Light-Cone Distribution Amplitudes of Bottom Baryons

Alexander Parkhomenko

P. G. Demidov Yaroslavl State University, Yaroslavl, Russia

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Outline

Introduction

Ordinary baryons – colorless systems containing three quarks

Heavy baryons H_Q – one heavy quark $Q = c, b$

Doubly-heavy baryons $H_{Q_1 Q_2}$ – two heavy quarks Q_1 and Q_2

Triply-heavy baryons $H_{Q_1 Q_2 Q_3}$ – all three heavy quarks

Convenient framework: Heavy Quark Effective Theory (HQET)

Features of the heavy-baryon system within HQET:

- heavy-quark spin S_Q is decoupled in the limit $m_Q \to \infty$
- classified by the total angular momentum j and parity p of the light-quark pair j^p

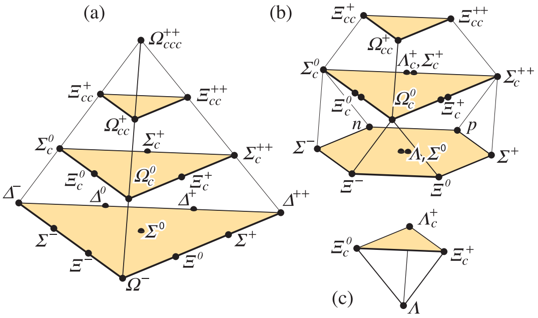
Similar features of the doubly-heavy-baryon system

Introduction

SU(4)-flavor multiplets for charmed baryons

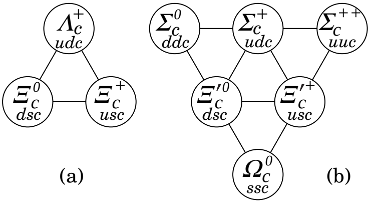
Decomposition into irreducible representations:

$$4 \times 4 \times 4 = 20 + 20'_1 + 20'_2 + \bar{4}$$

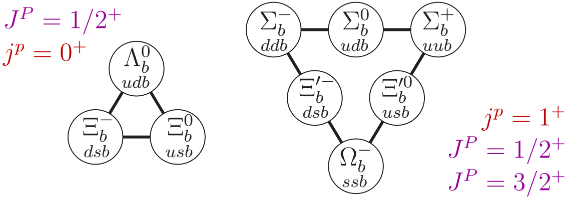


Introduction

Ground-state ($\ell = 0$) charmed baryons



Ground-state ($\ell = 0$) bottom baryons



Introduction

Experimental measurements [PDG, 2012] and theoretical predictions based on HQET [X. Liu et al., 2008] and Lattice QCD [R. Lewis et al., 2009] for masses of ground-state bottom baryons (in units of MeV)

Baryon	$I(J^P)$	j^p	Experiment	HQET	Lattice QCD
Λ_b	$0(1/2^+)$	0^+	5619.4 ± 0.7	5637^{+68}_{-56}	$5641 \pm 21^{+15}_{-33}$
Σ_b^+	$1(1/2^+)$	1^+	5811.3 ± 1.9	5809^{+82}_{-76}	$5795 \pm 16^{+17}_{-26}$
Σ_b^-	$1(1/2^+)$	1^+	5815.5 ± 1.8	5809^{+82}_{-76}	$5795 \pm 16^{+17}_{-26}$
Σ_b^{*+}	$1(3/2^+)$	1^+	5832.1 ± 1.9	5835^{+82}_{-77}	$5842 \pm 26^{+20}_{-18}$
Σ_b^{*-}	$1(3/2^+)$	1^+	5835.1 ± 1.9	5835^{+82}_{-77}	$5842 \pm 26^{+20}_{-18}$
Ξ_b^-	$1/2(1/2^+)$	0^+	5791.1 ± 2.2	5780^{+73}_{-68}	$5781 \pm 17^{+17}_{-16}$
Ξ_b^0	$1/2(1/2^+)$	0^+	5788 ± 5	5903^{+81}_{-79}	$5903 \pm 12^{+18}_{-19}$
Ξ_b'	$1/2(1/2^+)$	1^+		5903^{+81}_{-79}	$5903 \pm 12^{+18}_{-19}$
$\Xi_b'^*$	$1/2(3/2^+)$	1^+		5903^{+81}_{-79}	$5950 \pm 21^{+19}_{-21}$
Ω_b^-	$0(1/2^+)$	1^+	6071 ± 40	6036 ± 81	$6006 \pm 10^{+20}_{-19}$
Ω_b^*	$0(3/2^+)$	1^+		6063^{+83}_{-82}	$6044 \pm 18^{+20}_{-21}$

