Top quark production

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Lecture 1

- **Higher-order collinear and soft corrections**
- **Factorization, RGE, and Resummation**
- **Two-loop eikonal calculations**
- **Soft anomalous dimensions**
- **NNLL resummation and NNLO expansions**

Lecture 2

- **Cusp anomalous dimensions**
- **One- and two-loop eikonal diagrams**
- **Two-loop soft anomalous dimension matrix for top-pair production**

Lecture 3

- $t\bar{t}$ **production**
- $t\bar{t}$ **cross section at LHC and Tevatron**
- **Top** p_T **and rapidity distributions**
- **Single-top production**
- t**-channel and** s**-channel production**
- tW^- **and** tH^- **production**

Higher-order corrections

QCD corrections are significant for top-pair and single-top production
Soft-gluon corrections from emission of soft (low-energy) gluons
$\rightarrow$ arise from incomplete cancellations of infrared divergences between virtual diagrams and real diagrams with soft gluons

Soft terms $\left[\frac{\ln^k(s_4/m_t^2)}{s_4}\right]_+$ with $k \leq 2n-1$
and s_4 distance from threshold

double collinear and soft logarithms
also purely collinear terms $\frac{1}{m_t^2}\ln^k(s_4/m_t^2)$

Soft-gluon corrections are dominant near threshold
Resum (exponentiate) these corrections - factorization and RGE

NLL accuracy requires one-loop calculations in the eikonal approximation
Complete results at NNLL – two-loop soft anomalous dimension

NK, PRD 82, 114030 (2010); PRD 84, 011504 (2011) ($t\bar{t}$) NK, PRD81, 054028 (2010); PRD 82, 054018 (2010); PRD 83,091503 (2011) (single top)

Approximate NNLO double-differential cross section from expansion of resummed result

Factorization, RGE, and Resummation

Consider hadronic processes of the form

$$h_1(p_{h_1}) + h_2(p_{h_2}) \rightarrow t(p) + X$$

where h_1, h_2, are incoming hadrons and t denotes the observed top quark with X all additional final-state particles

The partonic processes are of the form

$$f_1(p_1) \; + \; f_2\,(p_2) \rightarrow t(p) \; + \; X$$

where f_1 and f_2 represent partons (quarks or gluons)

Define $s=(p_1+p_2)^2$, $t=(p_1-p)^2$, $u=(p_2-p)^2$

Also $s_4=s+t+u-\sum m^2$, where the sum is over the squared masses of all particles in the process

At threshold $s_4=0$

Factorization for the (differential) cross section

$$d\sigma_{h_1 h_2 \rightarrow tX} \quad = \quad \sum_{f_1,f_2} \int dx_1 \, dx_2 \, \phi_{f_1/h_1}(x_1,\mu_F) \, \phi_{f_2/h_2}(x_2,\mu_F) \, \hat{\sigma}_{f_1 f_2 \rightarrow tX}(s,t,u,\mu_F,\mu_R)$$

soft-gluon corrections appear as plus distributions in $\hat{\sigma}_{f_1 f_2 \to tX}$ of the form

$$\mathcal{D}_l(s_4) \equiv \left[\frac{\ln^l(s_4/m_t^2)}{s_4}\right]_+$$

and are defined through their integral with parton distribution functions

$$\int_0^{s_4\,max} ds_4\, \phi(s_4) \left[\frac{\ln^l(s_4/m_t^2)}{s_4}\right]_+ \equiv \int_0^{s_4\,max} ds_4 \frac{\ln^l(s_4/m_t^2)}{s_4}[\phi(s_4) - \phi(0)] + \frac{1}{l+1} \ln^{l+1}\left(\frac{s_{4\,max}}{m_t^2}\right)\phi(0)$$

Resummation follows from factorization properties of the cross section - performed in moment space

We define moments of the partonic cross section by

$\hat{\sigma}(N) = \int dz\, z^{N-1}\hat{\sigma}(z)$ **(PIM) or by** $\hat{\sigma}(N) = \int (ds_4/s)\, e^{-Ns_4/s}\hat{\sigma}(s_4)$ **(1PI)**

then the logarithms of N that appear in $\hat{\sigma}(N)$ exponentiate

write a factorized form for the moment-space infrared-regularized parton-parton scattering cross section $\sigma_{f_1 f_2 \to tX}(N,\epsilon)$,with $\epsilon = 4 - n$, which factorizes as the hadronic cross section

$$\sigma_{f_1 f_2 \to tX}(N,\epsilon) = \phi_{f_1/f_1}(N,\mu_F,\epsilon)\, \phi_{f_2/f_2}(N,\mu_F,\epsilon)\, \hat{\sigma}_{f_1 f_2 \to tX}(N,\mu_F,\mu_R)$$

with $\phi(N)=\int_0^1 dx\, x^{N-1}\phi(x)$

We factorize the initial-state collinear divergences, regularized by ϵ**, into the parton distribution functions,** ϕ**, and we thus obtain the perturbative expansion for the infrared-safe partonic short-distance function** $\hat{\sigma}$**.**

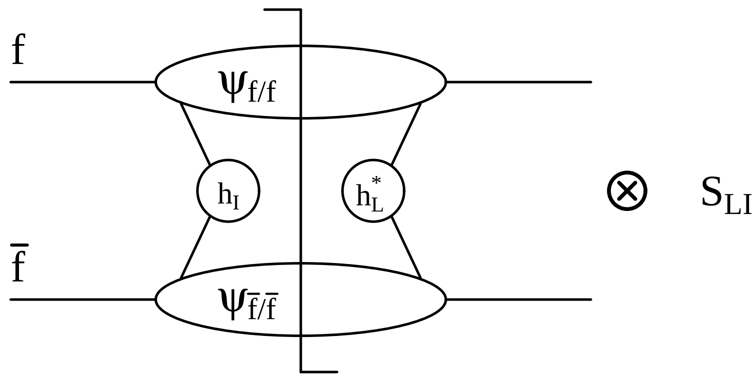
The partonic short-distance function $\hat{\sigma}$ **still has sensitivity to soft-gluon dynamics through its** N **dependence. We then refactorize the moments of the cross section**

$$\sigma_{f_1 f_2 \to tX}(N,\epsilon) = \psi_{f_1/f_1}(N,\mu_F,\epsilon)\; \psi_{f_2/f_2}(N,\mu_F,\epsilon)$$

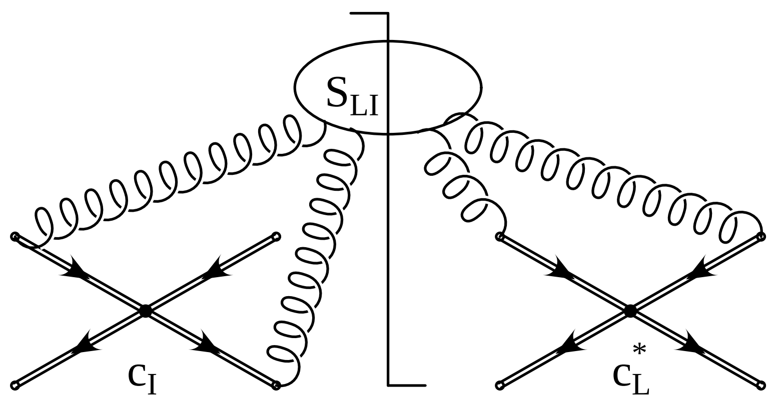
$$\times\, H_{IL}^{f_1 f_2 \to tX}\left(\alpha_s(\mu_R)\right)\; S_{LI}^{f_1 f_2 \to tX}\left(\frac{m_t}{N\mu_F},\alpha_s(\mu_R)\right)\prod_j J_j\left(N,\mu_F,\epsilon\right)+\mathcal{O}(1/N)$$

where the modified parton distributions ψ **are defined in the partonic center-of-mass at fixed energy and they absorb all the universal collinear singularities from the incoming partons**

H_{IL} **are** N**-independent hard components which describe the hard-scattering**
S_{LI} **is the soft gluon function for non-collinear soft gluons;**
it represents the coupling of soft gluons to the partons in the scattering
J **are jet functions**



(a)



(b)

The hard-scattering function involves contributions from the amplitude of the process and the complex conjugate of the amplitude, $H_{IL} = h^*_L \, h_I$

Comparing, we find

$$\hat{\sigma}_{f_1 f_2 \to tX}(N, \mu_F, \mu_R) = \frac{\psi_{f_1/f_1}(N, \mu_F, \epsilon) \, \psi_{f_2/f_2}(N, \mu_F, \epsilon)}{\phi_{f_1/f_1}(N, \mu_F, \epsilon) \, \phi_{f_2/f_2}(N, \mu_F, \epsilon)} \, H_{IL}^{f_1 f_2 \to tX}\left(\alpha_s(\mu_R)\right)$$
$$\times \, S_{LI}^{f_1 f_2 \to tX}\left(\frac{m_t}{N\mu_F}, \alpha_s(\mu_R)\right) \prod_j J_j\left(N, \mu_F, \epsilon\right)$$

All the factors are gauge and factorization scale dependent. The constraint that the product of these factors must be independent of the gauge and factorization scale results in the exponentiation of logarithms of N **in** ψ/ϕ **and** S_{LI}**.**

The soft matrix S_{LI} **depends on** N **through the ratio** $m_t/(N\mu_F)$**, and it requires renormalization as a composite operator. Its** N**-dependence can thus be resummed by renormalization group analysis. However, the product** $H_{IL} S_{LI}$ **needs no overall renormalization, because the UV divergences of** S_{LI} **are balanced by those of** H_{IL}**.**

$$
\begin{aligned}
H^0_{IL} &= \prod_{i=a,b} Z_i^{-1} \, \left(Z_S^{-1}\right)_{IC} H_{CD} \left[\left(Z_S^{\dagger}\right)^{-1}\right]_{DL} \\
S^0_{LI} &= (Z_S^{\dagger})_{LB} S_{BA} Z_{S,AI}
\end{aligned}
$$

where H^0 and S^0 denote the unrenormalized quantities, Z_i is the renormalization constant of the ith incoming partonic field, and Z_S is a matrix of renormalization constants, which describe the renormalization of the soft function. Z_S is defined to include the wave function renormalization necessary for the outgoing eikonal lines that represent any heavy quarks.

Thus S_{LI} satisfies the renormalization group equation

$$
\left(\mu \frac{\partial}{\partial \mu} + \beta(g_s)\frac{\partial}{\partial g_s}\right) S_{LI} = -(\Gamma_S^{\dagger})_{LB} S_{BI} - S_{LA} (\Gamma_S)_{AI}
$$

where β is the QCD beta function and $g_s^2 = 4\pi\alpha_s$. Γ_S is an anomalous dimension matrix that is calculated in the eikonal approximation by explicit renormalization of the soft function. It is given at one loop by

$$
\Gamma_S^{(1-loop)}(g_s) = -\frac{g_s}{2} \frac{\partial}{\partial g_s} \mathrm{Res}_{\epsilon \to 0} Z_S(g_s, \epsilon)
$$

Γ_S is the soft anomalous dimension - a matrix in color space and a function of kinematical invariants s, t, u

The process-dependent matrices Γ_S have been calculated at one loop for all $2 \to 2$ partonic processes

For quark-(anti)quark scattering, Γ_S is a 2×2 matrix
for quark-gluon scattering it is a 3×3 matrix
for gluon-gluon scattering it is an 8×8 matrix

Resummed cross section

$$\hat{\sigma}^{res}(N) = \exp\left[\sum_i E_i(N_i)\right] \exp\left[\sum_j E'_j(N')\right] \exp\left[\sum_{i=1,2} 2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu} \gamma_{i/i}\left(\tilde{N}_i, \alpha_s(\mu)\right)\right]$$
$$\times \, \mathrm{tr}\left\{H\left(\alpha_s\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}'} \frac{d\mu}{\mu} \Gamma_S^{\dagger}\left(\alpha_s(\mu)\right)\right] S\left(\alpha_s\left(\frac{\sqrt{s}}{\tilde{N}'}\right)\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}'} \frac{d\mu}{\mu} \Gamma_S\left(\alpha_s(\mu)\right)\right]\right\}$$

collinear and soft radiation from incoming partons

$$E_i(N_i) = \int_0^1 dz \frac{z^{N_i-1}-1}{1-z} \left\{ \int_1^{(1-z)^2} \frac{d\lambda}{\lambda} A_i\left(\alpha_s(\lambda s)\right) + D_i\left[\alpha_s((1-z)^2 s)\right] \right\}$$

purely collinear: replace $\frac{z^{N-1}-1}{1-z}$ **by** $-z^{N-1}$

Here $A_i = \frac{\alpha_s}{\pi} A_i^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 A_i^{(2)} + \cdots$
where
$A_i^{(1)} = C_i$ **with** $C_i = C_F = (N_c^2-1)/(2N_c)$ **for a quark or antiquark and** $C_i = C_A = N_c$ **for a gluon, with** N_c **the number of colors**
while $A_i^{(2)} = C_i K/2$ **with** $K = C_A\,(67/18 - \zeta_2) - 5n_f/9$ **where** n_f **is the number of quark flavors**
Here and below $\zeta_2 = \pi^2/6$, $\zeta_3 = 1.2020569\cdots$, $\zeta_4 = \pi^4/90$, $\zeta_5 = 1.0369278\cdots$
Also $D_i = (\alpha_s/\pi) D_i^{(1)} + (\alpha_s/\pi)^2 D_i^{(2)} + \cdots$
with $D_i^{(1)} = 0$ **in Feynman gauge (**$D_i^{(1)} = -C_i$ **in axial gauge)**
In Feynman gauge the two-loop result is

$$D_i^{(2)} = C_i C_A \left(-\frac{101}{54} + \frac{11}{6}\zeta_2 + \frac{7}{4}\zeta_3\right) + C_i n_f \left(\frac{7}{27} - \frac{\zeta_2}{3}\right)$$

collinear and soft radiation from outgoing massless quarks and gluons

$$E'(N') = \int_0^1 dz \frac{z^{N'-1}-1}{1-z} \left\{ \int_{(1-z)^2}^{1-z} \frac{d\lambda}{\lambda} A_i\left(\alpha_s\left(\lambda s\right)\right) + B_i\left[\alpha_s((1-z)s)\right] + D_i\left[\alpha_s((1-z)^2 s)\right] \right\}$$

where $B_j = (\alpha_s/\pi) B_j^{(1)} + (\alpha_s/\pi)^2 B_j^{(2)} + \cdots$ **with** $B_q^{(1)} = -3C_F/4$ **and** $B_g^{(1)} = -\beta_0/4$

β_0 **is the lowest-order** β**-function,** $\beta_0 = (11C_A - 2n_f)/3$

Also

$$B_q^{(2)} = C_F^2 \left(-\frac{3}{32} + \frac{3}{4}\zeta_2 - \frac{3}{2}\zeta_3\right) + C_F C_A \left(-\frac{1539}{864} - \frac{11}{12}\zeta_2 + \frac{3}{4}\zeta_3\right) + n_f C_F \left(\frac{135}{432} + \frac{\zeta_2}{6}\right)$$

$$B_g^{(2)} = C_A^2 \left(-\frac{1025}{432} - \frac{3}{4}\zeta_3\right) + \frac{79}{108} C_A n_f + C_F \frac{n_f}{8} - \frac{5}{108} n_f^2$$

factorization scale μ_F dependence controlled by the moment-space anomalous dimension of the $\overline{\rm MS}$ density $\phi_{i/i}$

$\gamma_{i/i} = -A_i \ln \tilde{N}_i + \gamma_i$

with the parton anomalous dimensions

$$\gamma_i = (\alpha_s/\pi)\gamma_i^{(1)} + (\alpha_s/\pi)^2 \gamma_i^{(2)} + \cdots$$

with $\gamma_q^{(1)} = 3C_F/4$**,** $\gamma_g^{(1)} = \beta_0/4$**,**

$$\gamma_q^{(2)} = C_F^2 \left(\frac{3}{32} - \frac{3}{4}\zeta_2 + \frac{3}{2}\zeta_3\right) + C_F C_A \left(\frac{17}{96} + \frac{11}{12}\zeta_2 - \frac{3}{4}\zeta_3\right) + n_f C_F \left(-\frac{1}{48} - \frac{\zeta_2}{6}\right)$$

and

$$\gamma_g^{(2)} = C_A^2 \left(\frac{2}{3} + \frac{3}{4}\zeta_3\right) - n_f \left(\frac{C_F}{8} + \frac{C_A}{6}\right)$$

The β function is

$$\beta(\alpha_s) \equiv \frac{1}{2\alpha_s}\frac{d\alpha_s}{d\ln\mu} = \mu\, d\ln g/d\mu = -\beta_0 \alpha_s/(4\pi) - \beta_1 \alpha_s^2/(4\pi)^2 + \cdots\,,$$

where $g^2 = 4\pi\alpha_s$**, with** $\beta_0 = (11C_A - 2n_f)/3$ **and** $\beta_1 = 34C_A^2/3 - 2n_f(C_F + 5C_A/3)$**.**

Note that

$$\alpha_s(\mu) = \alpha_s(\mu_R)\left[1 - \frac{\beta_0}{4\pi}\alpha_s(\mu_R)\ln\left(\frac{\mu^2}{\mu_R^2}\right) + \frac{\beta_0^2}{16\pi^2}\alpha_s^2(\mu_R)\ln^2\left(\frac{\mu^2}{\mu_R^2}\right) - \frac{\beta_1}{16\pi^2}\alpha_s^2(\mu_R)\ln\left(\frac{\mu^2}{\mu_R^2}\right) + \cdots\right]$$

H is the hard-scattering function while S is the soft-gluon function. We use the expansions

$$H = \alpha_s^{d_{\alpha_s}} H^{(0)} + \frac{\alpha_s^{d_{\alpha_s}+1}}{\pi} H^{(1)} + \frac{\alpha_s^{d_{\alpha_s}+2}}{\pi^2} H^{(2)} + \cdots$$

and

$$S = S^{(0)} + \frac{\alpha_s}{\pi} S^{(1)} + \frac{\alpha_s^2}{\pi^2} S^{(2)} + \cdots$$

Note that both H and S are matrices in color space and the trace is taken. At lowest order, the trace of the product of the hard matrices H and soft matrices S reproduces the Born cross section for each partonic process, $\sigma^B = \alpha_s^{d_{\alpha_s}} \mathrm{tr}[H^{(0)} S^{(0)}]$.

