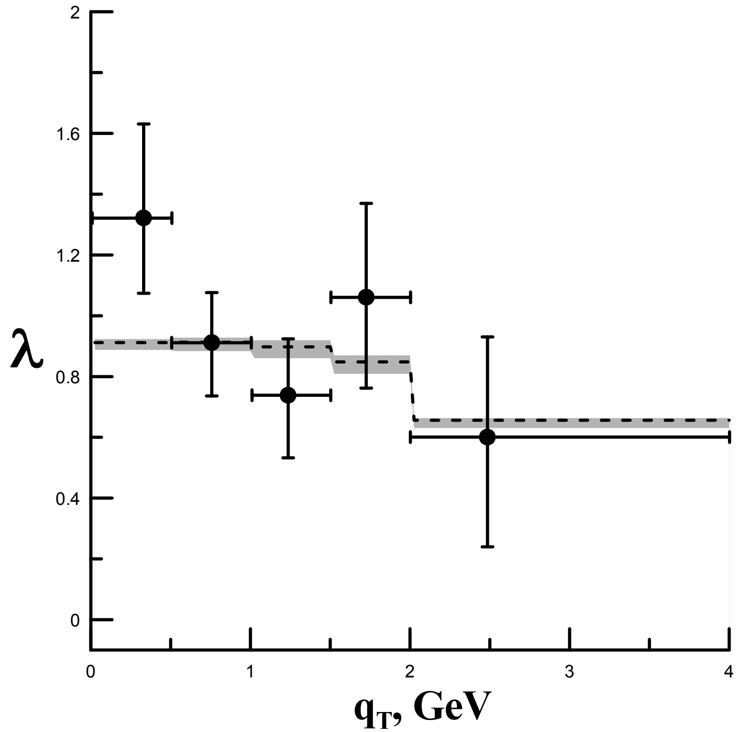


Figure 6: Angular coefficient λ as function of q_T. The histogram corresponds to LO calculation in PRA with KMR unintegrated PDFs. The data are from NuSea Collaboration.

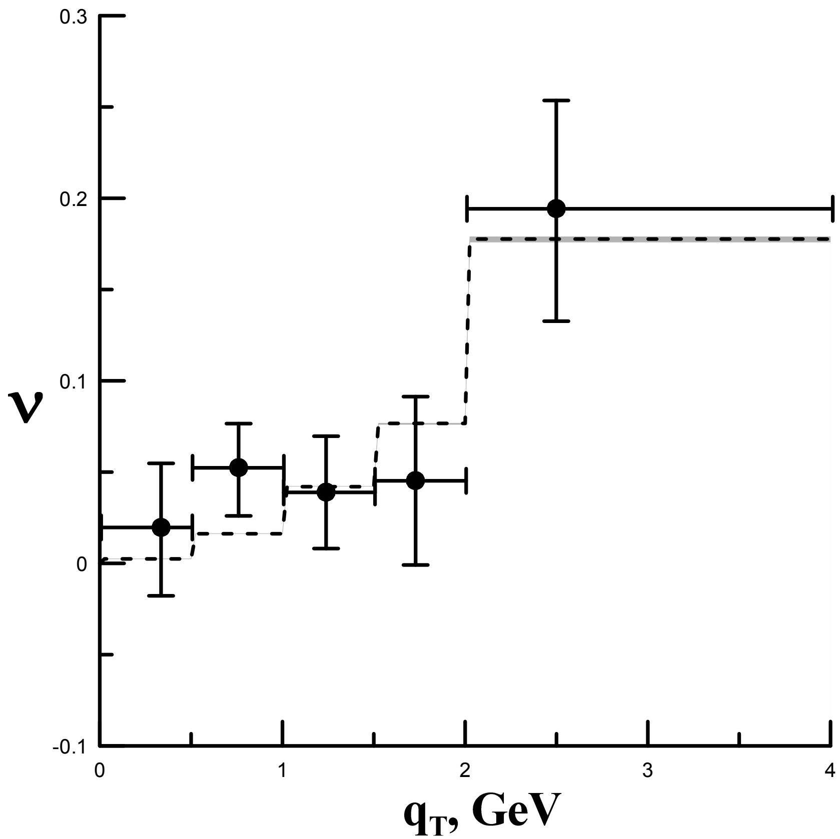


Figure 7: Angular coefficient ν as function of q_T. The histogram corresponds to LO calculation in PRA with KMR unintegrated PDFs. The data are from NuSea Collaboration.

Dijet hadroproduction

$$R + R \to g + g, \qquad R + R \to q + \bar{q},$$

$$Q + R \to q + g, \qquad Q + Q \to q + q, \qquad Q + Q' \to q + q',$$

$$Q + \bar{Q} \to q + \bar{q}, \qquad Q + \bar{Q} \to q' + \bar{q}', \qquad Q + \bar{Q} \to g + g,$$

At the LHC the dominant subprosess is $R + R \to g + g$.

Let us determine effective vertices:

$$\begin{aligned}
\Gamma_\mu^{(+-)}(q_1,q_2) &= 2\left[\left(q_1^+ + \frac{q_1^2}{q_2^-}\right)n_\mu^- - \left(q_2^- + \frac{q_2^2}{q_1^+}\right)n_\mu^+ + (q_2-q_1)_\mu\right], \\
\gamma_\mu^\pm(q,p) &= \gamma_\mu + \hat{q}\frac{n_\mu^\pm}{p^\pm}, \\
\gamma_\mu^{(+-)}(q_1,q_2) &= \gamma_\mu - \hat{q}_1\frac{n_\mu^-}{q_2^-} - \hat{q}_2\frac{n_\mu^+}{q_1^+},
\end{aligned}$$