Introduction

- Heavy baryons are copiously produced at the LHC
- Weak decays of bottom baryons induced by FCNC may give important information on physics beyond the SM
- LCDAs are the primary non-perturbative objects required for calculating decays into light particles based on the heavy quark expansion or within the method of Light-Cone Sum Rules (LCSRs)
- For a long time existing models for heavy baryons were motivated by the quark model
- Complete classification of the three-quark LCDAs of the Λ_b-baryon in QCD and main features of these LCDAs have been considered by V. Braun, P. Ball and E. Gardi (2008)
- Extension of this analysis for all ground-state bottom baryons have been done by A. Ali, C. Hambrock, A. P. and W. Wang (2013) and presented in this lecture

Light-Cone Distribution Amplitudes (LCDAs)

Light-cone distribution amplitudes of heavy baryons —
matrix elements of non-local light-ray operators
build off an effective heavy quark and two light quarks

- Similar in construction to *B*-meson LCDAs
- QCD description of nucleon LCDAs

Heavy Quark Symmetry $\Longrightarrow$ switch off the heavy-quark spin

$SU(3)_F$ antitriplet $\Longrightarrow$ scalar states with $J^P = j^p = 0^+$

$SU(3)_F$ sextets $\Longrightarrow$ axial-vector states with $J^P = j^p = 1^+$

LCDAs

$SU(3)_F$ antitriplet $J^P = j^p = 0^+$ scalar state

Non-local light-ray operators

$$\epsilon^{abc}\langle 0|\left(q_1^a(t_1 n)C\gamma_5 \not{n} q_2^b(t_2 n)\right) h_v^c(0)|H(v)\rangle = f_H^{(2)}\,\Psi_2(t_1,t_2)$$

$$\epsilon^{abc}\langle 0|\left(q_1^a(t_1 n)C\gamma_5 q_2^b(t_2 n)\right) h_v^c(0)|H(v)\rangle = f_H^{(1)}\,\Psi_3^s(t_1,t_2)$$

$$\epsilon^{abc}\langle 0|\left(q_1^a(t_1 n)C\gamma_5 i\sigma_{\bar{n}n} q_2^b(t_2 n)\right) h_v^c(0)|H(v)\rangle = 2f_H^{(1)}\,\Psi_3^\sigma(t_1,t_2)$$

$$\epsilon^{abc}\langle 0|\left(q_1^a(t_1 n)C\gamma_5 \not{\bar{n}} q_2^b(t_2 n)\right) h_v^c(0)|H(v)\rangle = f_H^{(2)}\Psi_4(t_1,t_2)$$

$q_i = u, d, s$ – light quark fields

C – charge conjugation matrix

$n^\mu,\ \bar{n}^\mu$ – two light-like vectors $(n\bar{n}) = 2$

Frame is adopted: $v^\mu = (n^\mu + \bar{n}^\mu)/2$

Light fields

Light-quark fields living on the light cone
assumed to be multiplied by the Wilson lines

$$q(tn) = [0, tn]\, q(tn) = \mathrm{P}\exp\left\{-ig_{\mathrm{st}}\, t \int_0^1 d\alpha\, n^\mu A_\mu(\alpha tn)\right\} q(tn)$$

Considered as generating function of formal expansion

$$q(tn) = \sum_{N=0}^{\infty} \frac{t^N}{N!}\, (n^\mu D_\mu)^N\, q(0)$$

The covariant derivative $D_\mu = \partial_\mu - ig_{\mathrm{st}} A_\mu$

Similar for the gluonic field

$$G_{\mu\nu}(tn) = [0, tn]\, G_{\mu\nu}(tn)$$

Heavy quark field

The heavy-quark field living on the light cone also includes the Wilson line but time-like [Korchemsky, Radushkin (1992)]

$$h_v(0) = \mathrm{P}\exp\left\{ i g_{\mathrm{st}} \int_{-\infty}^{0} d\alpha\, v^\mu\, A_\mu(\alpha v) \right\} \phi(-\infty)$$

Sterile field $\phi(-\infty)$ was introduced

LCDAs

Couplings $f_H^{(i)}$ are defined by local operators

$$\epsilon^{abc}\langle 0| \left(q_1^a(0) C\gamma_5 q_2^b(0)\right) h_v^c(0)|H(v)\rangle = f_H^{(1)}$$
$$\epsilon^{abc}\langle 0| \left(q_1^a(0) C\gamma_5 \not{v} q_2^b(0)\right) h_v^c(0)|H(v)\rangle = f_H^{(2)}$$

