Applications of QCD Sum Rules to Heavy Quark Physics

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Lecture 3:

Hadronic effects in $B \to K^{(*)}\ell^+\ell^-$,
$B \to K^*\gamma$ from LCSR's

$B \to K^{(*)} \ell^+ \ell^-$, the effective Hamiltonian

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) \Big|_{\mu \sim m_b}$$

- "direct" $b \to s\ell\ell$, $b \to s\gamma$ operators:

  $O_{9(10)} = \frac{\alpha_{em}}{4\pi} [\bar{s}_L \gamma_\mu b_L] \ell \gamma^\mu (\gamma_5) \ell$, $\quad C_9(m_b) \simeq 4.4$, $\quad C_{10}(m_b) \simeq = -4.7$,

  $O_{7\gamma} = -\frac{e m_b}{8\pi^2} [\bar{s} \sigma_{\mu\nu} (1 + \gamma_5) b] F^{\mu\nu}$, $\quad C_7(m_b) \simeq -0.3$

- quark-gluon operators, combined with quark e.m. current :

  $O_1^{(c)} = [\bar{s}_L \gamma_\rho c_L][\bar{c}_L \gamma^\rho b_L]$, $\quad C_1(m_b) \simeq 1.1$
  $O_2^{(c)} = [\bar{c}_L \gamma_\rho c_L][\bar{s}_L \gamma^\rho b_L]$, $\quad C_2(m_b) \simeq -0.25$

  $O_{8g} = -\frac{m_b}{8\pi^2} \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) b G^{\mu\nu}$, $\quad C_8(m_b) \simeq 0.2$

  O_{3-6} - quark-penguin operators , $\quad C_{3,4,5,6} < 0.03$

- the $\sim V_{ub} V_{us}^*$ part neglected

$B \to K^{(*)} \ell^+\ell^-$ decay amplitude

$$A(B \to K^{(*)}\ell^+\ell^-) = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i \langle K^{(*)}\ell^+\ell^- \mid O_i \mid B\rangle$$

- hadronic matrix elements:

$$A(B \to K^{(*)}\ell^+\ell^-) = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha_{em}}{2\pi} \Bigg[(\bar{\ell}\gamma^\rho\gamma_5\ell)\, C_{10} \langle K^{(*)}|\bar{s}\gamma_\rho(1-\gamma_5)b|B\rangle$$
$$+(\bar{\ell}\gamma^\rho\ell)\Big(C_9 \langle K^{(*)}|\bar{s}\gamma_\rho b|B\rangle + C_7 \frac{2m_b}{q^2} q^\nu \langle K^{(*)}|\bar{s}i\sigma_{\nu\rho}(1+\gamma_5)b|B\rangle$$
$$+\frac{8\pi^2}{q^2} \sum_{i=1,2,\dots,6,8} C_i\, \mathcal{H}_i^\rho \Big)\Bigg]$$

- include $B \to K^{(*)}$ form factors and nonlocal hadronic matrix elements

$$\boxed{\mathcal{H}_i^\rho(q,p) = \langle K^{(*)}(p)|i\int d^4x\, e^{iqx}\, T\{j^\rho_{em}(x), O_i(0)\}|B(p+q)\rangle},$$

- hereafter, consider the kaon final state, $B \to K\ell^+\ell^-$

Status of $B \to K$ form factors in QCD

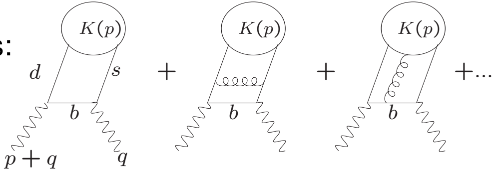
- lattice QCD at $q^2 \geq 15$ GeV^2, recent $n_f = 3$ results:
  [Fermilab-MILC 1211.1390 [hep-lat]; Cambridge-(MILC) 1101.2726[hep-ph], HPQCD-13

- QCD light-cone sum rules (LCSR), at $q^2 < 15$ GeV^2:

  correlation function = hadronic sum $\Rightarrow$ $\langle K|\, j\, |B\rangle$

  $\Uparrow$ {OPE, light-cone DAs} $\Downarrow$ {quark-hadron duality}

- LCSR with kaon DAs:



- soft-gluon (low virtuality) and hard-gluon effects enter separate terms of OPE
- alternative version of LCSR: B-meson DAs (HQET) and kaon interpolating current (LCSR in SCET)

