Applications of QCD Sum Rules to Heavy Quark Physics

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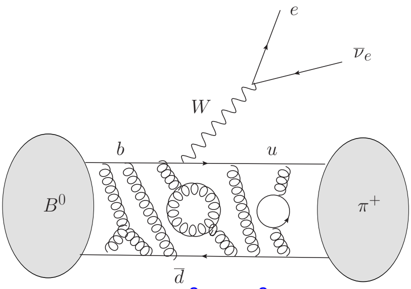
Lecture 2:

Light-Cone Sum Rules for Heavy-Light Form Factors

$B \to \pi l \nu_l$ and $|V_{ub}|$



- hadronic matrix element reduced to two form factors:

functions of the lepton pair invariant mass squared $q^2 = (p_e + p_\nu)^2$

$$\langle \pi^+(p)|\bar{u}\gamma_\mu b|B(p+q)\rangle = f^+_{B\pi}(q^2)\left[2p_\mu + \left(1 - \frac{m_B^2 - m_\pi^2}{q^2}\right)q_\mu\right] + f^0_{B\pi}(q^2)\frac{m_B^2 - m_\pi^2}{q^2}q_\mu ,$$

- form factors have to be calculated in nonperturbative QCD

  a perturbative mechanism (“factorization”) partially contributes

- an excellent source of $|V_{ub}|$ determination

$$\frac{d\Gamma(\bar{B}^0 \to \pi^+ l^- \nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} p_\pi^3 |f^+_{B\pi}(q^2)|^2 + O(m_l^2)$$

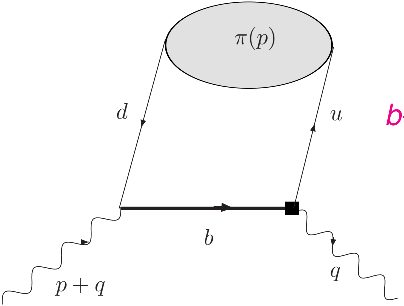
$0 < q^2 < (m_B - m_\pi)^2 \sim 26 \text{ GeV}^2$,

- in lattice QCD the $B \to \pi$ form factors accessible at $q^2 \geq 15 \text{ GeV}^2$,

The method of Light-Cone Sum Rules (LCSR)

- OPE in local operators with static condensates (e.g., three-point QCD sum rules) is not an adequate method for heavy-light form factors with "large recoil", i.e. at small q^2

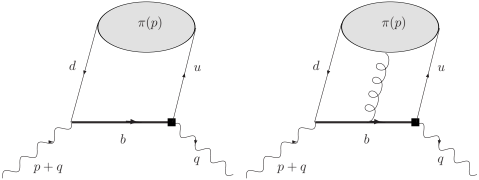
- the correlation function:



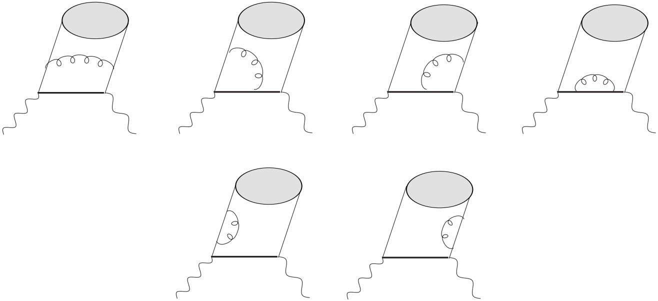
$q^2, (p+q)^2 \ll m_b^2,$
b-quark highly virtual $\Rightarrow$ $x^2 \sim 0$

$$F_\lambda(q,p) = i\int d^4x\, e^{iqx}\langle\pi(p)\mid T\{\bar{u}(x)\gamma_\lambda b(x), \bar{b}(0)i\gamma_5 d(0)\}\mid 0\rangle$$

Diagrams



- LO including soft, i.e. low-virtuality gluon



- NLO, perturbative $O(\alpha_s)$ contributions

Operator Product Expansion near the light-cone

$$F(q,p) = i\int d^4x\, e^{iqx}\Bigg\{ \Big[S_0(x^2,m_b^2,\mu) + \alpha_s S_1(x^2,m_b^2,\mu)\Big] \otimes \langle \pi(p) \mid \bar{u}(x)\Gamma d(0)\,|0\rangle|_\mu$$

$$+\int_0^1 dv\; \tilde{S}(x^2,m_b^2,\mu,v) \otimes \langle \pi(p) \mid \bar{u}(x)G(vx)\tilde{\Gamma} d(0)\} \mid 0\rangle|_\mu \Bigg\} + ...$$