Scale dependence of the couplings (NLO order):

$$f_H^{(i)}(\mu) = f_H^{(i)}(\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_1^{(i)}/\beta_0}\left[1 - \frac{\alpha_s(\mu_0)-\alpha_s(\mu)}{4\pi}\frac{\gamma_1^{(i)}}{\beta_0}\left(\frac{\gamma_2^{(i)}}{\gamma_1^{(i)}} - \frac{\beta_1}{\beta_0}\right)\right]$$

Example: Λ_b-baryon

NLO QCD sum rules [Groote et all., 1997]

$$f_{\Lambda_b}^{(1)}(\mu_0 = 1\ \mathrm{GeV}) \simeq f_{\Lambda_b}^{(2)}(\mu_0 = 1\ \mathrm{GeV}) \simeq 0.030 \pm 0.005\ \mathrm{GeV}^3$$

Supported by the non-relativistic constituent quark picture

LCDAs

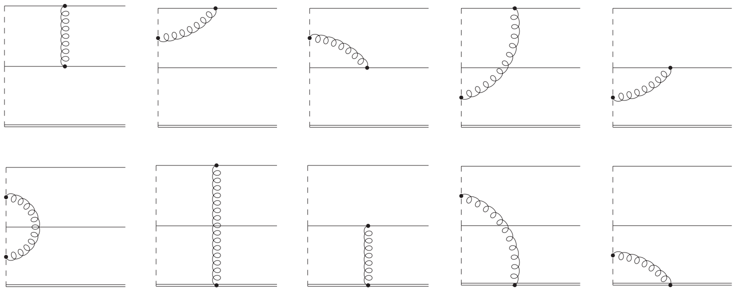
LCDAs $\Psi_i(t_1, t_2)$ are scale dependent

Fourier transrorm to the momentum space:

$$\begin{aligned}\Psi(t_1, t_2) &= \int_0^\infty d\omega_1 \int_0^\infty d\omega_2\, e^{-it_1\omega_1 - it_2\omega_2} \psi(\omega_1, \omega_2) \\ &= \int_0^\infty \omega\, d\omega \int_0^1 du\, e^{-i\omega(t_1 u + t_2 \bar{u})}\, \widetilde{\psi}(\omega, u)\end{aligned}$$

$\omega_1 = u\omega$, $\omega_2 = (1-u)\omega = \bar{u}\omega$ – energies of light quarks
LO evolution equation for $\psi_2(\omega_1, \omega_2; \mu)$: derived by identifying UV singularities of one-gluon-exchange diagrams

One-gluon exchange diagrams



LCDAs

Evolution equation is expressed in terms of two-particle kernels from evolution equations for B- and pseudoscalar mesons

$$\mu \frac{d}{d\mu} \psi_2(\omega_1, \omega_2; \mu) = -\frac{\alpha_s(\mu)}{2\pi} \frac{4}{3} \left\{ \int_0^\infty d\omega_1' \, \gamma^{\mathrm{LN}}(\omega_1', \omega_1; \mu) \, \psi_2(\omega_1', \omega_2; \mu) \right.$$
$$+ \int_0^\infty d\omega_2' \, \gamma^{\mathrm{LN}}(\omega_2', \omega_2; \mu) \, \psi_2(\omega_1, \omega_2'; \mu)$$
$$\left. - \int_0^1 dv \, V(u, v) \, \psi_2(v\omega, \bar{v}\omega; \mu) + \frac{3}{2} \psi_2(\omega_1, \omega_2; \mu) \right\}$$

Kernel $\gamma^{\mathrm{LN}}(\omega', \omega; \mu)$ controlling evolution of the B-meson LCDA

$V(u, v)$ is the ER-BL kernel

Term $3\psi_2/2$ results from $f_H^{(2)}$ renormalization subtraction

Evolution equation can be solved either numerically or semi-analytically [Braun et al., 2008]

LCDAs

$SU(3)_F$ sextet $\quad J^P = j^p = 1^+ \quad$ axial-vector state

Non-local light-ray operators (longitudinal polarization)

$$\epsilon^{abc}\langle 0|\left(q_1^a(t_1) C \not{n} q_2^b(t_2)\right) h_v^c(0)|H(v,\varepsilon)\rangle = (\bar{v}\varepsilon)\, f_H^{(2)}\, \Psi_2^{\parallel}(t_1,t_2)$$

$$\epsilon^{abc}\langle 0|\left(q_1^a(t_1) C\, q_2^b(t_2)\right) h_v^c(0)|H(v,\varepsilon)\rangle = (\bar{v}\varepsilon)\, f_H^{(1)}\, \Psi_3^{\parallel s}(t_1,t_2)$$