LCSR results

- $q^2 \leq 12 - 15\ \text{GeV}^2$ accessible,
  complementing the lattice FF's
- LCSR with kaon DAs, the recent update
  [A.K, Th.Mannel, A.Pivovarov, Y-M. Wang (2010)]
- employing z-parameterization:

$$F(q^2) = \frac{F(0)}{1 - q^2/m^2_{B_s(J^P)}} \Big\{ 1 + b_1 \Big(z(q^2, t_0) - z(0, t_0) + \frac{1}{2} \left[z(q^2, t_0)^2 - z(0, t_0)^2 \right] \Big) \Big\},$$

$B \to K$ form factor	$F^i_{BK}(0)$	b^i_1	$B_s(J^P)$	input at $q^2 < 12\ \text{GeV}^2$
f^+_{BK}	$0.34^{+0.05}_{-0.02}$	$-2.1^{+0.9}_{-1.6}$	$B^*_s(1^-)$	LCSR with K DA's
f^0_{BK}	$0.34^{+0.05}_{-0.02}$	$-4.3^{+0.8}_{-0.9}$	no pole	
f^T_{BK}	$0.39^{+0.05}_{-0.03}$	$-2.2^{+1.0}_{-2.00}$	$B^*_s(1^-)$	

Hadronic input in $B \to K\ell\ell$

$$A(B \to K\ell^+\ell^-) = \frac{G_F}{\sqrt{2}}\frac{\alpha_{em}}{\pi} V_{tb}V_{ts}^*\Bigg[\bar{\ell}\gamma_\mu\ell\, p^\mu\Bigg(C_9 f_{BK}^+(q^2)$$

$$+\frac{2(m_b+m_s)}{m_B+m_K}C_7^{eff} f_{BK}^T(q^2) + \sum_{i=1,2,\dots,6,8} C_i\,\mathcal{H}_i^{(BK)}(q^2)\Bigg) + \bar{\ell}\gamma_\mu\gamma_5\ell\, p^\mu C_{10} f_{BK}^+(q^2)\Bigg]$$

- the leading short-distance contributions determined by $B \to K$ form factors calculable in QCD
- remaining nonlocal matrix elements:

$$\mathcal{H}_i^{(BK)}(q^2) \sim \langle K(p)| i\int d^4x\, e^{iqx}\, T\{j^\rho_{em}(x), O_i(0)\}|B(p+q)\rangle$$

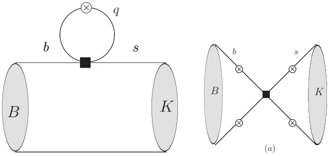
$j^\rho_{em} = \sum\limits_{q=u,d,s,c,b} Q_q\bar{q}\gamma^\rho q\,,$ the hierarchy $O_i = O_{1,2}^{(c)},\ O_{8g}, O_{3,4,5,6}^{(q)}, O_{1,2}^{(u)}$

$\Rightarrow$ corrections to fundamental short-distance coeff.:
$C_9 \to C_9 + \Delta C_9^{(BK,i)}(q^2)$ (q^2- and process-dependent)

- have to be estimated one by one

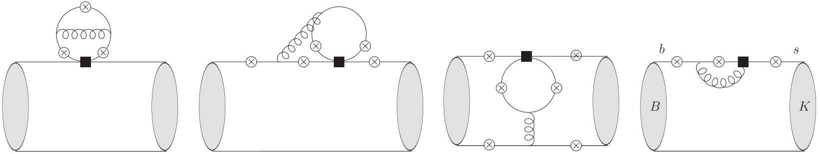
Are the nonlocal matrix elements calculable in QCD?

- LO, factorizable and weak annihilation

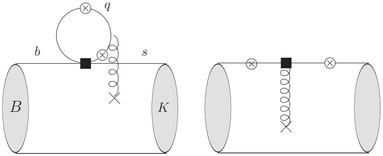


⊗ -virtual photon

- NLO, nonfactorizable ...



