Extracting $B \to K^*$ Form Factors from Data

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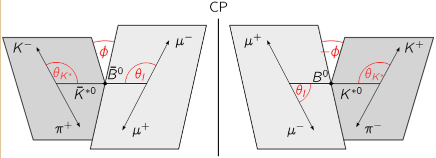
Dubna Physics of Heavy Quarks and Hadrons

in collaboration with:
G. Hiller, S. Schacht, R. Zwicky

- Overview & Motivation
- Framework
- Results

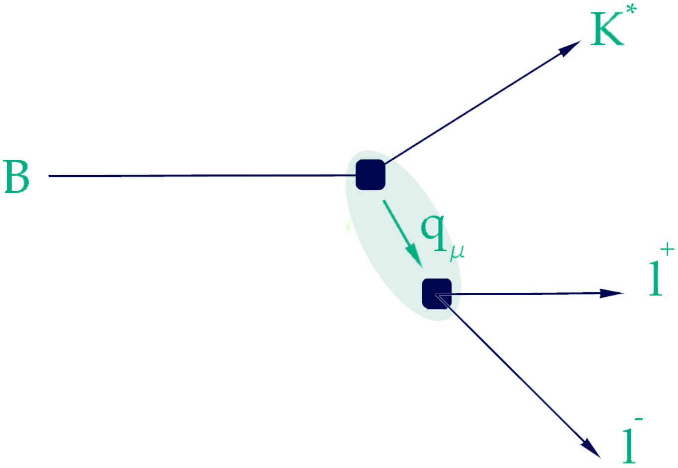
Overview & Motivation

$B \to K^* \mu^+ \mu^-$ [Slide from Davids lecture]

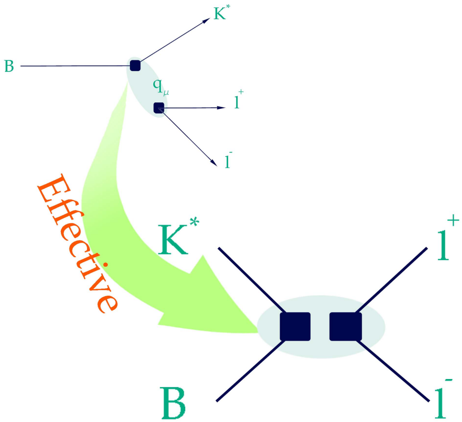


- 4-body decay: angular distribution with many observables sensitive to NP
- "self-tagging": sensitive to CP violation

FCNC B decays

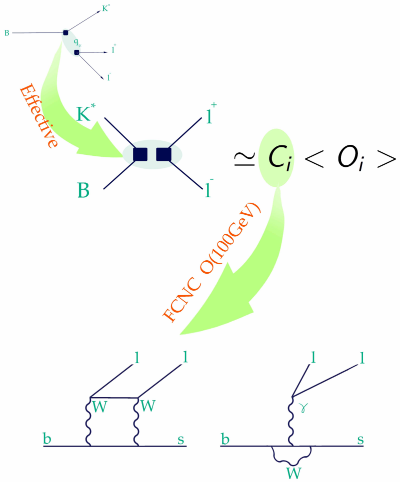


FCNC B decays [see lectures of Ahmed Ali & Andrey Grozin]



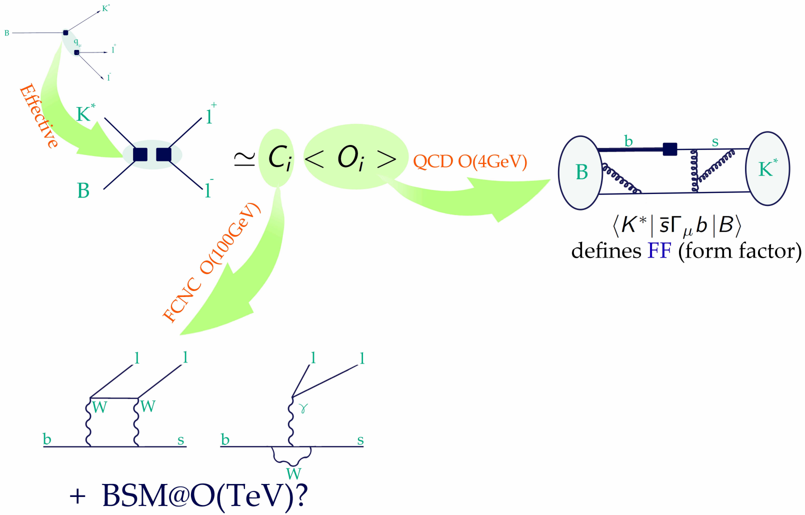
$$\simeq C_i < O_i >$$

FCNC B decays [see lectures of Ahmed Ali & Andrey Grozin]



+ BSM@O(TeV)?