$$\begin{aligned}
\Gamma^{\mu\nu+}(q_1,q_2) &= 2q_1^+ g^{\mu\nu} - (n^+)^\mu(q_1-q_2)^\nu - (n^+)^\nu(q_1+2q_2)^\mu + \frac{t_2}{q_1^+}(n^+)^\mu(n^+)^\nu, \\
\Gamma^{\mu\nu-}(q_1,q_2) &= 2q_2^- g^{\mu\nu} + (n^-)^\mu(q_1-q_2)^\nu - (n^-)^\nu(q_2+2q_1)^\mu + \frac{t_1}{q_2^-}(n^-)^\mu(n^-)^\nu,
\end{aligned}$$

and the triple-gluon vertex

$$\gamma_{\mu\nu\sigma}(q,p) = (q-p)_\sigma g_{\mu\nu} - (p+2q)_\mu g_{\nu\sigma} + (2p+q)_\nu g_{\mu\sigma}. \tag{6}$$

$$
\begin{aligned}
C_{RR,ab}^{gg,\ cd,\ \mu\nu}(q_1,q_2,k_1,k_2) &= g_s^2 \frac{q_1^+ q_2^-}{4\sqrt{t_1 t_2}} \Big(T_1 s^{-1} \Gamma^{(+-)_\sigma}(q_1,q_2)\gamma_{\mu\nu\sigma}(-k_1,-k_2) + \\
&+ T_3 t^{-1} \Gamma^{\sigma\mu-}(q_1,k_1-q_1)\Gamma^{\sigma\nu+}(k_2-q_2,q_2) - \\
&- T_2 u^{-1} \Gamma^{\sigma\nu-}(q_1,k_2-q_1)\Gamma^{\sigma\mu+}(k_1-q_2,q_2) - \\
&- T_1\big(n_\mu^- n_\nu^+ - n_\nu^- n_\mu^+\big) - T_2\big(2g_{\mu\nu} - n_\mu^- n_\nu^+\big) - T_3\big(-2g_{\mu\nu} + n_\nu^- n_\mu^+\big) + \\
&+ \Delta^{\mu\nu+}(q_1,q_2,k_1,k_2) + \Delta^{\mu\nu-}(q_1,q_2,k_1,k_2)\Big),
\end{aligned}
$$

where

$$
T_1 = f_{cdr} f_{abr}, \quad T_2 = f_{dar} f_{cbr}, \quad T_3 = f_{acr} f_{dbr}, \quad T_1 + T_2 + T_3 = 0,
$$

$$
\Delta^{\mu\nu+}(q_1,q_2,k_1,k_2) = 2t_2 n_\mu^+ n_\nu^+ \left(\frac{T_3}{k_2^+ q_1^+} - \frac{T_2}{k_1^+ q_1^+} \right),
$$

$$
\Delta^{\mu\nu-}(q_1,q_2,k_1,k_2) = 2t_1 n_\mu^- n_\nu^- \left(\frac{T_3}{k_1^- q_2^-} - \frac{T_2}{k_2^- q_2^-} \right),
$$

f^{abc} are structure constants of color gauge group SU(3), $g_s^2 = 4\pi\alpha_s$, and α_s is a strong-coupling constant.

$$C^{q\bar{q}}_{RR,\ ab}(q_1,q_2,k_1,k_2) = g_s^2\frac{q_1^+q_2^-}{4\sqrt{t_1t_2}}\bar{U}(k_1)\Big(-s^{-1}[T^a,T^b]\gamma^\sigma\Gamma^{(+-)}_\sigma(q_1,q_2)+$$

$$+ \quad t^{-1}T^aT^b\gamma^-(\hat{k}_1-\hat{q}_1)\gamma^+ + u^{-1}T^bT^a\gamma^+(\hat{k}_1-\hat{q}_2)\gamma^-\Big)V(k_2),$$

$$C^{qg,\ b,\ \mu}_{QR,\ a}(q_1,q_2,k_1,k_2) = \frac{1}{2}g_s^2\frac{q_2^-}{2\sqrt{t_2}}\bar{U}(k_1)\Big[\gamma^{(-)}_\sigma(q_1,k_1-q_1)t^{-1}\big(\gamma_{\mu\nu\sigma}(k_2,-q_2)n^+_\nu + t_2\frac{n^+_\mu n^+_\sigma}{k_2^+}\big)\times$$

$$\times \quad \big[T^a,T^b\big] - \gamma^+(\hat{q}_1-\hat{k}_2)^{-1}\gamma^{(-)}_\mu(q_1,-k_2)T^aT^b -$$

$$- \quad \gamma_\mu(\hat{q}_1+\hat{q}_2)^{-1}\gamma^{(-)}_\sigma(q_1,q_2)n^+_\sigma T^bT^a + \frac{2\hat{q}_1n^-_\mu}{k_1^-}\left(\frac{T^aT^b}{k_2^-}-\frac{T^bT^a}{q_2^-}\right)\Big],$$

$$C^{qq}_{QQ}(q_1,q_2,k_1,k_2) = g_s^2\Big(t^{-1}\bar{U}(k_2)\gamma^{(+)}_\sigma(q_2,k_2-q_2)T^c\otimes\bar{U}(k_1)\gamma^{(-)}_\sigma(q_1,k_1-q_1)T^c-$$

$$- \quad u^{-1}\bar{U}(k_1)\gamma^{(+)}_\sigma(q_2,k_1-q_2)T^c\otimes\bar{U}(k_2)\gamma^{(-)}_\sigma(q_1,k_2-q_1)T^c\Big),$$

$$C^{qq'}_{QQ'}(q_1,q_2,k_1,k_2) = g_s^2t^{-1}\bar{U}(k_2)\gamma^{(+)}_\sigma(q_2,k_2-q_2)T^c\otimes\bar{U}(k_1)\gamma^{(-)}_\sigma(q_1,k_1-q_1)T^c,$$