- $S_{0,1}$, $\tilde{S}$ - perturbative amplitudes, (b-quark propagators)
- vacuum-pion matrix elements - expanded near $x^2 = 0$

$\Rightarrow$ universal distribution amplitudes of π :

$$\langle \pi(q)|\bar{u}(x)[x,0]\gamma_\mu\gamma_5 d(0)|0\rangle_{x^2=0} = -iq_\mu f_\pi \int_0^1 du\, e^{iuqx}\varphi_\pi(u) + O(x^2)\,.$$

- the expansion goes over twists ($t \geq 2$)
- terms $\sim \tilde{S}$ suppressed by powers of $1/\sqrt{m_b\Lambda}$;

The OPE result

$$F(q^2,(p+q)^2) = \sum_{t=2,3,4,..} \int du\, T^{(t)}(q^2,(p+q)^2,m_b^2,\alpha_s,u,\mu)\, \varphi_\pi^{(t)}(u,\mu)$$

hard scattering amplitudes $\otimes$ pion light-cone DA

- LO twist 2,3,4 $q\bar{q}$ and $\bar{q}qG$ terms:

  [V.Belyaev, A.K., R.Rückl (1993); V.Braun, V.Belyaev, A.K., R.Rückl (1996)]

-NLO $O(\alpha_s)$ twist 2, (collinear factorization)

  [A.K., R.Rückl, S.Weinzierl, O. Yakovlev (1997); E.Bagan, P.Ball, V.Braun (1997);]

-NLO $O(\alpha_s)$ twist 3 (coll.factorization for asympt. DA)

  [P. Ball, R. Zwicky (2001); G.Duplancic,A.K.,B.Melic, Th.Mannel,N.Offen (2007)]

Distribution amplitudes (DA's) of the pion

- twist 2 DA: normalized with f_π,
  expansion in Gegenbauer polynomials

$$\varphi_\pi(u,\mu) = 6u(1-u)\left[1 + \sum_{n=2,4,..} a_n^\pi(\mu) C_n^{3/2}(2u-1)\right],$$

$$a_{2n}^\pi(\mu) \sim [Log(\mu/\Lambda_{QCD})]^{-\gamma_{2n}} \to 0 \quad \text{at} \;\; \mu \to \infty$$

[Efremov-Radyushkin-Brodsky-Lepage evolution]

Gegenbauer moments at low scale

- essential parameters: $a^{\pi}_{2,4}(\mu_0)$, determined from:
  - matching exp. pion form factors to LCSR,
  - two-point QCD sum rules,
  - lattice QCD

- $a^{\pi}_2 = 0.25 \pm 0.15$ (average. of recent determinations)

$a^{\pi}_2 + a^{\pi}_4 = 0.1 \pm 0.1$ (pion-photon form factor)

- remaining tw 3,4 DA parameters:

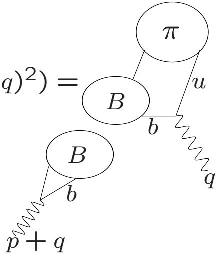
normalization constants and first moments,
determined mainly from two-point sum rules

[P. Ball, V.Braun, A.Lenz (2006)]

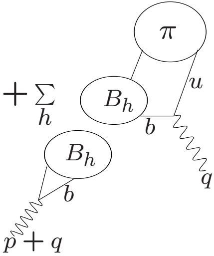
Derivation of LCSR

- Hadronic dispersion relation in the variable $(p+q)^2$:

$F(q^2,(p+q)^2) =$



$+\sum_h$



$$f_B f^{+}_{B\pi}(q^2) \qquad\qquad \sum_{B_h} \to duality\ (s_0^B)$$

$(q^2, (p+q)^2 \ll m_b^2\,)$

Derivation of LCSR

- matching OPE with disp. relation and using quark-hadron duality

$$[F((p+q)^2,q^2)]_{OPE} = \frac{m_B^2 f_B f^+_{B\pi}(q^2)}{m_B^2-(p+q)^2} + \int\limits_{s_0^B}^{\infty} ds \frac{[\mathrm{Im} F(s,q^2)]_{OPE}}{s-(p+q)^2}$$