FCNC B decays [see lectures of Ahmed Ali & Andrey Grozin]



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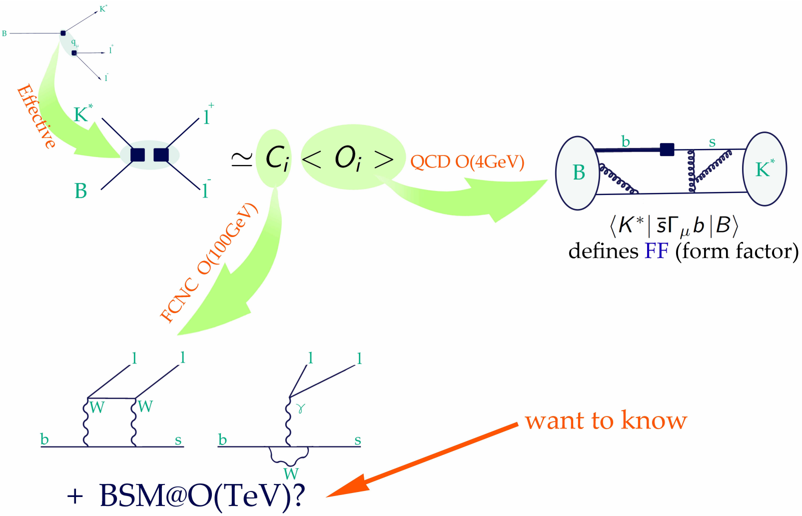

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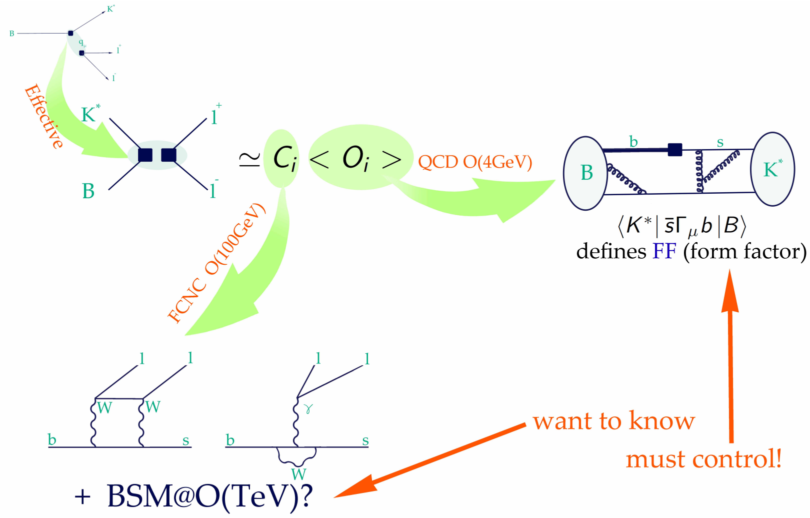
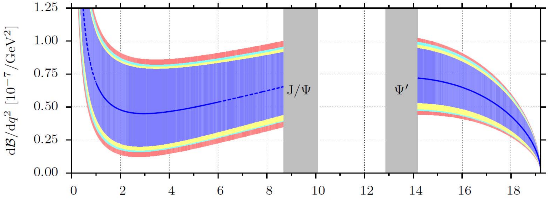


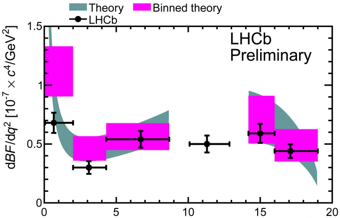
Motivation

Typical error budget ($BR(B \to K^* \mu^+ \mu^-)$):

• FF (form factor) • CKM • SD (short distance) • SL ($1/m_b$)



[Bobeth, Hiller. Dyk. JHEP 1007, (2010), 098.]



[LHCb-CONF-2012-008]

Theory determinations:

- LCSR (expansion in inverse powerse of light meson energy, $q^2 \lesssim 14\ \mathrm{GeV}^2$)
- Lattice (small hadronic momenta, control of discrete lattice $q^2 \gtrsim 14\ \mathrm{GeV}^2$)

Complementary & errors still large $\Rightarrow$ Independent knowledge wanted!

