Light and Heavy Hadrons in AdS/QCD

Valery Lyubovitskij

Institut für Theoretische Physik, Universität Tübingen
Kepler Center for Astro and Particle Physics, Germany

EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

in collaboration with

Thomas Gutsche
Ivan Schmidt
Alfredo Vega

PRD 87 (2013) 016017, 057001
PRD 86 (2012) 036007, 076003, 096004
PRD 83 (2011) 036001
PRD 82 (2010) 074022
PRD 80 (2009) 055014

"Physics of Heavy Quarks and Hadrons", 18 July 2013, Dubna/Russia

Outline

- Introduction
  - Motivation
  - Classification of AdS/QCD models
- Mesons in soft-wall AdS/QCD
  - Basic blocks of effective action
  - EOM for the bulk profiles: mass spectra and wave functions
  - Hadronic wave function
  - Light and heavy mesons: mass spectrum and decay constants
- Baryons in soft-wall AdS/QCD
  - Effective action
  - Inclusion of high Fock states
  - Inclusion of photons
  - Nucleons: electromagnetic form factors and parton distributions
  - Nucleon resonances: Roper resonance $N(1440)$

Introduction

- AdS/QCD $\equiv$ Holographic QCD (HQCD) – approximation to QCD: attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – anti-de Sitter (AdS) space

- HQCD models reproduce main features of QCD at low and high energies: chiral symmetry, confinement, power scaling of hadron form factors

- Motivation: AdS/CFT correspondence 1998 (Maldacena, Polyakov, Witten et al) Extra-Dimensional (ED) theories including gravity are holographically equivalent to gauge theories living on boundary of ED space

- Symmetry arguments: Conformal group acting in boundary theory isomorphic to $SO(4,2)$ – the isometry group of AdS_5 space

Introduction

- AdS metric   Poincaré form

  $$ds^2 = g_{MN}(z)\, dx^M dx^N = \frac{R^2}{z^2}\left(dx_\mu dx^\mu - dz^2\right)$$

- Metric Tensor $g_{MN}(z) = \epsilon^a_M(z)\epsilon^b_N(z)\eta_{ab}$

- Vielbein $\epsilon^a_M(z) = \frac{R}{z}\,\delta^a_M$

- Manifestly scale-invariant $x \to \lambda x,\ z \to \lambda z$.

- z – extra dimensional (holographic) coordinate; $z = 0$ is UV boundary

- Light-Front Holography (Brodsky-Teramond) $z \to \zeta$ with $\zeta^2 = \mathbf{b}_\perp^2 x(1-x)$

- $\mathbf{b}_\perp$ – impact-parameter separation between partons in hadron

- x – Bjorken scaling variable (fraction of longitudinal momentum of active quark)

Introduction

- Conformal group contains 15 generators:

  10 Poincaré (4 translations P_μ, 6 Lorentz transformations $M_{\mu\nu}$),
  5 conformal (4 conformal boosts K_μ, 1 dilatation D):

$M_{\mu\nu} = i(x_\mu \partial_\nu - x_\nu \partial_\mu)$	rotational symmetry
$D = i(x\,\partial)$	energy
$P_\mu = i\partial_\mu$	raising energy
$K_\mu = 2ix_\mu(x\,\partial) - ix^2\partial_\mu$	lowering energy

- Isomorphic to $SO(4,2)$ – the isometry group of AdS$_5$ space

- Fields in AdS$_5$ are classified by unitary, irreducible representations of $SO(4,2)$

- $SO(4,2)$ is decomposed with respect to $SO(4) \times SO(2)$
  $SO(4)$ is isomorphic to $SU(2) \times SU(2)$: use spins J_1 and J_2 for classification

- Irreducible representations $D(E_0, J_1, J_2)$ two spins J_1, J_2 and energy E_0
  (corresponds to Δ – conformal dimension of operators in CFT)

Introduction

- Scalar $D(E_0, 0, 0)$
- Vector $D\left(E_0, \frac{1}{2}, \frac{1}{2}\right)$
- Fermions of spin $J = 1/2$

  $D\left(E_0, 0, \frac{1}{2}\right) \oplus D\left(E_0, \frac{1}{2}, 0\right)$
- Fermions of spin $J = 3/2$

  $D\left(E_0, 1, \frac{1}{2}\right) \oplus D\left(E_0, \frac{1}{2}, 1\right)$
- Spin J totally symmetric tensor with $J \geq 2$

  $D\left(E_0, \frac{J}{2}, \frac{J}{2}\right)$
- Spin J totally symmetric spinor-tensor with $J \geq 5/2$

  $D\left(E_0, \frac{J+1/2}{2}, \frac{J-1/2}{2}\right) \oplus D\left(E_0, \frac{J-1/2}{2}, \frac{J+1/2}{2}\right)$

Introduction

- Action for scalar field

$$S_\Phi = \frac{1}{2} \int d^d x dz \, \sqrt{g} \Big(\partial_M \Phi(x,z) \partial^M \Phi(x,z) - m^2 \, \Phi(x,z) \Big)$$

- $g = |\mathrm{det} g_{MN}|$
- m – 5D mass, $m^2 R^2 = \Delta(\Delta - 4)$.
- Kaluza-Klein (KK) expansion $\Phi(x,z) = \sum_n \phi_n(x) \, \Phi_n(z)$
- Tower of KK modes $\phi_n(x)$ dual to 4-dimensional fields describing hadrons
- Bulk profiles $\Phi_n(z)$ — dual to hadronic wave functions
- With $-\partial_\mu \partial^\mu \phi_n(x) = M_n^2 \phi_n(x)$ follows
- Equation of motion for $\Phi_n(z)$

$$-z^{d+1} \partial_z \Big(z^{-d+1} \partial_z \Phi_n(z) \Big) + m^2 \Phi_n(z) = M_n^2 \Phi_n(z)$$

Introduction

- Scattering problem for AdS field gives information about propagation of external field from z to the boundary $z = 0$ — bulk-to-boundary propagator $\Phi_{\rm ext}(q,z)$ [Fourier-trasform of AdS field $\Phi_{\rm ext}(x,z)$]:

$$\Phi_{\rm ext}(q,z) = \int d^dx e^{-iqx}\, \Phi_{\rm ext}(x,z)$$

- $-z^{d+1}\partial_z\Big(z^{-d+1}\partial_z\Phi_{\rm ext}(q,z)\Big) + m^2\Phi_{\rm ext}(q,z) = q^2\Phi_{\rm ext}(q,z)$

- Hadron properties
$F_n(q^2) = \int\limits_0^\infty dz\Phi_{\rm ext}(q,z)\Phi_n^2(z)$

- Hadron structure is implemented by a nontrivial dependence of AdS fields on 5-th (holographic) coordinate

Introduction

- Top-down approaches Low-energy approximation of string theory trying to find a gravitational background with features similar to QCD (e.g. Sakai-Sugimoto model)
- Bottom-up approaches More phenomenological use the features of QCD to construct 5D dual theory including gravity on AdS space
- Towards to QCD:
  – Break conformal invariance and introduce confinement
- Hard-wall: Polchinski and Strassler, PRL 88 (2002) 031601
  AdS geometry is cutted by two branes UV $(z = \epsilon \to 0)$ and IR $(z = z_{\rm IR})$
  Analogue of quark bag model, linear dependence of hadron masses $M_n \sim J(L)$
- Soft-wall: Karch, Katz, Son, Stephanov, PRD 74 (2006) 015005
  Soft cuttoff of AdS space by dilaton field $e^{-\varphi(z)}$, $\varphi(z) = \kappa^2 z^2$
  Analytical solution of EOM, Regge behavior $M_n^2 \sim J(L)$

Mesons: scalar fields

- "Positive dilaton": Brodsky, Téramond

$$S_\Phi^+ = \frac{1}{2}\int d^dx dz\, \sqrt{g}\, e^{\varphi(z)} \left[\partial_M \Phi_+ \partial^M \Phi_+ - m^2\, \Phi_+^2\right]$$

- "Negative dilaton": Gutsche, Lyubovitskij, Schmidt, Vega PRD 85 (2012) 076003

$$S_\Phi^- = \frac{1}{2}\int d^dx dz\, \sqrt{g}\, e^{-\varphi(z)} \left[\partial_M \Phi_- \partial^M \Phi_- - (m^2 + U(z))\, \Phi_-^2\right]$$

Potential

$$U(z) = \frac{z^2}{R^2}\left(\varphi''(z) + \frac{1-d}{z}\varphi'(z)\right)$$

- "No-wall"

$$S_\Phi = \frac{1}{2}\int d^dx dz \sqrt{g} \left[\partial_M \Phi \partial^M \Phi - (m^2 + V(z))\Phi^2\right]$$

Potential

$$V(z) = \frac{z^2}{R^2}\left(\frac{1}{2}\varphi''(z) + \frac{1}{4}(\varphi'(z))^2 + \frac{1-d}{2z}\varphi'(z)\right)$$

- All 3 actions are equivalent

upon the field rescaling $\Phi_\pm = e^{\mp\varphi(z)/2}\Phi$ and $\Phi_\pm = e^{\mp\varphi(z)}\Phi_\mp$

Mesons: scalar fields

- Warping of the metric
- Instead of conformal metric $ds^2 = g_{MN}(z)\, dx^M dx^N = e^{2A(z)} \left(dx_\mu dx^\mu - dz^2 \right)$

  with $A(z) = \log(R/z)$
- Consider “warping metric” with $A_W(z)$
- Inclusion of the potential

$$W(z) = m^2 \left[e^{2A_W(z)} - e^{2A(z)} \right] + \frac{(d-1)^2}{4} \left[A'_W(z) - A'(z) \right]^2 + \frac{d-1}{2} \left[A''_W(z) - A''(z) \right]$$

makes it equivalent to the actions considered before.

Mesons: scalar fields

- KK expansion

$$\Phi(x,z) = \sum_n \psi_n(x)\,\Phi_n(z)$$

- Substitution

$$\Phi_n(z) = \left(\frac{R}{z}\right)^{1-d} \phi_n(z)$$

- Identify $\Delta = \tau = N + L$ (here $N = 2$ – number of partons in meson)

$$\left[-\frac{d^2}{dz^2} + \frac{4L^2-1}{4z^2} + \kappa^4 z^2 - 2\kappa^2\right]\phi_n(z) = M_n^2 \phi_n(z)$$

- Solutions: $\phi_{nL}(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L+1)}}\,\kappa^{L+1}\, z^{L+1/2}\, e^{-\kappa^2 z^2/2}\, L_n^L(\kappa^2 z^2)$

- $M_{nL}^2 = 4\kappa^2\left(n + \frac{L}{2}\right)$

- Massless pion $M_\pi^2 = 0$ for $n = L = 0$ Brodsky, Téramond

Mesons: scalar mesons

- Extension to arbitrrary twist $\tau = N + L$
- $\phi_{n\tau}(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\tau-1)}}\, \kappa^{\tau-1}\ z^{\tau-3/2}\ e^{-\kappa^2 z^2/2}\ L_n^{\tau-2}(\kappa^2 z^2)$
- $M^2_{n\tau} = 4\kappa^2\left(n + \frac{\tau}{2} - 1\right)$

Mesons: higher J boson fields

- $\Phi_J = \Phi_{M_1 \cdots M_J}(x,z)$ – a symmetric, traceless tensor:

  Brodsky, Téramond

  $$S_\Phi^+ = \frac{1}{2}\int d^dx dz\, \sqrt{g}\, e^{\phi(z)} \left(\partial_N \Phi_J^+ \partial^N \Phi^{J,+} - m_J^2\, \Phi_J^+ \Phi^{J,+}\right)$$

  Our

  $$S_\Phi^- = \frac{1}{2}\int d^dx dz\, \sqrt{g}\, e^{-\phi(z)} \left(\partial_N \Phi_J^- \partial^N \Phi^{J,-} - (m_J^2 + U_J(z))\, \Phi_J^- \Phi^{J,-}\right)$$