Noncollinear soft gluon emission controlled by the soft anomalous dimension Γ_S

Also

$$\Gamma_S = \frac{\alpha_s}{\pi} \Gamma_S^{(1)} + \frac{\alpha_s^2}{\pi^2} \Gamma_S^{(2)} + \cdots$$

determine Γ_S from coefficients of ultraviolet poles in dimensionally regularized eikonal diagrams

Γ_S is process-dependent; calculated at two loops

We are resumming $\ln^k N$ - we then expand to fixed order and invert to get $\ln^k(s_4/m_t^2)/s4$ terms

NLO expansion

$$\hat{\sigma}^{(1)} = \sigma^B \frac{\alpha_s(\mu_R)}{\pi} \{c_3 \, \mathcal{D}_1(s_4) + c_2 \, \mathcal{D}_0(s_4) + c_1 \, \delta(s_4)\} + \frac{\alpha_s^{d_{\alpha_s}+1}(\mu_R^2)}{\pi} \left[A^c \, \mathcal{D}_0(s_4) + T_1^c \, \delta(s_4)\right]$$

where σ^B is the Born term,

$$c_3 = \sum_i 2 \, A_i^{(1)} - \sum_j A_j^{(1)}$$

and c_2 is defined by $c_2 = c_2^\mu + T_2$,
with $c_2^\mu = -\sum_i A_i^{(1)} \ln(\mu_F^2/M^2)$ denoting the terms involving logarithms of the scale, and

$$T_2 = \sum_i \left[-2 \, A_i^{(1)} \, \ln\left(\frac{-t_i}{M^2}\right) + D_i^{(1)} - A_i^{(1)} \ln\left(\frac{M^2}{s}\right)\right] + \sum_j \left[B_j^{(1)} + D_j^{(1)} - A_j^{(1)} \, \ln\left(\frac{M^2}{s}\right)\right]$$

$$A^c = \mathrm{tr}\left(H^{(0)} \Gamma_S^{(1)\,\dagger} S^{(0)} + H^{(0)} S^{(0)} \Gamma_S^{(1)}\right)$$

$c_1 = c_1^\mu + T_1$ with c_1^μ denoting the terms involving logarithms of the scale

$$c_1^\mu = \sum_i \left[A_i^{(1)} \, \ln\left(\frac{-t_i}{M^2}\right) - \gamma_i^{(1)}\right] \ln\left(\frac{\mu_F^2}{M^2}\right) + d_{\alpha_s} \frac{\beta_0}{4} \ln\left(\frac{\mu_R^2}{M^2}\right)$$

NNLO expansion

$$
\begin{aligned}
\hat{\sigma}^{(2)} = \; & \sigma^B \frac{\alpha_s^2(\mu_R)}{\pi^2} \left\{ \frac{1}{2} c_3^2 \, \mathcal{D}_3(s_4) + \left[\frac{3}{2} c_3 c_2 - \frac{\beta_0}{4} c_3 + \sum_j \frac{\beta_0}{8} A_j^{(1)} \right] \mathcal{D}_2(s_4) \right. \\
& + \left[c_3 c_1 + c_2^2 - \zeta_2 c_3^2 - \frac{\beta_0}{2} T_2 + \frac{\beta_0}{4} c_3 \ln\left(\frac{\mu_R^2}{M^2}\right) + \sum_i 2 A_i^{(2)} - \sum_j A_j^{(2)} + \sum_j \frac{\beta_0}{4} B_j^{(1)} \right] \mathcal{D}_1(s_4) \\
& + \left[c_2 c_1 - \zeta_2 c_3 c_2 + \zeta_3 c_3^2 + \frac{\beta_0}{4} c_2 \ln\left(\frac{\mu_R^2}{s}\right) - \sum_i \frac{\beta_0}{2} A_i^{(1)} \ln^2\left(\frac{-t_i}{M^2}\right) \right. \\
& + \sum_i [\left(-2 A_i^{(2)} + \frac{\beta_0}{2} D_i^{(1)}\right) \ln\left(\frac{-t_i}{M^2}\right) + D_i^{(2)} + \frac{\beta_0}{8} A_i^{(1)} \ln^2\left(\frac{\mu_F^2}{s}\right) - A_i^{(2)} \ln\left(\frac{\mu_F^2}{s}\right)] \\
& \left. \left. + \sum_j [B_j^{(2)} + D_j^{(2)} - \left(A_j^{(2)} + \frac{\beta_0}{4} (B_j^{(1)} + 2 D_j^{(1)})\right) \ln\left(\frac{M^2}{s}\right) + \frac{3\beta_0}{8} A_j^{(1)} \ln^2\left(\frac{M^2}{s}\right)] \right] \mathcal{D}_0(s_4) \right\} \\
& + \frac{\alpha_s^{d_{\alpha_s}+2}(\mu_R)}{\pi^2} \left\{ \frac{3}{2} c_3 A^c \, \mathcal{D}_2(s_4) + \left[\left(2 c_2 - \frac{\beta_0}{2}\right) A^c + c_3 T_1^c + F^c \right] \mathcal{D}_1(s_4) \right. \\
& \left. + \left[\left(c_1 - \zeta_2 c_3 + \frac{\beta_0}{4} \ln\left(\frac{\mu_R^2}{s}\right)\right) A^c + c_2 T_1^c + F^c \ln\left(\frac{M^2}{s}\right) + G^c \right] \mathcal{D}_0(s_4) \right\}
\end{aligned}
$$

where

$$F^c = \mathrm{tr}\left[H^{(0)}\left(\Gamma_S^{(1)\,\dagger}\right)^2 S^{(0)} + H^{(0)} S^{(0)} \left(\Gamma_S^{(1)}\right)^2 + 2H^{(0)}\Gamma_S^{(1)\,\dagger} S^{(0)} \Gamma_S^{(1)}\right]$$

$$\begin{aligned} G^c \quad = \quad & \mathrm{tr}\left[H^{(1)}\Gamma_S^{(1)\,\dagger} S^{(0)} + H^{(1)} S^{(0)} \Gamma_S^{(1)} + H^{(0)}\Gamma_S^{(1)\,\dagger} S^{(1)} + H^{(0)} S^{(1)} \Gamma_S^{(1)} \right. \\ & \left. + H^{(0)}\Gamma_S^{(2)\,\dagger} S^{(0)} + H^{(0)} S^{(0)} \Gamma_S^{(2)}\right] \end{aligned}$$

and c_3, c_2, c_1, etc are from the NLO expansion

Two-loop universal quantities $A^{(2)}$, $B^{(2)}$, $D^{(2)}$ known

Two-loop process-dependent $\Gamma_S^{(2)}$ recently calculated for several processes

Eikonal approximation

Feynman rules for soft gluon emission simplify

$$\bar{u}(p)\,(-ig_s T_F^c)\,\gamma^\mu \frac{i(\not p + \not k + m)}{(p+k)^2 - m^2 + i\epsilon} \rightarrow \bar{u}(p)\,g_s T_F^c\,\gamma^\mu \frac{\not p + m}{2p\cdot k + i\epsilon} \quad = \quad \bar{u}(p)\,g_s T_F^c\,\frac{v^\mu}{v\cdot k + i\epsilon}$$

with $p \propto v$, T_F^c generators of SU(3)

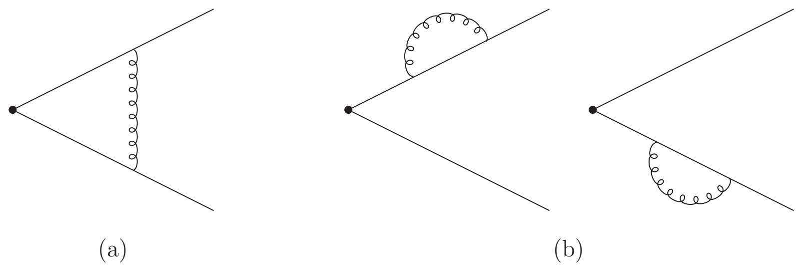
Perform calculation in momentum space and Feynman gauge

Complete two-loop results for

- **soft (cusp) anomalous dimension for $e^+e^- \rightarrow t\bar{t}$**
- **$t\bar{t}$ hadroproduction**
- **t-channel single top production**
- **s-channel single top production**
- **$bg \rightarrow tW^-$ and $bg \rightarrow tH^-$**
- **direct photon and W production at large Q_T**

Soft (cusp) anomalous dimension

One-loop eikonal diagrams



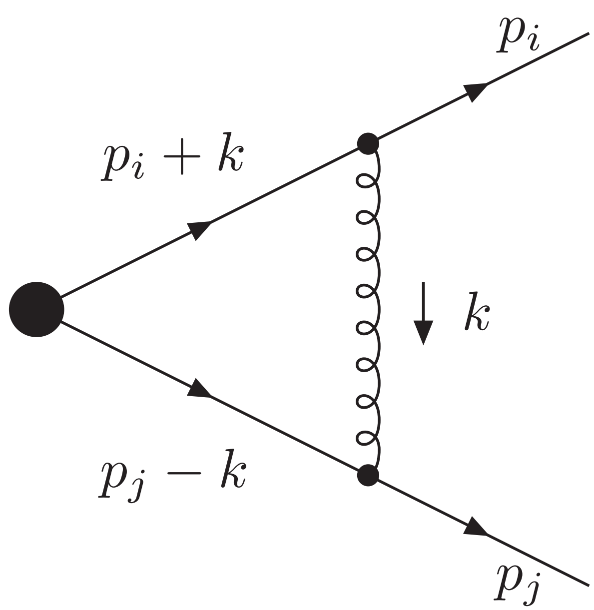
$$\Gamma_S \quad = \quad \frac{\alpha_s}{\pi}\Gamma_S^{(1)} + \frac{\alpha_s^2}{\pi^2}\Gamma_S^{(2)} + \cdots$$

The one-loop soft anomalous dimension, $\Gamma_S^{(1)}$, can be read off the coefficient of the ultraviolet (UV) pole of the one-loop diagrams

$$\Gamma_S^{(1)} = C_F \left[-\frac{(1+\beta^2)}{2\beta} \ln\left(\frac{1-\beta}{1+\beta}\right) - 1 \right] \quad \text{with} \quad \beta = \sqrt{1 - \frac{4m^2}{s}}$$

with $\beta = \sqrt{1 - \frac{4m^2}{s}}$

Example: one-loop vertex correction



$$I_{1a} = g_s^2 \int \frac{d^n k}{(2\pi)^n} \frac{(-i) g^{\mu\nu}}{k^2} \frac{v_i^{\mu}}{v_i \cdot k} \frac{(-v_j^{\nu})}{(-v_j \cdot k)}$$

Using Feynman parametrization, this can be rewritten as

$$I_{1a} = -2 i g_s^2 \frac{v_i \cdot v_j}{(2\pi)^n} \int_0^1 dx \int_0^{1-x} dy \int \frac{d^n k}{\left[x k^2 + y v_i \cdot k + (1-x-y) v_j \cdot k\right]^3}$$

which, after the integration over k, gives

$$I_{1a} = g_s^2\, v_i \cdot v_j\, 2^{6-2n}\, \pi^{-n/2}\, \Gamma\left(3-\frac{n}{2}\right) \int_0^1 dx\, x^{3-n}$$
$$\times \int_0^{1-x} dy \left[-y^2 v_i^2 - (1-x-y)^2 v_j^2 - 2y\, v_i \cdot v_j (1-x-y)\right]^{n/2-3}$$

After several manipulations, and with $n = 4-\epsilon$ and $\beta = \sqrt{1-4m^2/s}$,

$$I_{1a} = \frac{\alpha_s}{\pi} (-1)^{-1-\epsilon/2}\, 2^{5\epsilon/2}\, \pi^{\epsilon/2}\, \Gamma\left(1+\frac{\epsilon}{2}\right) (1+\beta^2) \int_0^1 dx\, x^{-1+\epsilon} (1-x)^{-1-\epsilon}$$
$$\times \left\{ \int_0^1 dz \left[4z\beta^2(1-z)+1-\beta^2\right]^{-1} - \frac{\epsilon}{2} \int_0^1 dz \frac{\ln\left[4z\beta^2(1-z)+1-\beta^2\right]}{4z\beta^2(1-z)+1-\beta^2} + \mathcal{O}\left(\epsilon^2\right) \right\}$$

integral over x contains both UV and IR singularities - isolate UV singularities

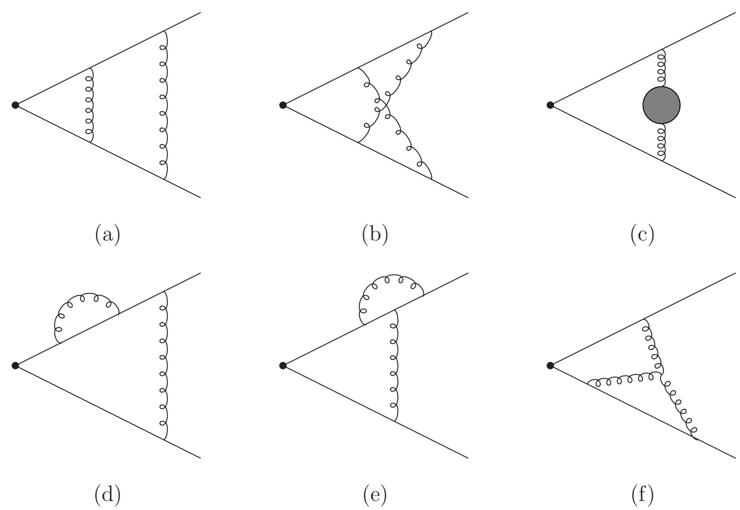
$$\int_0^1 dx\, x^{-1+\epsilon} (1-x)^{-1-\epsilon} = \frac{1}{\epsilon} + \mathrm{IR}$$

Then

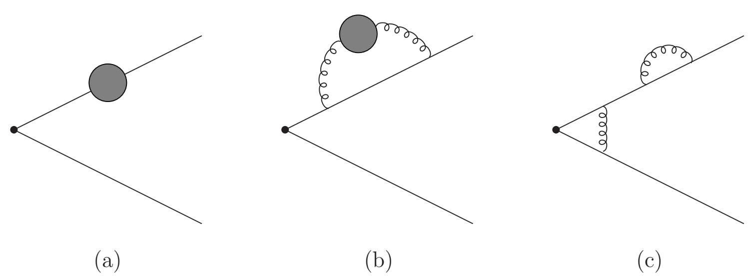
$$I_{1a}^{UV} = \frac{\alpha_s}{\pi} \frac{(1+\beta^2)}{2\beta} \frac{1}{\epsilon} \ln\left(\frac{1-\beta}{1+\beta}\right)$$

Two-loop eikonal diagrams

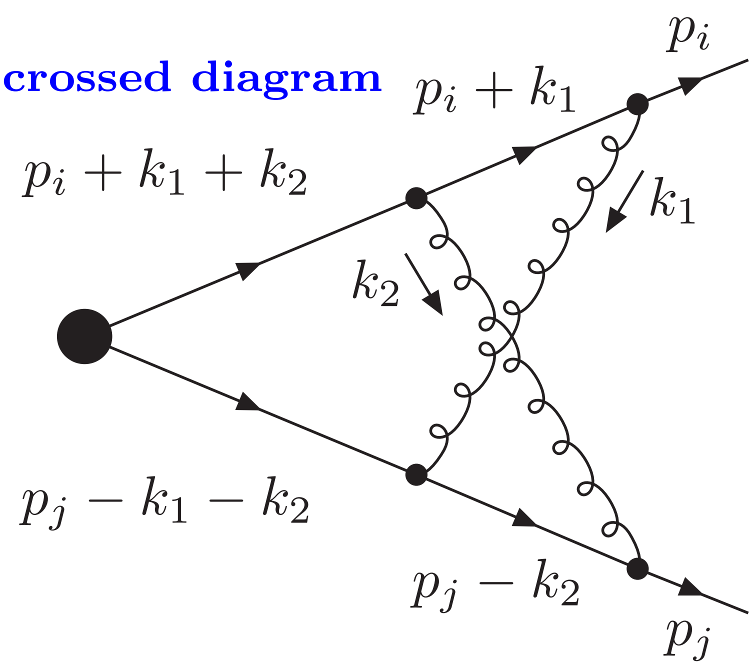
Vertex correction graphs



Heavy-quark self-energy graphs



Example: two-loop crossed diagram



$$I_{2b} = g_s^4 \int \frac{d^n k_1}{(2\pi)^n} \frac{d^n k_2}{(2\pi)^n} \frac{(-i) g^{\mu\nu}}{k_1^2} \frac{(-i) g^{\rho\sigma}}{k_2^2} \frac{v_i^\mu}{v_i \cdot k_1} \frac{v_i^\rho}{v_i \cdot (k_1 + k_2)} \frac{(-v_j^\nu)}{-v_j \cdot (k_1 + k_2)} \frac{(-v_j^\sigma)}{-v_j \cdot k_2}$$