$$\begin{aligned} C^{q\bar{q}}_{Q\bar{Q}}(q_1,q_2,k_1,k_2) &= g_s^2\Big(s^{-1}\bar{U}(k_1)\gamma_\sigma T^c V(k_2)\otimes\gamma^{(+-)}_\sigma(q_1,q_2)T^c + \\ &+ t^{-1}\bar{U}(k_1)\gamma^{(+)}_\sigma(q_2,k_2-q_2)T^c\otimes\gamma^{(-)}_\sigma(q_1,k_1-q_1)T^c V(k_2)\Big), \end{aligned}$$

$$C^{q'\bar{q}'}_{Q\bar{Q}}(q_1,q_2,k_1,k_2) = g_s^2 s^{-1}\bar{U}(k_1)\gamma_\sigma T^c V(k_2)\otimes\gamma^{(+-)}_\sigma(q_1,q_2)T^c,$$

$$\begin{aligned} C^{gg,\ ab,\ \mu\nu}_{Q\bar{Q}}(q_1,q_2,k_1,k_2) &= -g_s^2\Big(\gamma^{(+)}_\nu(q_2,-k_2)(\hat{k}_2-\hat{q}_2)^{-1}\gamma^{(-)}_\mu(q_1,-k_1)T^bT^a + \\ &+ \gamma^{(+)}_\nu(q_2,-k_1)(\hat{k}_1-\hat{q}_2)^{-1}\gamma^{(-)}_\mu(q_1,-k_2)T^aT^b + \\ &+ \gamma_{\mu\nu\sigma}(-k_1,-k_2)s^{-1}\gamma^{(+-)}_\sigma(q_1,q_2)[T^a,T^b] + \Delta^{ab}_{\mu\nu}(q_1,q_2)\Big), \end{aligned}$$

where

$$\Delta^{ab}_{\mu\nu}(q_1,q_2) = \frac{\hat{q}_1 n^-_\mu n^-_\nu}{q_2^-}\left(\frac{T^aT^b}{k_2^-}+\frac{T^bT^a}{k_1^-}\right) - \frac{\hat{q}_2 n^+_\mu n^+_\nu}{q_1^+}\left(\frac{T^aT^b}{k_1^+}+\frac{T^bT^a}{k_2^+}\right).$$

$$\overline{|\mathcal{M}|^2} = \pi^2 \alpha_S^2 A \sum_{n=0}^{4} W_n S^n,$$

where A and W_n are process-dependent functions of variables $s, t, u, a_1, a_2, b_1, b_2, t_1, t_2, S$.

$$k_1 = a_1 P_1 + b_1 P_2 + k_{1T}, \qquad k_2 = a_2 P_1 + b_2 P_2 + k_{2T}.$$

Applying a four-momentum conservation law one can find

$$x_1 = a_1 + a_2, \qquad x_2 = b_1 + b_2, \qquad q_{1T} + q_{2T} = k_{1T} + k_{2T}.$$

$$RR \to gg$$

$$\begin{aligned}
A &= \frac{18}{a_1 a_2 b_1 b_2 s^2 t^2 u^2 t_1 t_2}, \\
W_0 &= x_1 x_2 s^2 t u t_1 t_2 \big(x_1 x_2 (tu + t_1 t_2) + (a_1 b_2 + a_2 b_1) tu \big), \\
W_1 &= x_1 x_2 s t_1 t_2 \Bigg[t^2 u \Bigg(a_1 b_2 (a_2 b_2 + a_1 x_2)(t_1 + t_2) - a_2 b_1 (a_1 b_1 t_1 + a_2 b_2 t_2) + \\
&+ \big(x_2 (a_1^2 b_2 + a_2^2 b_1) + a_1 a_2 (b_1 - b_2)^2 \big) u + x_1 x_2 a_1 b_2 t \Bigg) \Bigg] + \Bigg[a_1 \leftrightarrow a_2, b_1 \leftrightarrow b_2, t \leftrightarrow u \Bigg], \\
W_2 &= a_1 a_2 b_1 b_2 t u \Bigg(x_1^2 x_2^2 \big[2(t_1 + t_2) \big(t^2 u + t_1 t_2 (s + u - t) \big) + tu \big((t_1 - t_2)^2 + t(u + 2t) \big) \big] + \\
&+ x_1 x_2 t t_1 t_2 \big(4(x_1 b_1 + x_2 a_2)(s + u) - (a_1 b_1 + a_2 b_2) u \big) + \\
&+ tu \big(x_1^2 b_2 (2 x_2 t - b_1 t_1) t_1 + x_2^2 a_1 (2 x_1 t - a_2 t_2 t_2) \big) \Bigg) + \Bigg(a_1 \leftrightarrow a_2, b_1 \leftrightarrow b_2, t \leftrightarrow u \Bigg),
\end{aligned} \tag{7}$$

$$
\begin{aligned}
W_3 &= x_1 x_2 a_1 a_2 b_1 b_2 \Big[t^2 u \Big(2a_1 b_2 \big(x_1 x_2 (t_1 + t_2)(2t - u - s) - (x_1 b_2 t_1 + x_2 a_1 t_2)(u + s) \big) + \\
&+ \big[x_1 t_1 \big(2(a_1 b_2^2 + a_2 b_1^2) + 3 x_1 b_1 b_2 \big) + x_2 t_2 \big(2(a_1^2 b_2 + a_2^2 b_1) + 3 a_1 a_2 x_2 \big) \big] u + \\
&+ 4 x_1 x_2 t \big((a_1 b_2 + a_2 b_1) u + a_1 b_2 t \big) \Big) \Big] + \Big[a_1 \leftrightarrow a_2, b_1 \leftrightarrow b_2, t \leftrightarrow u \Big], \\
W_4 &= x_1^2 x_2^2 a_1 a_2 b_1 b_2 \Big[t \Big(a_1 a_2 b_1 b_2 u (t_1 + t_2)(t - u - s) + (a_1 b_2 + a_2 b_1)^2 t u^2 - \\
&- 2 a_1 b_2 t (s + u)(2 a_2 b_1 u - a_1 b_2 s) \Big) \Big] + \Big[a_1 \leftrightarrow a_2, b_1 \leftrightarrow b_2, t \leftrightarrow u \Big].
\end{aligned}
$$