$$\epsilon^{abc}\langle 0|\left(q_1^a(t_1) C\, i\sigma_{\bar{n}n} q_2^b(t_2)\right) h_v^c(0)|H(v,\varepsilon)\rangle = 2\,(\bar{v}\varepsilon)\, f_H^{(1)}\, \Psi_3^{\parallel a}(t_1,t_2)$$

$$\epsilon^{abc}\langle 0|\left(q_1^a(t_1) C \not{\bar{n}} q_2^b(t_2)\right) h_v^c(0)|H(v,\varepsilon)\rangle = -\,(\bar{v}\varepsilon)\, f_H^{(2)}\, \Psi_4^{\parallel}(t_1,t_2)$$

$$\bar{v}^\mu = (\bar{n}^\mu - n^\mu)/2 \qquad (v\bar{v}) = 0 \qquad (\bar{v}\bar{v}) = -1$$

$$\varepsilon^\mu = \varepsilon_\parallel^\mu + \varepsilon_\perp^\mu \qquad \varepsilon_\parallel^\mu = \eta\bar{v}^\mu \qquad \sigma_{\bar{n}n} = i\,(\not{\bar{n}}\not{n} - \not{n}\not{\bar{n}})/2$$

LCDAs

$SU(3)_F$ sixtet $J^P = j^p = 1^+$ axial-vector state

Non-local light-ray operators (transverse polarization)

$$\epsilon^{abc}\langle 0|\left(q_1^a(t_1) C\,\gamma_\perp^\mu \not{n} q_2^b(t_2)\right) h_v^c(0)|H(v,\varepsilon)\rangle = f_H^{(2)}\,\Psi_2^\perp(t_1,t_2)\,\varepsilon_\perp^\mu$$

$$\epsilon^{abc}\langle 0|\left(q_1^a(t_1) C\,\gamma_\perp^\mu q_2^b(t_2)\right) h_v^c(0)|H(v,\varepsilon)\rangle = f_H^{(1)}\,\Psi_3^{\perp s}(t_1,t_2)\,\varepsilon_\perp^\mu$$

$$\epsilon^{abc}\langle 0|\left(q_1^a(t_1) C\,\gamma_\perp^\mu i\sigma_{\bar{n}n} q_2^b(t_2)\right) h_v^c(0)|H(v,\varepsilon)\rangle = 2 f_H^{(1)}\,\Psi_3^{\perp a}(t_1,t_2)\,\varepsilon_\perp^\mu$$

$$\epsilon^{abc}\langle 0|\left(q_1^a(t_1) C\,\gamma_\perp^\mu \not{\bar{n}} q_2^b(t_2)\right) h_v^c(0)|H(v,\varepsilon)\rangle = f_H^{(2)}\,\Psi_4^\perp(t_1,t_2)\,\varepsilon_\perp^\mu$$

$$\gamma_\perp^\mu = \gamma^\mu - \left(\not{\bar{n}}\not{n} + \not{n}\not{\bar{n}}\right)/2$$

LCDAs

Switching on the heavy quark spin

r.h.s. of matrix elements of all non-local operators must be multiplied on the Dirac spinor $U(v)$ of the heavy quark h_v

$$\not v\, U(v) = U(v) \qquad \overline{U}(v)\, U(v) = 1$$

Scalar state: $J^P = j^p = 0^+ \Longrightarrow J^P = 1/2^+ : \quad H(v) \equiv U(v)$

Axial-vector state: $J^P = j^p = 1^+ \Longrightarrow J^P = 1/2^+, J^P = 3/2^+$

$$\begin{aligned} \varepsilon_\mu\, U(v) &= \left[\varepsilon_\mu\, U(v) - \frac{1}{3}(\gamma_\mu + v_\mu) \not\varepsilon\, U(v)\right] + \frac{1}{3}(\gamma_\mu + v_\mu) \not\varepsilon\, U(v) \\ &\equiv R_\mu^{3/2}(v) + \frac{1}{3}(\gamma_\mu + v_\mu)\, H(v) \end{aligned}$$

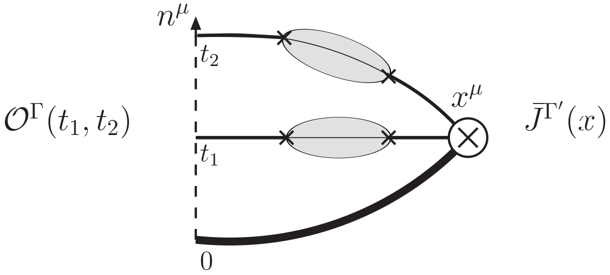
Rarita-Schwinger vector-spinor $R_\mu^{3/2}(v)$:

$$\not v\, R_\mu^{3/2}(v) = R_\mu^{3/2}(v), \quad v^\mu\, R_\mu^{3/2}(v) = 0, \quad \gamma^\mu\, R_\mu^{3/2}(v) = 0$$