Perform k_2 integral first

$$I_{2b} = -i \frac{\alpha_s^2}{\pi^2} 2^{-4+\epsilon} \pi^{-2+3\epsilon/2} \Gamma\left(1 - \frac{\epsilon}{2}\right) \Gamma(1+\epsilon)(1+\beta^2)^2 \int_0^1 dz \int_0^1 \frac{dy\,(1-y)^{-\epsilon}}{\left[2\beta^2(1-y)^2 z^2 - 2\beta^2(1-y)z - \frac{(1-\beta^2)}{2}\right]^{1-\epsilon/2}}$$

$$\times \int \frac{d^n k_1}{k_1^2 \, v_i \cdot k_1 \left[\left((v_i - v_j)z + v_j\right) \cdot k_1\right]^{1+\epsilon}}$$

Then proceed with the k_1 integral, and isolate UV and IR poles. After many steps

$$I_{2b}^{UV}=\frac{\alpha_s^2}{\pi^2}\frac{(1+\beta^2)^2}{8\beta^2}\frac{1}{\epsilon}\left\{-\ln\left(\frac{1-\beta}{1+\beta}\right)\left[\mathrm{Li}_2\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)+\zeta_2\right]-\frac{1}{3}\ln^3\left(\frac{1-\beta}{1+\beta}\right)+\mathrm{Li}_3\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)-\zeta_3\right\}$$

Include counterterms for all graphs and multiply with corresponding color factors
Determine two-loop soft anomalous dimension from UV poles of the sum of the graphs

$$\Gamma_S^{(2)}=\frac{K}{2}\,\Gamma_S^{(1)}+C_F C_A M_\beta=\frac{K}{2}\,\Gamma_S^{(1)}+C_F C_A\left\{\frac{1}{2}+\frac{\zeta_2}{2}+\frac{1}{2}\ln^2\left(\frac{1-\beta}{1+\beta}\right)\right.$$
$$-\frac{(1+\beta^2)^2}{8\beta^2}\left[\zeta_3+\zeta_2\ln\left(\frac{1-\beta}{1+\beta}\right)+\frac{1}{3}\ln^3\left(\frac{1-\beta}{1+\beta}\right)+\ln\left(\frac{1-\beta}{1+\beta}\right)\mathrm{Li}_2\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)-\mathrm{Li}_3\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)\right]$$
$$-\frac{(1+\beta^2)}{4\beta}\left[\zeta_2-\zeta_2\ln\left(\frac{1-\beta}{1+\beta}\right)+\ln^2\left(\frac{1-\beta}{1+\beta}\right)-\frac{1}{3}\ln^3\left(\frac{1-\beta}{1+\beta}\right)+2\ln\left(\frac{1-\beta}{1+\beta}\right)\ln\left(\frac{(1+\beta)^2}{4\beta}\right)\right.$$
$$\left.\left.-\mathrm{Li}_2\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)\right]\right\}$$

where $K=C_A(67/18-\zeta_2)-5n_f/9$

N. Kidonakis, Phys. Rev. Lett. 102, 232003 (2009), arXiv:0903.2561 [hep-ph]

The color structure of $\Gamma_S^{(2)}$ **involves only the factors** $C_F C_A$ **and** $C_F n_f$
In terms of the cusp angle (Korchemsky& Radyushkin) $\gamma=\ln[(1+\beta)/(1-\beta)]$ **we get** $\Gamma_S^{(1)}=C_F(\gamma\coth\gamma-1)$ **and**

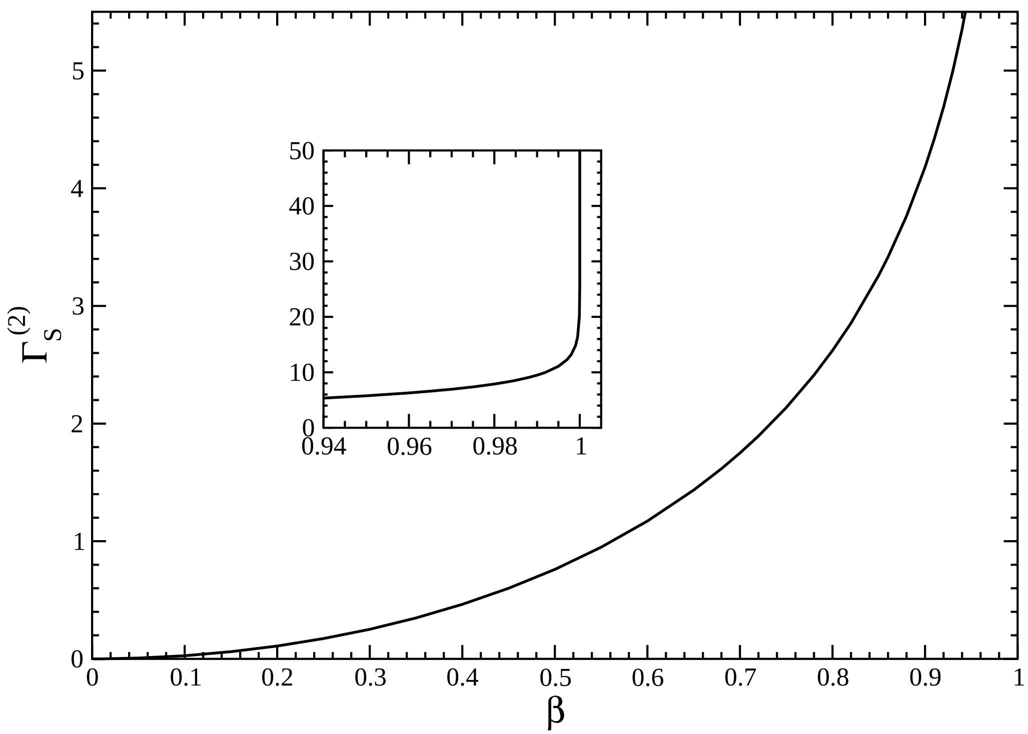
$$\Gamma_S^{(2)}=\frac{K}{2}\,\Gamma_S^{(1)}+C_F C_A\left\{\frac{1}{2}+\frac{\zeta_2}{2}+\frac{\gamma^2}{2}-\frac{1}{2}\coth^2\gamma\left[\zeta_3-\zeta_2\gamma-\frac{\gamma^3}{3}-\gamma\,\mathrm{Li}_2\left(e^{-2\gamma}\right)-\mathrm{Li}_3\left(e^{-2\gamma}\right)\right]\right.$$
$$\left.-\frac{1}{2}\coth\gamma\left[\zeta_2+\zeta_2\gamma+\gamma^2+\frac{\gamma^3}{3}+2\,\gamma\,\ln\left(1-e^{-2\gamma}\right)-\mathrm{Li}_2\left(e^{-2\gamma}\right)\right]\right\}$$

$\Gamma_S^{(2)}$ vanishes at $\beta=0$, the threshold limit
and diverges at $\beta=1$, the massless limit
If one quark is massless and one is massive

$$\Gamma_S^{(2)}=\frac{K}{2}\Gamma_S^{(1)}+C_F C_A \frac{(1-\zeta_3)}{4}$$

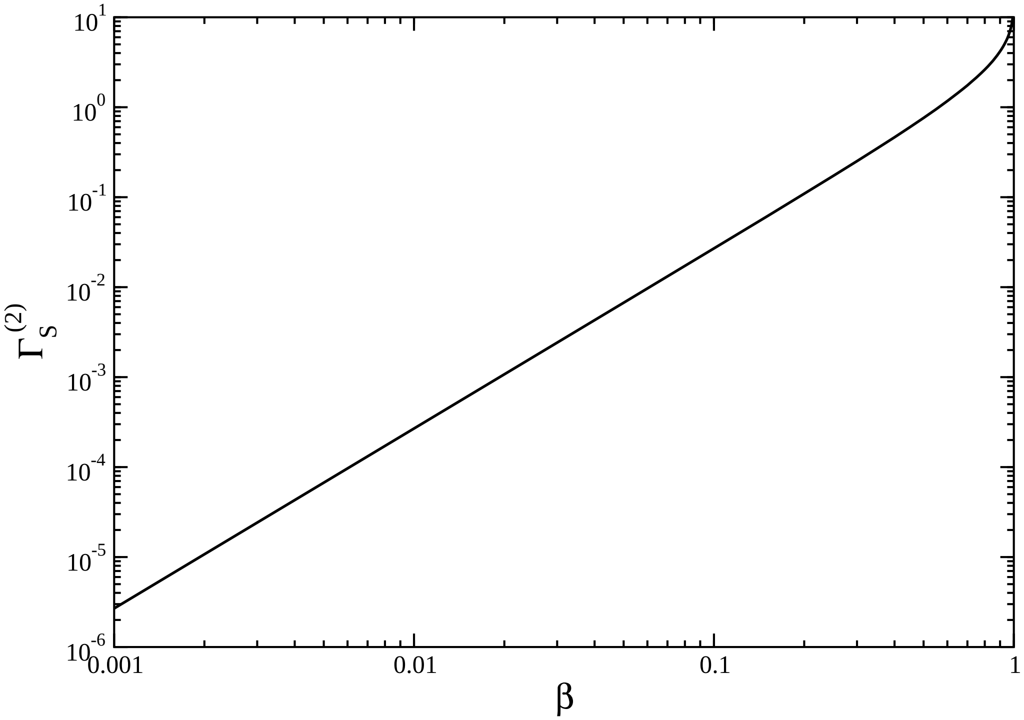
QCD processes: Color structure gets more complicated with more than two colored partons in the process

- Cusp anomalous dimension an essential component of other calculations



Two-loop soft anomalous dimension $\Gamma_S^{(2)}$ for $e^+e^- \to t\bar{t}$

$\Gamma_S^{(2)}$ **vanishes at $\beta = 0$, the threshold limit, and diverges at $\beta = 1$, the massless limit.**



Logarithmic plot of $\Gamma_S^{(2)}$ **for** $e^+e^- \to t\bar{t}$

Small and large β behavior of $\Gamma_S^{(2)}$

Small β behavior - expand around $\beta=0$

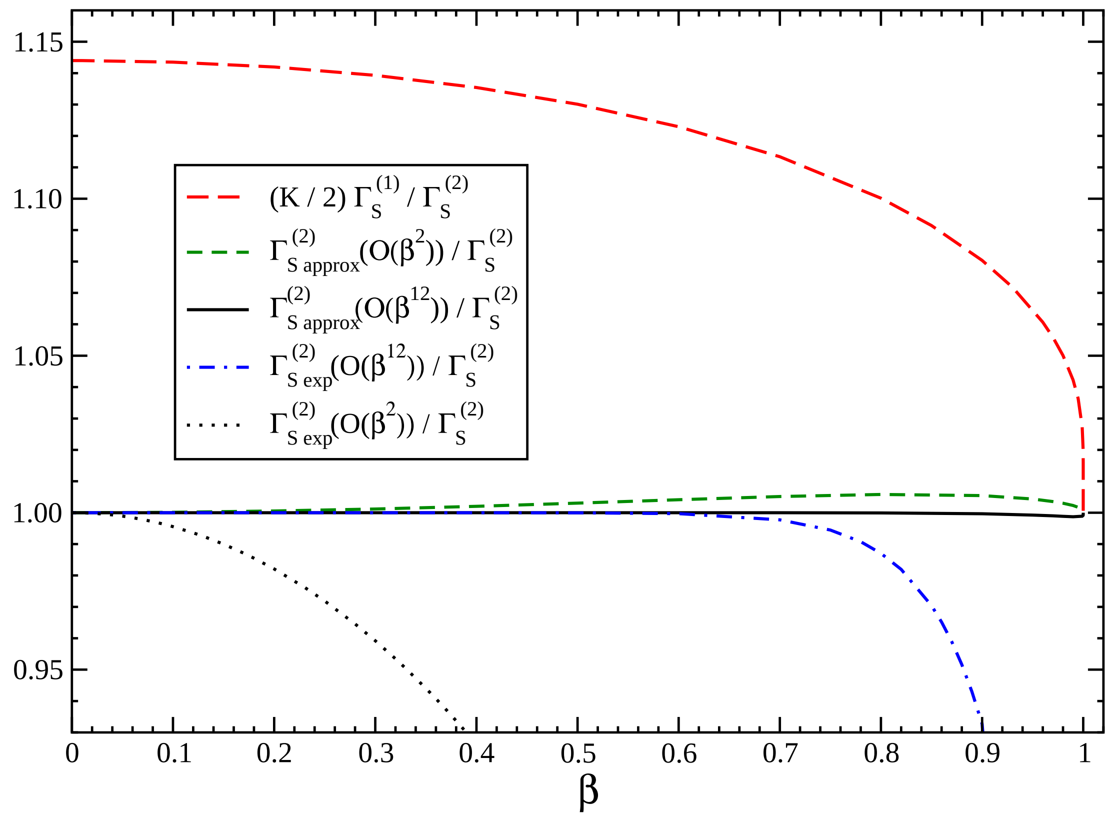
$$\Gamma_{S\,\mathrm{exp}}^{(2)} = -\frac{2}{27}\beta^2\left[C_F C_A(18\zeta_2-47)+5C_F n_f\right]+\mathcal{O}(\beta^4)$$

$\Gamma_S^{(2)}$ is an even function of β

Large β behavior - as $\beta \to 1$, $\Gamma_S^{(2)} \to \frac{K}{2}\Gamma_S^{(1)}$

Construct approximation for all β

$$\begin{aligned}
\Gamma_{S\,\mathrm{approx}}^{(2)} &= \Gamma_{S\,\mathrm{exp}}^{(2)}+\frac{K}{2}\Gamma_S^{(1)}-\frac{K}{2}\Gamma_{S\,\mathrm{exp}}^{(1)} \\
&= \frac{K}{2}\Gamma_S^{(1)}+C_F C_A\left(1-\frac{2}{3}\zeta_2\right)\beta^2+\mathcal{O}\left(\beta^4\right)
\end{aligned}$$



Expansions and approximations to $\Gamma_S^{(2)}$ for $e^+e^- \to t\bar{t}$

$\Gamma_{S\,\mathrm{approx}}^{(2)}$ is a remarkably good approximation to complete $\Gamma_S^{(2)}$

In general $\Gamma_S^{(2)} \neq \frac{K}{2}\Gamma_S^{(1)}$, i.e. more complicated than massless case, for all heavy quark processes ($e^+e^- \to t\bar{t}$ and heavy quark hadroproduction)

For $e^+e^- \to t\bar{t}$ soft logarithms of the form $\frac{\ln^{n-1}(\beta^2)}{\beta^2}$
NLO soft-gluon corrections

$$\sigma^{(1)} \quad = \quad \sigma^B \, \frac{\alpha_s}{\pi} \, 2\,\Gamma_S^{(1)} \, \frac{1}{\beta^2}$$

with σ^B the Born cross section
Second-order soft-gluon corrections

$$\sigma^{(2)} \quad = \quad \sigma^B \, \frac{\alpha_s^2}{\pi^2} \left\{ \left[4(\Gamma_S^{(1)})^2 - \beta_0\, \Gamma_S^{(1)}\right] \frac{\ln(\beta^2)}{\beta^2} + \left[2\,T_1\,\Gamma_S^{(1)} + 2\,\Gamma_S^{(2)}\right] \frac{1}{\beta^2} \right\}$$

with T_1 the NLO virtual corrections

At the Tevatron and the LHC the $t\bar{t}$ cross section receives most contributions in the region around $0.3 < \beta < 0.8$ which peak roughly around $\beta \sim 0.6$.