- $m_J^2 R^2 = (\Delta - J)(\Delta + J - d)$

- $\Delta = 2 + L$ and $\mu_J^2 = L^2 - (2-J)^2$ for $d = 4$

- Effective potential $U_J(z) = \frac{z^2}{R^2}\left[\varphi''(z) + \frac{1+2J-d}{z}\varphi'(z)\right]$

- $U_J(z) = 4\,\kappa^2 \left(\frac{z}{R}\right)^2 (J-1)$ for $d = 4$

Mesons: higher J boson fields

- KK decomposition $\Phi^{\nu_1 \cdots \nu_J}(x,z) = \sum_n \varphi_n^{\nu_1 \cdots \nu_J}(x)\, \Phi_{nJ}(z)$

- Substitution

$$\Phi_{nJ}(z) = \left(\frac{R}{z}\right)^{\frac{1-d}{2}} \varphi_{nJ}(z)$$

- Schrödinger EOM for $\Phi_{nJ}(z)$:

$$\left[-\frac{d^2}{dz^2} + U_J(z)\right]\varphi_{nJ}(z) = M_{nJ}^2 \varphi_{nJ}(z)$$

- Effective potential $U_J(z)$

$$U_J(z) = \kappa^4 z^2 + \frac{4a^2 - 1}{4z^2} + 2\kappa^2\left(b_J - 1\right).$$

- $$a = \frac{1}{2}\sqrt{d^2 + 4(\mu R)^2} = \Delta - \frac{d}{2}\,, \qquad b_J = J + \frac{4-d}{2}$$

Mesons: higher J boson fields

- Solutions:

$$\varphi_{nJ}(z) = \sqrt{\frac{2n!}{(n+a)!}}\, \kappa^{1+a}\, z^{1/2+a}\, e^{-\kappa^2 z^2/2} L_n^{a\,J}(\kappa^2 z^2)$$

$$M^2_{nJ} = 4\kappa^2\left(n + \frac{a+b_J}{2}\right)$$

- $a = L$ and $b_J = J$ at $d = 4$
- Finally

$$\varphi_{nJ}(z) = \sqrt{\frac{2n!}{(n+L)!}}\, \kappa^{1+L}\, z^{1/2+L}\, e^{-\kappa^2 z^2/2} L_n^{L}(\kappa^2 z^2)$$

$$M^2_{nJ} = 4\kappa^2\left(n + \frac{L+J}{2}\right)$$

- At $J(L) \to \infty$ $\quad M^2_{nJ} = 4\kappa^2(n+J)$
- Scaling $\Phi_{nJ} = z^{3/2}\,\varphi_{nJ} \sim z^{2+L}$ (not z^{2+J})

  $F_{\tau=2+L}(Q^2) \sim 1/(Q^2)^{\tau-1} \sim 1/(Q^2)^{L+1}$ independent on J

Mesons

- Interesting results deduced from the meson mass formula
- Two relations $\rho(770)$, $a_1(1270)$ and $f_0(600)$ mesons,

$$M_{a_1} = M_\rho \sqrt{2} = 2\kappa \,, \qquad M_{f_0} = M_\rho = \sqrt{2}\,\kappa$$

consistent with Weinberg, PRL 65, 1177 (1990) in limit of SBCS

- The same prediction relating the masses of ρ and a_1 mesons was also obtained by Weinberg, PRL 18, 507 (1967) on the basis of spectral function sum rules at $M_\pi = 0$
- Vector and axial-vector multiplets are not degenerate (even for higher values of J), because of the finite mass splitting of axial-vector and vector mesons states:

$$M_A^2 - M_V^2 = 2\kappa^2 \,.$$

- This means that we do not have parity doubling.

Meson: mass spectrum

- For the value $\kappa = 500$ MeV

$$
\begin{aligned}
M_\rho &= 721 \text{ MeV } (\text{data} : 775.49 \pm 0.34 \text{ MeV}) \,, \\
M_{a_1} &= 1010 \text{ MeV } (\text{data} : 1230 \pm 40 \text{ MeV})
\end{aligned}
$$

$M_{f_0} = 721$ MeV perfectly agrees with a model-independent result based on analyticity and unitarity of the S matrix:

$M_{f_0} = 735.0 \pm 6.1$ MeV
Surovtsev, Bydzovsky, Lyubovitskij, PRD 85, 036002 (2012)

Mesons: hadronic wave function

- Correspondence of holographic coordinate z to the impact variable ζ in LF suggested by Brodsky and Teramond

- Two parton case: $q_1\bar{q}_2$ mesons $z \to \zeta, \qquad \zeta^2 = \mathbf{b}_\perp^2 x(1-x)$
  ζ - impact variable; $\mathbf{b}_\perp$ - impact separation (conjugate to $\mathbf{k}_\perp$)

- Matching AdS/QCD and LF QCD

$$\psi_{nJ}(x,\zeta,m_1,m_2) = \psi_{\rm T}(\zeta)\cdot\psi_{\rm L}(x)\cdot\psi_{\rm A}(\varphi)$$

$\psi_T = \phi_{nJ}(\zeta)/\sqrt{\zeta}$ — transverse mode (from AdS/QCD)
$\psi_L = f(x,m_1,m_2)$ — longitudinal mode
$\psi_A = e^{im\varphi}/\sqrt{2\pi}$ — angular mode

$$\psi_{nJ}(x,\zeta,m_1,m_2) = \frac{\phi_{nL}(\zeta)}{\sqrt{2\pi\zeta}} f(x,m_1,m_2)e^{im\phi}$$

Mesons: hadronic wave function

- Modified meson mass formula

$$M_{nJ}^2 = 4\kappa^2 \Big(n + \frac{L+J}{2}\Big) + \int_0^1 dx \Big(\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\Big) f^2(x, m_1, m_2)$$

- Leptonic decay constants

$$f_M = \kappa \frac{\sqrt{6}}{\pi} \int_0^1 dx \sqrt{x(1-x)}\, f(x, m_1, m_2)$$

- Find $f(x, m_1, m_2)$ to fulfill the following constraints

- In sector of light quarks (consistency with ChPT):

  generate mechanism of explicit breaking of chiral symmetry
  Gell-Mann-Oakes-Renner (GMOR) $M_\pi^2 = 2\hat{m}\, B$
  Gell-Mann-Okubo (GMO) $4M_K^2 = M_\pi^2 + 3M_\eta^2$

Mesons: hadronic wave function

- In sector of heavy quarks (consistency with HQET)
- Leptonic decay constants

$f_{Q\bar{q}} \sim 1/\sqrt{m_Q}$ heavy–light mesons

$f_{Q\bar{Q}} \sim \sqrt{m_Q}$ heavy quarkonia

$f_{c\bar{b}} \sim m_c/\sqrt{m_b}$ at $m_c \ll m_b$

- Mass spectrum

  Expansion

$$\begin{aligned} M_{Q\bar{q}} &= m_Q + \bar{\Lambda} + \mathcal{O}(1/m_Q) \\ M_{Q\bar{Q}} &= 2m_Q + E + \mathcal{O}(1/m_Q) \end{aligned}$$

Splitting

$$M^V_{Q\bar{q}} - M^P_{Q\bar{q}} \sim \frac{1}{m_Q}$$

Light Mesons

- Following 't Hooft NPB 75 (1974) 461

$$f(x, m_1, m_2) = N\, x^{\alpha_1}\, (1-x)^{\alpha_2}$$

where N is the normalization constant

$$1 = \int_0^1 dx\, f^2(x, m_1, m_2)$$

α_1, α_2 are parameters fixed in order to get consistency with QCD.

- Light quark sector $\alpha_i = m_i/(2B)$

$$B = |\langle 0|\bar{u}u|0\rangle|/F_\pi^2$$

is the quark condensate parameter

- Leptonic decay constants in chiral limit

$$f_\pi = f_K = f_\rho = 3 f_\omega = \frac{3 f_\phi}{\sqrt{2}} = \kappa \frac{\sqrt{6}}{8}\,.$$

Heavy Mesons

- Heavy–light mesons $\alpha_Q = \alpha = \mathcal{O}(1)$

$$\alpha_q = \frac{2\alpha_Q}{m_Q}\left(1 + \frac{\bar{\Lambda}}{2m_Q}\right) - \frac{1}{2}\,.$$

Leds to

$$M_{Qq} = m_Q + \bar{\Lambda} + \mathcal{O}(1/m_Q)$$

$$M^2_{Q\bar{q}} = 4\kappa^2\left(n + \frac{L+J}{2}\right) + (m_Q + \bar{\Lambda})^2$$

$$f_{Q\bar{q}} = \frac{\kappa\sqrt{6}}{\pi}\,\frac{2\sqrt{\alpha}}{\alpha + \frac{3}{2}}\sqrt{\frac{\bar{\Lambda}}{m_Q}} \sim \sqrt{\frac{1}{m_Q}}$$

Heavy Mesons

- Heavy Quarkonia

$$\alpha_{Q_i} = \frac{m_{Q_i}}{4E}\left(1 - \frac{E}{2(m_{Q_1}+m_{Q_2})}\right) + \mathcal{O}\Big(\frac{1}{m_{Q_i}}\Big)$$

$$\kappa = \beta\left(\frac{\mu_{Q_1Q_2}}{E}\right)^{1/4}\left(\frac{m_{Q_1}+m_{Q_2}}{E}\right)^{1/2},$$

where $\beta = \mathcal{O}(1)$ and $\mu_{Q_1Q_2} = m_{Q_1}m_{Q_2}/(m_{Q_1}+m_{Q_2})$.

$$\begin{aligned} M^2_{Q_1\bar{Q}_2} &= 4\kappa^2\left(n + \frac{L+J}{2}\right) + (m_{Q_1}+m_{Q_2}+E)^2 \\ &- \frac{64\alpha_s^2 m_{Q_1}m_{Q_2}}{9\,(n+L+1)^2} \end{aligned}$$

and

$$f_{Q\bar{Q}} \sim \sqrt{m_Q}\,, \quad f_{c\bar{b}} \sim \frac{m_c}{\sqrt{m_b}}\,.$$

Mesons: choice of parameters

- Dilaton parameter $\kappa = 500$ MeV
- Current quark massses

$$m_{u/d} = 7 \text{ MeV}\,, \quad m_s = 24 m_{u/d} = 168 \text{ MeV}$$
$$m_c = 1.275 \text{ GeV}\,, \quad m_b = 4.18 \text{ GeV}$$

- Strong coupling constants

$$\alpha_s(c\bar{c}) = 0.45\,, \quad \alpha_s(c\bar{b}) = 0.383\,, \quad \alpha_s(b\bar{b}) = 0.27$$

- Parameters β

$$\beta(c\bar{c}) = 0.36 \text{ GeV}\,, \quad \beta(c\bar{b}) = 0.32 \text{ GeV}\,, \quad \beta(b\bar{b}) = 0.41 \text{ GeV}$$

Mesons: Results

Masses of light mesons

Meson	n	L	S	Mass [MeV]			
π	0,1,2,3	0	0	140	1010	1421	1738
K	0	0,1,2,3	0	495	1116	1498	1801
η	0,1,2,3	0	0	566	11494	1523	1822
$f_0[\bar{n}n]$	0,1,2,3	1	1	721	1233	1587	1876
$f_0[\bar{s}s]$	0,1,2,3	1	1	985	1404	1723	1993
$\rho(770)$	0,1,2,3	0	1	721	1233	1587	1876
$\omega(782)$	0,1,2,3	0	1	721	1233	1587	1876
$\phi(1020)$	0,1,2,3	0	1	985	1404	1723	1993
$a_1(1260)$	0,1,2,3	1	1	1010	1421	1738	2005