- inputs: $\overline{m}_b$, α_s, $\varphi_\pi^{(t)}(u)$, t=2,3,4;
  f_B - determined from two-point (SVZ) sum rule;
- uncertainties due to:
  - variation of (universal) input parameters,
  - quark-hadron duality

(suppressed with Borel transformation, controlled by the m_B calculation)

- LCSR contains *both* "soft" and "hard" contributions to $f_{B\pi}(q^2)$
- the method is used at finite m_b

Results for $B \to \pi$ form factors

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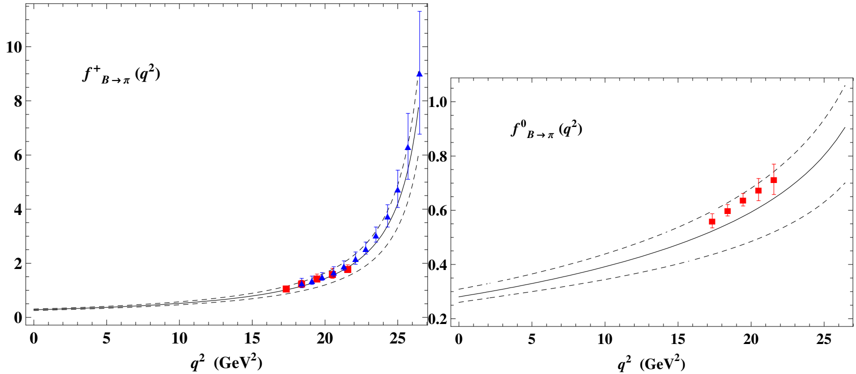
AK, T.Mannel,N.Offen,Y-M.Wang (2011)]



- $0 < q^2 < 12$ GeV^2 - LCSR, larger q^2 - extrapolation;

points: lattice QCD *[FNAL-MILC, HPQCD]*

- prediction of LCSR's was used by BaBar and Belle to extract $|V_{ub}|$

Extraction of $|V_{ub}|$

$$\Delta\zeta\,(0, q^2_{max} = 12 GeV^2) \equiv \frac{G_F^2}{24\pi^3}\int\limits_0^{q^2_{max}} dq^2 p_\pi^3 |f^+_{B\pi}(q^2)|^2 = \frac{1}{|V_{ub}|^2\tau_{B^0}}\int\limits_0^{q^2_{max}} dq^2 \frac{d\mathcal{B}(B\to\pi\ell\nu_\ell)}{dq^2}\,,$$

TABLE XII: Values of the CKM matrix element $|V_{ub}|$ based on rates of exclusive $\bar{B}\to X_u\ell^-\bar{\nu}_\ell$ decays and theoretical predictions of form factors within various q^2 ranges. The first uncertainty is statistical, the second is experimental systematic and the third is theoretical. The theoretical uncertainty for the ISGW2 model is not available.

X_u	Theory	q^2 GeV/c^2	$N^{\rm fit}$	$N^{\rm MC}$	$\Delta\mathcal{B}$ 10^{-4}	$\Delta\zeta$ ps^{-1}	$\lvert V_{ub}\rvert$ 10^{-3}
π^0	LCSR [33]	<12	119.6 ± 16.2	116.5	0.423 ± 0.057	$4.59^{+1.00}_{-0.85}$	$3.35\pm0.23\pm0.09^{+0.36}_{-0.31}$
	LCSR [34]	<16	168.2 ± 18.9	153.5	0.588 ± 0.066	$5.44^{+1.43}_{-1.43}$	$3.63\pm0.20\pm0.10^{+0.60}_{-0.40}$
	HPQCD [35]	>16	58.6 ± 10.5	57.6	0.196 ± 0.035	$2.02^{+0.55}_{-0.55}$	$3.44\pm0.31\pm0.09^{+0.59}_{-0.39}$
	FNAL [36]					$2.21^{+0.47}_{-0.42}$	$3.29\pm0.30\pm0.09^{+0.37}_{-0.30}$
π^+	LCSR [33]	<12	247.2 ± 18.9	233.1	0.808 ± 0.062	$4.59^{+1.00}_{-0.85}$	$3.40\pm0.13\pm0.09^{+0.37}_{-0.32}$
	LCSR [34]	<16	324.2 ± 22.6	305.1	1.057 ± 0.074	$5.44^{+1.43}_{-1.43}$	$3.58\pm0.12\pm0.09^{+0.59}_{-0.39}$
	HPQCD [35]	>16	141.3 ± 16.0	116.1	0.445 ± 0.050	$2.02^{+0.55}_{-0.55}$	$3.81\pm0.22\pm0.10^{+0.66}_{-0.43}$
	FNAL [36]					$2.21^{+0.47}_{-0.42}$	$3.64\pm0.21\pm0.09^{+0.40}_{-0.33}$