NNLO approximate cross sections include soft-gluon contributions

Top-antitop production in hadron colliders

The soft anomalous dimension matrix for $q(p_1) + \bar{q}(p_2) \rightarrow t(p_3) + \bar{t}(p_4)$ **in a color tensor basis consisting of singlet and octet exchange in the** s **channel,**

$$c_1 = \delta_{12}\delta_{34}\,, \qquad c_2 = T^c_{F\,21}\,T^c_{F\,34}\,.$$

is

$$\Gamma_{S\,q\bar{q}} = \left[\begin{array}{cc} \Gamma_{q\bar{q}\,11} & \Gamma_{q\bar{q}\,12} \\ \Gamma_{q\bar{q}\,21} & \Gamma_{q\bar{q}\,22} \end{array} \right]$$

Define $s = (p_1 + p_2)^2$**,** $t_1 = (p_1 - p_3)^2 - m^2$**,** $u_1 = (p_2 - p_3)^2 - m^2$
At one loop

$$\Gamma^{(1)}_{q\bar{q}\,11} = -C_F\,[L_\beta + 1] = \Gamma^{(1)}_S \qquad\qquad \Gamma^{(1)}_{q\bar{q}\,12} = \frac{C_F}{C_A}\ln\left(\frac{t_1}{u_1}\right)$$

$$\Gamma^{(1)}_{q\bar{q}\,21} = 2\ln\left(\frac{t_1}{u_1}\right)$$

$$\Gamma^{(1)}_{q\bar{q}\,22} = C_F\left[4\ln\left(\frac{t_1}{u_1}\right) - L_\beta - 1\right] + \frac{C_A}{2}\left[-3\ln\left(\frac{t_1}{u_1}\right) + \ln\left(\frac{t_1 u_1}{s m^2}\right) + L_\beta\right]$$

where $L_\beta = \frac{1+\beta^2}{2\beta}\ln\left(\frac{1-\beta}{1+\beta}\right)$ **with** $\beta = \sqrt{1 - 4m^2/s}$

Write the two-loop cusp anomalous dimension as $\Gamma_S^{(2)} = \frac{K}{2}\,\Gamma_S^{(1)} + C_F C_A M_\beta$. Then at two loops

$$\Gamma^{(2)}_{q\bar{q}\,11} = \frac{K}{2}\Gamma^{(1)}_{q\bar{q}\,11} + C_F C_A\, M_\beta = \Gamma_S^{(2)} \qquad \Gamma^{(2)}_{q\bar{q}\,12} = \frac{K}{2}\Gamma^{(1)}_{q\bar{q}\,12} - \frac{C_F}{2} N_\beta \ln\left(\frac{t_1}{u_1}\right)$$

$$\Gamma^{(2)}_{q\bar{q}\,21} = \frac{K}{2}\Gamma^{(1)}_{q\bar{q}\,21} + C_A N_\beta \ln\left(\frac{t_1}{u_1}\right) \qquad \Gamma^{(2)}_{q\bar{q}\,22} = \frac{K}{2}\Gamma^{(1)}_{q\bar{q}\,22} + C_A\left(C_F - \frac{C_A}{2}\right) M_\beta$$

with N_β a subset of terms of M_β

$$N_\beta = \frac{1}{2}\ln^2\left(\frac{1-\beta}{1+\beta}\right) - \frac{(1+\beta^2)}{4\beta}\left[\ln^2\left(\frac{1-\beta}{1+\beta}\right) + 2\ln\left(\frac{1-\beta}{1+\beta}\right)\ln\left(\frac{(1+\beta)^2}{4\beta}\right) - \mathrm{Li}_2\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)\right]$$

The soft anomalous dimension matrix for $g(p_1)+g(p_2) \to t(p_3)+\bar{t}(p_4)$ in a color tensor basis

$$c_1 = \delta^{12}\,\delta_{34}, \quad c_2 = d^{12c}\,T^c_{34}, \quad c_3 = i f^{12c}\,T^c_{34}$$

where d and f are the totally symmetric and antisymmetric $SU(3)$ invariant tensors

$$\Gamma_{S\,gg} = \left[\begin{array}{ccc} \Gamma_{gg\,11} & 0 & \Gamma_{gg\,13} \\ 0 & \Gamma_{gg\,22} & \Gamma_{gg\,23} \\ \Gamma_{gg\,31} & \Gamma_{gg\,32} & \Gamma_{gg\,22} \end{array}\right].$$

At one loop:

$$\begin{aligned}
\Gamma^{(1)}_{gg\,11} &= -C_F[L_\beta+1] = \Gamma^{(1)}_S \\
\Gamma^{(1)}_{gg\,31} &= 2\ln\left(\frac{t_1}{u_1}\right) \\
\Gamma^{(1)}_{gg\,13} &= \ln\left(\frac{t_1}{u_1}\right) \\
\Gamma^{(1)}_{gg\,22} &= -C_F[L_\beta+1] + \frac{C_A}{2}\left[\ln\left(\frac{t_1 u_1}{m^2 s}\right) + L_\beta\right] \\
\Gamma^{(1)}_{gg\,32} &= \frac{N_c^2-4}{2N_c}\ln\left(\frac{t_1}{u_1}\right) \\
\Gamma^{(1)}_{gg\,23} &= \frac{C_A}{2}\ln\left(\frac{t_1}{u_1}\right)
\end{aligned}$$

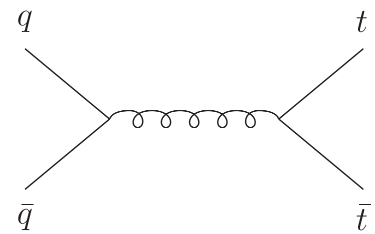
At two loops:

$$
\begin{aligned}
\Gamma^{(2)}_{gg\,11} &= \frac{K}{2}\Gamma^{(1)}_{gg\,11} + C_F C_A\, M_\beta = \Gamma^{(2)}_S \\
\Gamma^{(2)}_{gg\,31} &= \frac{K}{2}\Gamma^{(1)}_{gg\,31} + C_A N_\beta \ln\left(\frac{t_1}{u_1}\right) \\
\Gamma^{(2)}_{gg\,13} &= \frac{K}{2}\Gamma^{(1)}_{gg\,13} - \frac{C_A}{2} N_\beta \ln\left(\frac{t_1}{u_1}\right) \\
\Gamma^{(2)}_{gg\,22} &= \frac{K}{2}\Gamma^{(1)}_{gg\,22} + C_A \left(C_F - \frac{C_A}{2}\right) M_\beta \\
\Gamma^{(2)}_{gg\,32} &= \frac{K}{2}\Gamma^{(1)}_{gg\,32} \\
\Gamma^{(2)}_{gg\,23} &= \frac{K}{2}\Gamma^{(1)}_{gg\,23}
\end{aligned}
$$

Top-pair partonic processes at LO

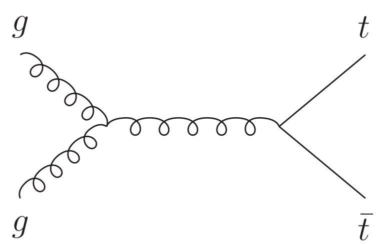
- $q\bar{q} \to t\bar{t}$

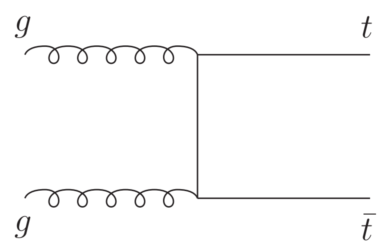
dominant at Tevatron

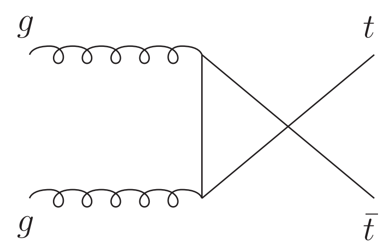


- $gg \to t\bar{t}$

dominant at LHC

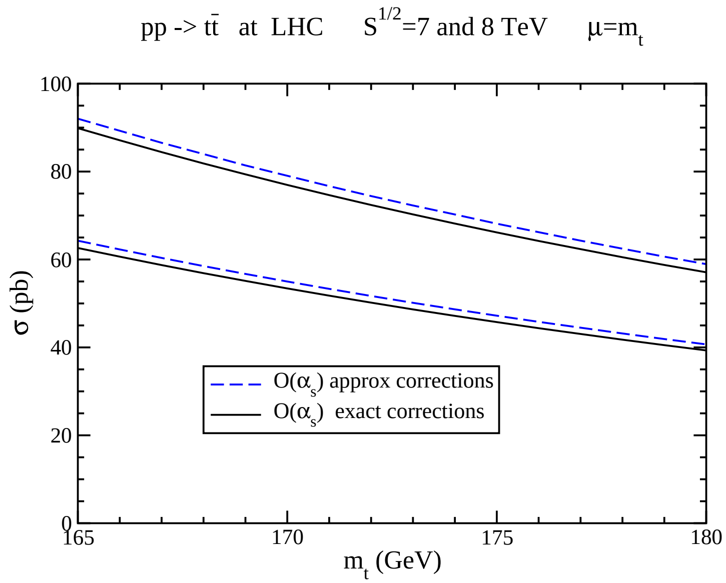


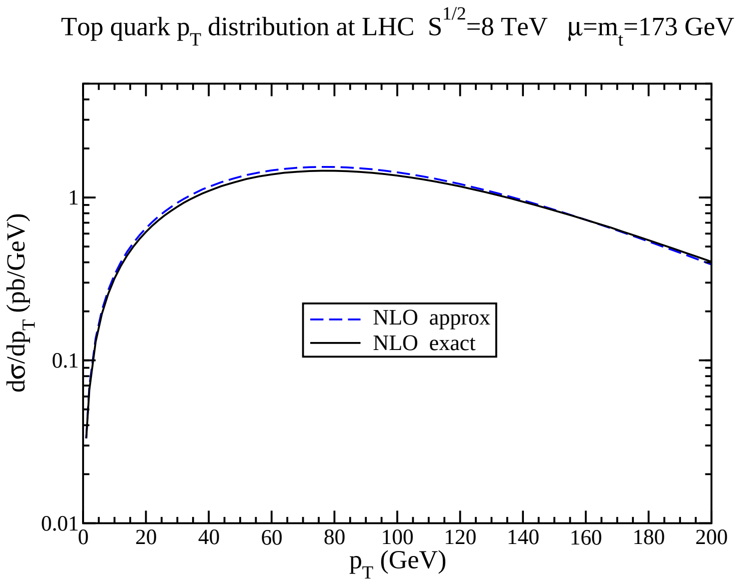




Threshold approximation

The approximation works very well both for total cross sections and differential distributions





excellent approximation:
less than 1% difference between NLO approximate and exact cross sections
Also excellent for differential distributions

For best prediction add NNLO approximate corrections to exact NLO cross section

Kinematics

We consider the hadronic process $p_{h_1} + p_{h_2} \to p_3 + p_4$ **with underlying partonic process** $p_1 + p_2 \to p_3 + p_4$

The hadronic invariants are $S = (p_{h1} + p_{h2})^2$**,** $T = (p_{h1} - p_3)^2$**,** $U = (p_{h2} - p_3)^2$

The partonic invariants are $s = (p_1 + p_2)^2$**,** $t = (p_1 - p_3)^2$**,** $u = (p_2 - p_3)^2$

also $s_4 = s + t + u - m_3^2 - m_4^2$

Note that $p_1 = x_1 p_{h1}$**,** $p_2 = x_2 p_{h2}$**,** $s = x_1 x_2 S$**,** $t - m_3^2 = x_1(T - m_3^2)$**,** $u - m_3^2 = x_2(U - m_3^2)$

$$\sigma_{p_{h1} p_{h2} \to p_3 p_4}(S) = \int_{T_{min}}^{T_{max}} dT \int_{U_{min}}^{U_{max}} dU \int_{x_{2min}}^{1} dx_2 \int_0^{s_{4max}} ds_4 \times \frac{x_1 x_2}{x_2 S + T - m_3^2} \, \phi(x_1) \, \phi(x_2) \, \frac{d^2 \hat{\sigma}_{p_1 p_2 \to p_3 p_4}}{dt \, du}$$

where

$$x_1 = \frac{s_4 - m_3^2 + m_4^2 - x_2(U - m_3^2)}{x_2 S + T - m_3^2},$$

$$T_{\substack{max \\ min}} = -\frac{1}{2}(S - m_3^2 - m_4^2) \pm \frac{1}{2}\sqrt{(S - m_3^2 - m_4^2)^2 - 4m_3^2 m_4^2}$$

$$U_{max} = m_3^2 + \frac{S \, m_3^2}{T - m_3^2}$$

$U_{min} = -S - T + m_3^2 + m_4^2$**,** $x_{2min} = (m_4^2 - T)/(S + U - m_3^2)$ **and** $s_{4max} = x_2(S + U - m_3^2) + T - m_4^2$

Kinematics with p_T and rapidity

We consider the hadronic process $p_{h_1} + p_{h_2} \to p_3 + p_4$ **with underlying partonic process** $p_1 + p_2 \to p_3 + p_4$

The hadronic invariants are $S = (p_{h1} + p_{h2})^2$, $T = (p_{h1} - p_3)^2$, $U = (p_{h2} - p_3)^2$

The partonic invariants are $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$, $u = (p_2 - p_3)^2$, $s_4 = s + t + u - m_3^2 - m_4^2$

Also, $U_1 = -\sqrt{S} m_T e^Y$ **and** $T_1 = -\sqrt{S} m_T e^{-Y}$ **with** $m_T = \sqrt{m_3^2 + p_T^2}$

$$\sigma_{p_{h1} p_{h2} \to p_3 p_4}(S) = \int_0^{p^2_{T\,max}} dp_T^2 \int_{Y^-}^{Y^+} dY \int_{x_1^-}^{1} dx_1 \int_0^{s_{4max}} ds_4 \times \frac{x_1 x_2 \, S}{x_1 S + U_1} \, \phi(x_1) \, \phi(x_2) \, \frac{d^2 \hat{\sigma}_{p_1 p_2 \to p_3 p_4}}{dt_1 \, du_1}$$

where

$$x_2 = \frac{s_4 - m_3^2 + m_4^2 - x_1 T_1}{x_1 S + U_1},$$

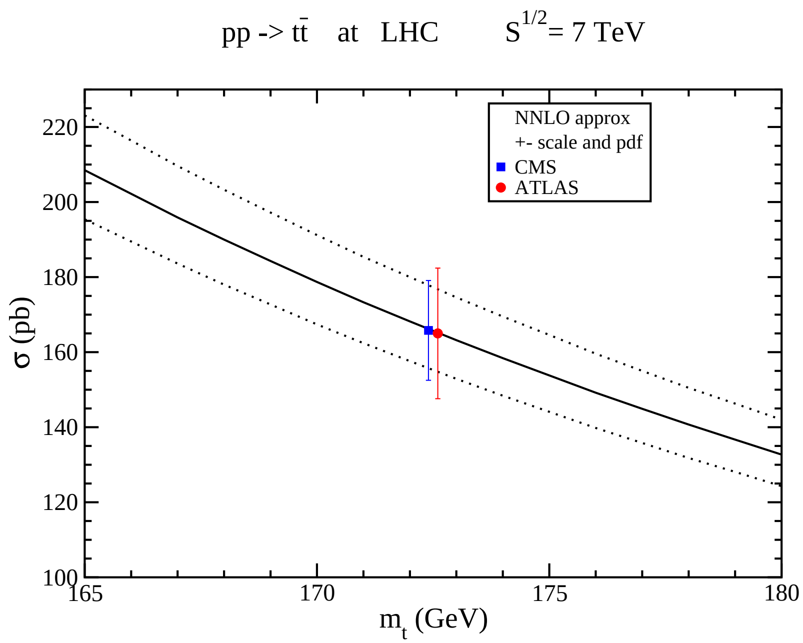
$$p^2_{T\,max} = \frac{(S - m_3^2 - m_4^2)^2 - 4 m_3^2 m_4^2}{4S}$$