- soft (low virtuality) gluons, nonfactorizable



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Use of OPE and effective theories

- at low q^2 (large recoil of $K^{(*)}$):

- $\mathcal{H}_i$ in $O(\alpha_s)$ obtained from QCD factoriz. (HQET/SCET) ($m_b \to \infty, E_{K^{(*)}} \sim m_b$)
- two-loop $b \to s$ factorizable diagrams taken from inclusive $B \to X_s \ell^+ \ell^-$ analysis *[H. Asatrian, C.Greub et al (2001)],*
- "nonspectator" contributions $\Rightarrow$ $B \to K^{(*)}$ FF's,
  "spectator " contributions factorized;
  nonpert. inputs B and $K^{(*)}$ DAs
  [M.Beneke, Th.Feldmann, D.Seidel (2001)], ...
- the magnitude of soft gluon effects ??
  (power-suppressed in OPE)

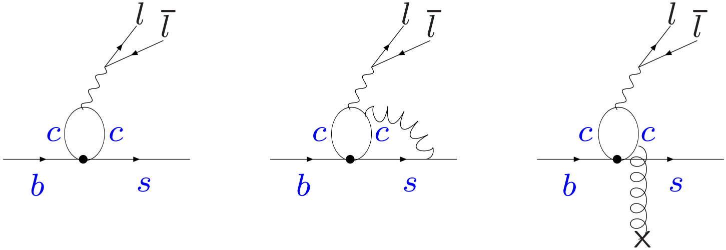
New approach to nonlocal hadronic matrix elements

- charm-loop effects in $B \to K^{(*)}\ell\ell$
  [A.K., Th. Mannel, A. Pivovarov and Yu-M. Wang, JHEP (2010)]

- complete analysis of $B \to K\ell\ell$
  [A.K., Th. Mannel and Yu-M. Wang, JHEP (2013)]

Charm-loops in $b \to s\ell^+\ell^-$ transitions

- a combination of the $(\bar{s}c)(\bar{c}b)$ weak interaction ($O_{1,2}$) and e.m.interaction $(\bar{c}c)(\bar{\ell}\ell)$ "mimicking FCNC"
- Charm-loop effect:



- similar effects:
  u, d, s, c, b-quark loops (quark-penguin operators O_{3-6}),
  u-loops from $O^u_{1,2}$ (CKM suppressed in $b \to s$),
  weak annihilation.
- In SM $A(B \to K^*\ell^+\ell^-)$ includes new hadronic matrix elements, not simply form factors

Charm loop turns charmonium

- at $q^2 \to m_{J/\psi}, \ldots$ an on-shell hadronic state:
  $B \to J/\psi K \otimes J/\psi \to \ell^+\ell^-$
- other ψ-levels (charmonia with $J^P = 1^-$),
  open-charm states $B \to \bar{D}DK \to K\ell^+\ell^-$,
  exotic charmonia?,
  ($\bar{c}c$ states with the masses up to $m_B - m_K^{(*)}$)
- $B \to \psi K$: the naive factorization fails, hinting at large nonfactorizable contributions
- to avoid huge backgr., the charmonium resonances are subtracted from the q^2-distribuiton data in $B \to K^{(*)}\ell^+\ell^-$
- the effect of intermediate/virtual $\bar{c}c$ states remains at $q^2 \ll m^2_{J/\psi}$ (nonperturbative at $q^2 \sim 4m_c^2$)
- Can we use the $\{$ loop $\oplus$ corrections $\}$ ansatz?

Isolating the charm-loop in the decay amplitude

- the contribution of $O_{1,2}$ and e.m. interaction:

$$A(B \to K^{(*)}\ell^+\ell^-)^{(O_{1,2})} = -(4\pi\alpha_{em}Q_c)\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^*\frac{\bar{\ell}\gamma^\mu\ell}{q^2}\mathcal{H}_\mu^{(B\to K^{(*)})},$$

- the relevant hadronic matrix element:

$$\mathcal{H}_\mu^{(B\to K^{(*)})}(p,q) = i\int d^4x e^{iq\cdot x}\langle K^{(*)}(p)|T\Big\{\bar{c}(x)\gamma_\mu c(x), \Big[C_1O_1(0)+C_2O_2(0)\Big]\Big\}|B(p+q)\rangle ,$$

- at small $q^2 \ll 4m_c^2$ use the operator-product expansion (OPE) for the T-product:

$$\mathcal{C}_\mu^a(q) = \int d^4x e^{iq\cdot x}T\Big\{\bar{c}(x)\gamma_\mu c(x), \bar{c}_L(0)\Gamma^a c_L(0)\Big\},$$