Method

(extracting FF from data)

FF Definition

Definition via currents, expansion in 3 Lorentz vectors ($\epsilon, k, p = k + q$):

$$
\begin{aligned}
\langle K^*(k,\epsilon)|\,\bar{s}\gamma_\mu b\,|B(p)\rangle &= \frac{2\,V}{m_B + m_{K^*}}\varepsilon_{\mu\rho\sigma\tau}\epsilon^{*\rho}p^\sigma k^\tau \\
\langle K^*(k,\epsilon)|\,\bar{s}\gamma_\mu\gamma_5 b\,|B(p)\rangle &= i\epsilon^{*\rho}\left[2A_0\,m_{K^*}\frac{q_\mu q_\rho}{q^2} + A_1\,(m_B + m_{K^*})(g_{\mu\rho} - \frac{q_\mu q_\rho}{q^2})\right. \\
&\qquad \left. - A_2\,q_\rho\left(\frac{(p+k)_\mu}{m_B + m_{K^*}} - \frac{m_B - m_{K^*}}{q^2}(p-k)_\mu\right)\right]
\end{aligned}
$$

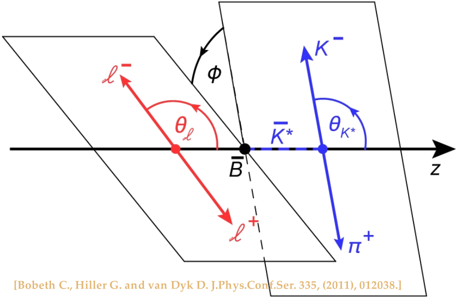
Redefinition: Projection on polarization of K^*:

$$
f_\perp = \mathcal{N}\frac{\sqrt{2\hat{s}\hat{\lambda}}}{1 + \hat{m}_{K^*}}V, \quad f_\parallel = \mathcal{N}\sqrt{2\hat{s}}\,(1 + \hat{m}_{K^*})\,A_1
$$

$$
f_0 = \mathcal{N}\frac{(1 - \hat{s} - \hat{m}_{K^*}^2)(1 + \hat{m}_{K^*})^2 A_1 - \hat{\lambda}\,A_2}{2\,\hat{m}_{K^*}(1 + \hat{m}_{K^*})}
$$

Full angular analysis Part 1

$$\frac{\mathrm{d}^4\Gamma}{\mathrm{d}q^2\mathrm{d}\cos\theta_l\mathrm{d}\cos\theta_{K^*}\mathrm{d}\phi} = \frac{3}{8\pi}J(q^2,\cos\theta_l,\cos\theta_{K^*},\phi)$$

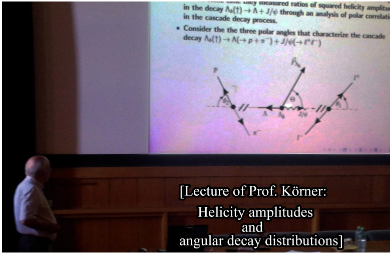


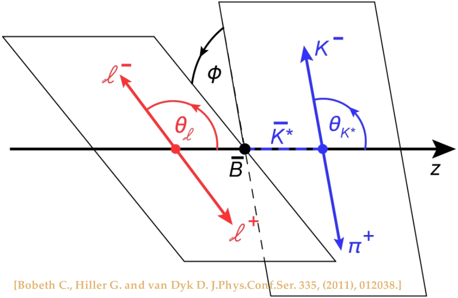
[Bobeth C., Hiller G. and van Dyk D. J.Phys.Conf.Ser. 335, (2011), 012038.]

$$\begin{aligned}J(q^2,\theta_l,\theta_{K^*},\phi) = {} & J_1^s\sin^2\theta_{K^*} + J_1^c\cos^2\theta_{K^*} + (J_2^s\sin^2\theta_{K^*} + J_2^c\cos^2\theta_{K^*})\cos 2\theta_l \\ & + J_3\sin^2\theta_{K^*}\sin^2\theta_l\cos 2\phi + J_4\sin 2\theta_{K^*}\sin 2\theta_l\cos\phi \\ & + J_5\sin 2\theta_{K^*}\sin\theta_l\cos\phi + J_6\sin^2\theta_{K^*}\cos\theta_l + J_7\sin 2\theta_{K^*}\sin\theta_l\sin\phi \\ & + J_8\sin 2\theta_{K^*}\sin 2\theta_l\sin\phi + J_9\sin^2\theta_{K^*}\sin^2\theta_l\sin 2\phi\end{aligned}$$