QCD Sum Rules

Models for LCDAs can be obtained using QCD sum rules

Correlation functions involve the non-local light-ray operators and a suitable local current



QCD Sum Rules

Heavy baryon local operators

$$\bar{J}^{\Gamma'}(x) = \epsilon^{abc}\left(\bar{q}_2^a(x)\,[A + B\,\not{v}]\,\Gamma' C^T \bar{q}_1^b(x)\right)\bar{h}_v^c(x)$$

Arbitrariness in the choice of local currents (variation in $A \in [0,1]$ and $B = 1 - A$) is adopted as an error estimate

Result are calculated for $A = B = 1/2$: supported by a constituent quark model picture [Braun et al., 2008]

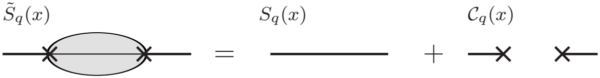
$$j^p = 0^+ \Longrightarrow \Gamma' = \gamma_5$$

$$j^p = 1^+ \Longrightarrow \Gamma' = \gamma_{\|},\ \gamma_{\perp}$$

QCD Sum Rules

Propagators of the light quark fields $\tilde{S}_q(x)$ are not free

To take effects of the QCD background inside baryons into account, method of non-local condensates is used



$$S_q(x) = \frac{i\not{x}}{2\pi^2 x^4} - \frac{m}{4\pi^2 x^2} \qquad \mathcal{C}_q(x) = \frac{1}{12}\langle \bar{q}(x) q(0) \rangle$$

QCD sum rules

General parametrization [Mikhailov, Radyushkin, 1986, 1992]

$$\mathcal{C}_q(x) = \langle \bar{q}q \rangle \int_0^\infty d\nu \, e^{\nu x^2/4} f(\nu)$$

Non-local condensate shape is chosen according to the model [Braun et el., 1994; Braun et al., 2003]

$$f(\nu) = \frac{\lambda^{a-2}}{\Gamma(a-2)} \nu^{1-a} e^{-\lambda/\nu}, \qquad a = 3 + \frac{4\lambda}{m_0^2}$$

Parameters included:

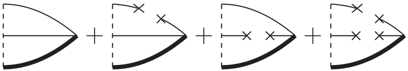
$\langle \bar{q}q \rangle$ — local quark condensate,
$\lambda = \langle \bar{q} D^2 q \rangle$ — correlation length,
$m_0^2 = \langle \bar{q} g_{\rm st} \sigma_{\mu\nu} G^{\mu\nu} q \rangle / \langle \bar{q}q \rangle$ — ratio of local mixed quark-gluon and quark condensates

QCD sum rules

Double Fourier transform of the correlation function

$$\Pi_{\Gamma\Gamma'}(\omega_1, \omega_2; E) = i \int_{-\infty}^{\infty} \frac{dt_1\, dt_2}{(2\pi)^2}\, e^{i(\omega_1 t_1 + \omega_2 t_2)} \int d^4x\, e^{-iE(vx)} \langle 0|\mathcal{O}^{\Gamma}(t_1, t_2)\, \bar{J}^{\Gamma'}(x)|0\rangle$$

In momentum space, correlation function reads diagrammatically

$\Pi(\omega, u; E) =$ 

Heavy quark condensate term is suppressed by $1/m_Q$

Sum rule reads

$$|f_H|^2\, \psi^{\Gamma}(\omega, u)\, e^{-\bar{\Lambda}_H/\tau} = \mathbb{B}[\Pi](\omega, u; \tau, s_0)$$