Expansion near the light-cone

- the dominant region in this T-product:$\langle x^2 \rangle \sim 1/(2m_c - \sqrt{q^2})^2$
- at $q^2 \ll 4m_c^2$: T- product of $\bar{c}c$-operators can be expanded near the light-cone $x^2 \sim 0$, schematically,

$$T\{\bar{c}(x)\gamma_\mu c(x), \bar{c}_L(0)\gamma_\rho c_L(0)\} = C_0^{\mu\rho}(x^2, m_c^2) + \text{two-gluon term} + ...$$

$$T\{\bar{c}(x)\gamma_\mu c(x), \bar{c}_L(0)\gamma_\rho \frac{\lambda^a}{2} c_L(0)\} = \int\limits_0^1 du C_1^{\mu\rho\alpha\beta}(x^2, m_c^2, u) G^a_{\alpha\beta}(ux) + ...$$

- after x-integration and taking hadronic matrix element: $O(C_1)/O(C_0) \sim O(C_{n+1})/O(C_n) \sim \Lambda^2_{QCD}/(4m_c^2 - q^2)$,
- but ! no local expansion possible in each term of LC OPE: $O(C_1) \sim \sum_{k=0}^{\infty}(q\Lambda_{QCD})^k/(4m_c^2 - q^2)^{k+1}$,

  $q \sim m_b/2$ and $m_b\Lambda_{QCD} \sim m_c^2$.

The resulting effective operators

- LO reduced to simple $\bar{c}c$ -loop,
  no difference between local and LC,

$$\mathcal{O}_\mu(q) = (q_\mu q_\rho - q^2 g_{\mu\rho}) \frac{9}{32\pi^2}\, g(m_c^2, q^2) \bar{s}_L \gamma^\rho b_L \,.$$

- gluon emission: use c-quark propagator near the light-cone in the external gluon field [I. Balitsky, V. Braun (1999)]
- define LC kinematics ($n_\pm$) in the rest-frame of B,
  $q \simeq (m_b/2) n_+$
- one-gluon emission yields a new nonlocal operator:

$$\widetilde{\mathcal{O}}_\mu(q) = \int d\omega\, I_{\mu\rho\alpha\beta}(q, m_c, \omega) \bar{s}_L \gamma^\rho \delta[\omega - \frac{(in_+\mathcal{D})}{2}] \widetilde{G}_{\alpha\beta} b_L \,,$$

The local OPE limit

- $\omega \to 0$ in the nonlocal operator, no derivatives of $G_{\mu\nu}$

$$\widetilde{\mathcal{O}}_{\mu}^{(0)}(q) = I_{\mu\rho\alpha\beta}(q)\bar{s}_L\gamma^{\rho}\widetilde{G}_{\alpha\beta}b_L \ ,$$

$$I_{\mu\rho\alpha\beta}(q, m_c) = (q_\mu q_\alpha g_{\rho\beta} + q_\rho q_\alpha g_{\mu\beta} - q^2 g_{\mu\alpha}g_{\rho\beta}) \times \frac{1}{16\pi^2}\int_0^1 dt \, \frac{t(1-t)}{m_c^2 - q^2 t(1-t)}$$

At $q^2 = 0$, the quark-gluon operator obtained
in $B \to X_s\gamma$ in *[M.Voloshin (1997)]*
in $B \to K^*\gamma$ *[A.K.,G. Stoll,R. Rueckl,D. Wyler(1997)]*

- the neccesity of resummation was discussed before
*[Z. Ligeti, L. Randall and M.B. Wise,(1997);
A.K. Grant, A.G. Morgan, S. Nussinov and R.D. Peccei (1997);
J. W. Chen, G. Rupak and M. J. Savage,(1997);
G. Buchalla, G. Isidori and S.J. Rey (1997)]*

Hadronic matrix elements for the charm-loop effect

- the LO: factorized $\bar{c}c$ loop

$$\left[\mathcal{H}_\mu^{(B\to K^{(*)})}(p,q)\right]_{fact} = \left(\frac{C_1}{3} + C_2\right)\langle K^{(*)}(p)|\mathcal{O}_\mu(q)|B(p+q)\rangle\,,$$

- reduced to $B \to K^{(*)}$ form factors, nothing new
- The gluon emission yields:

$$\left[\mathcal{H}_\mu^{(B\to K^{(*)})}(p,q)\right]_{nonfact} = 2C_1\langle K^{(*)}(p)|\widetilde{\mathcal{O}}_\mu(q)|B(p+q)\rangle\,.$$