Mesons: Results

Masses of heavy-light mesons and heavy quarkonia

Meson	J^{P}	n	L	S	Mass [MeV]			
$D(1870)$	0^-	0	0,1,2,3	0	1870	2000	2121	2235
$D^*(2010)$	1^-	0	0,1,2,3	1	2000	2121	2235	2345
$D_s(1969)$	0^-	0	0,1,2,3	0	1970	2093	2209	2320
$D_s^*(2107)$	1^-	0	0,1,2,3	1	2093	2209	2320	2425
$B(5279)$	0^-	0	0,1,2,3	0	5280	5327	5374	5420
$B^*(5325)$	1^-	0	0,1,2,3	1	5336	5374	5420	5466
$B_s(5366)$	0^-	0	0,1,2,3	0	5370	5416	5462	5508
$B_s^*(5413)$	1^-	0	0,1,2,3	1	5416	5462	5508	5553

Mesons: Results

Masses of heavy quarkonia

Meson	J^{P}	n	L	S	Mass [MeV]			
$\eta_c(2980)$	0^-	0,1,2,3	0	0	2975	3477	3729	3938
$\psi(3097)$	1^-	0,1,2,3	0	1	3097	3583	3828	4032
$\chi_{c0}(3415)$	0^+	0,1,2,3	1	1	3369	3628	3843	4038
$\chi_{c1}(3510)$	1^+	0,1,2,3	1	1	3477	3729	3938	4129
$\chi_{c2}(3555)$	2^+	0,1,2,3	1	1	3583	3828	4032	4219
$\eta_b(9390)$	0^-	0,1,2,3	0	0	9337	9931	10224	10471
$\Upsilon(9460)$	1^-	0,1,2,3	0	1	9460	10048	10338	10581
$\chi_{b0}(9860)$	0^+	0,1,2,3	1	1	9813	10110	10359	10591
$\chi_{b1}(9893)$	1^+	0,1,2,3	1	1	9931	10224	10471	10700
$\chi_{b2}(9912)$	2^+	0,1,2,3	1	1	10048	10338	10581	10808
$B_c(6277)$	0^-	0,1,2,3	0	0	6277	6719	6892	7025

Mesons: Results

Decay constants f_P (MeV) of pseudoscalar mesons

Meson	Data	Our
π^-	$130.4 \pm 0.03 \pm 0.2$	153
K^-	$156.1 \pm 0.2 \pm 0.8$	153
D^+	206.7 ± 8.9	207
D_s^+	257.5 ± 6.1	224
B^-	193 ± 11	163
B_s^0	$253 \pm 8 \pm 7$	170
B_c	$489 \pm 5 \pm 3$	489

Decay constants f_V (MeV) of vector mesons with open and hidden flavor

Meson	Data	Our	Meson	Data	Our
ρ^+	210.5 ± 0.6	216	ρ^0	154.7 ± 0.7	153
D^*	$245 \pm 20^{+3}_{-2}$	207	ω	45.8 ± 0.8	51
D_s^*	$272 \pm 16^{+3}_{-20}$	224	ϕ	76 ± 1.2	72
B^*	$196 \pm 24^{+39}_{-2}$	170	J/ψ	277.6 ± 4	223
B_s^*	$229 \pm 20^{+41}_{-16}$	170	$\Upsilon(1s)$	238.5 ± 5.5	170

Baryons in soft-wall model

- Baryons in soft-wall model: Forkel–Beyer–Frederico, Brodsky–Teramond, Abidin–Carlson, Gutsche–Lyubovitskij–Schmidt–Vega, ...

- SW holographic approach for baryons with inclusion of high Fock states dual to bulk fermion fields of higher dimension.

- Objective: Application to nucleon form factors, GPDs, nucleon resonances (Roper)

Baryons in soft-wall model

- Bulk fermion fields
  $\Psi_+(x,z)$ and $\Psi_-(x,z)$ dual to $\mathcal{O}_R = (p_R, n_R)$ and $\mathcal{O}_L = (p_L, n_L)$
- Bulk fermion mass $\pm m = \pm(\Delta - 3/2)$, where Δ - scaling dimension
- Scaling dimension $\equiv$ Twist-dimension $\tau = N + L$,
  N - number of partons, $L = \max|L_z|$
- Action for the fermion field of twist τ

$$S_\tau = \int d^4x dz \sqrt{g}\, e^{-\varphi(z)} \sum_{i=+,-} \bar{\Psi}_{i,\tau}(x,z)\, \hat{\mathcal{D}}_i(z)\, \Psi_{i,\tau}(x,z)\,,$$

$$\hat{\mathcal{D}}_\pm(z) = \frac{i}{2}\Gamma^M \overleftrightarrow{\partial}_M \mp \frac{m + \varphi(z)}{R}$$

- dilaton $\varphi(z) = \kappa^2 z^2$ (Regge behavior of hadron masses)
- metric $g_{MN}(z) = \epsilon^a_M(z)\epsilon^b_N(z)\eta_{ab}, \quad g = |\mathrm{det} g_{MN}|$
- vielbein $\epsilon^a_M(z) = e^{A(z)}\delta^a_M, \quad A(z) = \log(R/z)$ (conformal)
- interval $ds^2 = g_{MN}dx^M dx^N = e^{2A(z)}(g_{\mu\nu}dx^\mu dx^\nu - dz^2)$

Baryons in soft-wall model

- P-transformations

$$U_P^{-1}\Psi_{\tau,\pm}(t,\vec{x},z)U_P = \pm\gamma^0\gamma^5\,\Psi_{\tau,\mp}(t,-\vec{x},z)$$

$$U_P^{-1}\bar\Psi_{\tau,\pm}(t,\vec{x},z)U_P = \pm\bar\Psi_{\tau,\mp}(t,-\vec{x},z)\,\gamma^0\gamma^5$$

$$\pm U_P^{-1}\,\bar\Psi_{\pm,\tau}(t,\vec{x},z)\,\Psi_{\pm,\tau}(t,\vec{x},z)\,U_P = \mp\bar\Psi_{\mp,\tau}(t,-\vec{x},z)\,\Psi_{\mp,\tau}(t,\vec{x},z)\,,$$

$$U_P^{-1}\,S_\tau^{\pm}\,U_P = S_\tau^{\mp}$$

- C-transformations

$$U_C^{-1}\Psi_{\pm}(x,z)U_C = \mp\,C\,\gamma^5\,\bar\Psi^T_{\mp}(x,z)$$

$$U_C^{-1}\bar\Psi_{\pm}(x,z)U_C \;=\; \pm\Psi^T_{\mp}(x,z)\gamma^5 C$$

$$\pm\,U_C^{-1}\bar\Psi_{\pm}(x,z)\Psi_{\pm}(x,z)U_C = \mp\,\bar\Psi_{\mp}(x,z)\Psi_{\mp}(x,z)$$

$$U_C^{-1}\,S_\tau^{\pm}\,U_C = S_\tau^{\mp}$$

Baryons in soft-wall model

- Redefinition $\Psi_{i,\tau}(x,z) = e^{\varphi(z)/2-2A(z)}\psi_{i,\tau}(x,z)$
- Expansion on left- and right-chirality components (eigenstates of γ^5)
  $\psi_{i,\tau}(x,z) = \psi^L_{i,\tau}(x,z) + \psi^R_{i,\tau}(x,z)$
- Kaluza-Klein expansion

$$\psi^{L/R}_{i,\tau}(x,z) = \frac{1}{\sqrt{2}} \sum_n \psi^{L/R}_n(x)\, f^{L/R}_{i,\tau,n}(z)\,,$$

- Relations between bulk profiles

$$\begin{aligned} f^R_{\tau,n}(z) &\equiv f^R_{+,\tau,n}(z) = -f^L_{-,\tau,n}(z)\,, \\ f^L_{\tau,n}(z) &\equiv f^L_{+,\tau,n}(z) = f^R_{-,\tau,n}(z)\,. \end{aligned}$$

- EOM

$$\left[-\partial_z^2 + \kappa^4 z^2 + 2\kappa^2\left(m \mp \frac{1}{2}\right) + \frac{m(m\pm 1)}{z^2}\right] f^{L/R}_{\tau,n}(z) = M^2_{n\tau}\, f^{L/R}_{\tau,n}(z),$$

Baryons in soft-wall model

- Solutions

$$f^L_{\tau,n}(z) \;=\; \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\tau)}}\,\kappa^\tau\; z^{\tau-1/2}\; e^{-\kappa^2 z^2/2}\; L_n^{\tau-1}(\kappa^2 z^2)\,,$$

$$f^R_{\tau,n}(z) \;=\; \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\tau-1)}}\,\kappa^{\tau-1}\; z^{\tau-3/2}\; e^{-\kappa^2 z^2/2}\; L_n^{\tau-2}(\kappa^2 z^2)$$

$$\text{with}\quad \int\limits_0^\infty dz\, f^{L/R}_{\tau,n_1}(z)\, f^{L/R}_{\tau,n_2}(z) \;=\; \delta_{n_1 n_2}$$

and

$$M^2_{n\tau} = 4\kappa^2\Big(n+\tau-1\Big)$$

- Large N_c expansion

$$M_n = \sum_\tau c_\tau\, M_{n\tau} \sim \sum_\tau c_\tau \cdot \underbrace{\kappa}_{\sim\sqrt{N_c}} \cdot \underbrace{\sqrt{n+\tau-1}}_{\sim\sqrt{N_c}} \sim N_c$$

Baryons in soft-wall model

- Inclusion of high Fock states

$$S = \sum_\tau c_\tau \, S_\tau$$

c_τ - set of free parameters

- Integration over z using normalizazion condition for $f^{L/R}$

$$S \; = \; \int d^4x \, \bar\psi_n(x) \Big[\underbrace{\sum_\tau c_\tau}_{=1} \, i \, \not\partial - \underbrace{\sum_\tau c_\tau M_{n\tau}}_{=M_n} \Big] \psi_n(x) \, .$$

Correct normalization of kinetic term of 4D spinor field

$$\sum_\tau c_\tau = 1 \, , \qquad \sum_\tau c_\tau M_{n\tau} = M_n \quad (\text{baryon mass})$$

Electromagnetic stucture of nucleons

- Abidin-Carlson: First application of SW model (3q configurations)
- Coupling of bulk vector and fermion fields

$$\mathcal{L}_{\rm int}(x,z) = \sum_{i=+,-} \sum_{\tau} c_\tau \, \bar\Psi_{i,\tau}(x,z) \, \hat{\mathcal{V}}_i(x,z) \, \Psi_{i,\tau}(x,z)$$

$$\hat{\mathcal{V}}_\pm(x,z) = \underbrace{Q_N \, \Gamma^M V_M(x,z)}_{\text{min. coupling}} \pm \underbrace{\frac{i}{4} \eta_V \, [\Gamma^M, \Gamma^N] \, V_{MN}(x,z))}_{\text{nonmin. coupling}}$$

$$\langle p'|J^\mu(0)|p\rangle \; = \; \bar u(p') \, \left[\gamma^\mu F_1^N(t) + \frac{i}{2m_N} \sigma^{\mu\nu} \, q_\nu F_2^N(t)\right] u(p)$$

$$\begin{aligned}
F_1^p(Q^2) &= C_1(Q^2) + \eta_V^p C_2(Q^2) \\
F_2^p(Q^2) &= \eta_V^p C_3(Q^2) \\
F_1^n(Q^2) &= \eta_V^n C_2(Q^2) \\
F_2^n(Q^2) &= \eta_V^n C_3(Q^2)
\end{aligned}$$

Electromagnetic stucture of nucleons

- $V(Q,z)$ – propagator of trans. massless vector field (analogue of EM field)

$$V(Q,z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right)$$

$$V(0,z) = 1\,, \qquad V(Q,0) = 1\,, \qquad V(Q,\infty) = 0\,.$$

Electromagnetic stucture of nucleons

Matveev-Muradyan-Tavkhelidze-Brodsky-Farrar quark-counting rules at large Q^2

- $$C_1(Q^2) = \frac{1}{2} \int\limits_0^\infty dz\, V(Q,z) \sum_\tau c_\tau \left([f_\tau^L(z)]^2 + [f_\tau^R(z)]^2 \right)$$
  $$= \sum_\tau c_\tau\, B(a+1,\tau)\, \left(\tau + \frac{a}{2}\right) \sim \sum_\tau \frac{c_\tau}{a^{\tau-1}}$$

- $$C_2(Q^2) = \frac{1}{2} \int\limits_0^\infty dz z\, \partial_z V(Q,z) \sum_\tau c_\tau \left([f_\tau^L(z)]^2 - [f_\tau^R(z)]^2 \right)$$
  $$= a \sum_\tau c_\tau\, B(a+1,\tau+1)\, \frac{a(\tau-1)-1}{\tau} \sim \sum_\tau \frac{c_\tau}{a^{\tau-1}}$$

- $$C_3(Q^2) = 2m_N \int\limits_0^\infty dz z\, V(Q,z) \sum_\tau c_\tau\, f_\tau^L(z) f_\tau^R(z)$$
  $$= \frac{2m_N}{\kappa} \sum_\tau c_\tau\, (a+1+\tau)\, B(a+1,\tau+1)\, \sqrt{\tau-1} \sim \sum_\tau \frac{c_\tau}{a^\tau}$$

- $a = \frac{Q^2}{4\kappa^2}$, $\qquad B(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$ is the beta function.