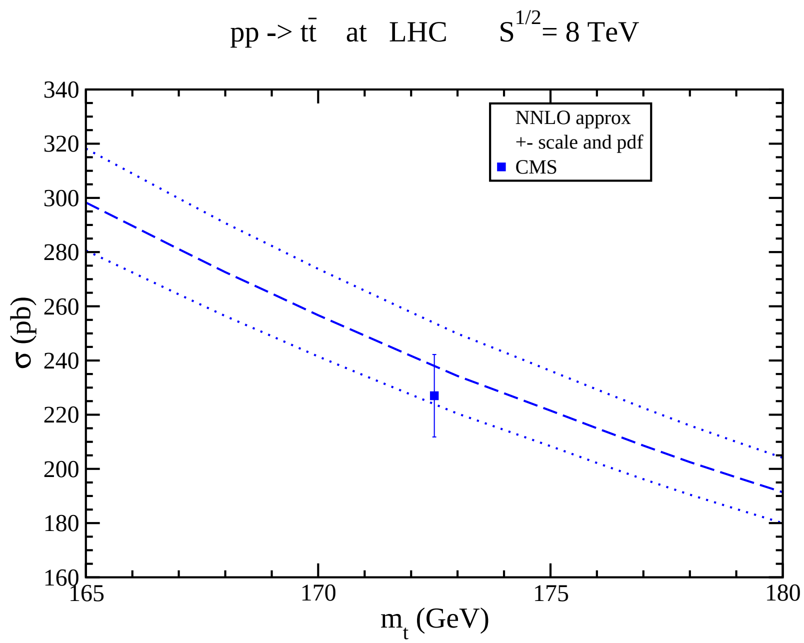
$$Y^{\pm} = \pm \frac{1}{2} \ln \frac{1+\sqrt{1-\frac{4m_T^2}{S[1+(m_3^2-m_4^2)/S]^2}}}{1-\sqrt{1-\frac{4m_T^2}{S[1+(m_3^2-m_4^2)/S]^2}}}$$

$$x_1^- = \frac{-(U_1+m_3^2-m_4^2)}{S+T_1}$$

$$s_{4max} = x_1(S+T_1)+U_1+m_3^2-m_4^2$$

$t\bar{t}$ cross section at the LHC





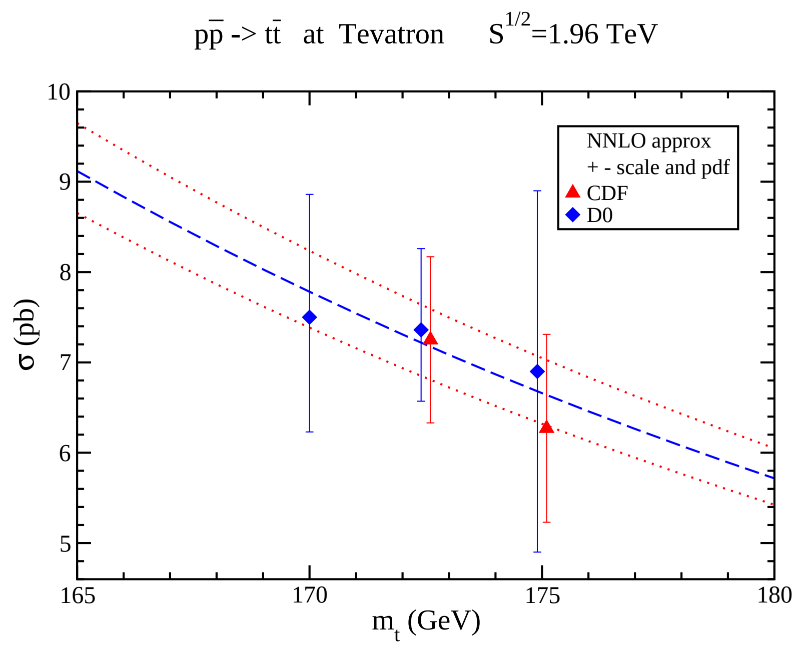
$$\sigma_{t\bar{t}}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV},\ 7\,{\rm TeV}) \quad = \quad 163^{+7}_{-5} \pm 9 \text{ pb}$$

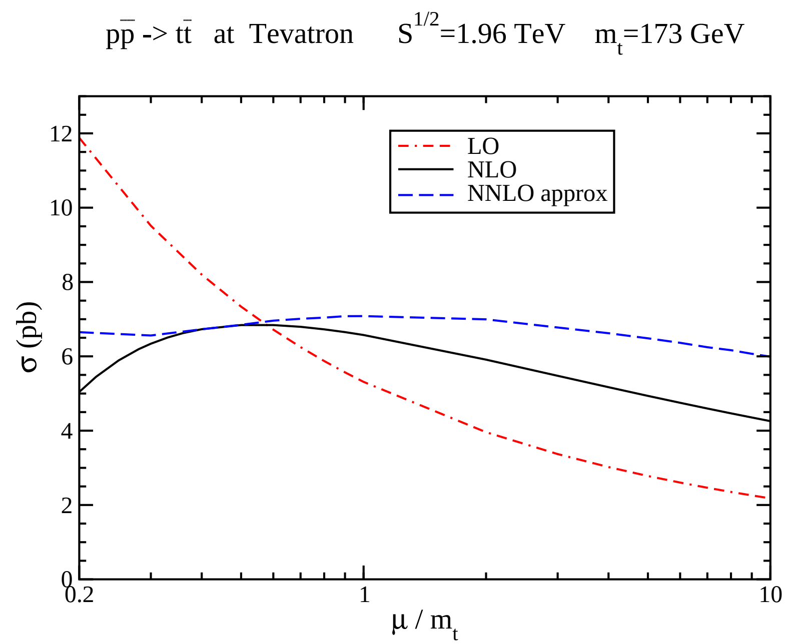
$$\sigma_{t\bar{t}}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV},\ 8\,{\rm TeV}) \quad = \quad 234^{+10}_{-7} \pm 12 \text{ pb}$$

$$\sigma_{t\bar{t}}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV}, 14\,{\rm TeV}) \quad = \quad 920^{+50}_{-39}{}^{+33}_{-35} \text{ pb}$$

NNLO approx: enhancement over NLO (same pdf) is $\sim 8\%$

$t\bar{t}$ cross section at the Tevatron





$$\sigma_{t\bar{t}}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV},\, 1.96\,{\rm TeV}) \quad = \quad 7.08^{+(0.20)}_{-0.24}\,{}^{+0.36}_{-0.27}\ {\rm pb}$$

scale pdf

NNLO approx: $\sim 8\%$ enhancement over NLO
scale dependence greatly reduced

Differences between various resummation/NNLO approx approaches

Total vs differential cross section
moment-space pQCD vs SCET

Name	Observable	Soft limit
single-particle-inclusive (1PI)	$d\sigma/dp_T dy$	$s_4 = s + t_1 + u_1 \rightarrow 0$
pair-invariant-mass (PIM)	$d\sigma/dM_{t\bar{t}} d\theta$	$(1-z) = 1 - M^2_{t\bar{t}}/s \rightarrow 0$
production threshold	σ	$\beta = \sqrt{1 - 4m_t^2/s} \rightarrow 0$

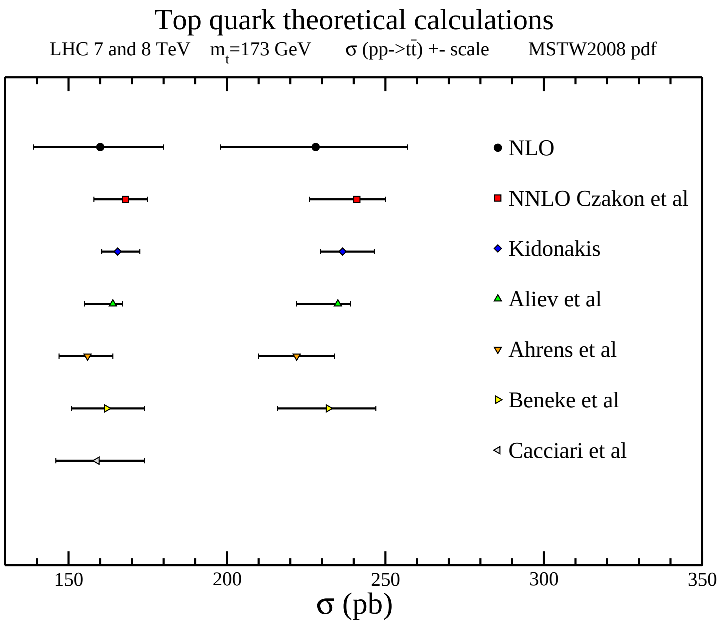
The more general approach is double-differential
$\rightarrow p_T$ and rapidity distributions

total-only approaches are limit/special case (absolute vs partonic threshold)

For differential calculations, further differences arise from how the relation $s + t_1 + u_1 = 0$ is used in the plus-distribution coefficients, how subleading terms are treated, damping factors, etc.

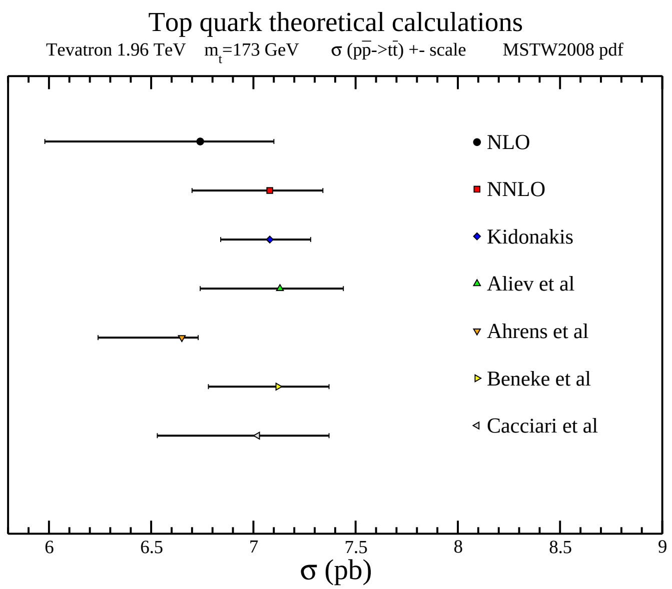
see N. Kidonakis and B.D. Pecjak, Eur. Phys. J C 72, 2084 (2012)

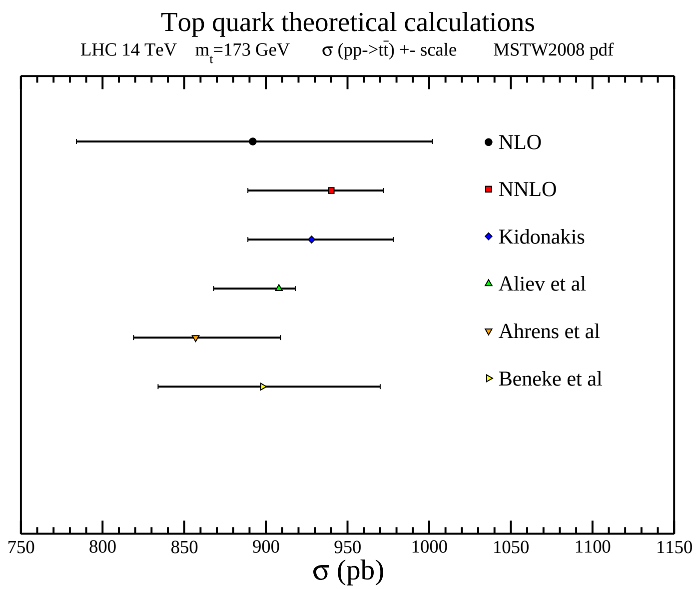
for details and review



Comparison of various NNLO approx approaches
all with the same choice of parameters

Kidonakis, PRD 82, 114030 (2010) differential-pQCD
Aliev *et al*, CPC 182, 1034 (2011) total-pQCD
Ahrens *et al*, PLB 703, 135 (2011) differential-SCET
Beneke *et al*, NPB 855, 695 (2012) total-SCET
Cacciari *et al*, PLB 710, 612 (2012) total-pQCD





Varying degree of success of the various approaches

The Kidonakis PRD 82 result is very close to the exact NNLO:
both the central values and the scale uncertainty are nearly the same
true for all collider energies and top quark masses

This was expected from comparison to NLO, and comparison of 1PI and PIM results at NNLO in 2003

(PRD 68, N. Kidonakis & R. Vogt; see also discussion in PRD78 and PRD82)

less than 1% difference between NLO approximate and exact cross sections at both NLO and NNLO

In near future add approximate NNNLO

(see N. Kidonakis PRD 73,034001 (2006) for early NNNLO results)

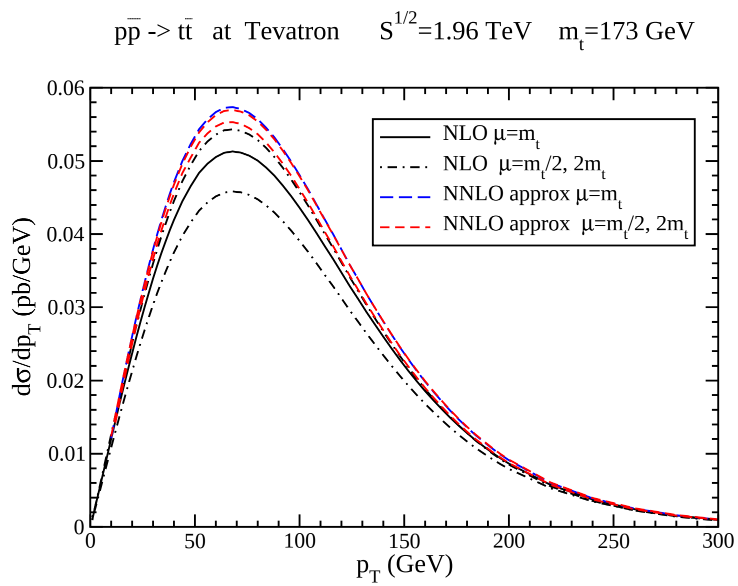
This is the only calculation for partonic threshold at the double differential cross section level using the standard moment-space resummation in pQCD

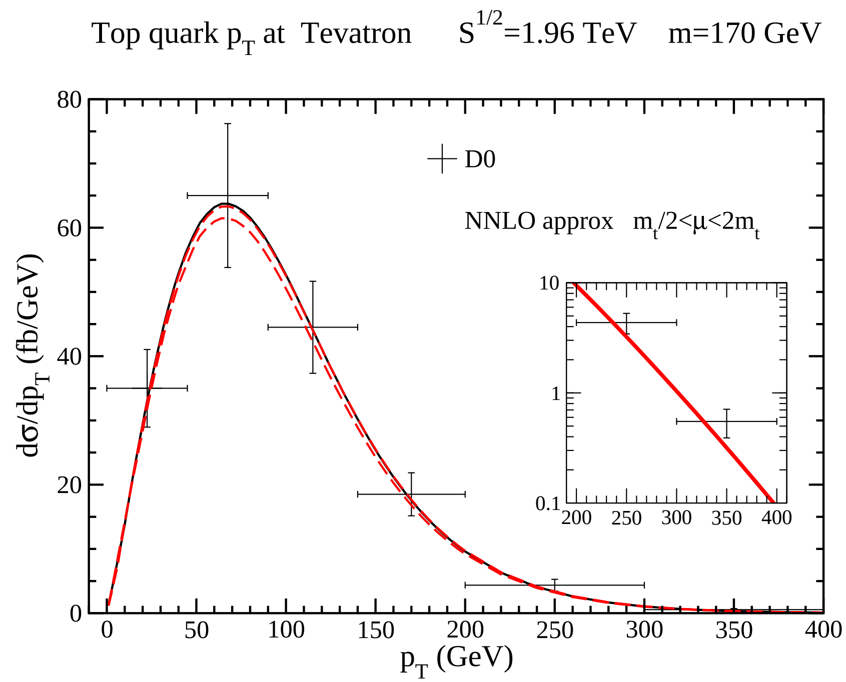
stability of the theoretical NNLO approx result over the past decade

the reliability of the NNLO approximate result and near-identical value to exact NNLO is very important for several reasons

- **provides confidence of application to other processes (single-top, W, etc)**
- **used as background for many analyses (Higgs, etc)**
- **means that we have near-exact NNLO p_T and rapidity distributions**