- new hadronic matrix element

$$\langle K^{(*)}(p)|\bar{s}_L\gamma^\rho\delta[\omega - \frac{(in_+\mathcal{D})}{2}]\widetilde{G}_{\alpha\beta}b_L|B(p+q)\rangle\,,$$

Charm-loop effect in $B \to K\ell^+\ell^-$

- The invariant amplitude:

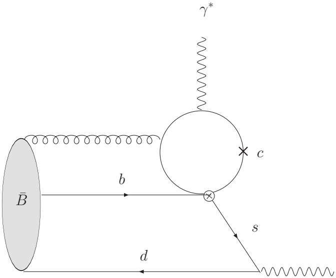
$$\left[\mathcal{H}_\mu^{(B\to K)}(p,q)\right]_{fact.+nonfact.} = \left[(p\cdot q)q_\mu - q^2 p_\mu\right] \times \left[\left(\frac{C_1}{3}+C_2\right)A(q^2) + 2C_1\widetilde{A}(q^2)\right]$$

- the factorizable part $A(q^2) = \frac{9}{32\pi^2}\, g(m_c^2, q^2) f_{BK}^+(q^2)$
- Wilson coefficients enhance the nonfact. part $C_1/3 + C_2 \ll C_1$
- need nonperturbative QCD methods to calculate the form factor $f_{BK}^+(q^2)$ and the nonfactorizable amplitude $\widetilde{A}(q^2)$
- use one and the same LCSR approach

LCSR for the soft-gluon hadronic matrix element

- the correlation function:

$$\mathcal{F}^{(B\to K)}_{\nu\mu}(p,q) = i\int d^4y e^{ip\cdot y}$$
$$\langle 0|T\{j^K_\nu(y)\widetilde{\mathcal{O}}_\mu(q)\}|B(p+q)\rangle\,,$$



- hadronic dispersion relation in the kaon channel

$$\mathcal{F}^{(B\to K)}_{\nu\mu}(p,q) = \frac{if_K p_\nu}{m_K^2 - p^2}[(p\cdot q)q_\mu - q^2 p_\mu]\tilde{A}(q^2) + \int_{s_h}^{\infty} ds\,\frac{\tilde{\rho}_{\mu\nu}(s,q^2)}{s-p^2}$$

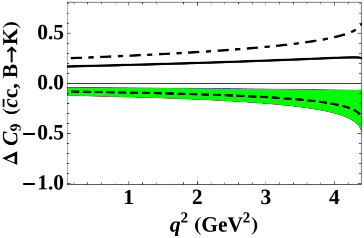
Charm-loop effect in $B \to K\ell^+\ell^-$ in terms of ΔC_9

- the effective coefficient $C_9(\mu = m_b) \simeq 4.4$
  a process-dependent correction to be added:

$$\Delta C_9^{(\bar{c}c, B\to K)}(q^2) = \frac{32\pi^2}{3}\frac{\mathcal{H}^{(B\to K)}(q^2)}{f^+_{BK}(q^2)}$$

$$= (C_1 + 3C_2)\, g(m_c^2, q^2) + 2C_1 \frac{32\pi^2}{3}\frac{\tilde{A}(q^2)}{f^+_{BK}(q^2)}$$

$\Delta C_9(0) = 0.17^{+0.09}_{-0.18}$,
$(\mu = m_b = 4.2\text{GeV})$



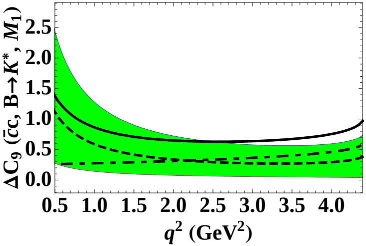
Charm-loop effect for $B \to K^* \ell^+ \ell^-$

- factorizable part determined by the three $B \to K^*$ form factors $V^{BK*}(q^2)$, $A_1^{BK^*}(q^2)$, $A_2^{BK^*}(q^2)$,
- three kinematical structures for the nonfactorizable part:

$$\Delta C_9^{(\bar{c}c, B\to K^*, V)}(q^2) = (C_1 + 3C_2)\, g(m_c^2, q^2) \\ -2C_1 \frac{32\pi^2}{3} \frac{(m_B + m_{K^*})\widetilde{A}_V(q^2)}{q^2\, V^{BK^*}(q^2)} \,,$$