Electromagnetic stucture of nucleons

Choice of free parameters

- $\kappa = 383\ \mathrm{MeV}\,, \quad c_3 = 1.25\,, \quad c_4 = 0.16\,, \quad g_V = 0.3$
- c_5 is expressed through c_3 and c_4

$$c_5 = 1 - c_3 - c_4 = -0.41$$

- c_3, c_4 are constrained by the nucleon mass
- κ is fixed by the nucleon mass and nucleon electromagnetic radii
- Nonminimal couplings $\eta_V^{p,n}$ from nucleon magnetic moments

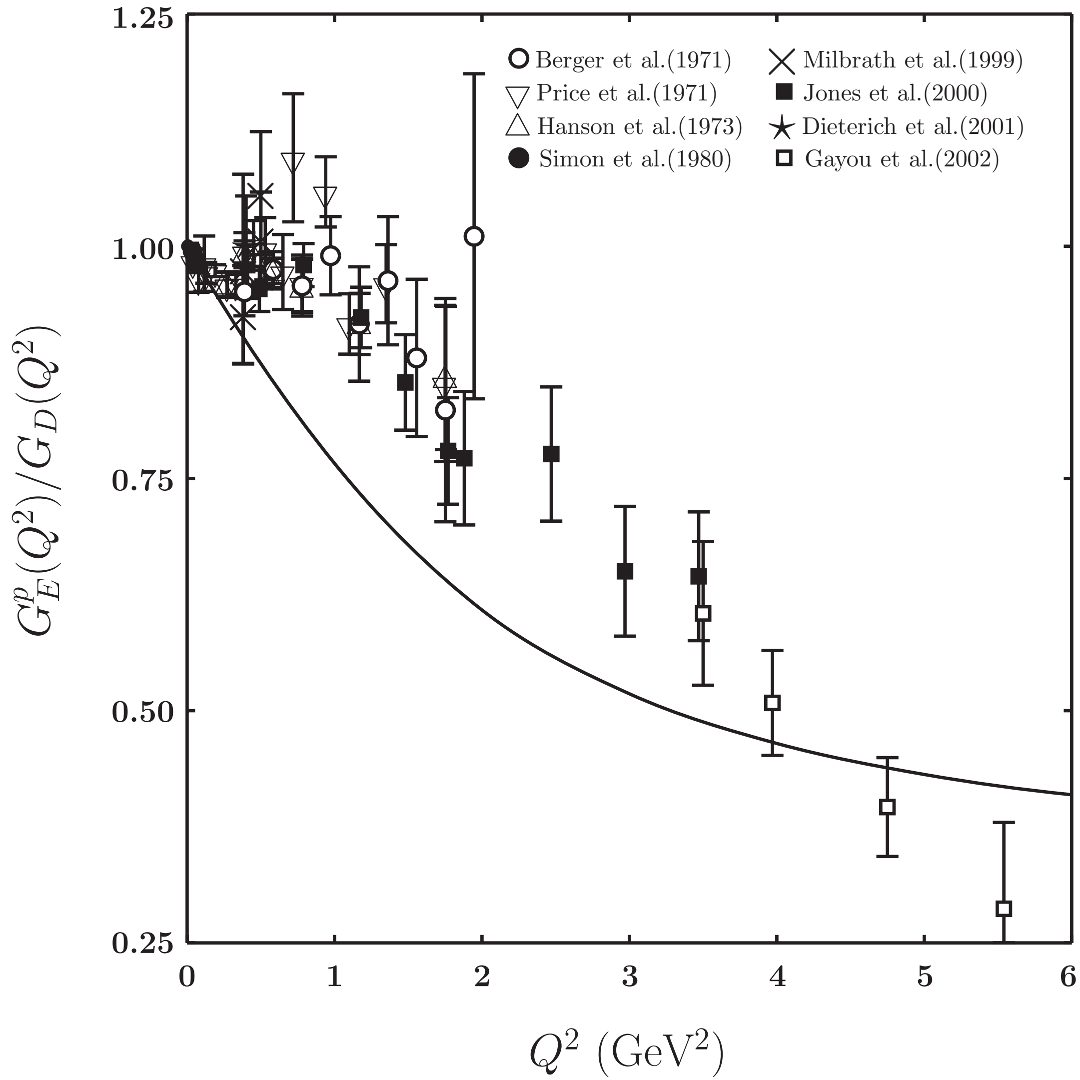
$$\eta_V^p = \frac{\kappa\,(\mu_p - 1)}{2m_N\,C_0} = 0.30\,, \quad \eta_V^n = \frac{\kappa\,\mu_n}{2m_N\,C_0} = -0.32\,, \quad C_0 = \sqrt{2}\,c_3 + \sqrt{3}\,c_4 + 2c_5$$

Electromagnetic stucture of nucleons

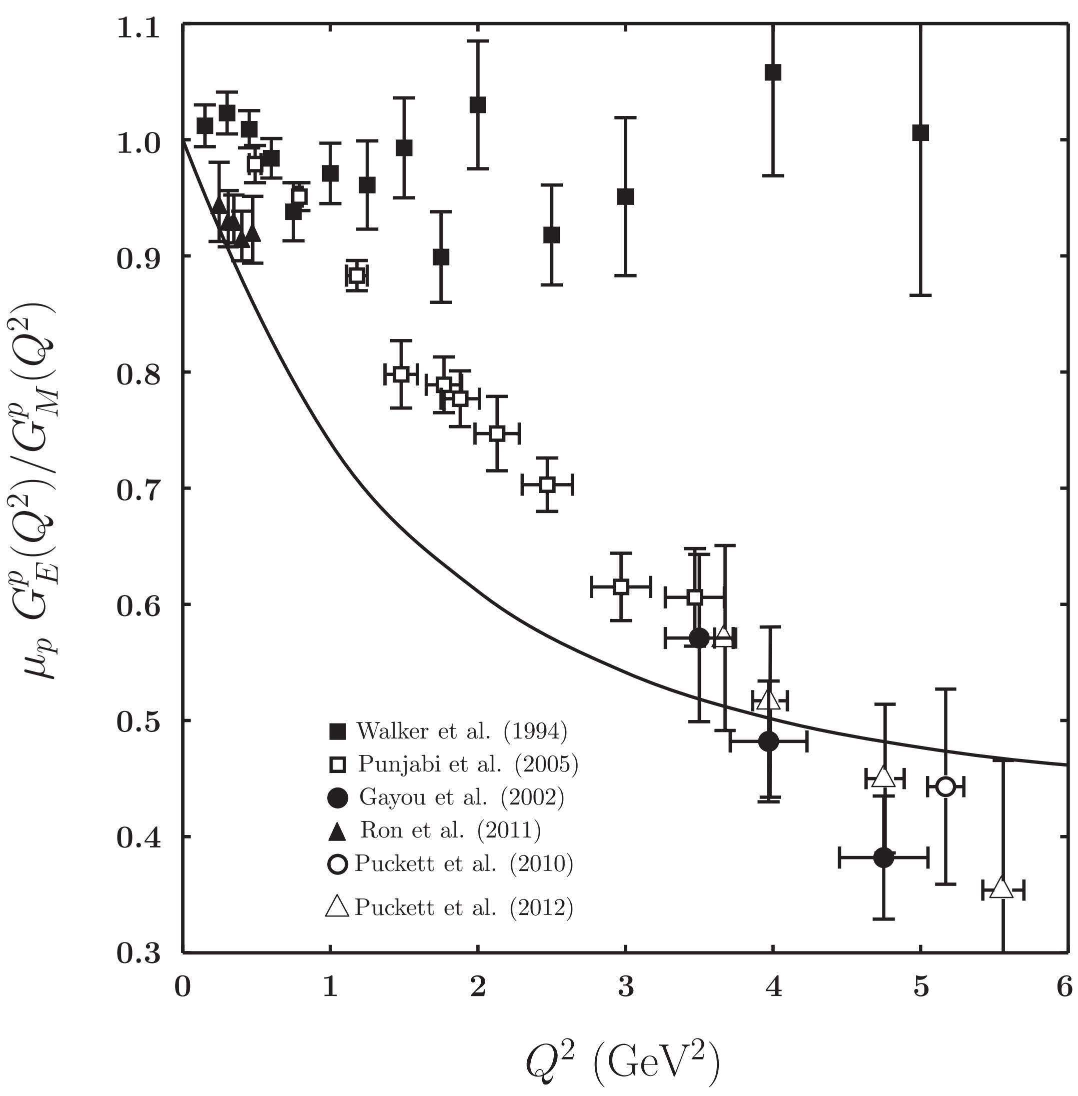
Mass and electromagnetic properties of nucleons

Quantity	Our results	Data
m_p (GeV)	0.93827	0.93827
μ_p (in n.m.)	2.793	2.793
μ_n (in n.m.)	-1.913	-1.913
r_E^p (fm)	0.840	0.8768 $\pm$ 0.0069
$\langle r_E^2 \rangle^n$ (fm^2)	-0.117	-0.1161 $\pm$ 0.0022
r_M^p (fm)	0.785	0.777 $\pm$ 0.013 $\pm$ 0.010
r_M^n (fm)	0.792	0.862$^{+0.009}_{-0.008}$
r_A (fm)	0.667	0.67$\pm$0.01