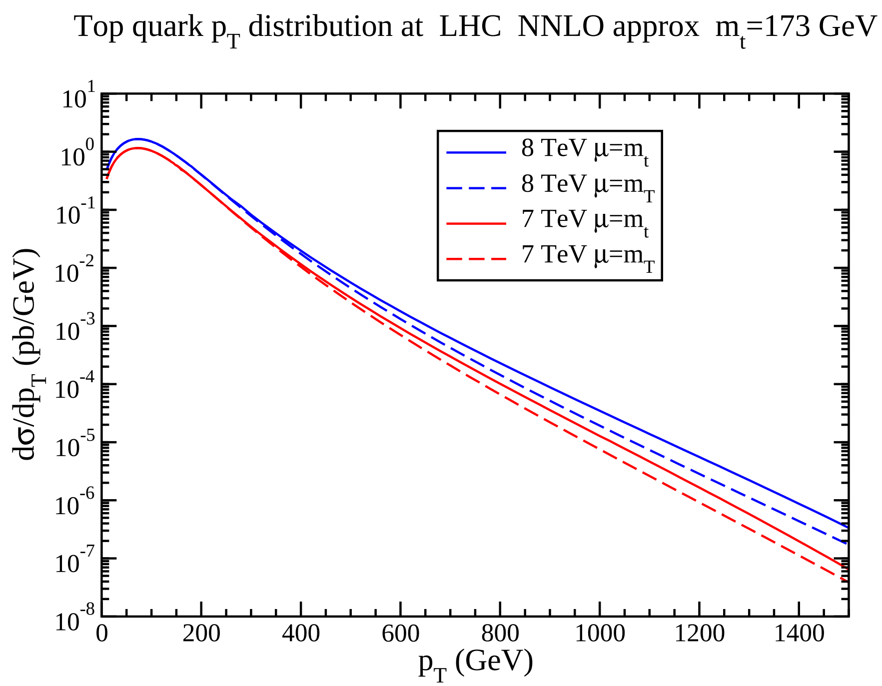
Top quark p_T distribution at Tevatron

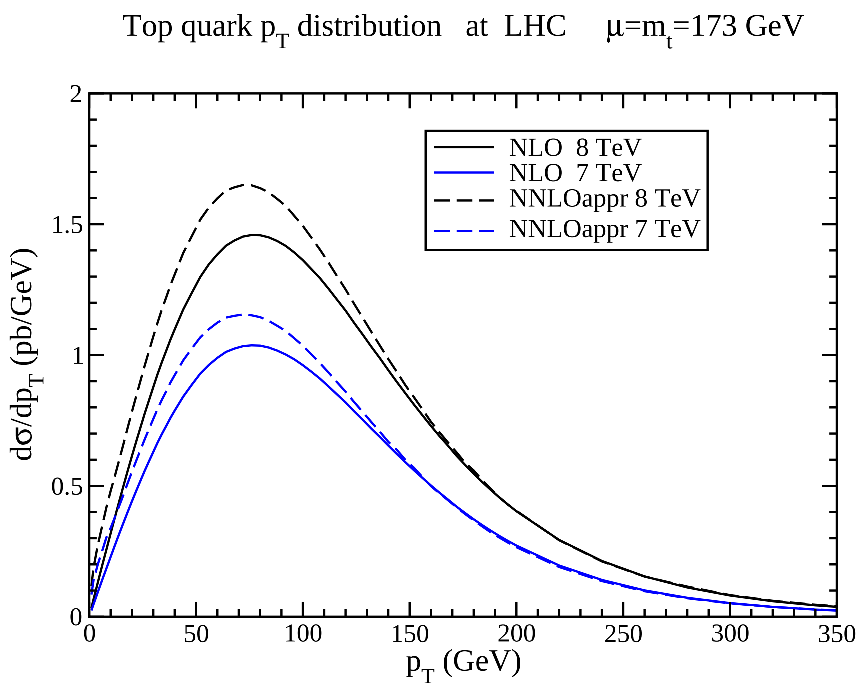




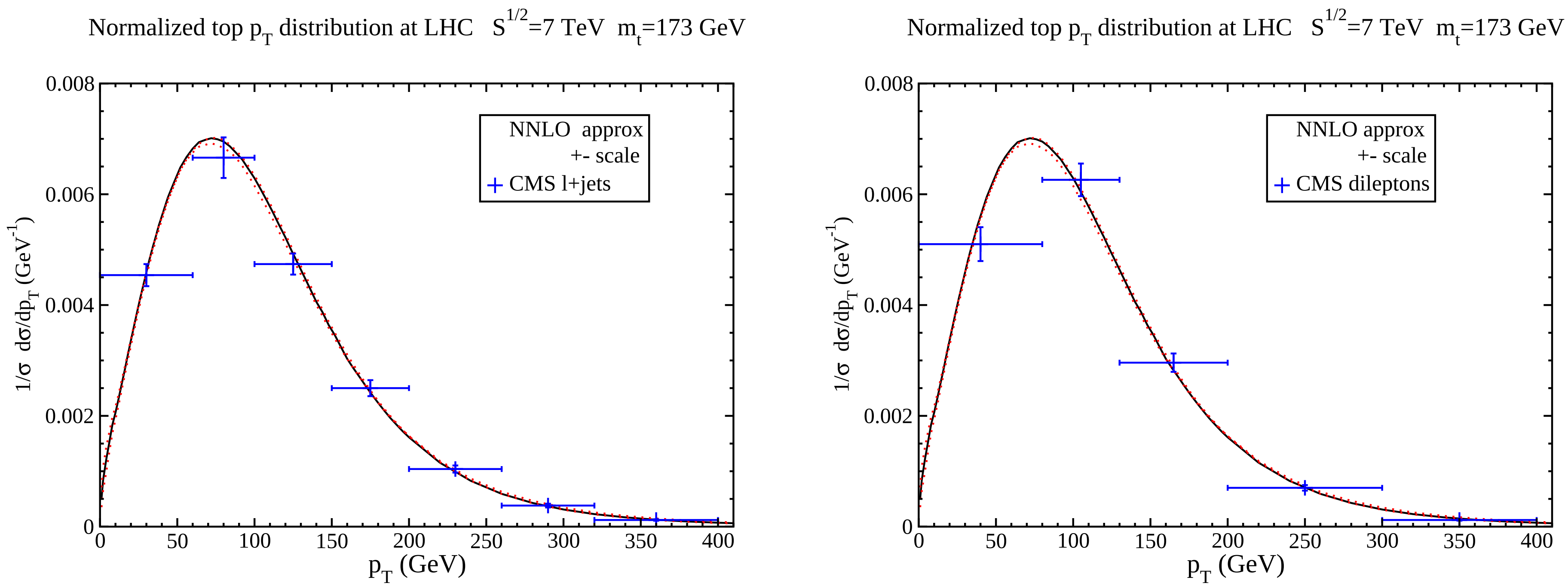
Excellent agreement of NNLO approx results with D0 data

Top quark p_T distribution at the LHC



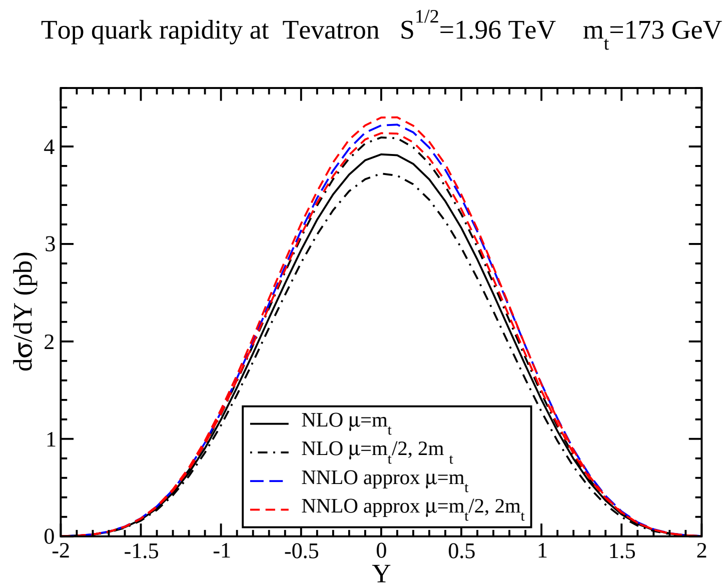


Normalized top quark p_T distribution at the LHC



Excellent agreement with CMS data at 7 TeV; also at 8 TeV

Top quark rapidity distribution at Tevatron



Top Forward-backward asymmetry

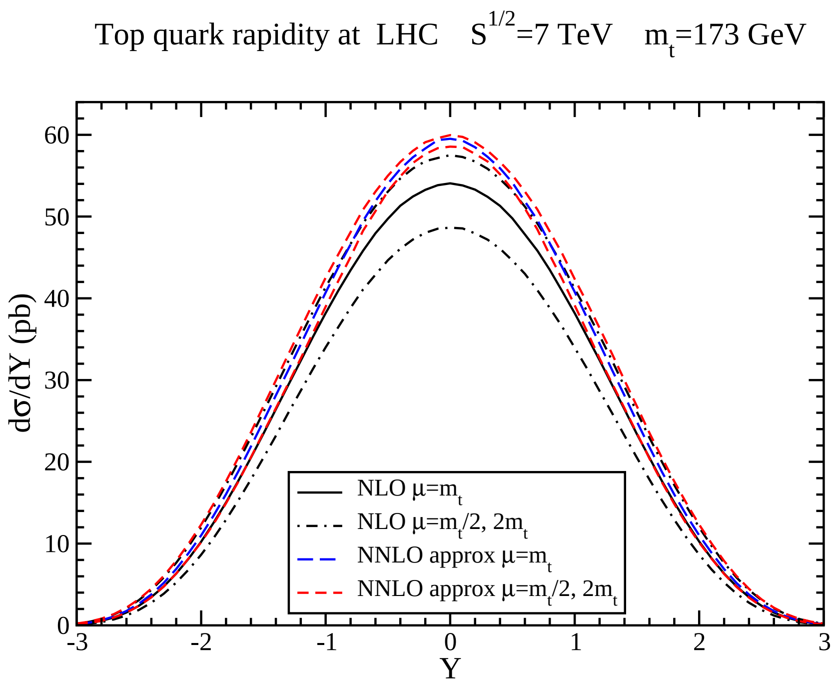
$$A_{\rm FB} = \frac{\sigma(Y>0)-\sigma(Y<0)}{\sigma(Y>0)+\sigma(Y<0)}$$

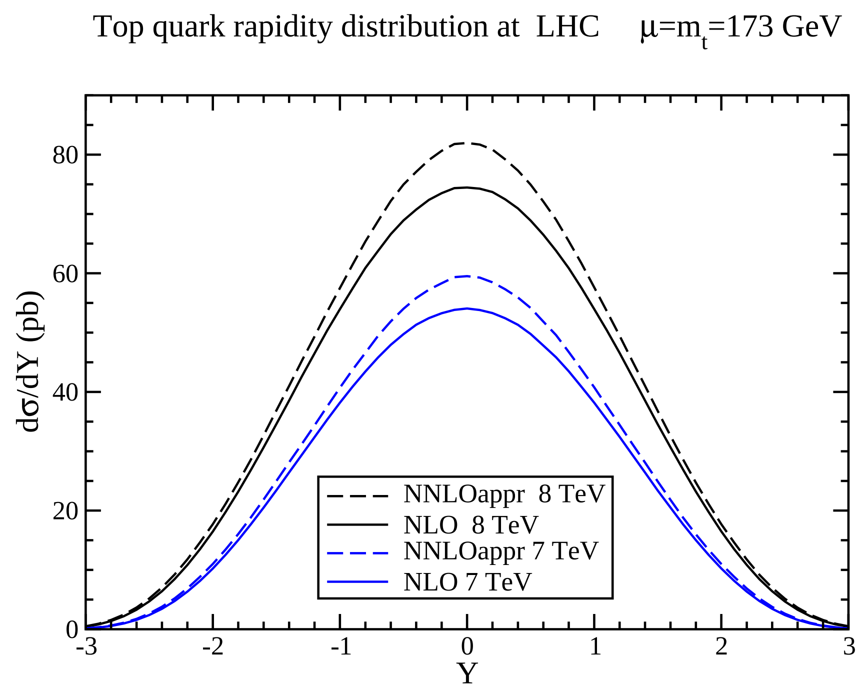
Asymmetry significant at the Tevatron

Theoretical result at Tevatron: $A_{\rm FB} = 0.052^{+0.000}_{-0.006}$

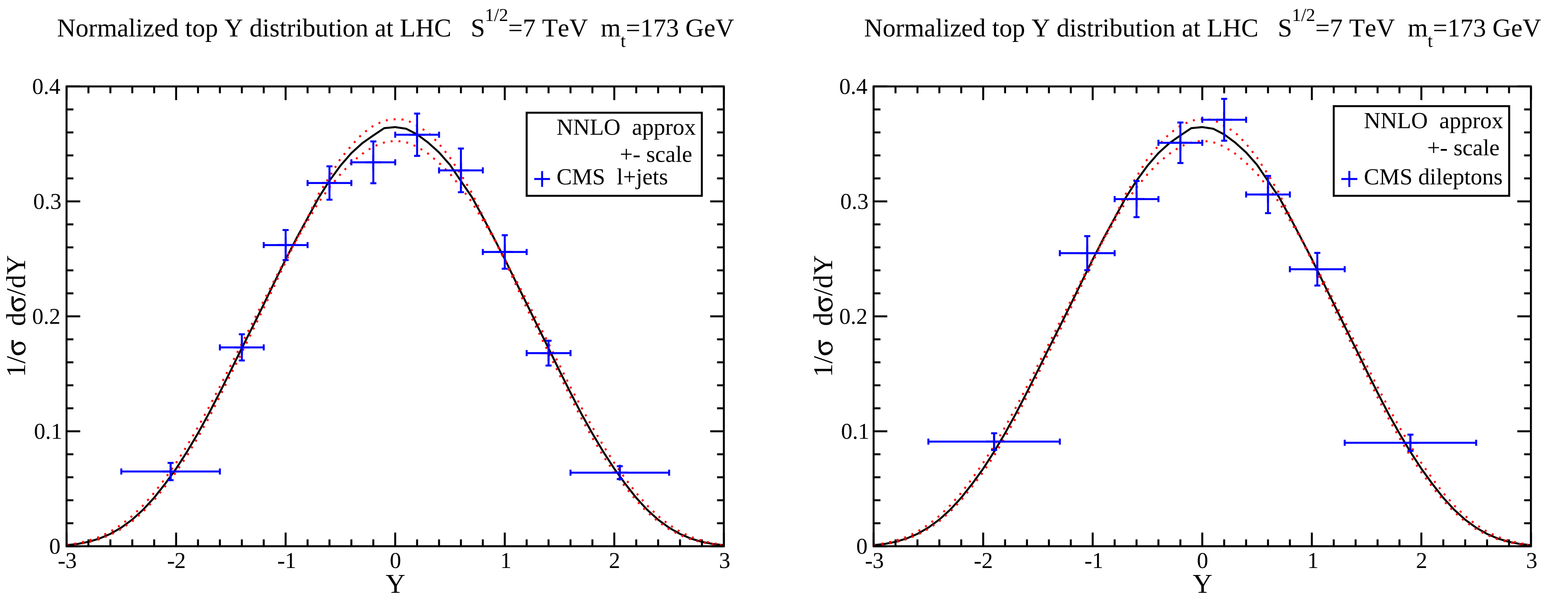
smaller than observed values

Top quark rapidity distribution at LHC





Normalized top quark rapidity distribution at LHC

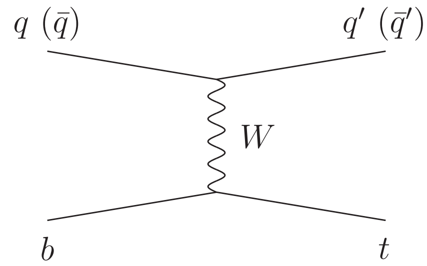


Excellent agreement with CMS data at 7 TeV; also at 8 TeV

Single-top partonic processes at LO

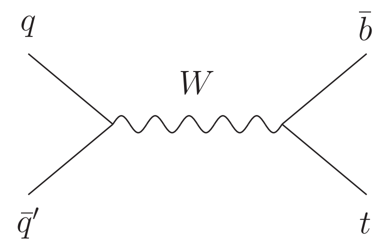
- t **channel:** $qb \to q't$ **and** $\bar{q}b \to \bar{q}'t$

dominant at Tevatron and LHC



- s **channel:** $q\bar{q}' \to \bar{b}t$

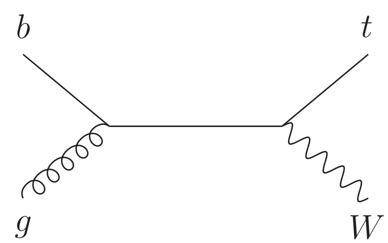
small at Tevatron and LHC

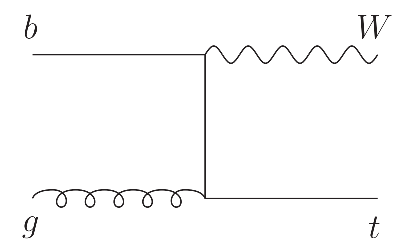


- **associated** tW **production:** $bg \to tW^-$

very small at Tevatron, significant at LHC

Related process: $bg \to tH^-$





Single top quark production - t channel

Dominant single top production channel at both Tevatron and LHC energies

Soft anomalous dimension for t-channel single top production

One loop

$$\Gamma_{S\,11}^{(1)} = C_F \left[\ln\left(\frac{-t}{s}\right) + \ln\left(\frac{m_t^2 - t}{m_t\sqrt{s}}\right) - \frac{1}{2}\right]$$

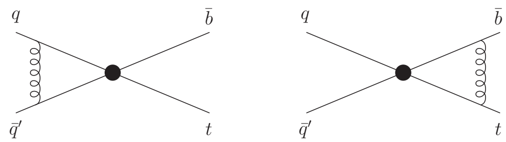
$$\Gamma_{S\,21}^{(1)} = \ln\left(\frac{u(u-m_t^2)}{s(s-m_t^2)}\right) \qquad \Gamma_{S\,12}^{(1)} = \frac{C_F}{2N_c}\,\Gamma_{S\,21}^{(1)}$$

Two loops

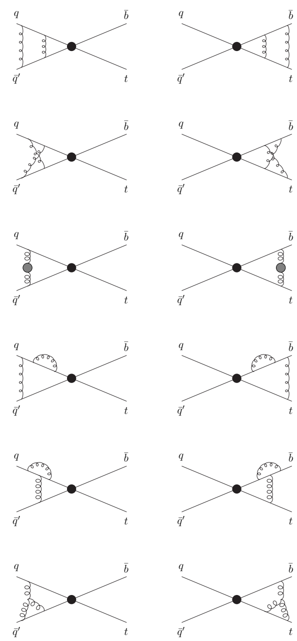
$$\Gamma_{S\,11}^{(2)} = \frac{K}{2}\Gamma_{S\,11}^{(1)} + C_F C_A \frac{(1-\zeta_3)}{4}$$