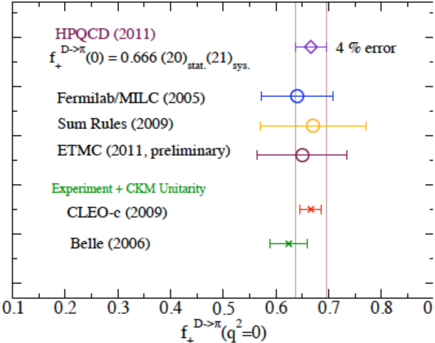
Belle Collab 1306.2781 [hep-ex]

LCSR results on $D \to \pi, K$ form factors

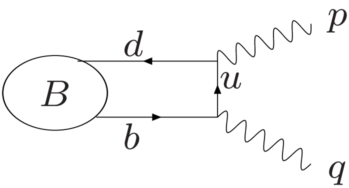
[Ch. Klein, A.K., Th. Mannel, N. Offen (2009)]

- simply replacing b quark to c quark in the correlation function
- $c \to d, s$ flavour-changing transitions
  using CLEO collaboration results on $D \to \pi(K) e \nu_e$ decays

  $|V_{cd}| = 0.219 \pm 0.005 \pm 0.004\ ^{+0.016}_{-0.010}\,, \quad |V_{cs}| = 1.03 \pm 0.08\ ^{+0.08}_{-0.06}\,,$



LCSR with B-meson distribution amplitudes



- on-shell *B* meson state and pion interpolating current

  [A.K., T. Mannel, N.Offen,2005]

- advantage: pseudoscalar,vector, ... light mesons are treated similarly via duality approximation

- a similar approach: LCSR for $B \to \pi$ in SCET

  [F.De Fazio, Th. Feldmann and T. Hurth, (2005)]

B-meson DA's

$$\langle 0|\bar{q}_{2\alpha}(x)[x,0]h_{v\beta}(0)|\bar{B}_v\rangle = -\frac{if_B m_B}{4}\int\limits_0^\infty d\omega e^{-i\omega v\cdot x}\left[(1+\not v)\left\{\phi_+^B(\omega) - \frac{\phi_+^B(\omega)-\phi_-^B(\omega)}{2v\cdot x}\not x\right\}\gamma_5\right]_{\beta\alpha}$$

- defined in HQET;

[A.Grozin, M.Neubert (1997); M.Beneke,Th.Feldmann (2001)]

key input parameter: the inverse moment

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty d\omega \frac{\phi_+^B(\omega,\mu)}{\omega}$$

- QCD sum rules in HQET: $\lambda_B(1\text{ GeV}) = 460 \pm 110$ MeV

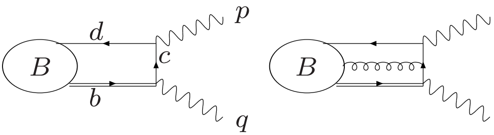
  [V.Braun, D.Ivanov, G.Korchemsky,2004]
- all $B \to \pi, K^{(*)}, \rho$ form factors calculated
- so far the uncertainties are larger than for original LCSR's