- nonfactorizable part enhances the effect, $1/q^2$ factor

$\Delta C_9^{(\bar{c}c, B\to K^*, V)}(1.0\text{GeV}^2) = 0.7^{+0.6}_{-0.4}$
$\Delta C_9^{(\bar{c}c, B\to K^*, A_1)}(1.0\text{GeV}^2) = 0.8^{+0.6}_{-0.4}$
$\Delta C_9^{(\bar{c}c, B\to K^*, A_2)}(1.0\text{GeV}^2) = 1.1^{+1.1}_{-0.7}$



Charm-loop effect in $B \to K^*\gamma$

- By-product of our calculation for $B \to K^*\ell^+\ell^-$ at $q^2 = 0$
- factorizable part vanishes,
  nonfactorizable part yields a correction to $C_7^{eff}(m_b) \simeq -0.3$
  in the two inv. amplitudes:

  $$C_7^{eff} \to C_7^{eff} + [\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}]_{1,2}\,,$$

  $$\left[\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}\right]_1 \simeq \left[\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}\right]_2 = (-1.2^{+0.9}_{-1.6}) \times 10^{-2}\,,$$

- the previous results in the local OPE limit , LCSR with K^* DA:

  $$[\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}]_1^{BZ} = (-0.39 \pm 0.3) \times 10^{-2}\,,$$
  $$[\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}]_2^{BZ} = (-0.65 \pm 0.57) \times 10^{-2}\,. \qquad (2)$$

  [P.Ball, G. W. Jones and R. Zwicky (2007)]

- our result in the local limit is closer to 3-point sum rule estimate: [A.K.,G. Stoll,R. Rueckl,D. Wyler(1997)]

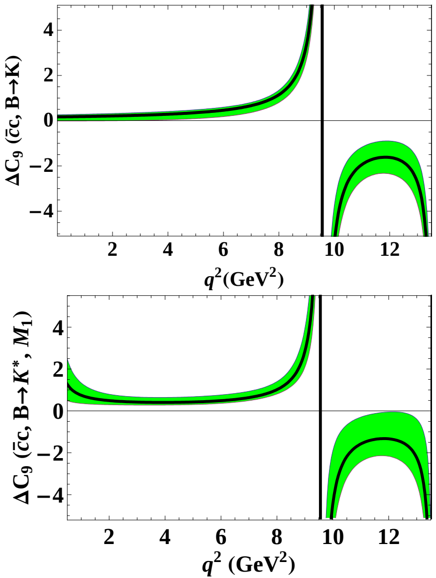
Accessing large q^2 with dispersion relation

- analyticity of the hadronic matrix element in q^2, dispersion relation:

$$\mathcal{H}^{(B\to K)}(q^2) = \mathcal{H}^{(B\to K)}(0) + q^2 \Big[\sum_{\psi=J/\psi,\psi(2S)} \frac{f_\psi A_{B\psi K}}{m_\psi^2 (m_\psi^2 - q^2 - i m_\psi \Gamma_\psi^{tot})} + \int_{4m_D^2}^{\infty} ds \frac{\rho(s)}{s(s-q^2-i\epsilon)} \Big]$$

- absolute values of the residues $|f_\psi A_{B\psi K}|$ from exp. data
- the integral over $\rho(s)$ fitted as an effective pole
  no attempt to use semi-local duality
- complex phases neglected, destructive interference preferable, small integral over $\rho(s)$
- previous uses of dispersion relation: only factorizable part, positive residues, k-factors to accomodate data
  [F. Krüger, L. Sehgal (1997); A. Ali, P. Ball, L. T. Handoko, G. Hiller (2000)]

Charm-loop effect at large q^2

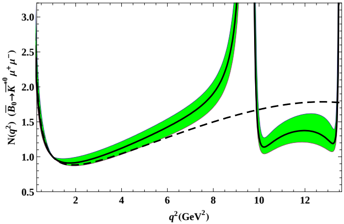


solid- central input,
green-shaded - uncertainties

► the dispersion relation ansatz coincides with OPE result at $q^2 < 4.0$ GeV^2 and is valid up to $s = 4m_D^2$ (at $q^2 < m_{J/\psi}^2$ largely independent of higher-states ansatz)

