# QCD jets production at proton - proton collision in peripheral kinematics

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# Talk Plan


1. Introduction
2. Motivation
3. Heavy Fermion and Quark Production
4. Emission of two gluon jets
5. Conclusion
6. Appendix

# Motivation

Processes in fragmentation region of proton - heavy ion (electron) collisions with creation of hadronic jets are considered in frames of QCD. Creation of one and two gluon jets as well as heavy quark jets initiated by gluons and light quarks inside the proton are considered. The differential cross section on the energy fractions and the transversal momenta of gluons and quarks from the jets are considered. The "reflection" kinematics of the light quark (gluon) jet, accompanied by heavy quark pair is discussed.

# Introduction

In papers of seventies (Kuraev E. A., Lipatov L.N., Yad Fiz. v.20,(1974),112; Budnev V.,Ginzburg I.,Meledin G. and Serbo V.G., Phys.Rep.15C(1975),181; Baier V.N.,Fadin V.S., Khoze V.A. and Kuraev E.A., Phys.Rep.v 78(1981),293; Kuraev E. A., Lipatov L.N., Yad Fiz. v.16,(1972),1060; Kuraev E.A., Lipatov L.N., Merenkov N.P., Fadin V.S. and Khoze V.A., Yad. Fyz. v 19,(1974),331.) a series of processes in fragmentation region which take place in colliding electron-positron beams was performed. In Weizsacker-Williams (WW) approximation the spectral distributions and the contributions to the total cross section was obtained. Below we apply the developed technique to the processes of emission of real hard jets created by quarks and gluons inside the proton. In chapter I we consider the processes with initial light quark converting to one and two gluon jets. Compared with QED processes of single and double bremsstrahlung processes a new mechanism must be considered, associated with a channel with decal of gluon to two gluons in process of two gluon jets production.

# Introduction

First we remind the general Sudakov technique approach to study the peripheral kinematics of QED process $e+\mu\to e+\mu+l+\bar{l}$ of creation heavy charged lepton pair in high-energy electron-muon scattering

$$
\begin{aligned}
e(p_2)+\mu_-(p_1)\to e(p_2')+\mu_-(p_1')+l_-(q_-)+l_+(q_+),\\
p_2^2=p_2^{'2}=m_e^2, p_1^2=p_1^{'2}=m^2, q_\pm^2=M^2,\\
s=(p_1+p_2)^2\approx 2p_1p_2>>M^2>>m^2>>m_e^2.
\end{aligned}
\tag{1}
$$

Peripheral kinematics or the fragmentation of quark $q$ region defined as

$$
s>>-q^2=-(p_2-p_2')^2\sim M^2. \tag{2}
$$

It is convenient to use Sudakov parametrization of momenta. For this aim we introduce two light-like 4 vectors constructed from the momenta of initial particles $\tilde{p}_2=p_2-p_1(m_e^2/s), \tilde{p}_1=p_1-p_2(m^2/s)$

$$
\begin{aligned}
q=\alpha\tilde{p}_2+\beta\tilde{p}_1+q_\perp, q_\pm=\alpha_\pm\tilde{p}_2+x_\pm\tilde{p}_1+q_{\perp\pm},\\
p_1'=\alpha'\tilde{p}_2+x\tilde{p}_1+p_\perp,\\
a_\perp p_1=a_\perp p_2=0, a_\perp^2=-\vec{a}^2<0,
\end{aligned}
\tag{3}
$$

# Figures

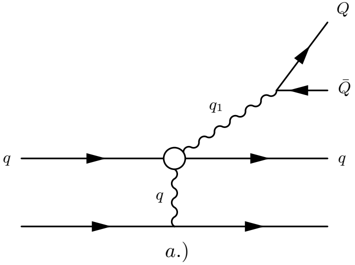


Figure: Production of heavy quark pair

# Introduction

with $\vec{a}$ is two-dimensional vector transversal to the beams axis (direction of $\vec{p}_1$, center of mass reference frame implied), $x, x_{\pm}$-energy fractions of the scattered $\mu$-meson and heavy lepton pair, $x + x_- + x_+ = 1$. According to the energy-momentum conservation law we have as well

$$\vec{q} = \vec{p} + \vec{q}_- + \vec{q}_+, \\ \alpha = \alpha' + \alpha_+ + \alpha_-. \quad (4)$$

The on mass shell condition for the scattered electron $p_2'^2 - m_e^2 = 0$, written in terms of Sudakov variables reads as (one must take into account the relation $2p_2\tilde{p}_2 = m_e^2$)

$$(p_2 - q)^2 - m_e^2 = s\alpha\beta - \vec{q}^2 - m_e^2\alpha - s\beta = 0, \\ s\beta = -\frac{\vec{q}^2 + m_e^2\alpha}{1-\alpha}. \quad (5)$$