Observables $J_i = J_i(q^2)$ (angular coefficients)
see [Kruger, Sehgal, Sinha, Sinha, 00], [Kruger, Matias, 05], [Bobeth, Hiller, Dyk, 11]

Full angular analysis Part 1





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& + J_3 \sin^2\theta_{K^*} \sin^2\theta_l \cos 2\phi + J_4 \sin 2\theta_{K^*} \sin 2\theta_l \cos\phi \\
& + J_5 \sin 2\theta_{K^*} \sin\theta_l \cos\phi + J_6 \sin^2\theta_{K^*} \cos\theta_l + J_7 \sin 2\theta_{K^*} \sin\theta_l \sin\phi \\
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Full angular analysis Part 2

In terms of transversity amplitudes ($\simeq$ projection amplitude on l^+l^- spin):

$$J_1^s = \frac{3}{4}\Big\{\frac{(2+\beta_l^2)}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + (L \to R)\right] + \frac{4m_l^2}{q^2}\mathrm{Re}\left(A_\perp^L A_\perp^{R\,*} + A_\parallel^L A_\parallel^{R*}\right)\Big\},$$

$$J_1^c = \frac{3}{4}\Big\{|A_0^L|^2 + |A_0^R|^2 + \frac{4m_l^2}{q^2}\left[|A_t|^2 + 2\mathrm{Re}(A_0^L A_0^{R\,*})\right]\Big\},$$

$$J_2^s = \frac{3\beta_l^2}{16}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + (L \to R)\right],$$

$$J_2^c = -\frac{3\beta_l^2}{4}\left[|A_0^L|^2 + (L \to R)\right], \quad J_3 = \frac{3}{8}\beta_l^2\left[|A_\perp^L|^2 - |A_\parallel^L|^2 + (L \to R)\right],$$

$$J_4 = \frac{3}{4\sqrt{2}}\beta_l^2\left[\mathrm{Re}(A_0^L A_\parallel^{L\,*}) + (L \to R)\right], \quad J_5 = \frac{3\sqrt{2}}{4}\beta_l\left[\mathrm{Re}(A_0^L A_\perp^{L\,*}) - (L \to R)\right],$$

$$J_6 = \frac{3}{2}\beta_l\left[\mathrm{Re}(A_\parallel^L A_\perp^{L\,*}) - (L \to R)\right], \quad J_7 = \frac{3\sqrt{2}}{4}\beta_l\left[\mathrm{Im}(A_0^L A_\parallel^{L\,*}) - (L \to R)\right],$$

$$J_8 = \frac{3}{4\sqrt{2}}\beta_l^2\left[\mathrm{Im}(A_0^L A_\perp^{L\,*}) + (L \to R)\right], \quad J_9 = \frac{3}{4}\beta_l^2\left[\mathrm{Im}(A_\parallel^{L\,*} A_\perp^L) + (L \to R)\right]$$

and

$$\beta_l = \sqrt{1 - \frac{4m_l^2}{q^2}} \qquad (1)$$

Full angular analysis Part 3

Constructing observables to use data efficiently ([LHCb 2013] 883 ± 34 events):

- partial width in q^2:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = 2J_1^s + J_1^c - \frac{2J_2^s + J_2^c}{3} = |A_0^L|^2 + |A_\perp^L|^2 + |A_\parallel^L|^2 + (L \leftrightarrow R)$$

- Longitudinal polarized K^* fraction:

$$F_\mathrm{L} = \frac{J_1^c - \frac{1}{3}J_3^c}{\mathrm{d}\Gamma/\mathrm{d}q^2} = \frac{|A_0^L|^2 + |A_0^R|^2}{\mathrm{d}\Gamma/\mathrm{d}q^2}$$

- Transverse asymmetrie $A_T^{(2)}$:

$$A_T^{(2)} = \frac{1}{2}\frac{J_3}{J_2^s} = \frac{|A_\perp^L|^2 + |A_\perp^R|^2 - |A_\parallel^L|^2 - |A_\parallel^R|^2}{|A_\perp^L|^2 + |A_\perp^R|^2 + |A_\parallel^L|^2 + |A_\parallel^R|^2}$$

... and many others

Short distance-free observables

Observables through full angular analysis (BR in transversity amplitudes $A_i^{L/R}$)

$$\mathrm{d}\Gamma/\mathrm{d}q^2 = |A_0^L|^2 + |A_\perp^L|^2 + |A_\parallel^L|^2 + (L \leftrightarrow R)$$

Fraction of longitudinal K^*'s & transverse asymmetry [Kruger, Matias, 05]:

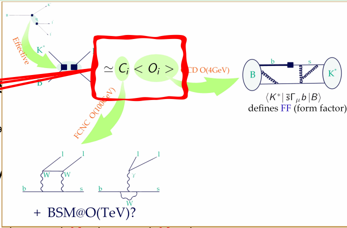
$$F_L = \frac{|A_0^L|^2 + |A_0^R|^2}{\mathrm{d}\Gamma/\mathrm{d}q^2} \qquad A_T^{(2)} = \frac{|A_\perp^L|^2 + |A_\perp^L|^2 - |A_\parallel^L|^2 - |A_\parallel^R|^2}{|A_\perp^L|^2 + |A_\perp^L|^2 + |A_\parallel^L|^2 + |A_\parallel^R|^2}$$

Short distance-free observables

Observables through full angular analysis (BR in trans

$$\mathrm{d}\Gamma/\mathrm{d}q^2 = |A_0^L|^2 + |A_\perp^L|^2 + |A_\parallel^L|^2 + (L \leftrightarrow$$

Fraction of longitudinal K^*'s & transverse asymmetry



$$F_L = \frac{|A_0^L|^2 + |A_0^R|^2}{\mathrm{d}\Gamma/\mathrm{d}q^2} \qquad A_T^{(2)} = \frac{|A_\perp^L|^2 + |A_\perp^L|^2 - |A_\parallel|^2 - |A_\parallel|^2}{|A_\perp^L|^2 + |A_\perp^L|^2 + |A_\parallel^L|^2 + |A_\parallel^R|^2}$$

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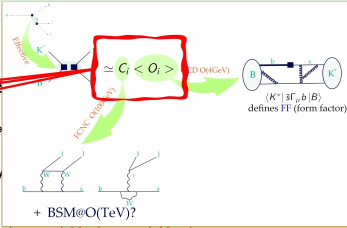

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$$A_i^{L,R} = \pm i \left\{ \mathcal{C}_9^{\text{eff}} \mp C_{10} + \frac{2 m_b m_B}{q^2} \mathcal{C}_7^{\text{eff}} \right\} f_i, \qquad i = \perp, \parallel, 0$$

calculation [Bobeth, Hiller, Dyk, 10]

- HQET LO $1/m_b$
- $q^2 \sim \mathcal{O}(m_b^2)$

$$F_L = \frac{\cancel{\rho_{SD}(q^2)} f_0{}^2}{\cancel{\rho_{SD}(q^2)} (f_0{}^2 + f_\perp{}^2 + f_\parallel{}^2)} \qquad A_T^{(2)} = \frac{\cancel{\rho_{SD}(q^2)} (f_\perp{}^2 - f_\parallel{}^2)}{\cancel{\rho_{SD}(q^2)} (f_\perp{}^2 + f_\parallel{}^2)}$$

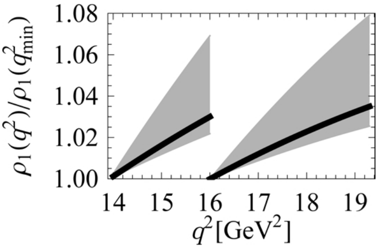
locally Short Distance drops out!

Short distance control

Experiment binned$\Rightarrow$ BSM through SD physics in binning:

$$F_L = \frac{\int_{\text{bin}} \mathrm{d}q^2 \rho_{\text{SD}}(q^2) f_0{}^2}{\int_{\text{bin}} \mathrm{d}q^2 \rho_{\text{SD}}(q^2) \left(f_0{}^2 + f_\perp{}^2 + f_\parallel{}^2\right)} \qquad (A_T(2) \text{ same})$$

$$\rho_{\text{SD}}(q^2) = \left| \mathcal{C}_9^{\text{eff}} + \frac{2 m_b m_B}{q^2} \mathcal{C}_7^{\text{eff}} \right|^2 + |\mathcal{C}_{10}|^2$$



Variation (conservative):

$-0.4 < C_7 < -0.3$ & $2 < |\mathcal{C}_{10}| < 5$

$\Rightarrow$ few % from BSM

Parametrization of FF

Parametrization of FF in terms of fit parameters $\Rightarrow$ Series Expansion (SE)
(maximize theoretical input $\Leftrightarrow$ minimize fit parameters)