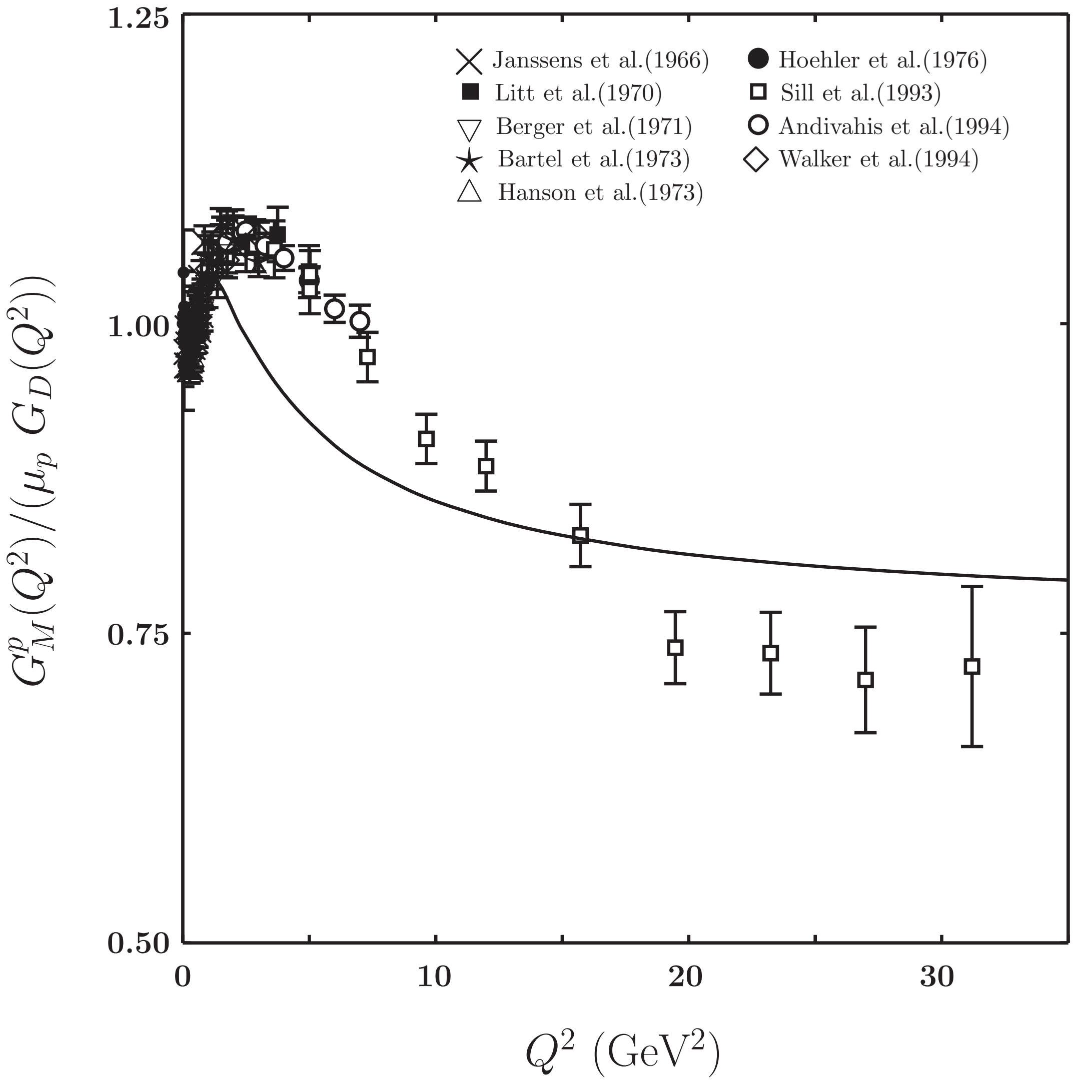
Electromagnetic stucture of nucleons



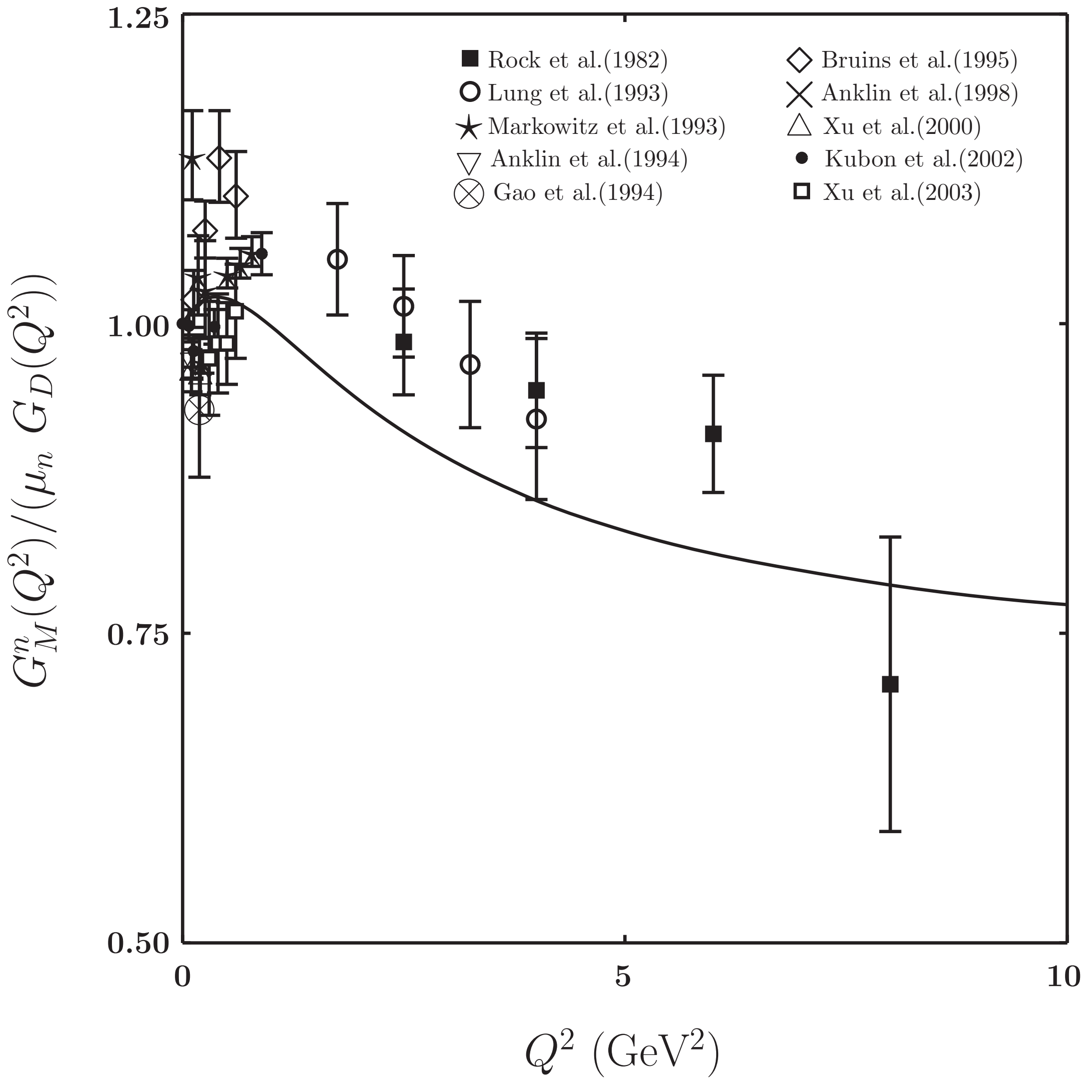
Electromagnetic stucture of nucleons



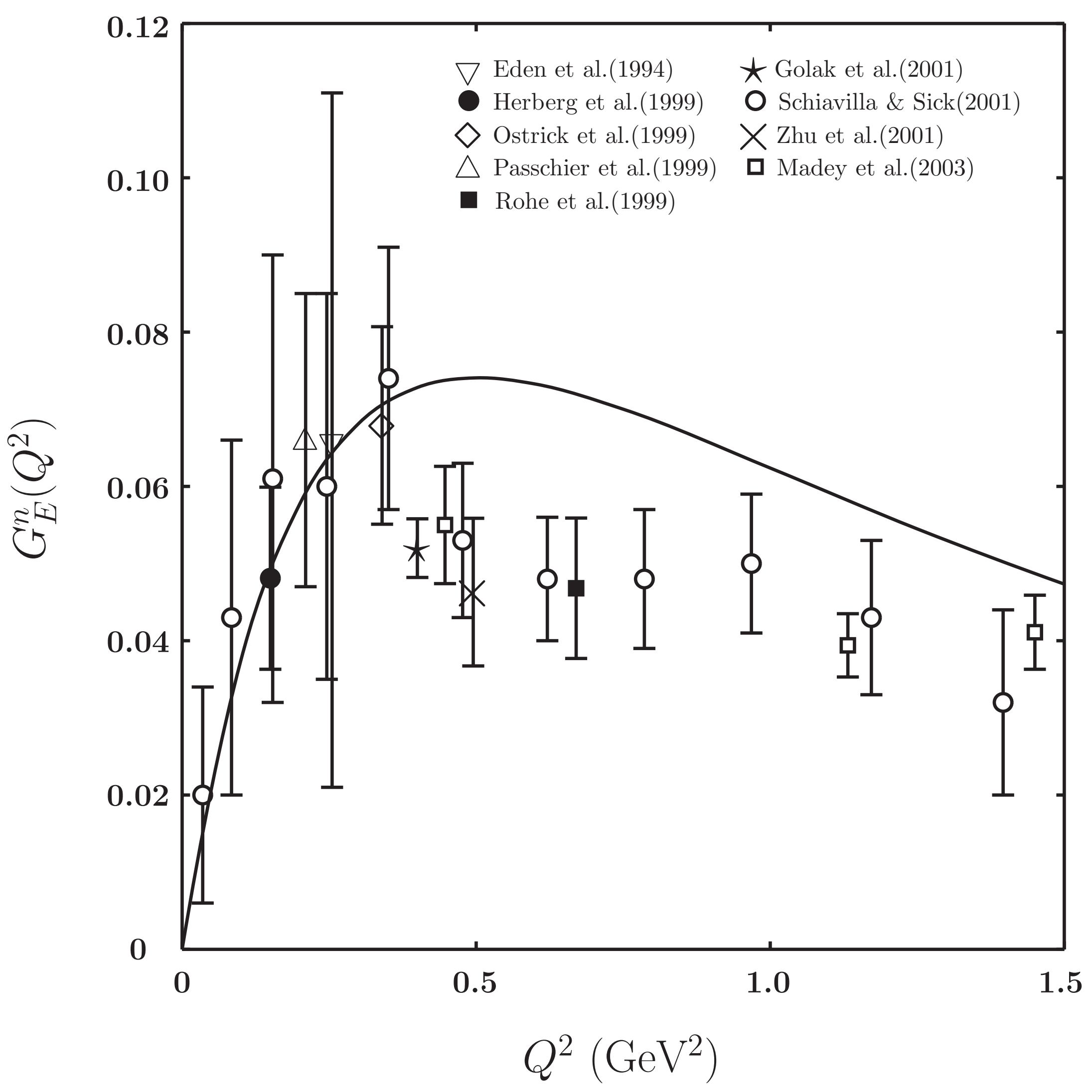
Electromagnetic stucture of nucleons



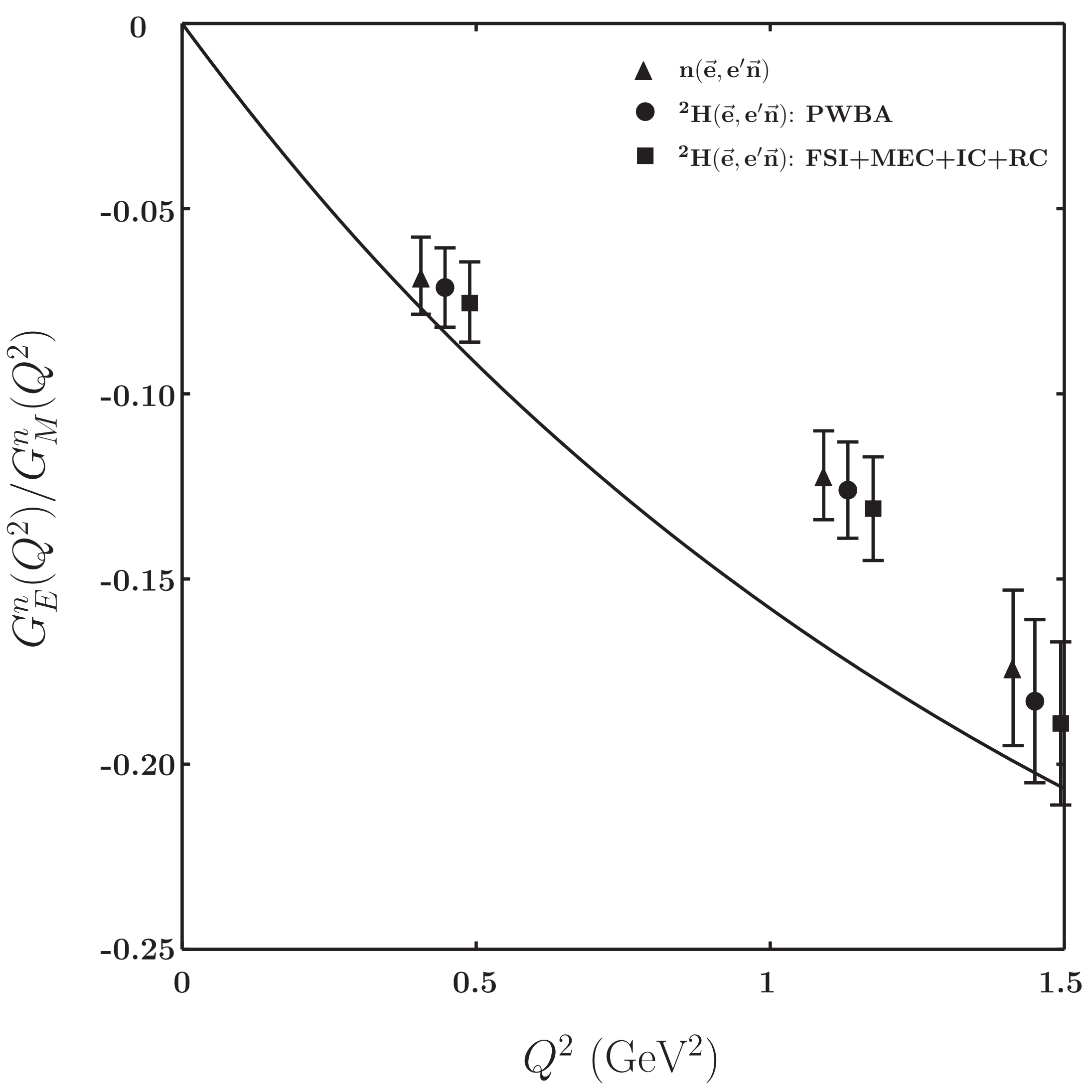
Electromagnetic stucture of nucleons



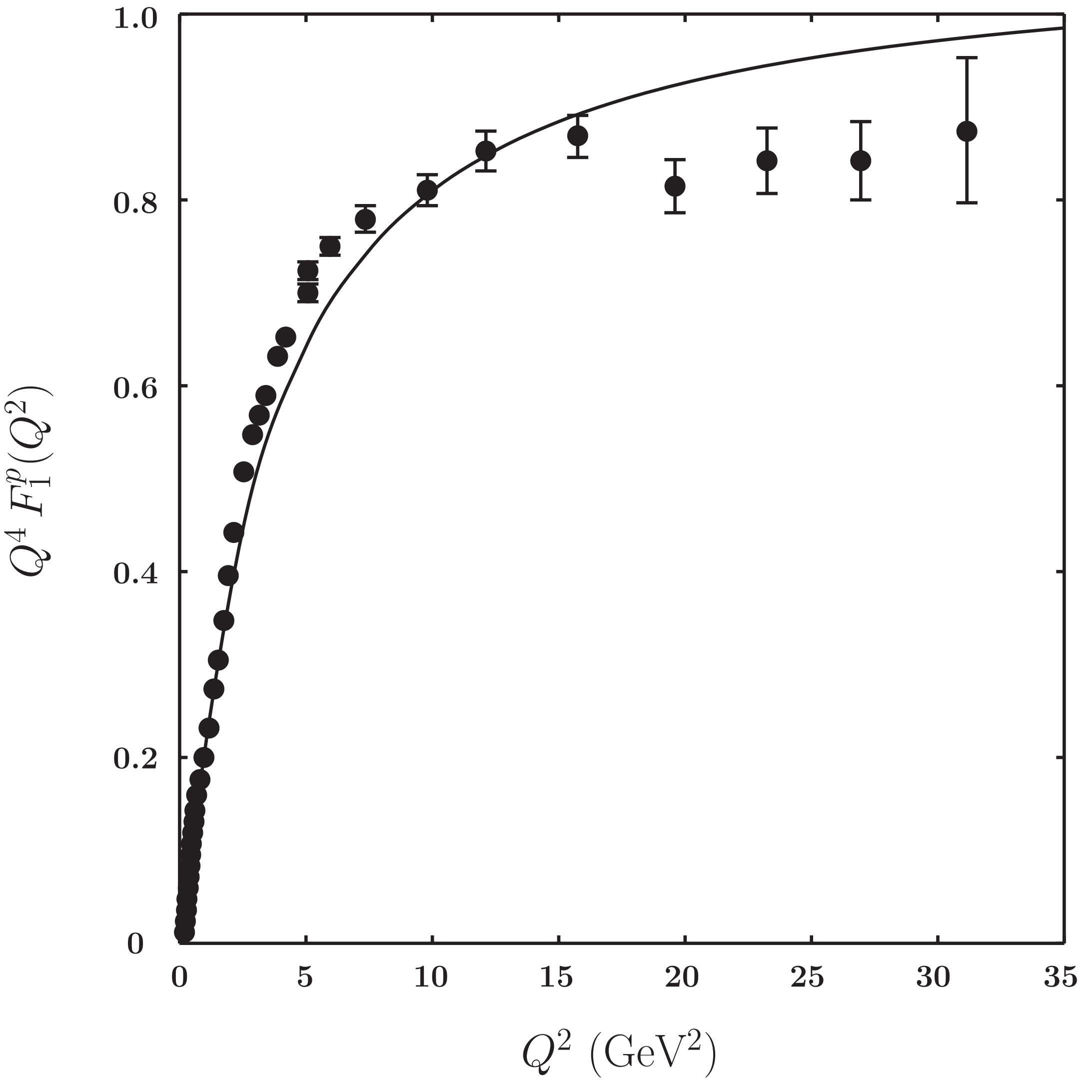
Electromagnetic stucture of nucleons



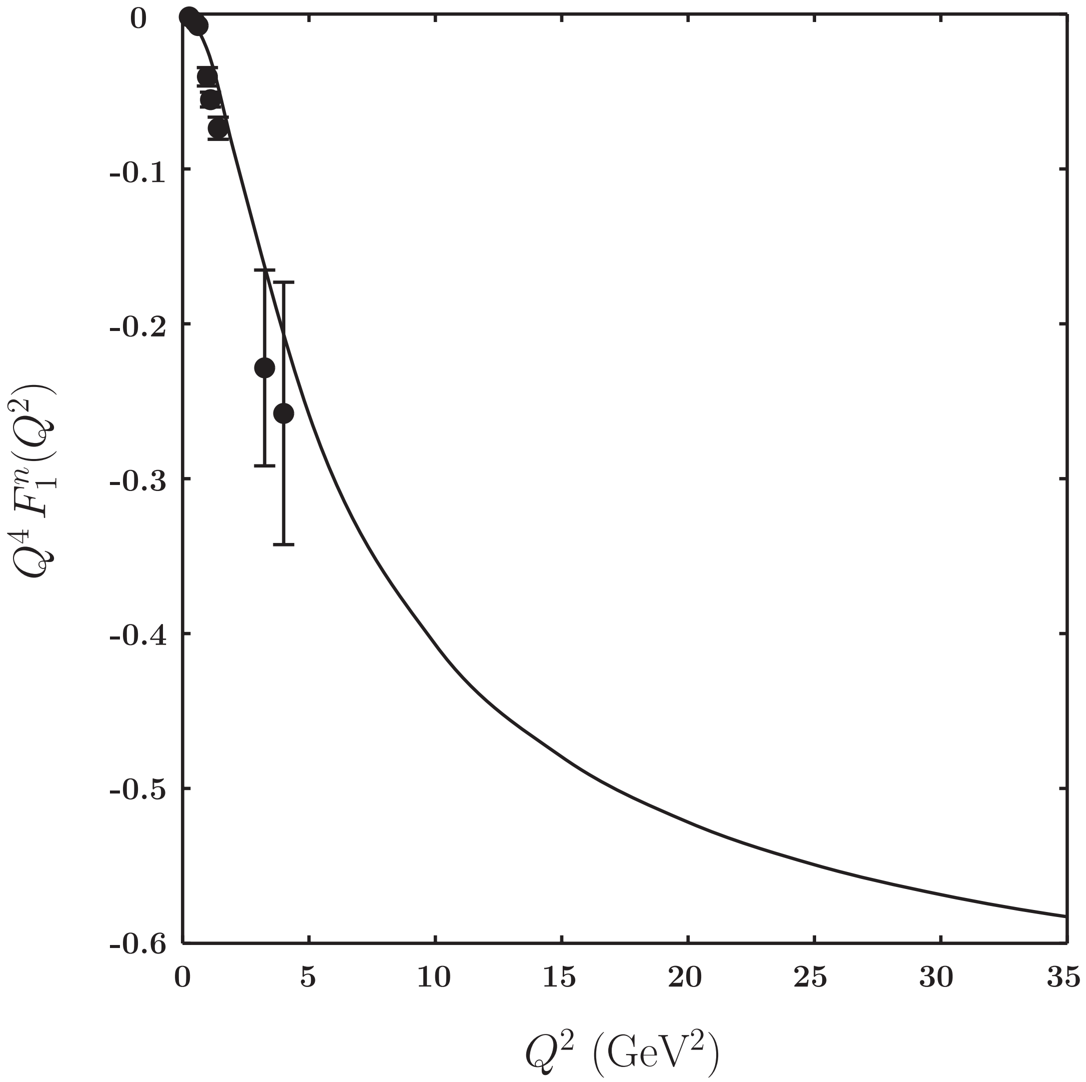