$\mathbb{B}$ means the Borel-transform, $\bar{\Lambda}_H = m_H - m_Q$

s_0 – momentum cutoff from applying the quark-hadron duality

QCD sum rules

Analytic result for leading-twist transverse LCDA at $\mu_0 = 1$ GeV

$$f_H^{(2)} \left[A\, f_H^{(1)} + B\, f_H^{(2)} \right] \tilde{\psi}_2^{SR}(\omega, u)\, \mathrm{e}^{-\bar{\Lambda}/\tau} =$$

$$\frac{3\tau^4}{2\pi^4} \left[B\hat{\omega}^2\, u\bar{u} + A\,\hat{\omega}\,(\hat{m}_2 u + \hat{m}_1 \bar{u}) \right] E_1(2\hat{s}_\omega)\mathrm{e}^{-\hat{\omega}}$$

$$- \frac{\langle \bar{q}_1 q_1 \rangle \tau^3}{\pi^2} \left[A\hat{\omega}\bar{u} + B\hat{m}_2 \right] f(2\tau\omega u)\, E_{2-a}(2\hat{s}_\kappa)\, \mathrm{e}^{-\hat{\omega}}$$

$$- \frac{\langle \bar{q}_2 q_2 \rangle \tau^3}{\pi^2} \left[A\hat{\omega} u + B\hat{m}_1 \right] f(2\tau\omega\bar{u})\, E_{2-a}(2\hat{s}_{\bar{\kappa}})\, \mathrm{e}^{-\hat{\omega}}$$

$$+ \frac{2B}{3} \langle \bar{q}_1 q_1 \rangle \langle \bar{q}_2 q_2 \rangle\, \tau^2\, f(2\tau\omega u)\, f(2\tau\omega\bar{u})\, E_{3-2a}(2\hat{s}_{\kappa\bar{\kappa}})\, \mathrm{e}^{-\hat{\omega}},$$

The following function was introduced

$$E_a(x) = \frac{1}{\Gamma(a+1)} \int_0^x dt\, t^a \mathrm{e}^{-t} = 1 - \frac{\Gamma(a+1, x)}{\Gamma(a+1)}$$

$$\bar{\Lambda} = m_H - m_b, \quad s_\omega = s_0 - \omega/2, \quad \kappa = \lambda/(2u\omega\tau), \quad \bar{\kappa} = \lambda/(2\bar{u}\omega\tau)$$

$$\hat{\omega} = \omega/(2\tau), \quad \hat{s}_\omega = s_\omega/(2\tau), \quad \hat{m}_{1,2} = m_{1,2}/(2\tau)$$

$$\hat{s}_\kappa = \hat{s}_\omega - \kappa/2, \quad \hat{s}_{\bar{\kappa}} = \hat{s}_\omega - \bar{\kappa}/2, \quad \hat{s}_{\kappa\bar{\kappa}} = \hat{s}_\omega - \kappa/2 - \bar{\kappa}/2$$

QCD sum rules

Normalization of symmetric LCDAs ($t = 2, 3s, 4$)

$$\int_0^{2s_0} \omega d\omega \int_0^1 du\, \tilde{\psi}_t^{\rm SR}(\omega, u) \equiv 1$$

Normalization of antisymmetric LCDAs ($t = 3\sigma$) can be fixed by

$$\int_0^{2s_0} \omega d\omega \int_0^1 du\, C_1^{1/2}(2u-1)\, \tilde{\psi}_t^{\rm SR}(\omega, u) \equiv 1$$

Here, $C_n^m(x)$ are the Gegenbauer polinomials

QCD sum rules

QCD sum rules constrain certain moments

$$\langle f(\omega, u)\rangle_k \equiv \int_0^{2s_0} \omega d\omega \int_0^1 du\, f(\omega, u)\, \tilde{\psi}_t^{\mathrm{SR}}(\omega, u)$$

Numerical values of the parameters

$\bar{\Lambda}_{\Lambda_b}$	0.8 GeV	$s_0^{(\Lambda_b)}$	1.2 GeV
$\bar{\Lambda}_{\Xi_b}$	1.0 GeV	$s_0^{(\Xi_b)}$	1.3 GeV
τ	0.6 ± 0.2	m_s (1 GeV)	128 ± 21 MeV
$\langle \bar{q}q \rangle$ (1 GeV)	$-(242^{+28}_{-19})$ MeV^3	$\langle \bar{s}s \rangle / \langle \bar{q}q \rangle$	0.8 ± 0.3
m_0^2	0.8 ± 0.2 GeV^2	λ	0.16 GeV^2

Numerical analysis

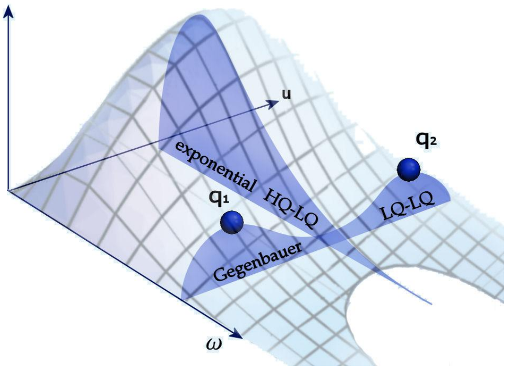
Numerical values of first several moments