One find for $q^2 = s\alpha\beta - \vec{q}^2$:

$$q^2 = -\frac{\vec{q}^2 + \alpha^2 m_e^2}{1-\alpha} \approx -(\vec{q}^2 + \frac{s_1^2}{s^2}m_e^2). \quad (6)$$

# Introduction

We conclude that for the case $s_1 \neq 0$ virtual photon have a space-like 4-vector, and besides $|q^2| > q^2_{min} = m_e^2(s_1/s)^2$. The quantity $s_1 = 2qp_1 = (p_1' + q_+ + q_-)^2 - q^2 - m^2$ in WW approximation $\vec{q} = 0$ coincide with invariant mass square of jet moving into the initial quark momenta direction. Using the on mass shell conditions for momenta of the scattered quark and the created pair of heavy quarks

$$p_1'^2 = s\alpha' x - \vec{p}^2 = m_q^2 = m^2, q_\pm^2 = s\alpha_\pm x_\pm - \vec{q}_\pm^2 = M^2, x + x_+ + x_- = 1, \quad (7)$$

we find (in WW approximation)

$$s_1 = \frac{1}{xx_+x_-}[x_-(1-x_-)\vec{q}_+^2 + x_+(1-x_+)\vec{q}_-^2 + 2x_-x_+\vec{q}_-\vec{q}_+ + m^2x_+x_- + x(1-x)M^2]. \quad (8)$$

Matrix element can be written as

$$M = \frac{G}{q^2} g^{\mu\nu} J_\mu^{(q)}(p_1) J_\nu^{(e)}(p_2), \quad (9)$$

with $G$ is product of coupling constants (for our problem $G = 16\pi\alpha^2$) and $J^{(q,e)}$-the currents associated with quark and electron blocks of the relevant Feynman diagram.

# Introduction

Main contribution arises from the longitudinal components of the tensor $g^{\mu\nu} = g_{\mu\nu\perp} + (2/s)(p_2^\mu p_1^\nu + p_2^\nu p_1^\mu)$:

$$g^{\mu\nu} \approx \frac{2}{s} p_2^\mu p_1^\nu. \tag{10}$$

So we obtain for the squared module of summed on spin states of matrix element

$$\sum |M|^2 = 4G^2 s^2 (q^2)^2 \Phi^{(e)} \Phi^{(\mu)}, \;\; \Phi^{(\mu)} = \sum |\frac{1}{s} J_\lambda^{(\mu)} p_2^\lambda|^2,$$
$$\Phi^{(e)} = \sum |\frac{1}{s} J_\sigma^{(e)} p_1^\sigma|^2. \tag{11}$$

The quantities $\Phi^{(e,\mu)}$ (so called impact factors) remains finite in the limit of high energies $s \to \infty$. In particular

$$\Phi^{(e)} = \sum |\frac{1}{s} \bar{u}(p_2') \hat{p}_1 u(p_2)|^2 = 2. \tag{12}$$

The quark current obey the gauge condition

$$q_\mu J^\mu(p_1) \approx (\alpha p_2 + q_\perp)_\mu J^{(q)}(p_1) = 0. \tag{13}$$

# Introduction

Using this relation we obtain for our process

$$\sum |M|^2 = 8\frac{s^2}{s_1^2}\frac{G^2\vec{q}^2}{(q^2)^2}\sum |\frac{1}{s}J_\lambda^{(q)}e^\lambda|^2, \tag{14}$$

with $\vec{e} = \vec{q}/|\vec{q}$ can be interpreted as a polarization vector of virtual photon. The symbol $\approx$ we use means the power accuracy of our calculations, namely we omit contributions of order

$$1 + O(\frac{M^2}{s}, \frac{\vec{q}^2}{s}) \tag{15}$$

compared with ones of order of unity. To obtain the differential cross section

$$d\sigma^{eq\to ejet_q} = \frac{1}{8s}\sum |M|^2 d\Gamma_{2+n}, \tag{16}$$

we must rearrange the phase volume of final state (electron remains to be spectator, whereas the scattered mu-meson is accompanied with $n$ particles)

$$d\Gamma_{2+n} = (2\pi)^4\delta^4(p_1 + p_2 - p_1' - p_2' - \sum q_i)\frac{d^3p_1'}{2E_1'(2\pi)^3}\frac{d^3p_2'}{2E_2'(2\pi)^3}\Pi_i\frac{d^3q_i}{2E_i(2\pi)^3}, \tag{17}$$

including the additional variable $q$ as

$$d\Gamma_{2+n} \to d\Gamma_{2+n} d^4 q \delta^4(p_2 - q - p_2'). \quad (18)$$

We use Sudakov variables:

$$d^4 q = \frac{s}{2} d\alpha d\beta d^2 \vec{q}; \frac{d^3 q_\pm}{2E_\pm} = \frac{s}{2} d\alpha_\pm dx \pm d^2 \vec{q}_\pm \delta(s\alpha_\pm x_\pm - \vec{q}_\pm^2 - M^2) \quad (19)$$