Electromagnetic stucture of nucleons



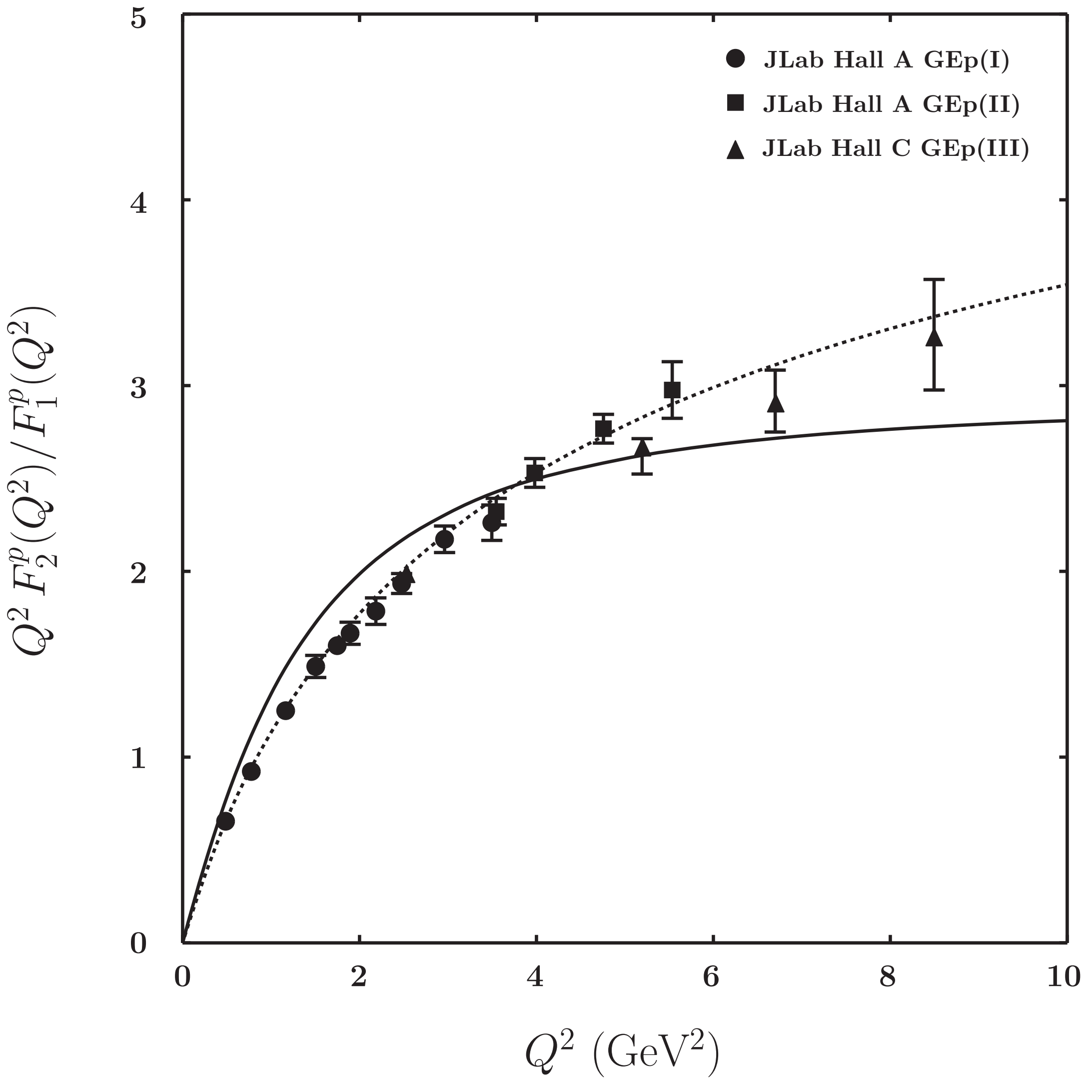
Electromagnetic stucture of nucleons



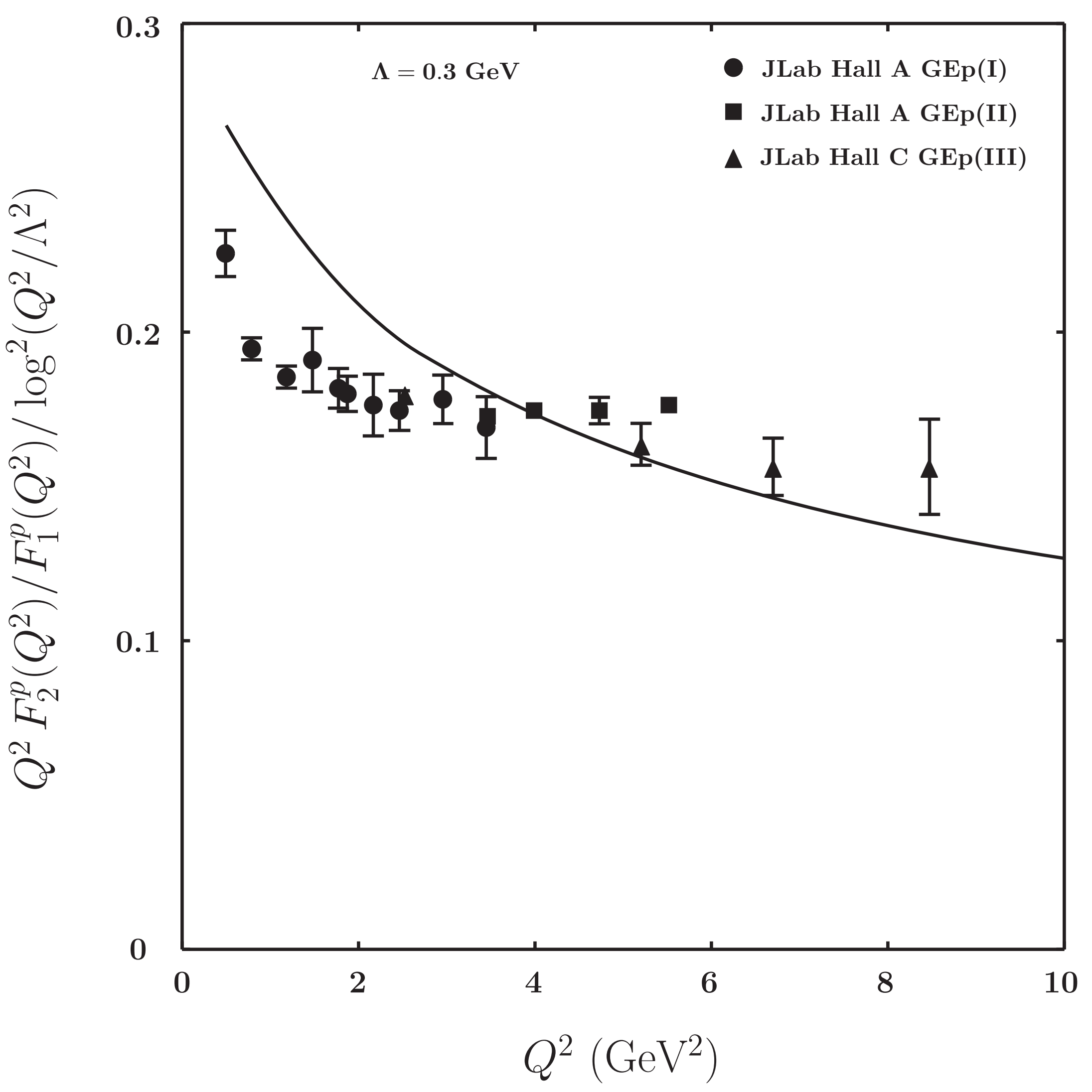
Electromagnetic stucture of nucleons



Electromagnetic stucture of nucleons



Electromagnetic stucture of nucleons



GPDs

Hadronic form factor is given by

$$F_\tau(Q^2) = \int_0^\infty dz\, \varphi_\tau^2(z)\, V(Q^2, z^2) = \int_0^1 dx\, \mathcal{H}_\tau(x, Q^2)\,,$$