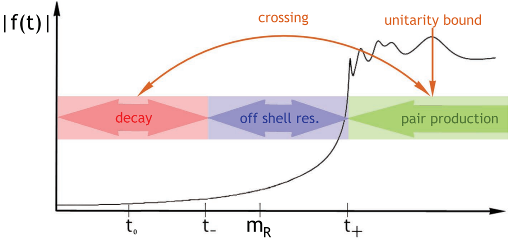
- analytic continuation of $q^2 \to t$
- crossing
- unitarity bound
- resonances coupling to currents

$$f_i(t) = \Theta_i(t, m_R) \sum_k \boxed{\alpha_{i,k}}\, z^k(t)$$

$$z(t) \equiv \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

$$t_\pm = (m_B \pm m_{K^*})^2$$

$0 \le t_0 < t_+$ mapping point



[Boyd, Grinstein, Lebed, 95], [Boyd, Savage, 97], [Caprini, Lellouch, Neubert 98], [Becher, Hill, 06] , [Bourrely, Caprini, Lellouch, 09], [Bharucha, Feldmann, Wick 10]

We use SE @ LO:

$$f_\perp = \boxed{\alpha_{\perp,0}}\, \Lambda(t, m_{1^-}^2)\sqrt{-z(t,0)}\sqrt{z(t,t_-)} \qquad f_\parallel = \boxed{\alpha_{\parallel,0}}\, \Lambda(t, m_{1^+}^2)\sqrt{-z(t,0)}$$

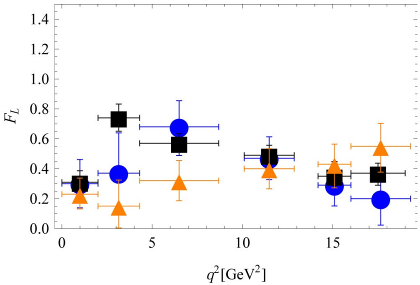
$$f_0 = \boxed{\alpha_{0,0}}\, \Lambda(t, m_{1^+}^2)$$

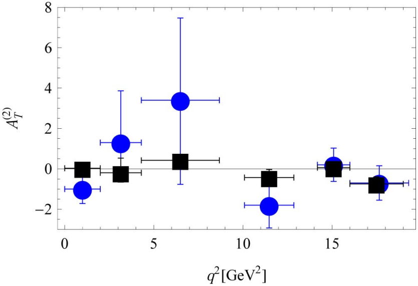
(for $\Theta_i(t, m_R)$ & $\Lambda(t, m_R^2)$), see [CH, Hiller, PRL 12]

Data on F_L & $A_T^{(2)}$

Recent data:

■ LHCb [LHCb, 12]
● CDF [CDF, 12]
▲ BaBar [BaBar, S. Akar, 12]





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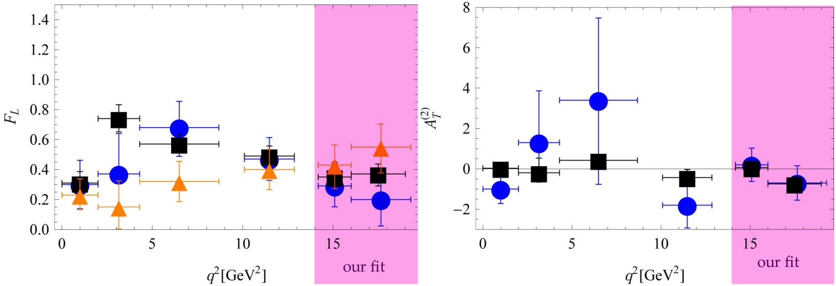
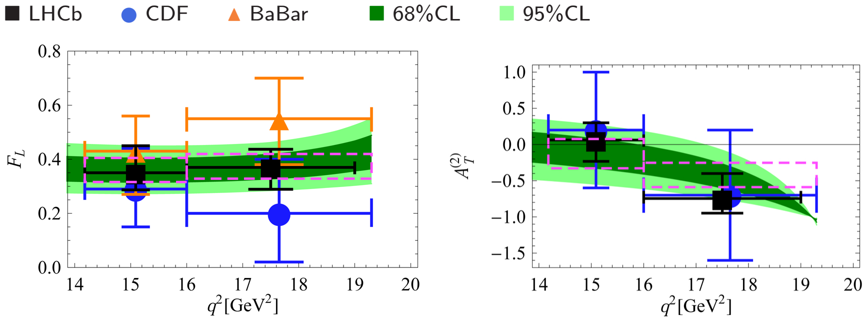


- fit only $q^2 \gtrsim 14$ GeV^2

Results
(χ^2 fit)

- 1st order SE
- 2nd order SE (in preparation)