Performing the integrations on "small" Sudakov variables $\alpha, \alpha_\pm$, we obtain

$$d\Gamma_{2+n} = \frac{1}{sx}(2\pi)^4 (2\pi)^{-3(2+n)} 2^{-n-1} d^2 \vec{q} \Pi_1^n \frac{dx_i}{v_i} d^2 \vec{q}_i. \quad (20)$$

It can be noted that the cross section do not depend on $s$ at large $s$ and tends to zero in the limit of zero recoil momentum of the spectator electron $\vec{q} \to 0$. The last property is the consequence of gauge invariance of the theory. The cross section being integrated by the recoil momentum reveals the so called WW enhancement factor

$$L = \int\limits_0^{Q^2} \frac{\vec{q}^2 d\vec{q}^2}{(\vec{q}^2 + m_2^2 \alpha^2)^2} = \ln \frac{Q^2 s^2}{m_2^2 s_1^2} - 1, \;\; Q^2 \sim M^2. \quad (21)$$

# Heavy Fermion and Quark Production

For the case of production of pair of heavy fermions in a fragmentation region of initial quark we have for phase volume

$$d\Gamma_4 = \frac{1}{8s}(2\pi)^{-8}\frac{dx_+dx_-}{xx_+x_-}d^2\vec{q}d^2\vec{q}_+d^2\vec{q}_- = \frac{1}{2^{11}\pi^5 s}d\vec{q}^2\frac{d\phi}{2\pi}d\gamma_4,$$
$$d\gamma_4 = \frac{d^2\vec{q}_+d^2\vec{q}_-}{\pi^2}\frac{dx_+dx_-}{xx_+x_-}. \qquad (22)$$

Differential cross section in WW approximation have a form

$$d\sigma^{QED} = \frac{2\alpha^4}{\pi s_1^2}LRd\gamma_4, \quad R = \frac{\sum|M|^2}{\vec{q}^2}|_{\vec{q}=0}. \qquad (23)$$

Here

$$\sum|M|^2 = \frac{1}{(q_1^2)^2}S_1 + \frac{1}{(q_2^2)^2}S_2 + \frac{2}{q_1^2q_2^2}S_3,$$
$$q_1^2 = (q_+ + q_-)^2, q_2^2 = (p_1 - p_1')^2, \qquad (24)$$

with

# Heavy Fermion and Quark Production

$$S_1 = \frac{1}{4} Sp(\hat{p}_1' + m) O_\lambda (\hat{p}_1 + m) \bar{O}_\eta \cdot \frac{1}{4} Sp(\hat{q}_- + M)\gamma_\lambda(\hat{q}_+ - M)\gamma_\eta,$$
$$S_2 = \frac{1}{4} Sp(\hat{p}_1' + m) \gamma_\lambda (\hat{p}_1 + m) \gamma_\sigma \cdot \frac{1}{4} Sp(\hat{q}_- + M) R_\lambda(\hat{q}_+ - M)\bar{R}_\sigma,$$
$$S_3 = \frac{1}{4} Sp(\hat{p}_1' + m) O_\lambda (\hat{p}_1 + m) \gamma_\sigma \cdot \frac{1}{4} Sp(\hat{q}_- + M)\gamma_\lambda(\hat{q}_+ - M)\bar{R}_\sigma,$$
$$O_\lambda = \hat{p}_2(\hat{p}_1' - \hat{q} + m)\gamma_\lambda \frac{1}{d_1} + \gamma_\lambda(\hat{p}_1 + \hat{q} + m) p_2 \frac{1}{d};$$
$$\bar{O}_\eta = \gamma_\eta(\hat{p}_1' - \hat{q} + m)\hat{p}_2 \frac{1}{d_1} + \hat{p}_2(\hat{p}_1 + \hat{q} + m)\gamma_\eta \frac{1}{d};$$
$$R_\lambda = \gamma_\lambda(-\hat{q}_+ + \hat{q} + M)\hat{p}_2 \frac{1}{\chi_+} + \hat{p}_2(\hat{q}_- - \hat{q} + M)\gamma_\lambda \frac{1}{\chi_-};$$
$$\bar{R}_\sigma = \hat{p}_2(-\hat{q}_+ + \hat{q} + M)\gamma_\sigma \frac{1}{\chi_+} + \gamma_\sigma(\hat{q}_- - \hat{q} + M)\hat{p}_2 \frac{1}{\chi_-}; \qquad (25)$$

$$d = (p_1 + q)^2 - m^2; \;\; d_1 = (p_1' - q)^2 - m^2; \;\; \chi_+ = (q - q_+)^2 - M^2,$$
$$\chi_- = (q_- - q)^2 - M^2. \qquad (26)$$