$$RR \to q\bar{q}$$

$$
\begin{aligned}
A &= \frac{S}{3s^2t^2u^2t_1t_2}, \\
W_0 &= 18x_1x_2st^2u^2t_1t_2, \\
W_1 &= tu\Big(-18tu\big((x_1b_1t_1 - a_1x_2t_2) + x_1x_2(u+t_2)\big)\big((x_1b_1t_1 - a_1x_2t_2) - x_1x_2(t+t_1)\big) + \\
&+ x_1x_2s\big[9\big((t-u)(a_1b_2 - a_2b_1)t_1t_2 - x_1x_2stu\big) - x_1x_2s(t_1t_2 - tu)\big]\Big), \\
W_2 &= x_1x_2tu\big[9\big(2(x_1b_1t_1 - x_2a_1t_2) - x_1x_2(t + t_1 - u - t_2)\big) \times \\
&\times \big(2(a_2b_1 - a_1b_2)tu - (a_1b_2t - a_2b_1u)s\big) - 7x_1x_2s^2(a_2b_1u + a_1b_2t)\big], \\
W_3 &= -x_1^2x_2^2\big[18tu(a_2b_1 - a_1b_2)\big((a_2b_1 - a_1b_2)tu + (a_2b_1u - a_1b_2t)s\big) + \\
&+ 2s^2(4a_2^2b_1^2u^2 + 4a_1^2b_2^2t^2 - a_1a_2b_1b_2tu)\big], \\
W_4 &= 0.
\end{aligned}
$$

$$QR \to qg$$

$$\begin{aligned}
A &= -\frac{8x_1}{9a_2b_1b_2st^2u^2t_2}, \\
W_0 &= -9x_2t_2stu\big(x_2t_1t_2 + (x_2+b_2)tu\big), \\
W_1 &= tu\Bigg(-a_2b_1b_2tu\big[8\big(b_1(s+t) - b_2u\big) + x_2s\big] + \\
&+ \big[9x_2^2\big(a_2(b_1-b_2)s^2 + (a_2b_1 + a_1b_2)st - a_1b_2tu\big) + \\
&+ 9x_2a_2u\big((b_1^2 - 2b_1b_2 - b_2^2)s - b_1b_2u\big) + \\
&+ x_2a_2b_1t\big((b_1-b_2)s + b_1t\big) + 2a_2b_1tu(b_1^2 + b_1b_2 + 4b_2^2)\big]t_2 + \\
&+ x_2\big((a_2b_1t + 9a_2x_2s)(b_1-b_2) + 9b_1b_2u(a_1-a_2)\big)t_2^2\Bigg),
\end{aligned} \tag{8}$$

$$\begin{aligned}
W_2 &= a_2b_1b_2x_2tu\Big(9\big(a_1t(x_2s+b_2u)-a_2u(x_2s-b_2u)-2a_2b_1u(t+s)\big)+\\
&+ b_1t(a_1t+a_2u)-2a_1b_2t(s+u)+\\
&+ \Big[9\big(x_1x_2(s+u)+2a_1b_1u+x_2a_1(s-3u)\big)+(b_1x_1-2a_1b_2)t\Big]t_2\Big),\\
W_3 &= a_2b_1b_2x_2^2\Big(9\big[s\big(a_1^2b_2t^2-2a_2^2b_1u^2+a_1a_2ut(b_1-b_2)\big)+a_2tu^2(a_1b_2-a_2b_1)\big]-\\
&- t^2\big(u(a_1b_2-a_2b_1)+a_1b_2s\big)\Big),\\
W_4 &= 0.
\end{aligned}$$

$$QQ \to qq$$

$$\begin{aligned}
A &= \frac{64x_1x_2}{27a_1a_2b_1b_2t^2u^2}, \\
W_0 &= x_1x_2st\big[t_1t(3a_2b_1 - x_1b_2) + t_2t(3a_2b_1 - x_2a_1) + t_1t_2(x_2a_2 - x_1b_2) - x_1x_2t^2 + \\
&+ st\big(6(a_1b_1 + a_2b_2) + 5(2a_2b_1 + a_1b_2)\big)\big], \\
W_1 &= t\big[t_1x_1a_2b_2\big(6b_1t(a_2b_1 - a_1b_2) - x_2s(x_1b_1 + a_2b_1 - a_1b_2)\big) + \\
&+ t_2x_2a_1b_1\big(6a_2t(a_2b_1 - a_1b_2) - x_1s(x_2a_2 + a_2b_1 - a_1b_2)\big) + \\
&+ 6x_1x_2a_2b_1(a_2b_1 - a_1b_2)t^2 + x_1x_2a_2b_1s^2(a_2b_1 - a_1b_2 + 6x_1x_2) + \\
&+ x_1x_2st\big((a_1b_2 - a_2b_1)^2 + a_1b_2(a_1b_1 + a_2b_2) - 2a_2b_1(2a_2b_1 + x_1b_2 + x_2a_1)\big)\big], \\
W_2 &= x_1x_2a_2b_1\big(6t^2(a_2b_1 - a_1b_2)^2 + 3x_1x_2a_2b_1s^2 + st(x_1b_1 + x_2a_2)(a_1b_2 - a_2b_1)\big), \\
W_3 &= W_4 = 0.
\end{aligned}$$