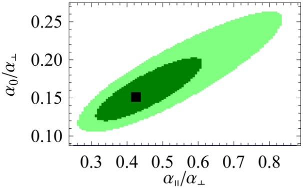
Numerical Results



- simultaneous fit $\chi^2_{\rm min} = 3.8$
- Only ratios $\Rightarrow$ 2 fit parameters:

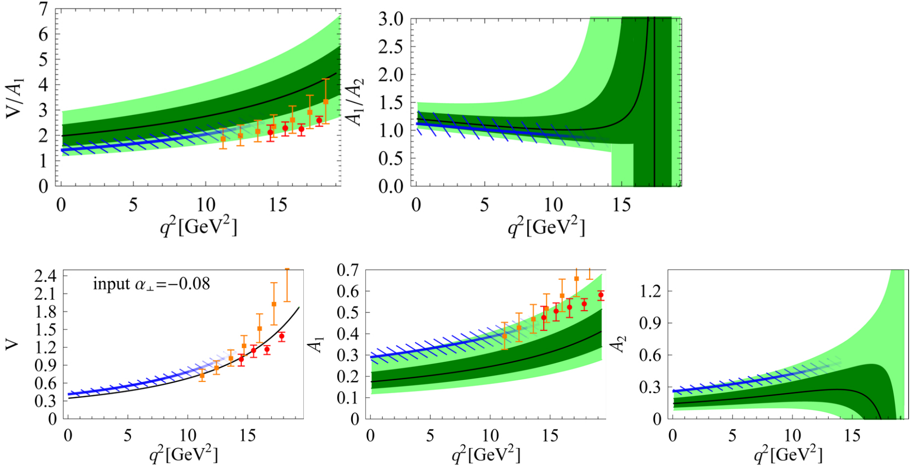
$$\alpha_\parallel/\alpha_\perp = 0.43^{+0.11}_{-0.08} \qquad \alpha_0/\alpha_\perp = 0.15^{+0.03}_{-0.02}$$

[CH, Hiller, PRL 12]



- LHCb dominates fit

Fitted Distributions



LCSR: [Ball, Zwicky, 05]

Lattice: ● [Liu, Meinel, Hart, Horgan, Muller, Wingate, 10]

■ [Becirevic, Lubicz, Mescia, 07]

Next step - new input

[CH, G. Hiller, S. Schacht, R. Zwicky, in preparation]

- 2nd order SE (needs more input at $q^2 = 0$)
- new data [Atlas, 13] ● & [CMS, 13] ●
- include theory constraints at $q^2 = 0$ for V/A_1 (rfit scheme):
  - none
  - LEL (large energy limit, $m_b \ll \Lambda_{QCD}$):

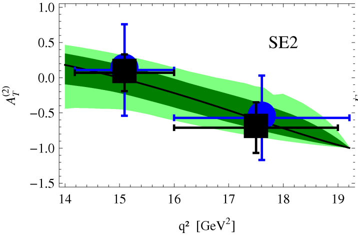
$$\left.\frac{V(q^2)}{A_1(q^2)}\right|_{q^2=0} \approx \left.\frac{(m_B + m_{K^*})^2}{2m_B E_{K^*}}\right|_{E_{K^*}=\frac{(m_B+m_{K^*})^2}{2m_B}} = 1.33 \pm 0.40$$

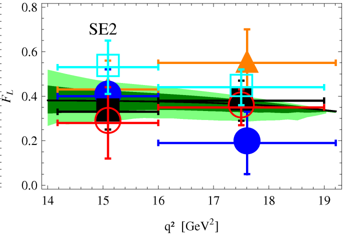
  - LCSR

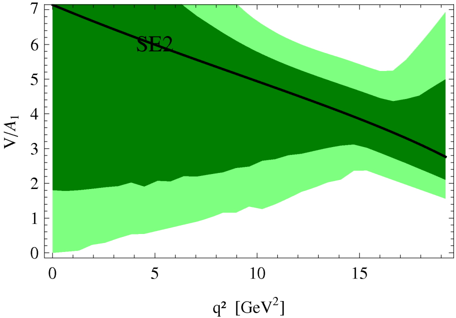
$$\left.\frac{V(q^2)}{A_1(q^2)}\right|_{q^2=0} = 1.31 \pm 0.10 \qquad \left.\frac{A_2(q^2)}{A_1(q^2)}\right|_{q^2=0} = 0.83 \pm 0.08$$

  - all input

Next step (preliminary)