# Heavy Fermion and Quark Production

Here we distinguish two mechanisms of the heavy quark pair creation-the so called "bremsstrahlung mechanism" (see Fig. 1a) and the "two-photon" one (Fig. 1b). The quantities $S_1, S_2, S_3$ corresponds to the squares of the relevant amplitudes and their interference. Using the on mass shell conditions for the momenta of the fragments of quark jet, given above we can express the bilinear combinations of momenta in terms of their energy fractions $x, x_\pm$ and the transversal 2-component momenta $\vec{q}_\pm$.

For the case of processes on quark we must take into account the quark color degrees of freedom.

$$d\sigma^{QCD} = C_{col} \frac{\alpha_s^2}{\alpha^2} d\sigma^{QED}, \qquad (27)$$

with $C_{col} = (N^2 - 1)/(4N^2)$, where we take into account as well the averaging on the color of quarks factor. Expressions of all necessary bilinear combinations of momenta of the problem in terms of Sudakov's variables are listed in Appendix A. Different distributions can be obtained using Monte-Carlo programs, using the explicit expression for $R$, which is presented in Appendix B.

# Figures

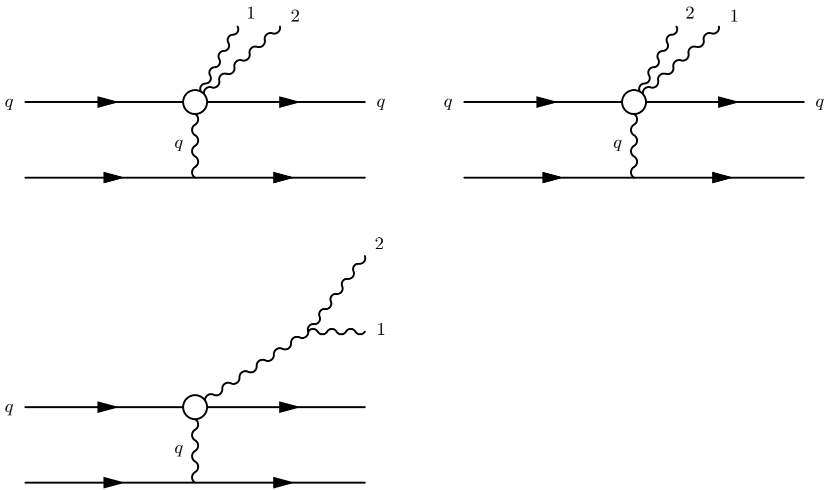


Figure: Emission of two gluons

# Heavy Fermion and Quark Production

One of important ones is the distribution on energy fractions of heavy quarks. Using the linear transformations of the transverse momenta of quarks (explicit form is given in Appendix) one can write down quantities $s_1, q_1^2, q_2^2$ to diagonal form which provide the possibility to perform the analytic integration for the case when no restriction on the values of quark transversal momenta is implied. Sudakov's parametrization method provides to express the cross section in terms of positively-definite scalar products of momenta of on mass shell particles and sign-definite expressions for squares of momenta of virtual particles. Remind that using the traditional Green functions imply Feynman prescription for the denominators $m^2 \to m^2 - i0$ and working with sign-indefinite expressions for scalar products. The results, expressed in terms of Sudakov variables turns out to be extremely convenient for numeric calculations.
It can be applied as well beyond WW approximation. In the case when the WW enhance factor $L$ is large enough, the result obtained provides the accuracy

$$1 + O(\frac{1}{L}), \tag{28}$$

which typically guarantee the accuracy on the level of several percents.