$$QQ' \to qq'$$

$$A = \frac{64 x_1 x_2}{9 a_2 b_1 t^2}, \qquad W_0 = 2t^2, \quad W_1 = 2a_2 b_1 t, \quad W_2 = a_2^2 b_1^2, \quad W_3 = 0, \quad W_4 = 0.$$

$$Q\bar{Q} \to q\bar{q}$$

$$\begin{aligned}
A &= \frac{64}{27 x_1 x_2 a_2 b_1 s^2 t^2}, \\
W_0 &= x_1 x_2 s t \big[t_1 t (3a_2 b_1 - x_1 b_2) + t_2 t (3 a_2 b_1 - x_2 a_1) + t_1 t_2 (x_2 a_2 - x_1 b_2) - x_1 x_2 t^2 + \\
&+ st\big(6(a_1 b_1 + a_2 b_2) + 5(2 a_2 b_1 + a_1 b_2)\big)\big], \\
W_1 &= t\big[t_1 x_1 a_2 b_2 \big(6 b_1 t (a_2 b_1 - a_1 b_2) - x_2 s (x_1 b_1 + a_2 b_1 - a_1 b_2)\big) + \\
&+ t_2 x_2 a_1 b_1 \big(6 a_2 t (a_2 b_1 - a_1 b_2) - x_1 s (x_2 a_2 + a_2 b_1 - a_1 b_2)\big) + \\
&+ 6 x_1 x_2 a_2 b_1 (a_2 b_1 - a_1 b_2) t^2 + x_1 x_2 a_2 b_1 s^2 (a_2 b_1 - a_1 b_2 + 6 x_1 x_2) + \\
&+ x_1 x_2 s t \big((a_1 b_2 - a_2 b_1)^2 + a_1 b_2 (a_1 b_1 + a_2 b_2) - 2 a_2 b_1 (2 a_2 b_1 + x_1 b_2 + x_2 a_1)\big)\big], \\
W_2 &= x_1 x_2 a_2 b_1 \big(6 t^2 (a_2 b_1 - a_1 b_2)^2 + 3 x_1 x_2 a_2 b_1 s^2 + st (x_1 b_1 + x_2 a_2)(a_1 b_2 - a_2 b_1)\big), \\
W_3 &= W_4 = 0.
\end{aligned}$$

$$Q\bar{Q} \to q'\bar{q}'$$

$$\begin{aligned}
A &= \frac{64}{9x_1x_2s^2}, \\
W_0 &= -x_1x_2s(t+u), \\
W_1 &= 2\big(a_1^2b_1b_2(t_1+u) + a_1^2b_2^2u + a_1a_2b_1^2(t+t_2) + a_1a_2b_1b_2(t_1+t_2-s) + \\
&+ a_1a_2b_2^2(t_2+u) + a_2^2b_1^2t + a_2^2b_1b_2(t_1+t)\big), \\
W_2 &= 2x_1x_2(a_2b_1 - a_1b_2)^2, \\
W_3 &= W_4 = 0.
\end{aligned}$$

$$Q\bar{Q} \to gg$$

$$\begin{aligned}
A &= \frac{1}{27x_1x_2Sa_1a_2b_1b_2s^2t^2u^2}, \\
W_0 &= 2x_1x_2s^2tu\Big[\big(x_1x_2 - 9(a_1b_2 + a_2b_1)\big)(t+u)tu - x_1x_2t_1t_2(t_1+t_2)\Big] + \Big[a_1 \leftrightarrow a_2, b_1 \leftrightarrow b_2, t \leftrightarrow u\Big] \\
W_1 &= -2x_1x_2stu\Bigg(x_1x_2(a_1-a_2)(b_1-b_2)(t_1+t_2)t_1t_2 + 2t\Big[x_1x_2(x_1b_2t_2^2 + x_2a_1t_1^2) + \\
&+ t_1t_2\big((x_1x_2 + x_1b_2 + x_2a_1)(a_1b_1 + a_2b_2) + (8a_2b_1 - a_1b_2)(a_2b_1 - a_1b_2)\big)\Big] + \\
&+ x_1x_2\big(2x_1x_2 - 17(a_1b_2 + a_2b_1)\big)(t_1+t_2)tu + 2\big(x_1^2(x_2+b_1)(b_2 - 8b_1) + \\
&+ x_2^2a_1(a_2 - 8a_1) + a_1b_1(25x_1x_2 - 72a_2b_2)\big)t^2u + 2t^2(x_1x_2 - 9a_2b_1)(x_1b_2t_2 + x_2a_1t_1)\Bigg) + \\
&+ \Bigg(a_1 \leftrightarrow a_2, b_1 \leftrightarrow b_2, t \leftrightarrow u\Bigg),
\end{aligned}$$