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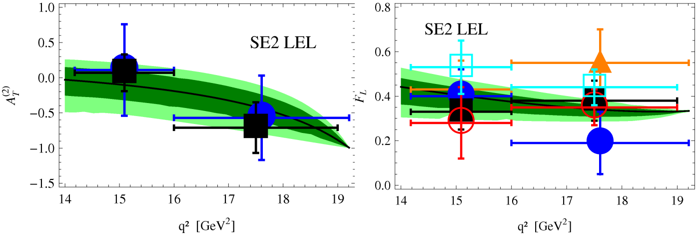

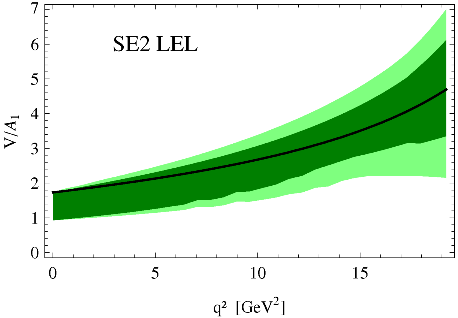

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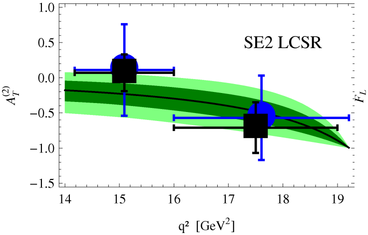
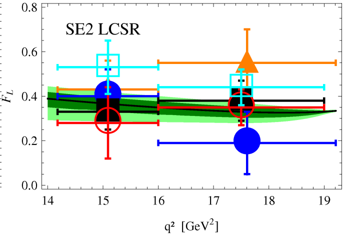




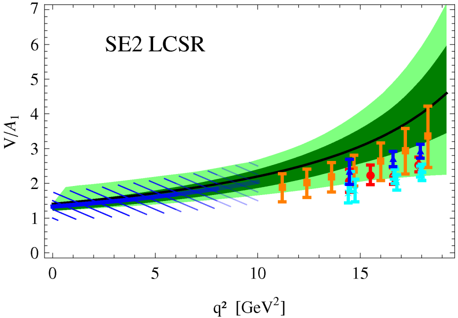
LEL:

Lattice:

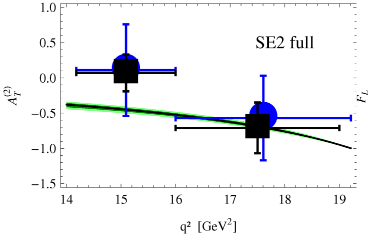
● [Liu, Meinel, Hart, Horgan, Muller, Wingate, 10]

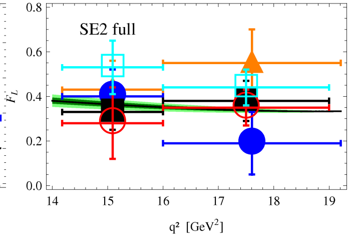
■ [Becirevic, Lubicz, Mescia, 07]

● (● est. from T-FF) [Wingate preliminary]



Next step (preliminary)





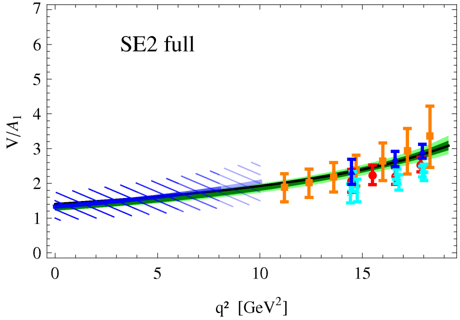
LEL:

Lattice:

[Liu, Meinel, Hart, Horgan, Muller, Wingate, 10]

[Becirevic, Lubicz, Mescia, 07]

(est. from T-FF)
[Wingate preliminary]



Conclusions

- few d.o.f.-fit to first-time available data works surprisingly well
- overall agreement with LCSR & lattice

Outlook

- better data from LHC $\Rightarrow$ more bins $\Rightarrow$ better control
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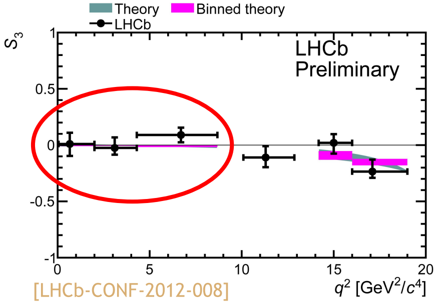
Thank You!

About right-handed currents

Low q^2 region experimentally consistent with 0
$\Longrightarrow$ right-handed currents from BSM negligible.



[LHCb-CONF-2012-008]