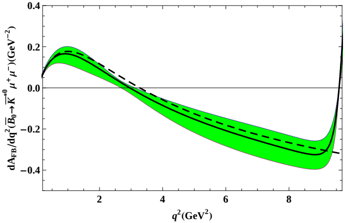
Observables for $B \to K^* \ell^+ \ell^-$

- differential distribution in q^2 with (solid) and without (dashed) charm-loop effect



- forward-backward asymmetry : $q_0^2 = 2.9^{+0.2}_{-0.3} \mathrm{GeV}^2$

  $\sim$ 10% larger without nonfactorizable correction



All nonlocal effects in $B \to K\ell^+\ell^-$

- all operators $O_{1,2}$, O_{8g}, O_{3-6} included
- quark-loop soft-gluon effects at $q^2 < 0$ calculated
- soft-gluon emission from gluonic penguin operator (new LCSR calculation)
- hard-gluon effects estimated employing QCDF (partly cross-checked with LCSRs)
- LO weak annihilation (small effect)
- dispersion relation includes $V = \rho, \omega, \phi$ in addition to $V = J/\psi, \psi'$; the parameters fitted with $q^2 < 0$ calculation and with measured $BR(B \to VK)$.

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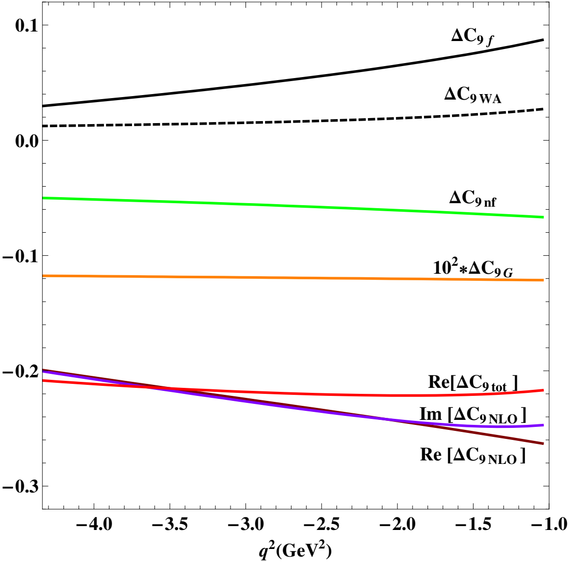
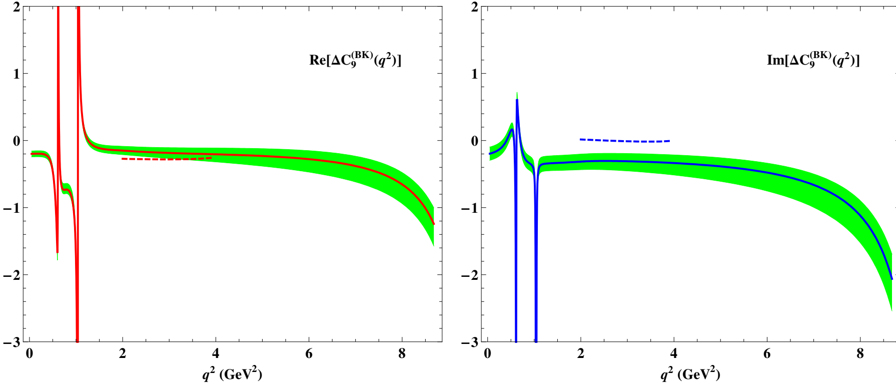
- $\Delta C_9^{(BK)}(q^2 < 0)$



$\Delta C_9(q^2)$ below J/ψ region



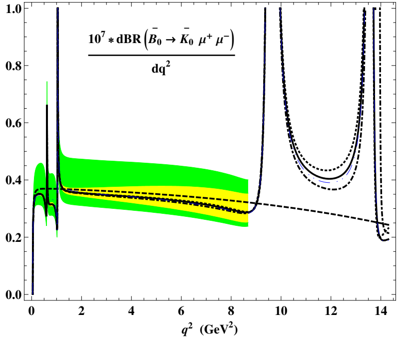
the red (blue) solid curve corresponds to the Re (Im) part obtained from the hadronic dispersion relation, fitted to the QCD calculation at q2 < 0 (central input, default parametrization). The shaded areas indicate the uncertainties. The dashed curves correspond to the prediction of QCDF.