$$
\begin{aligned}
W_2 &= \Big[-2tu\Big(-x_1^2x_2^4a_1a_2(t_1+t_2)^2t_1 + 2x_1x_2a_1tt_1(t_1+t_2)\big[x_2^2\big(9a_1^2b_1 - x_1(x_1x_2+a_1b_2)\big) + \\
&+ x_1(8b_1-b_2)\big(x_2^2(a_2-a_1)+a_2b_2^2\big)\big] + \\
&+ x_1tut_1\big(36x_1a_1a_2b_2^3(b_2-2x_2) + 5a_1a_2x_1x_2^2b_2^2 - x_2^4a_2(x_1+a_1)(2a_1-7a_2) + \\
&+ 9x_2^2b_2(x_1^3b_2 - 2x_2a_1^3 + 4x_1^2x_2a_1) - 4x_1x_2^3b_2(a_1^2+4x_1^2)\big) + 2x_1x_2a_1t^2t_1 \times \\
&\times \big[9a_2\big(b_2^2(3x_1b_1-2x_2a_1) - x_2^2b_2(x_1-3a_1) + x_2^3a_2\big) - 2x_1x_2b_2(a_1x_2+a_2b_2) - x_1x_2^3a_2\big] + \\
&+ x_1x_2t^2u\big(a_1^3b_2(7b_2^2+10b_1b_2-4b_1^2) + a_1^2a_2b_1(39b_2^2+30b_1b_2-2b_1^2) + \\
&+ 14a_1a_2^2b_1^2(x_2+2b_2) + 8a_2^3b_1^3\big) + x_1x_2a_1b_2t^3\big(18a_2b_1(a_1x_2+x_1b_2) - x_1x_2(a_1b_1+x_1b_2)\big)\Big) + \\
&+ \Big(a_1 \leftrightarrow b_2, a_2 \leftrightarrow b_1, t_1 \leftrightarrow t_2\Big)\Big] + \Big[a_1 \leftrightarrow a_2, b_1 \leftrightarrow b_2, t \leftrightarrow u\Big], \\
W_3 &= -4x_1x_2a_1a_2b_1b_2\Big(t^2\big[8x_1x_2a_1b_2(s+u)^2 - u(s+u)\big((9a_2b_1+7a_1b_2)(a_1b_1+a_2b_2) + \\
&+ 2a_1b_2(17a_2b_1-a_1b_2)\big) + 2a_1u^2\big(4b_1(a_1b_2+a_2b_1) + b_2(13a_1b_2-5a_2b_1)\big)\big]\Big) + \\
&+ \Big(a_1 \leftrightarrow a_2, b_1 \leftrightarrow b_2, t \leftrightarrow u\Big), \\
W_4 &= 0.
\end{aligned}
$$

$$\frac{d\sigma(pp \to ggX)}{dk_{1T}dy_1dk_{2T}dy_2d\Delta\varphi} = \frac{k_{1T}k_{2T}}{16\pi^3}\int dt_1 \int d\phi_1 \Phi_g^p(x_1, t_1, \mu^2)\Phi_g^p(x_2, t_2, \mu^2)\frac{\overline{|\mathcal{M}(RR \to gg)|^2}}{(x_1x_2S)^2},$$

where $k_{1,2T}$ and $y_{1,2}$ are final gluon transverse momenta and rapidities, respectively, and $\Delta\varphi$ is an azimuthal angle enclosed between the vectors $\vec{k}_{1T}$ and $\vec{k}_{2T}$,

$$x_1 = (k_1^0 + k_2^0 + k_1^z + k_2^z)/\sqrt{S}, \quad x_2 = (k_1^0 + k_2^0 - k_1^z - k_2^z)/\sqrt{S},$$

$$k_{1,2}^0 = k_{1,2T}\cosh(y_{1,2}), \quad k_{1,2}^z = k_{1,2T}\sinh(y_{1,2}).$$