$$\mathcal{H}_\tau(x, Q^2) = q_\tau(x)\, f_\tau(x, Q^2)$$

Here

$$f_\tau(x, Q^2) = \frac{1}{(\tau+1)\,\Gamma(\tau-1)} \int_0^\infty dt t^{\tau-2}\, e^{-t}(2+t)\, V(Q^2, t(1-x))$$

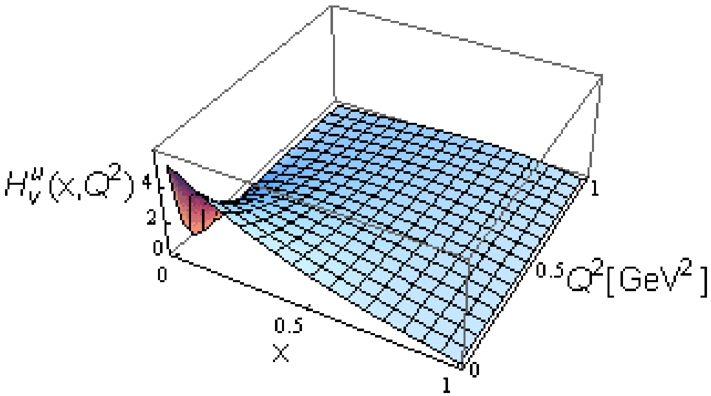
$$V(Q^2, t(1-x)) \to V(Q^2, 0) \equiv 1$$

as required by model-independent result and $f_\tau(x, Q^2) \to 1$

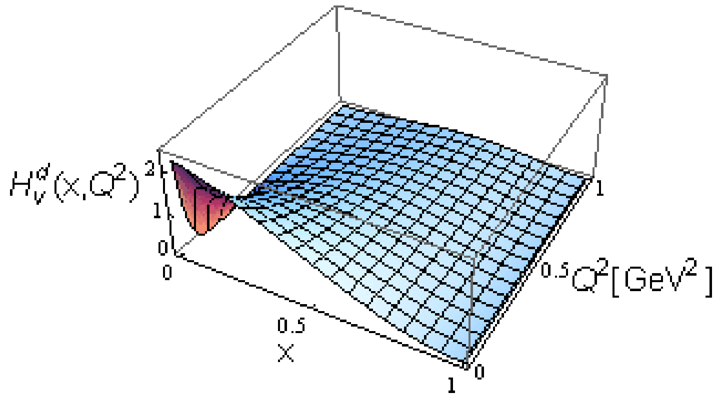
The GPD $H_\tau(x, Q^2)$ and PDF $q_\tau(x)$ have correct behavior at $x \to 1$