# Heavy Fermion and Quark Production

Matrix element of process

$$l_1(p_1) + l_2(p_2) \to l_1(p_1') + l_2(p_2') + Q_-(q_-) + Q_+(q_+),$$
$$p_1^2 = p_1'^2 = m^2; p_2^2 = p_2'^2 = m_2^2, q_\pm^2 = M^2 \tag{29}$$

in kinematic region of $l_1$ particle fragmentation can be written beyond the WW approximation (—$\vec{q}|$ is not small) terms of "effective vertex" functions

$$M^{l_1 l_2 \to l_1 (Q\bar{Q}) l_2} = \frac{2}{q^2}\left[\frac{1}{s}\bar{l}_2(p_2')\hat{p}_1 l_2(p_2)\right] \cdot$$
$$\left[\frac{1}{q_1^2}\bar{u}(p_1')Q_\mu u(p_1)\bar{u}(q_-)\gamma_\mu v(q_+) + \frac{1}{q_2^2}\bar{u}(q_-)R_\mu v(q_+)\bar{u}(p_1')\gamma_\mu u(p_1)\right], \tag{30}$$

with

$$Q_\mu = \frac{x_+ x_-}{d_1 d_1'}[s x \gamma_\mu (d_1' - d_1) + x d_1' \gamma_\mu \hat{q}\hat{p}_2 + d_1 \hat{p}_2 \hat{q}\gamma_\mu],$$
$$R_\mu = \frac{x}{d_2 d_2'}[s x_+ x_- \gamma_\mu (d_2' - d_2) - x_- d_2' \hat{p}_2 \hat{q}\gamma_\mu - x_+ d_2 \gamma_\mu \hat{q}\hat{p}_2],$$
$$\vec{q} = \vec{p}' + \vec{q}_- + \vec{q}_+, \tag{31}$$

# Heavy Fermion and Quark Production

$$d_1 = d_0 - 2xx_-\vec{q}(\vec{p}' + \vec{q}_-) + \vec{q}^2 xx_-(1 - x_+);$$
$$d_1' = d_0 - 2xx_-\vec{q}\vec{q}_- + x_-(1 - x_-)(\vec{q}^2 - 2\vec{q}\vec{p}');$$
$$d_2' = d_0 - 2xx_-\vec{q}\vec{p}' + x(1 - x)(\vec{q}^2 - 2\vec{q}\vec{q}_-);$$
$$d_2 = d_0 = M^2 x(1 - x) + m^2 x_+ x_-(1 - x) +$$
$$\vec{p}'^2 x_-(1 - x_-) + \vec{q}_-^2 x(1 - x) + 2x_-\vec{p}'\vec{q}_-. \quad (32)$$

In WW approximation we have $d_1 = d_1' = d_2 = d_2' = d_0$ and

$$Q_\mu = \frac{1}{sxd_0^2} v_\mu; \quad R_\mu = \frac{1}{sx_+x_-d_0^2}\rho_\mu;$$
$$v_\mu = -2(\vec{q}\vec{p})\gamma_\mu + x(d_0/s)\gamma_\mu\hat{q}\hat{p}_2 + (d_0/s)\hat{p}_2\hat{q}\gamma_\mu;$$
$$\rho_\mu = 2(\vec{q}\vec{r})\gamma_\mu - x_-(d_0/s)\gamma_\mu\hat{q}\hat{p}_2 + x_+(d_0/s)\hat{p}_2\hat{q}\gamma_\mu,$$
$$k_1^2 = \frac{1}{x_+x_-}[M^2(1 - x)^2 + \vec{r}^2], \vec{r} = x_+\vec{q}_- - x_-\vec{q}_+;$$

$$k_2^2 = -\frac{1}{x}[m^2(1 - x)^2 + \vec{p}^2], \vec{p} = -\vec{q}_+ - \vec{q}_-, q^2 = -(\vec{q}^2 + m_2^2\alpha_0^2), \alpha_0 = \frac{d_0}{sx_+x_-}. \quad (33)$$

# Heavy Fermion and Quark Production

The differential cross section of heavy quark production by the bremsstrahlung mechanism is (we imply the pair created move close to the initial quark motion $\vec{p}_1$ or $z$-axis)

$$d\sigma_{br} = \frac{2\alpha_s^2\alpha^2}{\pi} C_{color} \frac{S_{br}}{(q^2 k_1^2)^2 d_0^4 x^2} d\gamma_4;$$

$$d\sigma_{\gamma\gamma} = \frac{2\alpha_s^2\alpha^2}{\pi} C_{color} \frac{S_{\gamma\gamma}}{(q^2 k_2^2)^2 d_0^4 (x_+ x_-)^2} d\gamma_4;$$

$$d\sigma_{odd} = \frac{4\alpha_s^2\alpha^2}{\pi} C_{color} \frac{S_{odd}}{(q^2)^2 k_1^2 k_2^2 d_0^4 x x_+ x_-} d\gamma_4; \quad d\gamma_4 = \frac{d^2 q d^2 q_+ d^2 q_-}{\pi^3} \frac{dx_+ dx_-}{x x_+ x_-}. \quad (34)$$