Form factors from LCSR with B-meson DA's

form factor	this work	LCSR with light-meson DA's [P.Ball and R.Zwicky(2004),(2005)]
$f^+_{B\pi}(0)$	0.25±0.05	0.258±0.031 (0.28 ± 0.03) [AK, T.Mannel,N.Offen,Y-M.Wang (2011)]
$f^+_{BK}(0)$	0.31±0.04	0.301±0.041±0.008
$f^T_{B\pi}(0)$	0.21±0.04	0.253±0.028
$f^T_{BK}(0)$	0.27±0.04	0.321±0.037±0.009
$V^{B\rho}(0)$	0.32±0.10	0.323±0.029
$V^{BK^*}(0)$	0.39±0.11	0.411±0.033±0.031
$A_1^{B\rho}(0)$	0.24±0.08	0.242±0.024
$A_1^{BK^*}(0)$	0.30±0.08	0.292±0.028±0.023
$A_2^{B\rho}(0)$	0.21±0.09	0.221±0.023
$A_2^{BK^*}(0)$	0.26±0.08	0.259±0.027±0.022
$T_1^{B\rho}(0)$	0.28±0.09	0.267±0.021
$T_1^{BK^*}(0)$	0.33±0.10	0.333±0.028±0.024

LCSR for $B \to D^{(*)}$ form factors

[S.Faller, A.K.,Ch.Klein, Th.Mannel, [hepph]]



(a) (b)

- virtual *c* quark in the correlator with *B*-meson DA
- $B \to D, B \to D^*$ form factors near maximal recoil

(not directly accessible in HQET)

$B \to D$ form factors

$$\frac{\langle D(p)|\bar{c}\gamma_\mu b|\bar{B}(p+q)\rangle}{\sqrt{m_B m_D}} = (v+v')_\mu\, h_+(w) + (v-v')_\mu\, h_-(w)$$

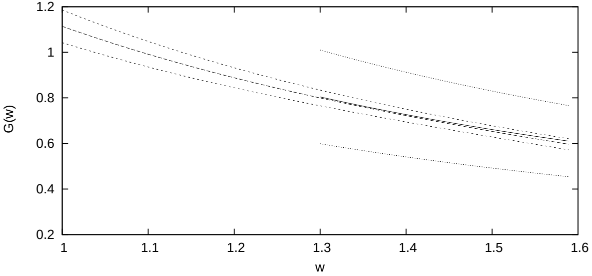
$$w = v\cdot v' = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2\, m_B\, m_{D^{(*)}}}\,,$$

$$\frac{d\Gamma(\bar{B}\to D l\bar{\nu}_l)}{dw} = \frac{G_F^2|V_{cb}|^2}{48\pi^3}(m_B+m_D)^2 m_D^3 (w^2-1)^{3/2}|\mathcal{G}(w)|^2\,.$$

the two form factors $h_\pm$ are combined within a single function:

$$\mathcal{G}(w) = h_+(w) - \frac{1-r}{1+r}\, h_-(w)\,.$$

Result for $B \to D$ form factors

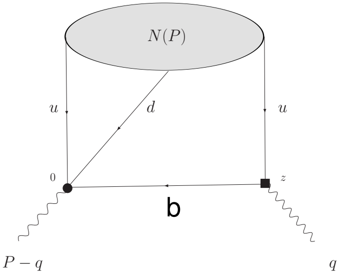


LCSR prediction at $w \sim w_{max}$ compared with BaBar(2008) data fitted to Caprini-Lelloch-Neubert-parametrization

- $B \to D^*$ form factors calculated in the same region

$\Lambda_b \to p$ form factors from LCSR

[A.K., Ch.Klein, Th.Mannel, Y.-M. Wang arXiV:1108.2971]



- vacuum-to-nucleon correlation function:

$$\Pi_{\mu(5)}(P,q) = i\int d^4z\, e^{iq\cdot z}\langle 0|T\left\{\eta_{\Lambda_b}(0), \bar{b}(z)\gamma_\mu(\gamma_5)u(z)\right\}|N(P)\rangle\,.$$

- $q^2 \ll m_b^2$, $(P-q)^2 \ll m_b^2$, $P^2 = m_N^2$,
- Λ_b interpolating 3-quark current, we use

$$\eta^{(\mathcal{P})}_{\Lambda_b} = (u\,C\,\gamma_5\,d)\,b,\ \ \eta^{(\mathcal{A})}_{\Lambda_b} = (u\,C\,\gamma_5\gamma_\lambda\,d)\,\gamma^\lambda\,b\,.$$