$$H_\tau(x, Q^2) \sim q_\tau(x) \sim (1-x)^\tau$$

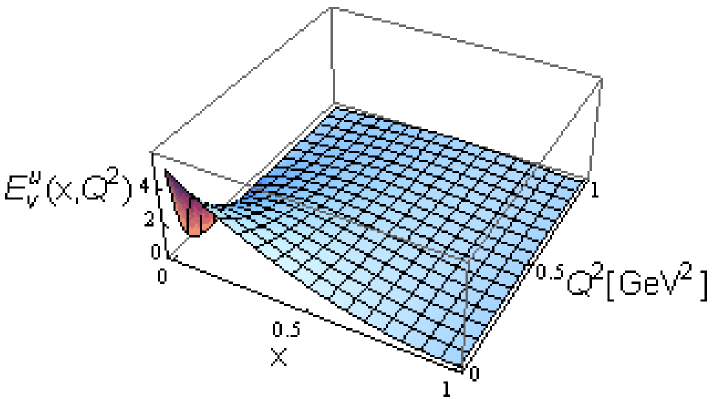
Nucleon GPDs



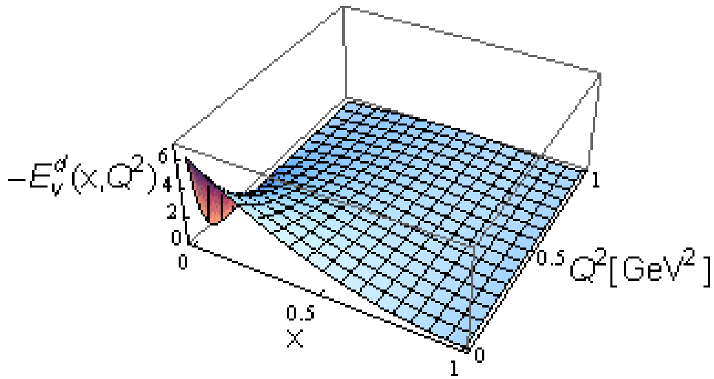
$H_v^u(x,Q^2)$



$H_v^d(x,Q^2)$

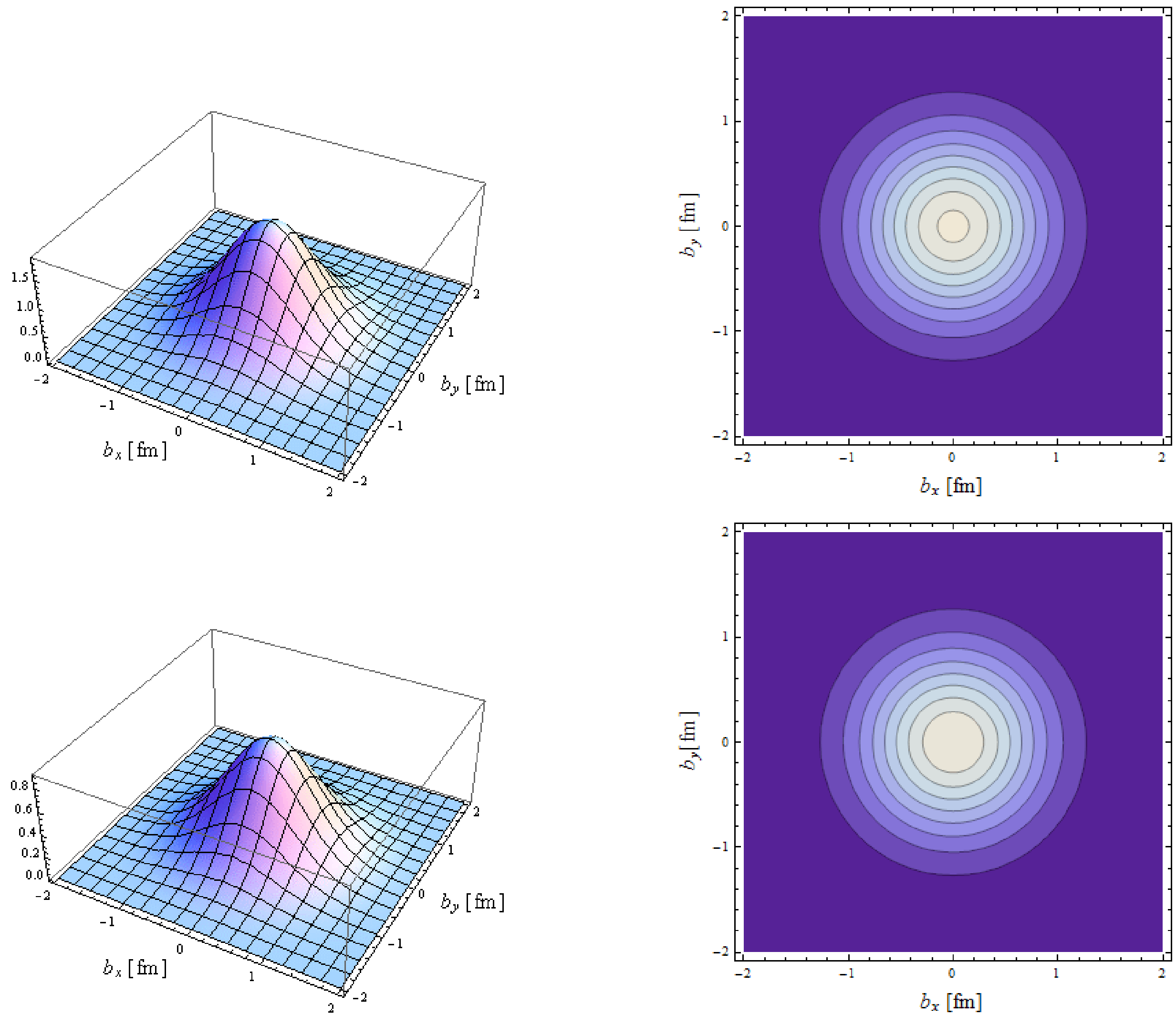


$E_v^u(x,Q^2)$



$E_v^d(x,Q^2)$

Nucleon GPDs



Plots for $q(x, \mathbf{b}_\perp)$ for $x = 0.1$: $u(x, \mathbf{b}_\perp)$ - upper pannels, $d(x, \mathbf{b}_\perp)$ - lower pannels

Roper resonance $N(1440)$

- Put $n = 1$ and get solutions dual to Roper:

$$M_{\mathcal{R}} \simeq 1440 \text{ MeV}$$

- $N \to R + \gamma$ transition

$$M^\mu = \bar{u}_{\mathcal{R}} \left[\gamma_\perp^\mu \, F_1(q^2) \, + \, i\sigma^{\mu\nu} \frac{q_\nu}{M_{\mathcal{R}}} \, F_2(q^2) \right] u_N \, , \quad \gamma_\perp^\mu = \gamma^\mu - q^\mu \frac{\not{q}}{q^2}$$

- Helicity amplitudes

$$H_{\pm\frac{1}{2}0} \quad = \quad \sqrt{\frac{Q_-}{Q^2}} \left(F_1 M_+ - F_2 \frac{Q^2}{M_{\mathcal{R}}} \right)$$

$$H_{\pm\frac{1}{2}\pm 1} \quad = \quad -\sqrt{2Q_-} \left(F_1 + F_2 \frac{M_+}{M_{\mathcal{R}}} \right)$$

- Alternative set [Weber, Capstick, Copley et al]

$$A_{1/2} \quad = \quad -b \, H_{\frac{1}{2}1} \, , \quad S_{1/2} \, = \, b \, \frac{|\mathbf{p}|}{\sqrt{Q^2}} \, H_{\frac{1}{2}0} \, ,$$

$$Q_\pm \quad = \quad M_\pm^2 + Q^2 \, , \quad M_\pm = M_{\mathcal{R}} \pm M_N \, , \quad b \, = \, \sqrt{\frac{\pi\alpha}{2EM_{\mathcal{R}} M_N}}$$

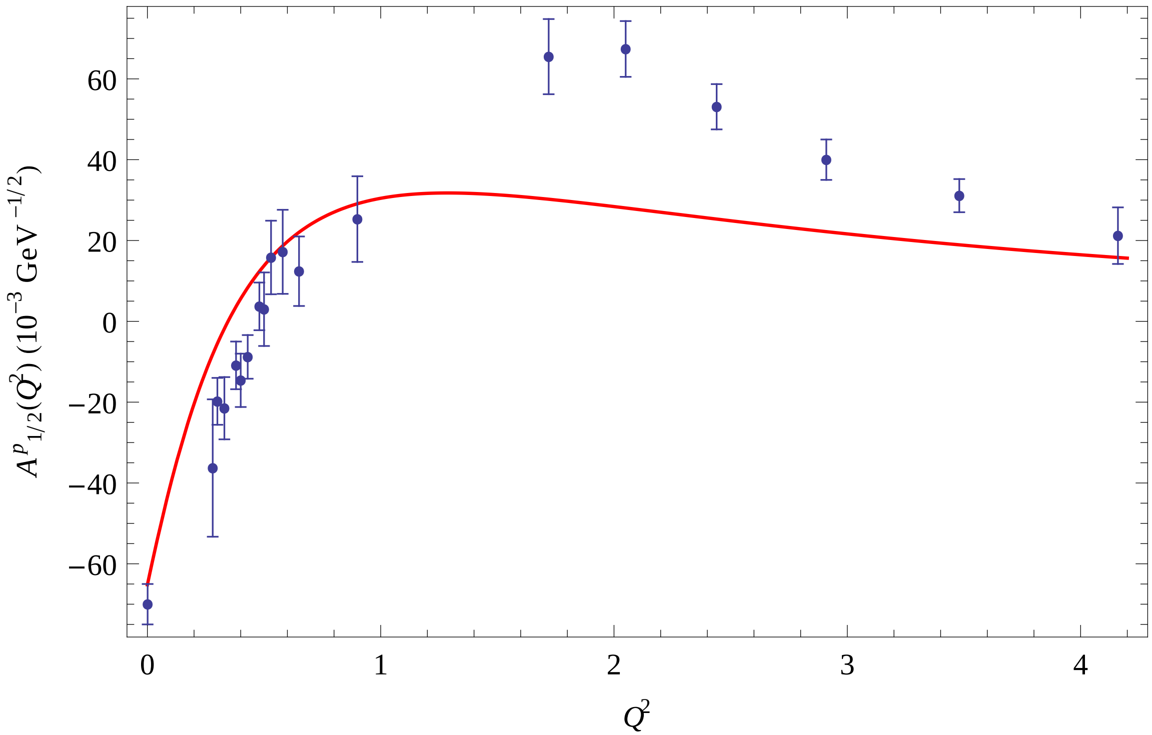
Roper resonance $N(1440)$

Helicity amplitudes $A^N_{1/2}(0)\,,\ S^N_{1/2}(0)$

Quantity	Our results	Data
$A^p_{1/2}(0)$ (GeV$^{-1/2}$)	-0.065	-0.065 $\pm$ 0.004
$A^n_{1/2}(0)$ (GeV$^{-1/2}$)	0.040	0.040 $\pm$ 0.010
$S^p_{1/2}(0)$ (GeV$^{-1/2}$)	0.040	
$S^p_{1/2}(0)$ (GeV$^{-1/2}$)	-0.040	

Roper resonance $N(1440)$

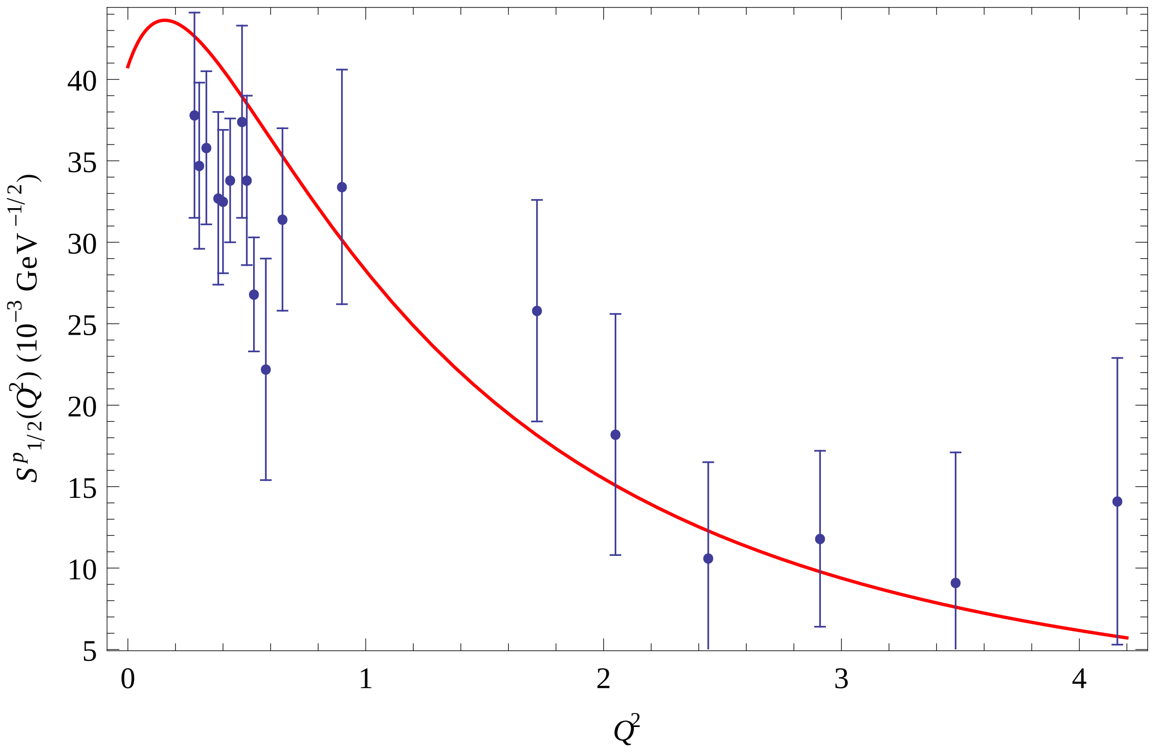
Helicity amplitude $A^p_{1/2}(Q^2)$



Data: CLAS Coll at JLab, Mokeev et al, 1205.3948 [nucl-ex]

Roper resonance $N(1440)$

Helicity amplitude $S^p_{1/2}(Q^2)$



Data: CLAS Coll at JLab, Mokeev et al, 1205.3948 [nucl-ex]

Summary

- AdS/QCD $\equiv$ Holographic QCD (HQCD) – approximation to QCD: attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – anti-de Sitter (AdS) space
- HQCD models reproduce main features of QCD at low and high energies
- Soft–wall holographic approach – covariant and analytic model for hadron structure with confinement at large distances and conformal behavior at short distances
- Mesons, baryons, exotics from unified point view and including high Fock states
- Future work: nucleon TMDs, DVCS