H_Q	t	$\langle\omega^{-1}\rangle$	$\langle C_1^{3/2}\rangle$	$\langle\omega^{-1}C_1^{3/2}\rangle$	$\langle C_2^{3/2}\rangle$	$\langle\omega^{-1}C_2^{3/2}\rangle$
Λ_b	2	$1.65^{+0.91}_{-0.47}$	0	0	$1.00^{+0.54}_{-1.03}$	$0.61^{+0.76}_{-1.45}$
Ξ_b	2	$1.61^{+0.71}_{-0.42}$	$0.10^{+0.10}_{-0.06}$	$0.08^{+0.07}_{-0.04}$	$0.98^{+0.49}_{-0.82}$	$0.69^{+0.63}_{-1.07}$

H_Q	t	$\langle\omega^{-1}\rangle$	$\langle C_1^{1/2}\rangle$	$\langle\omega^{-1}C_1^{1/2}\rangle$	$\langle C_2^{1/2}\rangle$	$\langle\omega^{-1}C_2^{1/2}\rangle$
Λ_b	3s	$2.16^{+0.70}_{-0.36}$	0	0	$-0.032^{+0.022}_{-0.041}$	$-0.29^{+0.14}_{-0.27}$
	3σ	0	1	$1.54^{+0.14}_{-0.22}$	0	0
	4	$2.84^{+0.88}_{-0.46}$	0	0	$-0.108^{+0.035}_{-0.018}$	$-0.41^{+0.08}_{-0.15}$
Ξ_b	3s	$2.08^{+0.50}_{-0.29}$	$0.11^{+0.10}_{-0.06}$	$0.063^{+0.080}_{-0.047}$	$0.87^{+0.08}_{-0.14}$	$0.84^{+0.27}_{-0.45}$
	3σ	$0.00054^{+0.00033}_{-0.00054}$	1	$1.51^{+0.12}_{-0.19}$	$0.054^{+0.033}_{-0.054}$	$0.098^{+0.061}_{-0.098}$
	4	$2.73^{+0.61}_{-0.35}$	$0.12^{+0.09}_{-0.05}$	$0.05^{+0.09}_{-0.05}$	$0.55^{+0.18}_{-0.11}$	$0.99^{+0.16}_{-0.09}$

Numerical analysis

Model functions for the *b*-baryon LCDAs, composed of the exponential part for the heavy-light interaction and the Gegenbauer polynomials for the light-light interaction



Numerical analysis

Proposed simple models for LCDAs at the scale $\mu_0 = 1$ GeV

$$\tilde{\psi}_2(\omega, u) = \omega^2 u(1-u) \sum_{n=0}^{2} \frac{a_n^{(2)}}{\epsilon_n^{(2)\,4}} C_n^{3/2}(2u-1)\, \mathrm{e}^{-\omega/\epsilon_n^{(2)}},$$

$$\tilde{\psi}_{3s}(\omega, u) = \frac{\omega}{2} \sum_{n=0}^{2} \frac{a_n^{(3)}}{\epsilon_n^{(3)\,3}} C_n^{1/2}(2u-1)\, \mathrm{e}^{-\omega/\epsilon_n^{(3)}},$$

$$\tilde{\psi}_{3\sigma}(\omega, u) = \frac{\omega}{2} \sum_{n=0}^{3} \frac{b_n^{(3)}}{\eta_n^{(3)\,3}} C_n^{1/2}(2u-1)\, \mathrm{e}^{-\omega/\eta_n^{(3)}},$$

$$\tilde{\psi}_4(\omega, u) = \sum_{n=0}^{2} \frac{a_n^{(4)}}{\epsilon_n^{(4)\,2}} C_n^{1/2}(2u-1)\, \mathrm{e}^{-\omega/\epsilon_n^{(4)}},$$

Numerical analysis

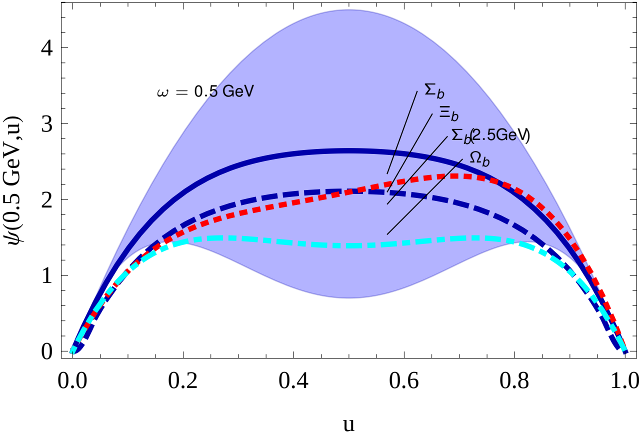
Estimates of the parameters entering the theoretical models for the heavy baryon LCDAs