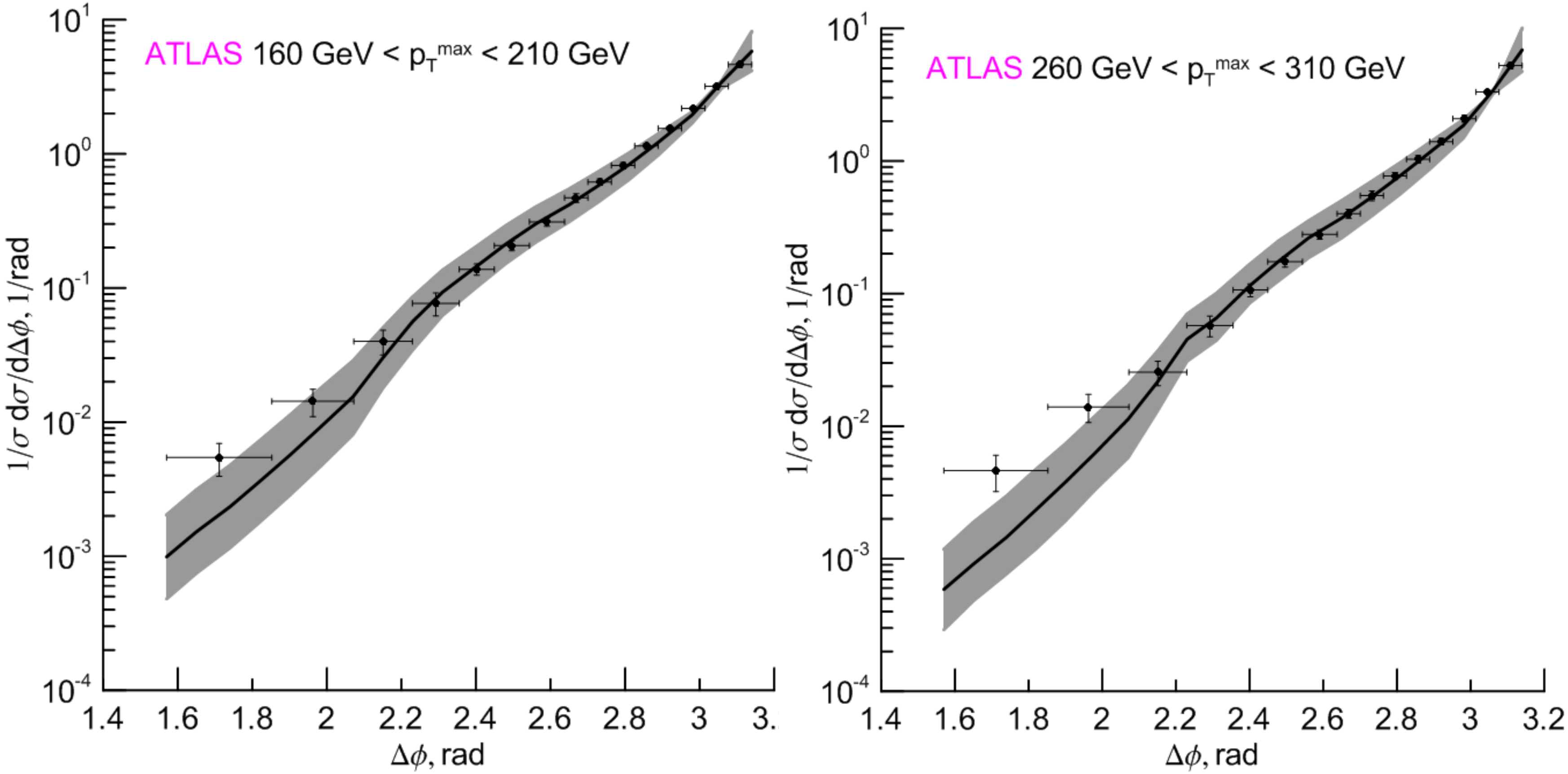


Figure 8: LHC, $\sqrt{S} = 7$ TeV, ATLAS. Solid lines - KMR unPDF.

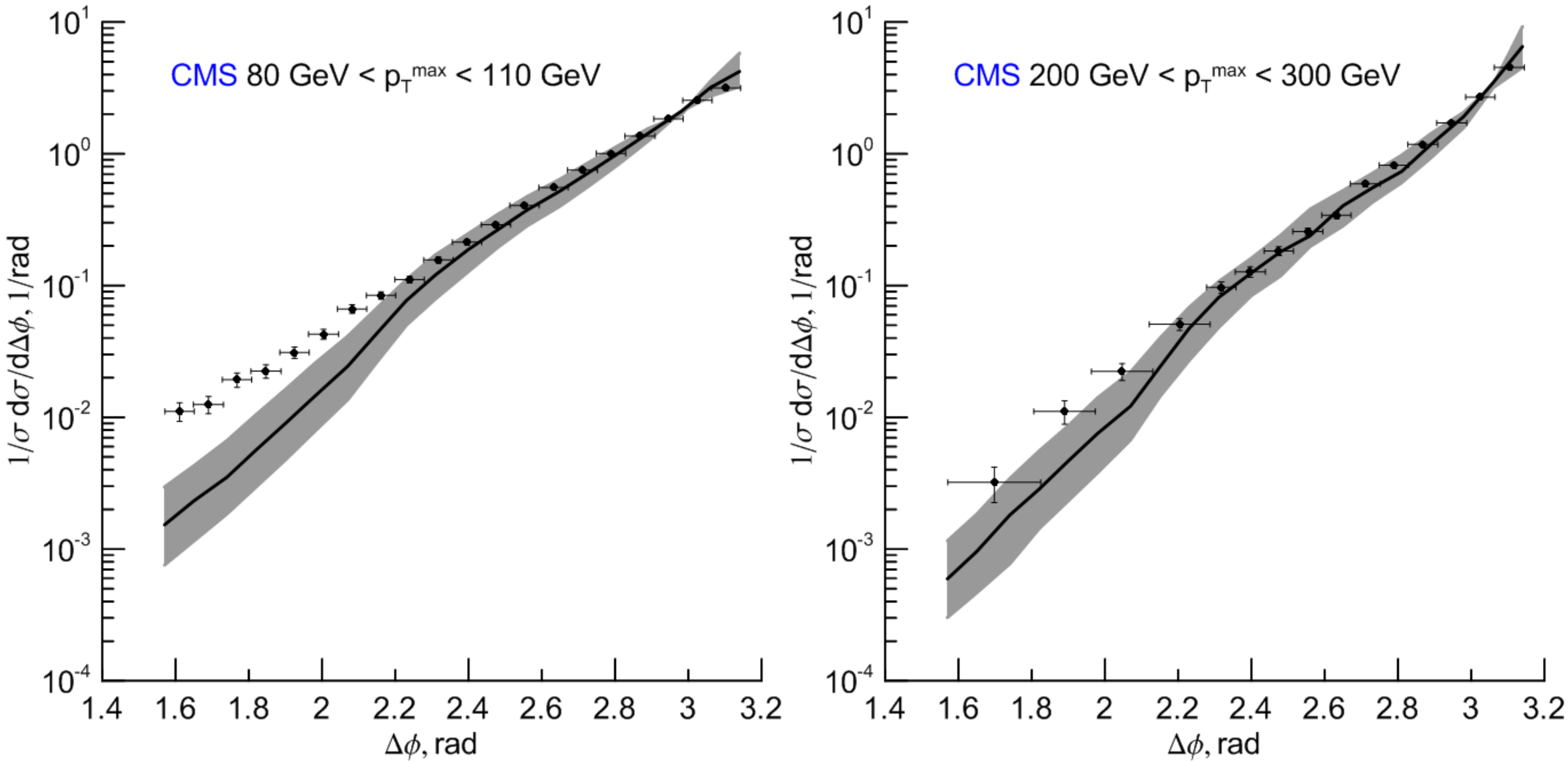


Figure 9: LHC, $\sqrt{S} = 7$ TeV, CMS. Solid lines - KMR unPDF.

Certainly, the precise comparison of theoretical predictions in the LO parton Reggeization approach should be made when we separate only the two-jet production in the central rapidity region. In fact, the data include multi-jet production contribution. The ATLAS Collaboration presents the $\Delta\varphi-$distributions for different number of final-state jets. Using these data, we can extract $F(\Delta\varphi)$ for two-jet production only, as a difference between number of events: $n(2) = n(\geq 2) - n(\geq 3)$ or
$\sigma(2)F(\Delta\varphi, 2) = \sigma(\geq 2)F(\Delta\varphi, \geq 2) - \sigma(\geq 3)F(\Delta\varphi, \geq 3)$, for the kinematic domain of $p_T^{max} > 100$ GeV and $|y| < 0.8$.