$d\mathrm{BR}(B \to K\mu^+\mu^-)/dq^2$ and bins

solid (dotted) lines - central input, default (alternative) parametrization for the dispersion integrals.

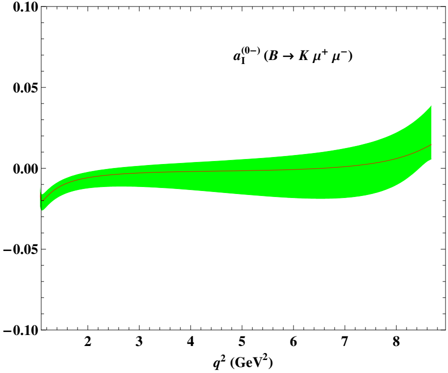
long-dashed line -the width calculated without nonlocal hadronic effects.

The green (yellow) shaded area indicates the uncertainties including (excluding) the one from the $B \to K$ FF normalization.



$[q^2_{min}, q^2_{max}]$	Belle	CDF	LHCb	LHCb	this work
[0.05, 2.0]	$0.81^{+0.18}_{-0.16} \pm 0.05$	$0.33 \pm 0.10 \pm 0.02$	$0.21^{+0.27}_{-0.23}$	$0.56 \pm 0.05 \pm 0.03$	$0.71^{+0.22}_{-0.08}$
[2.0, 4.3]	$0.46^{+0.14}_{-0.12} \pm 0.03$	$0.77 \pm 0.14 \pm 0.05$	$0.07^{+0.25}_{-0.21}$	$0.57 \pm 0.05 \pm 0.02$	$0.80^{+0.27}_{-0.11}$
[4.3, 8.68]	$1.00^{+0.19}_{-0.08} \pm 0.06$	$1.05 \pm 0.17 \pm 0.07$	1.2 ± 0.3	$1.00 \pm 0.07 \pm 0.04$	$1.39^{+0.53}_{-0.22}$
[1.0, 6.0]	$1.36^{+0.23}_{-0.21} \pm 0.08$	$1.29 \pm 0.18 \pm 0.08$	$0.65^{+0.45}_{-0.35}$	$1.21 \pm 0.09 \pm 0.07$	$1.76^{+0.60}_{-0.23}$

Isospin asymmetry: $\bar{B}^0 \to \bar{K}^0 \ell^+ \ell^-$ vs $B^- \to K^- \ell^+ \ell^-$



- integrated over $1.0 < q^2 < 6.0$ GeV^2.

Belle (2009)	BaBar (2012)	LHCb (2012)	this work
$-0.41^{+0.25}_{-0.20} \pm 0.07$	$-0.41 \pm 0.25 \pm 0.01$	$-0.35^{+0.23}_{-0.27}$	$(-0.4)\% \div (-0.3)\%$

Towards complete analysis of $B \to K^* \ell^+ \ell^-$

- the main challenge: $B \to K^*$ form factors:

only quenched lattice QCD,
LCSR with K^* DAs with $\Gamma_K^* = 0$ (sort of "quenched") [P.Ball, R.Zwicky(2004)] LCSR with B DA's have a large uncertainty; [AK,T.Mannel, A.Pivovarov, YM.Wang(2010)]

- plans to extend the LCSR approach to

$B \to K\pi\ell\ell$ form factors (embedding K^* and other resonances)

- $\Delta C_9/C_9$ for $B \to K^* \ell^+ \ell^-$ are generally larger at small q^2

(due to $1/q^2$ multiplying $\mathcal{H}_i^{BK^*}$)

Summary

- the nonlocal hadronic amplitudes in $B \to K^{(*)}\ell^+\ell^-, K^*\gamma$ are not accessible on the lattice
- with light-cone OPE and LCSR's $\oplus$ hadronic dispersion relations, these effects can be systematically accounted in the region $q^2 <$few GeV^2
- in future not only soft-gluon but also the hard-gluon effects can be also calculated from the sum rules (correlation functions with multiloop/multiscale diagrams), currently QCD factorization $\oplus$ inclusive $b \to s$ perturbative diagrams used
- charm loops in $B \to K^{(*)}\ell^+\ell^-, K^*\gamma$ yield an important correction to C_9
- $B \to K\ell^+\ell^-$ allows for a more accurate and complete book-keeping of the hadronic input (both form factors and nonlocal effects) than $B \to K^*\ell^+\ell^-, K^*\gamma$