H_Q	t	$\varepsilon_0^{(t)}$	$\varepsilon_1^{(t)}$	$\varepsilon_2^{(t)}$	$a_1^{(t)}$	$a_2^{(t)}$
Λ_b	2	$0.201^{+0.143}_{-0.059}$	0	$0.551^{+0.550}_{-0.356}$	0	$0.391^{+0.279}_{-0.279}$
	3	$0.232^{+0.047}_{-0.056}$	0	$0.055^{+0.010}_{-0.020}$	0	$-0.161^{+0.108}_{-0.207}$
	4	$0.352^{+0.067}_{-0.083}$	0	$0.262^{+0.116}_{-0.132}$	0	$-0.541^{+0.173}_{-0.090}$
Ξ_b	2	$0.207^{+0.073}_{-0.063}$	$0.461^{+0.620}_{-0.284}$	$0.469^{+0.560}_{-0.559}$	$0.058^{+0.058}_{-0.034}$	$0.380^{+0.189}_{-0.319}$
	3	$0.218^{+0.043}_{-0.047}$	$0.877^{+0.820}_{-0.152}$	$0.049^{+0.005}_{-0.012}$	$0.037^{+0.032}_{-0.019}$	$-0.027^{+0.016}_{-0.027}$
	4	$0.378^{+0.065}_{-0.080}$	$2.291^{+2.291}_{-0.842}$	$0.286^{+0.130}_{-0.150}$	$0.039^{+0.030}_{-0.018}$	$-0.090^{+0.037}_{-0.021}$

H_Q	t	$\eta_1^{(t)}$	$\eta_2^{(t)}$	$\eta_3^{(t)}$	$b_2^{(t)}$	$b_3^{(t)}$
Λ_b	3	$0.324^{+0.054}_{-0.026}$	0	$0.633^{+0.092}_{-0.050}$	0	$-0.240^{+0.240}_{-0.147}$
Ξ_b	3	$0.218^{+0.043}_{-0.047}$	$0.877^{+0.820}_{-0.152}$	$0.049^{+0.005}_{-0.012}$	$0.037^{+0.032}_{-0.019}$	$-0.027^{+0.016}_{-0.027}$

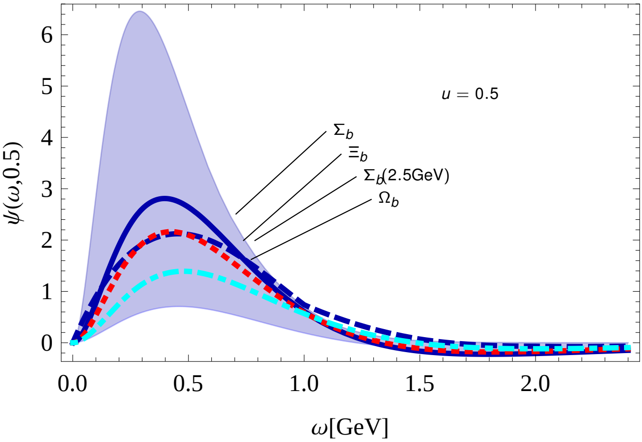
Numerical analysis

Twist-2 LCDAs of Σ (blue), Ξ (red) and Ω (cyan) baryons at the energy scales $\mu_0 = 1$ GeV estimated within the range $A \in [0, 1]$



Numerical analysis

Twist-2 LCDAs of Σ (blue), Ξ (red) and Ω (cyan) baryons at the energy scales $\mu_0 = 1$ GeV estimated within the range $A \in [0, 1]$



Renormalization of higher twist operators

Renormalization of heavy-light light-ray operators up to twist-three was performed by Knoedlseder, Offen (2011)

Used the spinor formalism applied to QCD by Braun, Manashov, etc.

Corresponding evolution equations for the twist-three operators are written explicitly

Classification of the four-partical (with three quarks and gluon) baryonic operators was not presented

Conclusions

- The total set of the non-local light-ray operators for the ground-state heavy baryons is constructed in the framework of HQET
- Their matrix elements between the heavy-baryon state and vacuum determine the LCDAs of different twist through the diquark operator
- First several moments are calculated within the method of QCD sum rules
- Simple theoretical models for the LCDAs have been proposed and their parameters are fitted based on the QCD sum rules estimations
- $SU(3)_F$ breaking effects are of order 10%