Here

$$S_{br} = [q_{+\nu} q_{-\mu} + q_{+\mu} q_{-\nu} - \frac{1}{2} k_1^2 g_{\mu\nu}] \frac{1}{4} Sp \hat{p}_1' v_\mu \hat{p}_1 v_\nu^+,$$

$$S_{\gamma\gamma} = [p_{1\nu} p_{1\mu}' + p_{1\mu} p_{1\nu}' - \frac{1}{2} k_2^2 g_{\mu\nu}] \frac{1}{4} Sp(\hat{q}_+ M) \rho_\mu (\hat{q}_+ - M) \rho_\nu^+,$$

$$S_{odd} = \frac{1}{4} Sp(\hat{q}_+ M) \rho_\mu (\hat{q}_+ - M) v_\nu^+ \frac{1}{4} Sp \hat{p}_1' \gamma_\mu \hat{p}_1 \rho_\nu. \quad (35)$$

# Emission of two gluon jets

Matrix element of process of two gluon jets in peripheral quark-colorless fermion target collision (see Fig.2)

$$q(p_1) + Y(p_2) \to q(p_1') + Y(p_2') + g(k_1) + g(k_2), \qquad (36)$$

have a form

$$M = \frac{32s\alpha\alpha_s}{q^2} J^q J^Y, J^q = \bar{u}(p_1')[A_{12}R_2 + A_{21}R_1 + (R_1 - R_2)A_3]u(p_1), \qquad (37)$$

with $J^Y = (1/s)\bar{Y}(p_2')\hat{p}_1 Y(p_2)$. When this quantity square is summed by spin states one obtain $\sum |J^Y|^2 = 2$. Besides $R_1 = (t^a t^b)_{r_2 r_1}, R_2 = (t^b t^a)_{r_2 r_1}$, with $r_2(r_1)$ describe the color states of the scattered (initial) quark.

# Emission of two gluon jets

Here

$$A_{12} = -\frac{1}{d_1\chi_1}\hat{p}_2(\hat{p}_1' - \hat{q} + m)\hat{e}^b(\hat{p}_1 - \hat{k}_1 + m)\hat{e}^a$$
$$-\frac{1}{\chi_2'\chi_1}\hat{e^b}(\hat{p}_1' + \hat{k}_2 + m)\hat{p}_2(\hat{p}_1 - \hat{k}_1 + m)\hat{e}^a +$$
$$\frac{1}{d\chi_2'}\hat{e}^b(\hat{p}_1' + \hat{k}_2 + m)\hat{e}^a(\hat{p}_1 + \hat{q} + m)\hat{p}_2, \quad (38)$$
$$A_{21} = -\frac{1}{d_1\chi_2}\hat{p}_2(\hat{p}_1' - \hat{q} + m)\hat{e}^a(\hat{p}_1 - \hat{k}_2 + m)\hat{e}^b$$
$$-\frac{1}{\chi_1'\chi_2}\hat{e^a}(\hat{p}_1' + \hat{k}_1 + m)\hat{p}_2(\hat{p}_1 - \hat{k}_2 + m)\hat{e}^b +$$
$$\frac{1}{d\chi_1'}\hat{e}^a(\hat{p}_1' + \hat{k}_1 + m)\hat{e}^b(\hat{p}_1 + \hat{q} + m)\hat{p}_2, \quad (39)$$
$$A_3 = -\frac{2}{(k_1+k_2)^2}V\mu[\frac{1}{d_1}\hat{p}_2(\hat{p}_1' - \hat{q} + m)\gamma_\mu + \frac{1}{d}\gamma_\mu(\hat{p}_1 + \hat{q} + m)],$$
$$V_\mu = e_\mu^b(k_2 e^a) + k_{1\mu}(e^a e^b) - e_\mu^a(k_1 e^b). \quad (40)$$

# Emission of two gluon jets

To work with the irreducible color states we use the projectors in color space

$$C_1 = \frac{1}{\sqrt{N(N^2-1)}}\delta^{ab}\delta_{r_2 r_1};$$
$$C_2 = \sqrt{\frac{2N}{(N^2-1)(N^2-4)}}d^{abc}(t^c)_{r_2 r_1};$$
$$C_3 = \sqrt{\frac{2}{N(N^2-1)}}f^{abc}(t^c)_{r_2 r_1} \quad (41)$$

these projectors obey the equations

$$C_i\tilde{C}_j = \delta_{ij}, i, j = 1, 2, 3. \quad (42)$$

Here $(\tilde{A})_{r_1 r_2} = (A)_{r_2 r_1}$ and summation on $a, b$ implied. The expansion on irreducible color representations is

$$R = C_1(R\tilde{C}_1) + C_2(R\tilde{C}_2) + C_3(R\tilde{C}_3). \quad (43)$$

# Emission of two gluon jets

For our case

$$R_1 = \sqrt{\frac{N^2-1}{4N}}[C_1 + \sqrt{\frac{N^2-4}{2}}C_2 + \frac{N}{\sqrt{2}}C_3],$$
$$R_2 = \sqrt{\frac{N^2-1}{4N}}[C_1 + \sqrt{\frac{N^2-4}{2}}C_2 - \frac{N}{\sqrt{2}}C_3]. \quad (44)$$

As a check we have

$$(R_1\tilde{R}_1) = (R_1\tilde{R}_1) = Trt^at^bt^bt^a = NC_F^2;$$
$$(R_1\tilde{R}_2) = Trt^at^bt^at^b = -\frac{1}{2}C_F. \quad (45)$$

These relations are fulfilled.