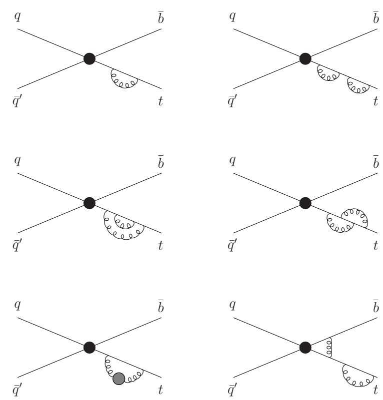
Single top quark production - s channel

One-loop eikonal diagrams



Two-loop eikonal diagrams



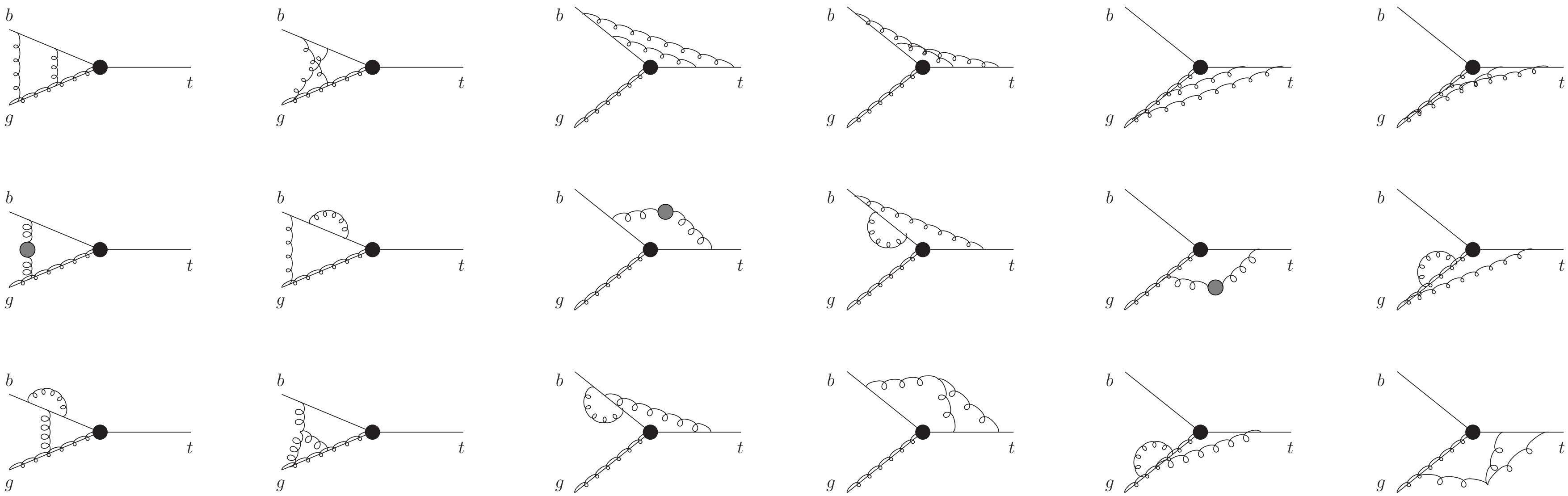


Soft anomalous dimension for s-channel single top production

$$\Gamma_{S\,11}^{(1)} = C_F \left[\ln\left(\frac{s-m_t^2}{m_t\sqrt{s}}\right) - \frac{1}{2}\right], \qquad \Gamma_{S\,11}^{(2)} = \frac{K}{2}\Gamma_{S\,11}^{(1)} + C_F C_A \frac{(1-\zeta_3)}{4}$$

Associated production of a top quark with a W^- or H^-

Two-loop eikonal diagrams (+ extra top-quark self-energy graphs)

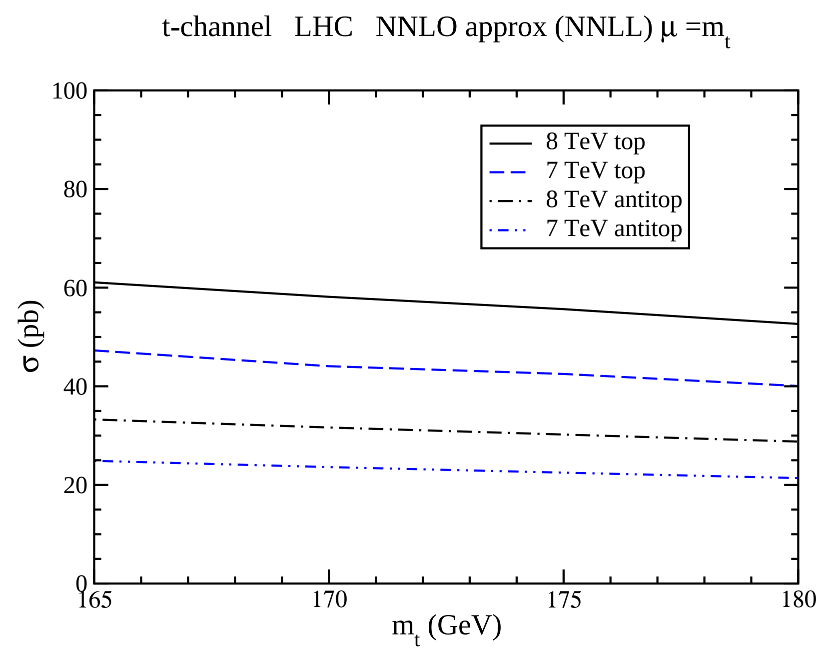


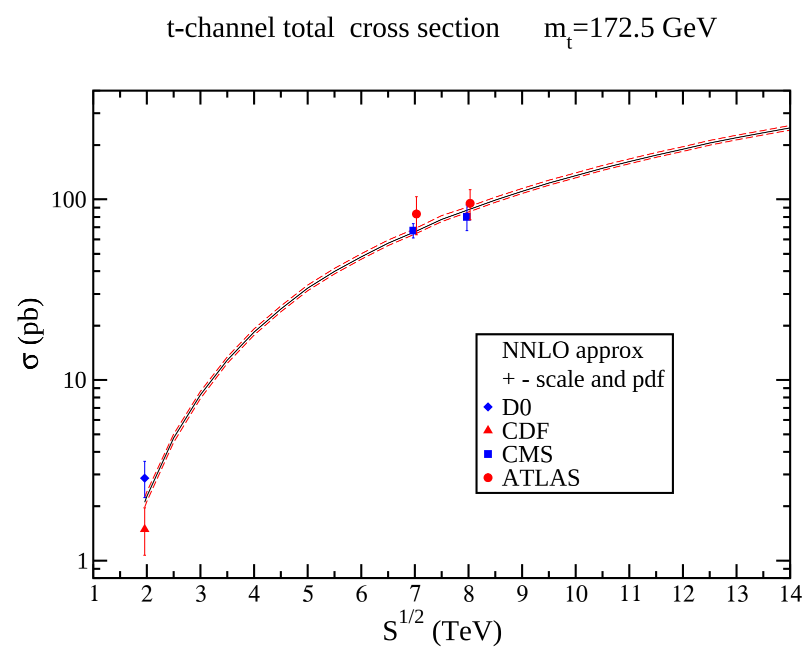
Soft anomalous dimension for $bg \to tW^-$ (or $bg \to tH^-$)

$$\Gamma^{(1)}_{S,\,tW^-} = C_F \left[\ln\left(\frac{m_t^2 - t}{m_t \sqrt{s}}\right) - \frac{1}{2} \right] + \frac{C_A}{2} \ln\left(\frac{m_t^2 - u}{m_t^2 - t}\right)$$

$$\Gamma^{(2)}_{S,\,tW^-} = \frac{K}{2} \Gamma^{(1)}_{S,\,tW^-} + C_F C_A \frac{(1-\zeta_3)}{4}$$

Single top t-channel cross sections at LHC





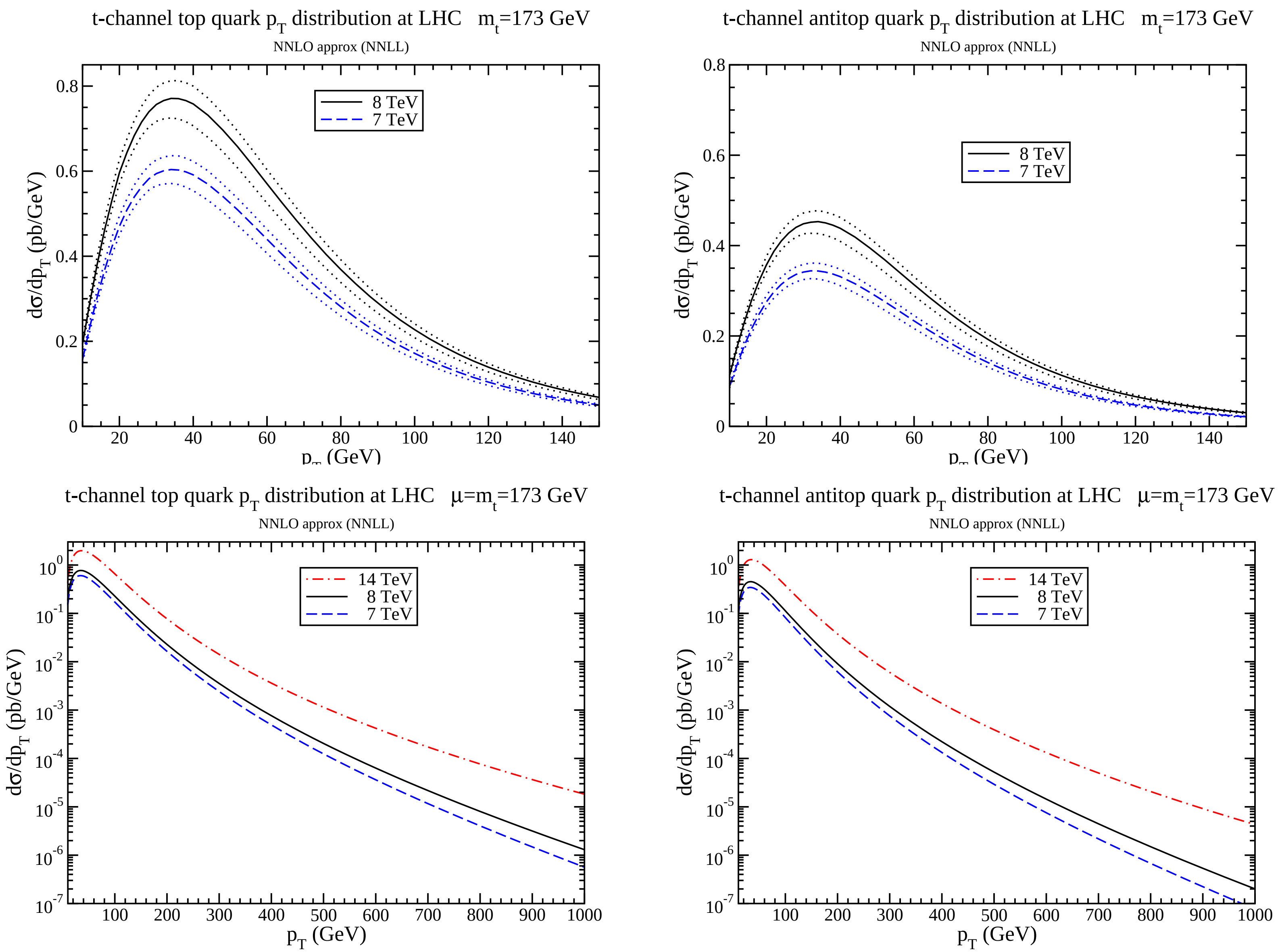
$m_t = 173$ **GeV**

LHC	t	$\bar{t}$	Total (pb)
7 TeV	$43.0^{+1.6}_{-0.2} \pm 0.8$	$22.9 \pm 0.5^{+0.7}_{-0.9}$	$65.9^{+2.1}_{-0.7}{}^{+1.5}_{-1.7}$
8 TeV	$56.4^{+2.1}_{-0.3} \pm 1.1$	$30.7 \pm 0.7^{+0.9}_{-1.1}$	$87.2^{+2.8}_{-1.0}{}^{+2.0}_{-2.2}$
14 TeV	$154^{+4}_{-1} \pm 3$	$94^{+2}_{-1}{}^{+2}_{-3}$	$248^{+6}_{-2}{}^{+5}_{-6}$

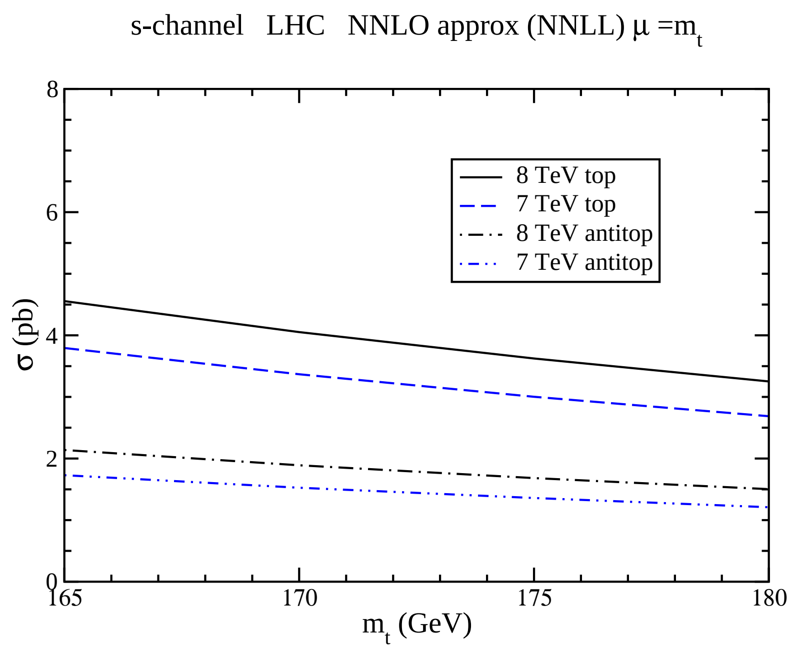
$\pm$ **scale** $\pm$ **pdf errors with MSTW2008 NNLO pdf 90% CL**

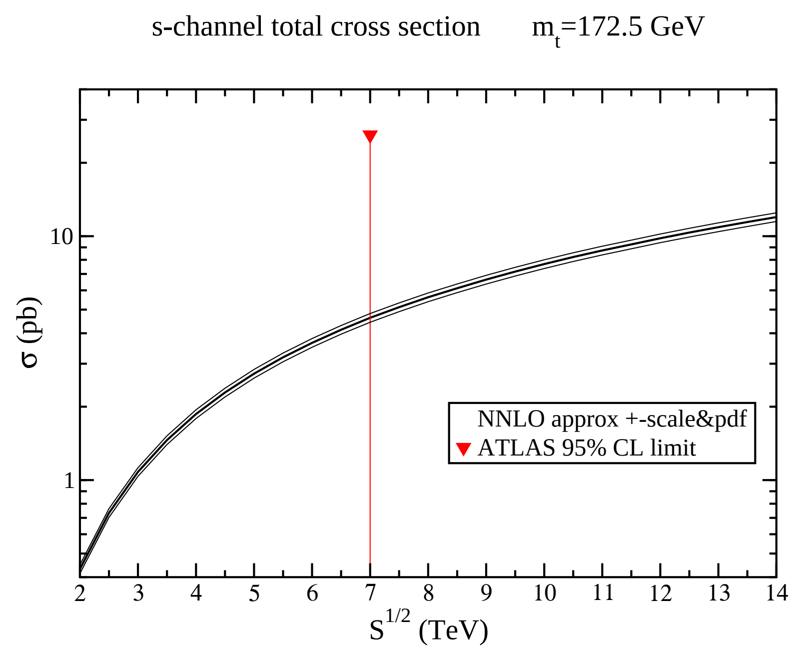
ratio $\sigma(t)/\sigma(\bar{t}) = 1.88^{+0.11}_{-0.09}$ **at 7 TeV - compares well with ATLAS result** $1.81^{+0.23}_{-0.22}$

t-channel top and antitop p_T distributions at LHC



Single top s-channel cross sections at LHC



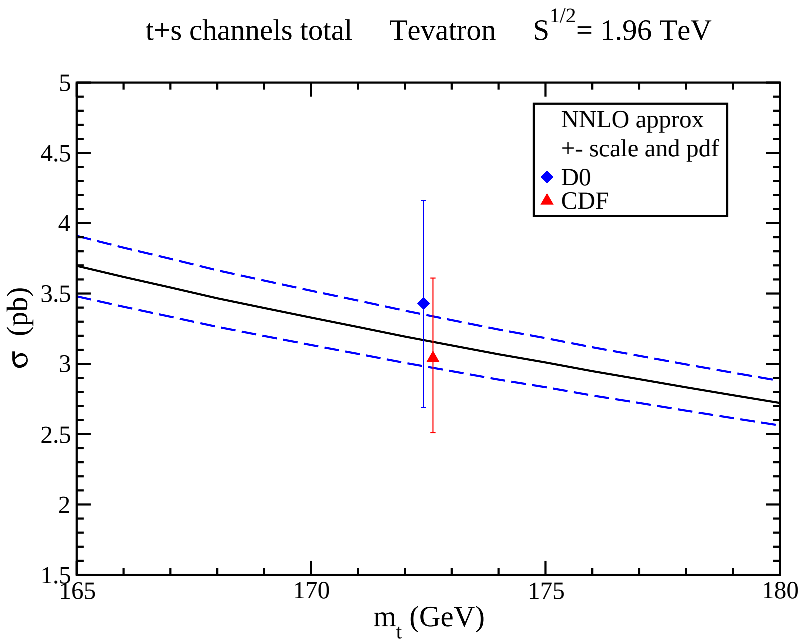


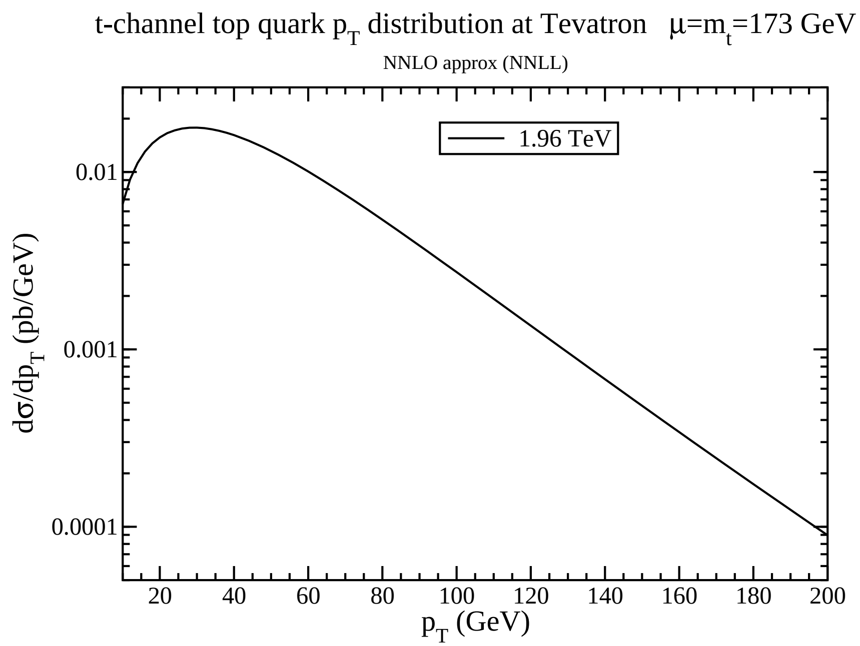
$m_t = 173$ **GeV**

LHC	t	$\bar{t}$	Total (pb)
7 TeV	$3.14 \pm 0.06^{+0.12}_{-0.10}$	$1.42 \pm 0.01^{+0.06}_{-0.07}$	$4.56 \pm 0.07^{+0.18}_{-0.17}$
8 TeV	$3.79 \pm 0.07 \pm 0.13$	$1.76 \pm 0.01 \pm 0.08$	$5.55 \pm 0.08 \pm 0.21$
14 TeV	$7.87 \pm 0.14^{+0.31}_{-0.28}$	$3.99 \pm 0.05^{+0.14}_{-0.21}$	$11.86 \pm 0.19^{+0.45}_{-0.49}$