Nucleon Distribution Amplitudes (DA's)

[V.Braun, A.Lenz et al (2000-2009)],

- definition, schematically ($z^2 \sim 0$):

$$\langle 0|\epsilon^{ijk} u^i_\alpha(0) u^j_\beta(z) d^k_\gamma(0)|N(P)\rangle = \sum_t \mathcal{S}^t_{\alpha\beta\gamma}$$

$$\times \int dx_1 dx_2 dx_3 \delta(1 - \sum_{i=1}^{3} x_i) e^{-ix_2 P\cdot z} F_t(x_i,\mu)\,,$$

- twist expansion: 27 DA's of twist 3,4,5,6
- coefficients and normalization parameters determined from 2-point sum rules
- proton e.m. form factors were calculated from LCSR

Accessing the $\Lambda_b \to p$ form factors

- hadronic dispersion relation, schematically

$$\Pi_{\mu(5)}(P,q) = \frac{\langle 0|\eta_{\Lambda_b}|\Lambda_b\rangle\langle\Lambda_b|\bar{b}\gamma_\mu(\gamma_5)u|N\rangle}{m_{\Lambda_b}^2 - (P-q)^2}$$

$$+\frac{\langle 0|\eta_{\Lambda_b}|\Lambda_b^*\rangle\langle\Lambda_b^*|\bar{b}\gamma_\mu(\gamma_5)u|N\rangle}{m_{\Lambda_b^*}^2 - (P-q)^2} + \int\limits_{s_0^h}^{\infty} ds\, \frac{\rho_{\mu(5)}(s,q^2)}{s-(P-q)^2}$$

- 6 form factors, standard definitions (cf nucleon β decay):

$$\langle\Lambda_b(P-q)|\bar{b}\gamma_\mu\, u|N(P)\rangle = \bar{u}_{\Lambda_b}(P-q)\left\{ f_1(q^2)\,\gamma_\mu + i\frac{f_2(q^2)}{m_{\Lambda_b}}\,\sigma_{\mu\nu}q^\nu + \frac{f_3(q^2)}{m_{\Lambda_b}}\,q_\mu \right\} u_N(P)\,,$$

$$0 \le q^2 \le (m_{\Lambda_b}^2 - m_N^2)\,, \quad \gamma_\mu \to \gamma_\mu\gamma_5\,,\; f_i(q^2) \to g_i(q^2)$$

- decay constant of Λ_b from two-point sum rules

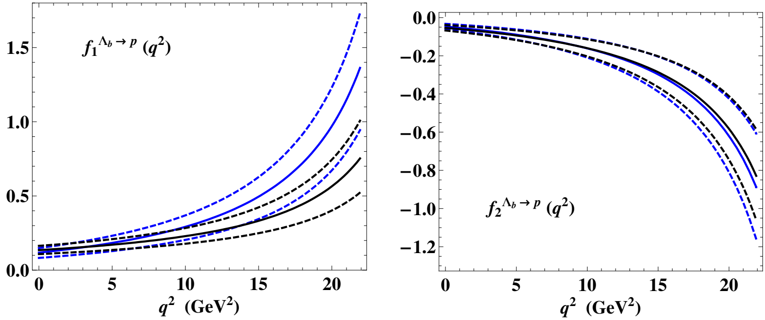
LCSR in detail

- specific problems for baryon QCD sum rules
  - the contributions of Λ_b^* , ($J^P = 1/2^-$ state,
  $m_{\Lambda_b^*} - m_{\Lambda_b} \sim 200 - 300$ MeV
  we used linear combinations of kinematical structures in the correlation function to eliminate Λ_b^*

  - baryon interpolating current: multiple choice
  we used pseudoscalar and axial currents
- replace b by c in LCSR $\Rightarrow \Lambda_c \to N$ form factors
  (used to calculate strong couplings)
- inputs:
  finite m_b , a few universal parameters of nucleon DA's ,
  two-point sum rules for η_{Λ_b} currents:

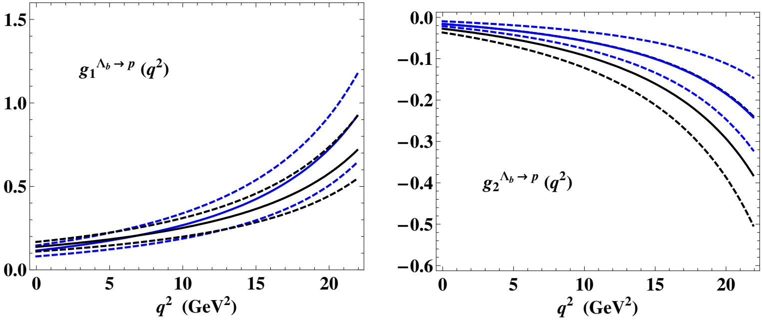
$$\lambda_{\Lambda_b}^{(\mathcal{A})} = 1.27^{+0.35}_{-0.34} \times 10^{-2}\ \mathrm{GeV}^2\,,\ \ \lambda_{\Lambda_b}^{(\mathcal{P})} = 1.09^{+0.31}_{-0.30} \times 10^{-2}\ \mathrm{GeV}^2\,,$$

Numerical results for the $\Lambda_b \to p$ vector form factors



- $q^2 \leq 11$ GeV^2 direct calculation from LCSR, at larger q^2 z-parameterization and extrapolation
- reasonable agreement between sum rules with different baryon currents

Numerical results for the axial-vector $\Lambda_b \to p$ form factors

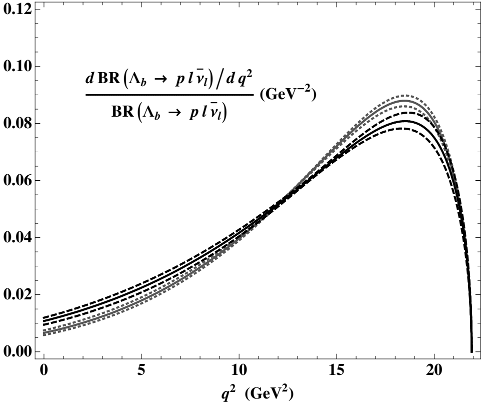


The width of $\Lambda_b \to p\ell\nu_\ell$ decay

- can be used to extract $|V_{ub}|$

$$\frac{d\Gamma}{dq^2}(\Lambda_b \to pl\nu_l) = \frac{G_F^2 m_{\Lambda_b}^3}{192\pi^3}|V_{ub}|^2\left\{k_1(q^2, m_{\Lambda_b}, m_N)|f_1(q^2)|^2 + ...\right\}$$

- normalized q^2-spectrum



The width of $\Lambda_b \to p\ell\nu_\ell$ decay

- partially integrated width: pure prediction of LCSR

$$\Delta\zeta(0, q^2_{max}) = \frac{1}{|V_{ub}|^2} \int_0^{q^2_{max}} dq^2 \, \frac{d\Gamma}{dq^2}(\Lambda_b \to p l \nu_l)$$
$$= 5.5^{+2.5}_{-2.0} \,\mathrm{ps}^{-1} \left(= 5.6^{+3.2}_{-2.9} \,\mathrm{ps}^{-1} \right)$$

  for axial-vector (pseudoscalar) interpolating current of Λ_b

- improvements in the future possible: nucleon DA parameters, α_s corrections

How accurate are QCD sum rules

- two main sources of uncertainties:

  (I) OPE truncated, inputs uncertain

- a reasonable accuracy achieved in 2-point correlators, due to progress in multiloop calculations,

- α_s, quark masses, quark/gluon condensates, DA's: accuracy slowly improving

- in LCSR's: only NLO $t \leq 4$ available, twist expansion demands additional studies, *B* meson DA's not sufficiently well studied yet

(II) hadronic sum approximated with quark-hadron duality

- not easy to estimate the "systematic" error related with the effective threshold s_0:
fixing s_0 by adjusting the hadron mass

- a better solution: experimental information on excited states $\Rightarrow$ the hadronic spectral function

- theoretical information on the spectrum
(string-like hadronic models)

- the accuracy of lattice QCD calculation already in the nearest future cannot be achieved by QCD sum rules
- but: there are hadronic matrix elements where even a 30-40% accuracy would be sufficient, and they are not accessible on the lattice

stay tuned for the last lecture