# Emission of two gluon jets

It can be checked that matrix element obey gauge invariance, namely it turns to zero if one replace $p_2 \to q$ and $e_i(k_i) \to k_i$. The expression for the matrix element at $R_1 = R_2 = 1$, coincide with the QED result (Kuraev E.A., Lipatov L.N., Merenkov N.P., Fadin V.S. and Khoze V.A., Yad. Fyz. v 19,(1974),331). So the summed on color and spin states matrix element squared can be written as

$$\sum |M|^2 = \frac{32s^2(16\pi^2\alpha\alpha_s)^2}{(q^2)^2}\frac{N^2-1}{4N}F,$$

$$F = (1+\frac{N^2-4}{2})\frac{1}{4}Sp\hat{p}_1'(A_{12}+A_{21})\hat{p}_1(A_{12}+A_{21})^+ + \frac{N^2}{2}\frac{1}{4}Sp\hat{p}_1'(A_{12}-A_{21}+2A_3)\hat{p}_1(A_{12}-A_{21}+2A_3)^+. \quad (46)$$

The differential cross section is

$$d\sigma^{qY\to qggY} = \frac{(\alpha\alpha_s)^2}{8\pi}\frac{N^2-1}{N^3}\frac{F}{(q^2)^2}d\gamma_4. \quad (47)$$

Further details are presented in Appendix.

# Conclusion

In conclusion we remind a remarkable property of the kinematics of processes in the fragmentation region. It is known as a "cumuljative" phenomena. It consist in the events with production of a heavy quark-anti-quark pair, accompanied by the "reflected" scattered parents light particle.It was known in processes of production of muon-anti-muon pair in the fragmentation region of electron in electron-positron collisions. It turns out that the electron, "accompanied" the pair created in the kinematic region near the threshold move in the direction, opposite to the initial electron direction. For the case of production of a a heavy quark-anti-quark pair by one of the valence quarks from the initial proton the parent (light) quark effectively reflected. So the jet created by this quark in fact is two jets-one consist from the pair created and the two spectator quarks from the initial proton and another is the one moving in the opposite direction, created by the "reflected" quark. To see it let consider the kinematics of peripheral process $q(p_1) + q(p_2) \to Q(p_a) + \bar{Q}(p_b) + q(p_1')$. Using Sudakov parametrization (3) with

$$\tilde{p}_1 = E(1,1,0,0), \tilde{p}_2 = E(1,-1,0,0), p_\perp = (0,0,\vec{p}), \tag{48}$$

we obtain for 4-momentum of the scattered quark

$$\frac{1}{E} p_1' = \frac{m^2 + \vec{p}^2}{xs}(1,-1,0,0) + x(1,1,0,0) + (0,0,\vec{p}). \tag{49}$$

# Figures

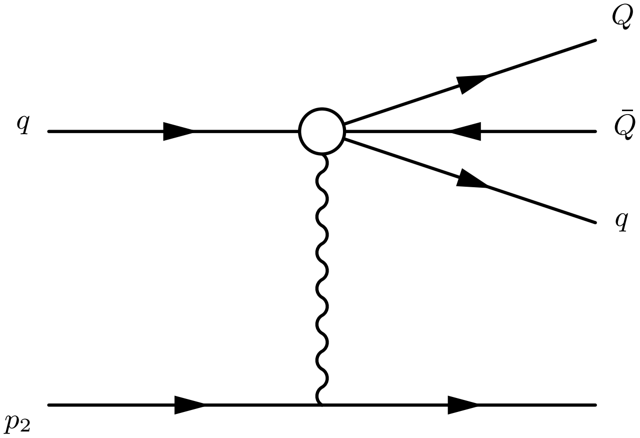


Figure: To "cummulation effect"

# Conclusion

Comparing it's component along $z$ axis from the first and second terms we find that for

$$x - \frac{m^2 + \vec{p}^2}{4E^2 x} < 0, \vec{p}^2 = (\vec{p}_a - \vec{p}_b)^2, \quad (50)$$

the "reflection" phenomena take place. For instance, assuming $\vec{p}^2 \sim M^2 >> m^2$ we have $x < (M/2E) \sim 1$. This situation can be realized near the threshold of heavy pair production.