NNLO approx: enhancement over NLO (same pdf) is $\sim 10\%$

t- and s-channel single top production at Tevatron

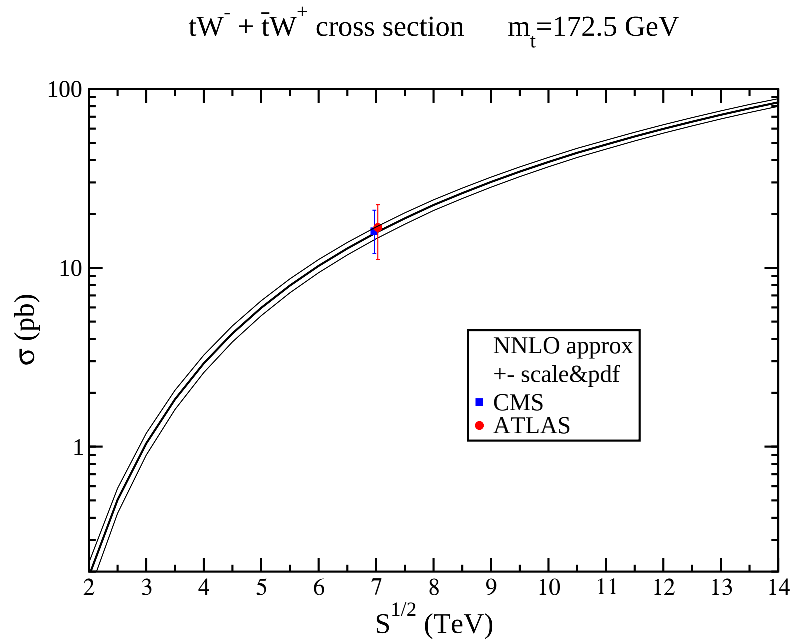


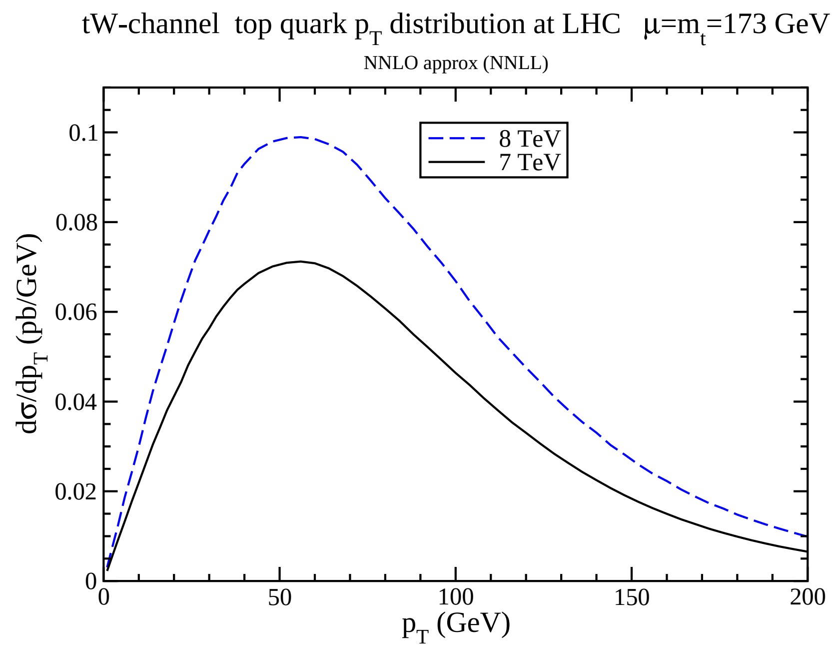


$m_t = 173$ **GeV**

Tevatron	Total (pb) at 1.96 TeV
t-channel	$2.08^{+0.00}_{-0.04} \pm 0.12$
s-channel	$1.05^{+0.00}_{-0.01} \pm 0.06$
$t+s$ sum	$3.13^{+0.00}_{-0.05} \pm 0.18$

Associated tW^- production at the LHC





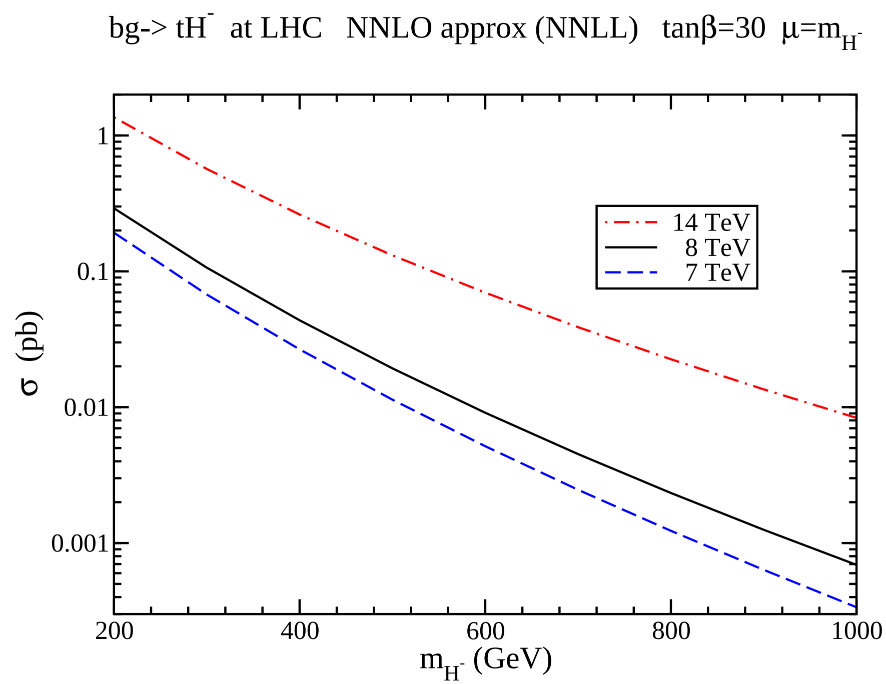
$m_t = 173$ **GeV**

LHC	tW^- (pb)
7 TeV	$7.8 \pm 0.2^{+0.5}_{-0.6}$
8 TeV	$11.1 \pm 0.3 \pm 0.7$
14 TeV	$41.8 \pm 1.0^{+1.5}_{-2.4}$

NNLO approx corrections increase NLO cross section by $\sim 8\%$

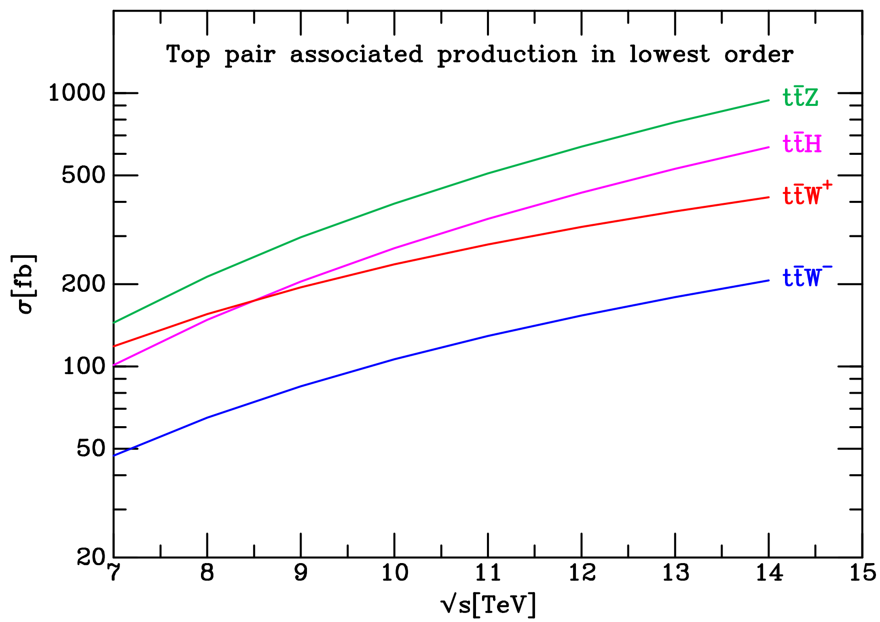
Cross section for $\bar{t}W^+$ production is identical

Associated production of a top quark with a charged Higgs



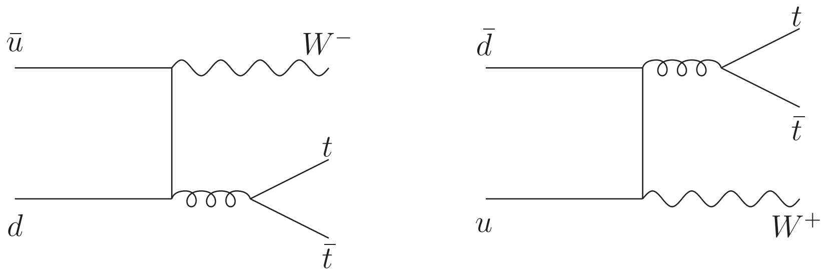
NNLO approx corrections increase NLO cross section by ~ 15 **to** $\sim 20\%$

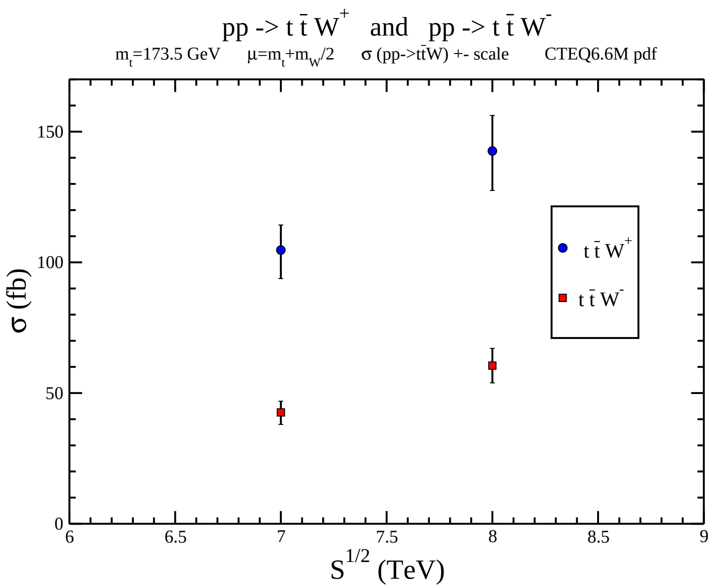
Associated production of a $t\bar{t}$ pair with bosons



Campbell and Ellis, JHEP 1207 (2012) 052

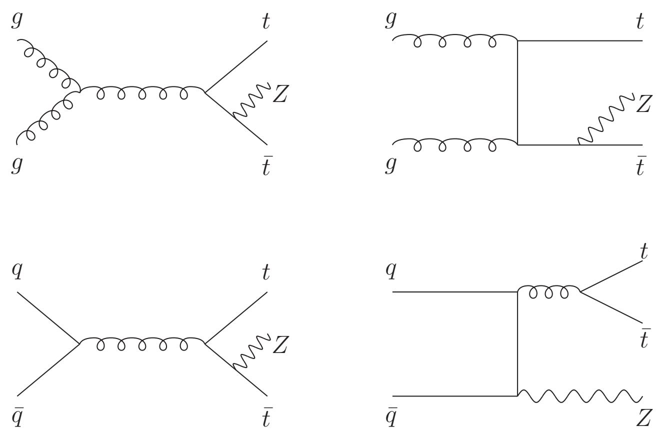
$t\bar{t}W$ production

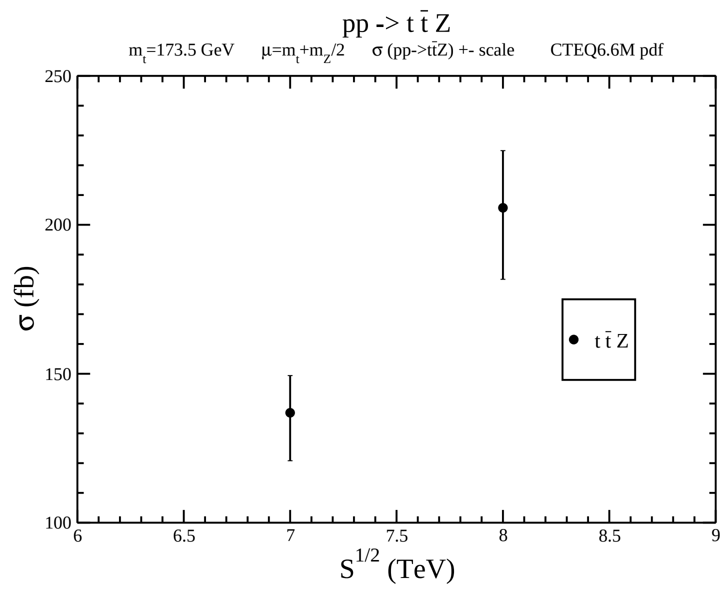




Garzelli, Kardos, Papadopoulos, Trocsanyi, JHEP 1211 (2012) 056

$t\bar{t}Z$ production





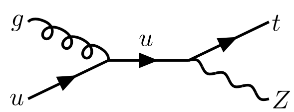
Garzelli, Kardos, Papadopoulos, Trocsanyi, JHEP 1211 (2012) 056

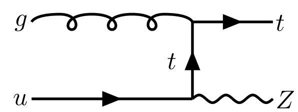
FCNC processes

Single-top production via flavor-changing neutral currents
Anomalous couplings in Lagrangian, e.g.

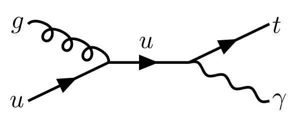
$$\Delta \mathcal{L}^{eff} = \frac{1}{\Lambda}\, \kappa_{tqV}\, e\, \bar{t}\, \sigma_{\mu\nu}\, q\, F_V^{\mu\nu} + h.c.$$

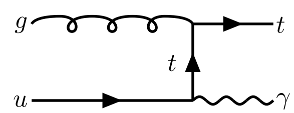
$gu \rightarrow tZ$



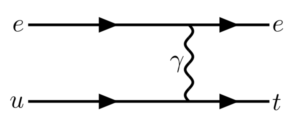


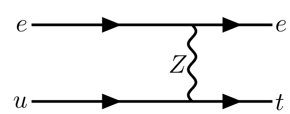
$gu \rightarrow t\gamma$





$eu \rightarrow et$





decrease of scale dependence, significant corrections over LO

Future studies at LHC energies and for other couplings

Summary

- **Resummation of soft-gluon corrections**
- **NNLL resummation / NNLO expansions**
- **Two-loop eikonal calculations**
- **Soft anomalous dimension matrices**
- $t\bar{t}$ **production cross section**
- **top quark** p_T **and rapidity distributions**
- **single top cross sections and** p_T **distributions**
- **NNLO approx corrections are very significant**
- **excellent agreement with LHC and Tevatron data**