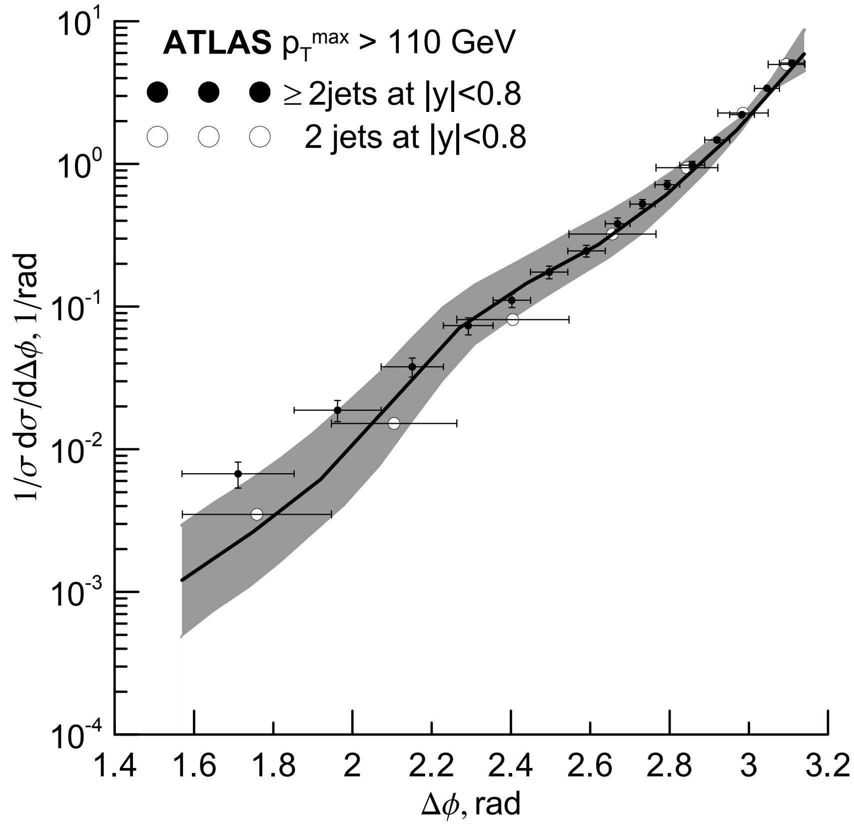


Figure 10: Normalized $F(\Delta\phi)$ distribution for 2 (open circles) and ≥ 2 (black circles) jets with $p_T > 100$ GeV, $|y| < 0.8$, $p_T^{max} > 110$ GeV and $\sqrt{S} = 7$ TeV. The data are from the ATLAS Collaboration. The curve corresponds to LO parton Reggeization approach.

We see, the theoretical prediction nicely agrees with data for $F(\Delta\varphi, 2)$ distribution. So, if the last one would be extracted for different regions of p_T^{max}, we can make a more precise comparison of our predictions with experimental data.

Summarizing results of a present analysis for dijet production at the LHC, we find a strong difference of theoretical interpretation of azimuthal decorrelation between leading and subleading jets, in the collinear parton model and in the parton Reggeization approach. In the first case, an azimuthal decorrelation at different values of $\Delta\varphi$ is provided by hard $2 \to 3$ ($3\pi/4 < \Delta\varphi < \pi$), $2 \to 4$ ($\pi/2 < \Delta\varphi < 3\pi/4$) partonic subprocesses, correspondingly. The explanation of data in the region of $\Delta\varphi < \pi/2$ in the framework of collinear parton model becomes possible only because of initial-state radiation and hadronization effects, and an agreement of theory expectations and data is achieved using MC generators only.

Oppositely, in the parton Reggeization approach, the azimuthal decorrelation is explained by the coherent parton emission during the QCD-evolution, which is described by the transverse-momentum dependent PDFs of Reggeized partons. Already in the LO approximation, at the level of $2 \to 2$ subprocesses with Reggeized partons, we can account the main part of decorrelation effect in dijet production, and we obtain a full description of data in $b\bar{b}-$pair production.

Conclusion and discussion

Conclusion:

- PRA = Lipatov's Effective Theory + High-energy Factorization + KMR unPDF $\Rightarrow$ $t-$channel exchange dominance has been proved at LO in α_s (tree amplitudes)
- Next step, NLO amplitudes + NLO unPDF, would be done.

Discussion

There are another high-energy approaches: k_T-factorization, off-shell Monte-Carlo simulation, ...

1) Initial quarks are introduced correctly (gauge-invariant) only in PRA !

2) k_t- factorization prescription for off-shell gluon polarization vector:

$$\epsilon^\mu(q_T) = \frac{q_{T\mu}}{\sqrt{\vec{q}_T^2}}, \qquad P^\mu M_\mu \sim q_T^\mu M_\mu, \qquad q^\mu M_\mu = (xP^\mu + q_T^\mu)M_\mu = 0,$$

$$C_{RR}^{\mu,g} = \epsilon^\alpha(q_{1T})\epsilon^\beta(q_{2T})g_{\alpha\beta\mu}(q_1, q_2, q_1 + q_2), \qquad C_{RR}^{q\bar{q}} = \epsilon^\alpha(q_{1T})\epsilon^\beta(q_{2T})M_{\alpha\beta}(gg \to q\bar{q})$$

$$C_{RR}^{gg,\mu\nu} \neq \epsilon^\alpha(q_{1T})\epsilon^\beta(q_{2T})M_{\alpha\beta}^{\mu\nu}(gg \to gg), \qquad C_{RQ}^{gq,\mu} \neq \epsilon^\alpha(q_{1T})M_\alpha^\mu(gq \to gq)$$

Thank you for attention!