Basing on formulae given above the energy-energy correlations of the jets in the final state can be constructed. It consist in construction of average of product of the energy fractions of the heavy quarks. As well the azimuthal angles correlation which is the average of $4(\vec{q}_{+i})(\vec{q}_{-j})/s$ can be investigated. Energy spectra, the total cross sections and the sum rules for different processes in fragmentation region can be investigated in full analogy with QED program for colliding $eY = (eX)Y$ (Baier V.N.,Fadin V.S., Khoze V.A. and Kuraev E.A.,Phys.Rep.v 78(1981),293).

The approach developed here can be used for description of jets in the fragmentation region with creation of $K, \bar{K}$ states when the heavy strange quark and anti-quark are created. As well the jets originated from $D, \bar{D}$ and $B, \bar{B}$ can be considered.

# Appendix

Expressions for scalar products of momenta become more simple in WW approximation $\vec{q} = \vec{p} + \vec{q}_+ + \vec{q}_- = 0$

$$q_1^2 = (q_+ + q_-)^2 = \frac{1}{x_+ x_-}[M^2 \bar{x}^2 + (x_- \vec{q}_+ - x_+ \vec{q}_-)^2];$$
$$q_2^2 = (p_1 - p_1')^2 = -\frac{1}{x}[\vec{p}^2 + \bar{x}^2 m_e^2];$$
$$d = (p_1 + q)^2 - m^2 = s_1; \;\; d_1 = (p_1' - q)^2 - m^2 = -s_1 x;$$
$$\chi_+ = (q - q_+)^2 - M^2 = -s_1 x_+, \;\; \chi_- = (q_- - q)^2 - M^2 = -s_1 x_-, \qquad (51)$$

with $\bar{x} = 1 - x, x_+ + x_- + x = 1$. In terms of the variables

$$\vec{\rho} = \frac{1}{x_+ x_-}(x_- \vec{q}_+ - x_+ \vec{q}_-), \quad \vec{\sigma} = \frac{1}{\bar{x}}(\vec{q}_+ + \vec{q}_-),$$
$$\vec{q}_+ = \frac{x_+}{\bar{x}}[x_- \vec{\rho} + \bar{x}\vec{\sigma}], \quad \vec{q}_- = \frac{x_-}{\bar{x}}[-x_+ \vec{\rho} + \bar{x}\vec{\sigma}], \qquad (52)$$

# Appendix

three quadratic forms $s_1, q_1^2, q_2^2$ can be written in diagonal form simultaneously

$$q_1^2 = x_+x_-[(\frac{\bar{x}}{x_+x_-})^2 M^2 + \vec{\rho}^2];$$
$$q_2^2 = -\frac{\bar{x}^2}{x}[m_e^2 + \vec{\sigma}^2];$$
$$s_1 = \frac{1}{x}[m_e^2 + \frac{\bar{x}}{x_+x_-}M^2 + \frac{xx_+x_-}{\bar{x}}\vec{\rho}^2 + \bar{x}\vec{\sigma}^2]. \tag{53}$$

# Appendix

Other scalar products are

$$2q_+q_- = x_+x_-\vec{\rho}^2 + \frac{x_+^2+x_-^2}{x_+x_-}M^2; \;\; 2p_1p_1' = \frac{1}{x}[\bar{x}^2\vec{\sigma}^2 + m_e^2(1+x^2)],$$
$$2p_1q_+ = \frac{x_+}{\bar{x}^2}(x_-\vec{\rho}+\bar{x}\vec{\sigma})^2 + m_e^2x_+ + \frac{1}{x_+}M^2,$$
$$2p_1q_- = \frac{x_-}{\bar{x}^2}(-x_+\vec{\rho}+\bar{x}\vec{\sigma})^2 + m_e^2x_- + \frac{1}{x_-}M^2,$$
$$2p_1'q_+ = \frac{x_+}{x\bar{x}^2}(x_-x\vec{\rho}+\bar{x}\vec{\sigma})^2 + m_e^2\frac{x_+}{x} + \frac{x}{x_+}M^2,$$
$$2p_1'q_- = \frac{x_-}{x\bar{x}^2}(-x_+x\vec{\rho}+\bar{x}\vec{\sigma})^2 + m_e^2\frac{x_-}{x} + \frac{x}{x_-}M^2. \tag{54}$$

Computing the relevant determinant we find

$$d^2\vec{q}_+d\vec{q}_- = x_+^2x_-^2d^2\vec{\rho}d^2\vec{\sigma}. \tag{55}$$