

# The multiquark states in the b-quark sector - II/II

Christian Hambrock

TU Dortmund

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**Dubna 2013**

# Exotica

Belle & others [Zupanc et al. 09] , [Bondar et al., PRL 12] , [Liu et al., 13] , [Ablikim et al., 13]

State	$M$ (MeV)	$\Gamma$ (MeV)	$J^{PC}$	Decay Modes	Production Modes	Also observed by	date
$Y_s(2175)$	$2175 \pm 8$	$58 \pm 26$	$1^{--}$	$\phi f_0(980)$	$e^+e^-$ (ISR) $J/\psi \rightarrow \eta Y_s(2175)$	BaBar*, BESII	2006
$X(3872)$	$3871.4 \pm 0.6$	$< 2.3$	$1^{++}$	$\pi^+\pi^-J/\psi$ , $\gamma J/\psi, DD^*$	$B \rightarrow KX(3872), p\bar{p}$	BaBar	2003
$Z(3900)$	$3899 \pm 6$	$46 \pm 22$	$1^+$	$\pi^\pm J/\psi$	$Z(4260) \rightarrow Z(3900)\pi$	BESIII*	2013
$X(3915)$	$3914 \pm 4$	$28^{+12}_{-14}$	$0/2^{++}$	$\omega J/\psi$	$\gamma\gamma \rightarrow X(3915)$		2009
$Z(3930)$	$3929 \pm 5$	$29 \pm 10$	$2^{++}$	$D\bar{D}$ $DD^*$ (not $D\bar{D}$ )	$\gamma\gamma \rightarrow Z(3940)$		2009
$X(3940)$	$3942 \pm 9$	$37 \pm 17$	$0^{?+}$	or $\omega J/\psi$	$e^+e^- \rightarrow J/\psi X(3940)$		2005
$Y(3940)$	$3943 \pm 17$	$87 \pm 34$	$?^{?+}$	$\omega J/\psi$ (not $DD^*$ )	$B \rightarrow KY(3940)$	BaBar	2005
$Y(4008)$	$4008^{+82}_{-49}$	$226^{+97}_{-80}$	$1^{--}$	$\pi^+\pi^-J/\psi$	$e^+e^-$ (ISR)		2005
$Z(4020)$	$4022 \pm 3$	$6 \pm 4$	$1^+$	$\pi^\pm J/\psi$	$Z(4260) \rightarrow Z(4020)\pi$	BESIII* (only)	2013
$Z(4025)$	$4026 \pm 5$	$25 \pm 10$	$1^+$	$\pi^\pm J/\psi$	$Z(4260) \rightarrow Z(4025)\pi$	BESIII* (only)	2013
$X(4160)$	$4156 \pm 29$	$139^{+113}_{-65}$	$0^{?+}$	$D^*\bar{D}^*$ (not $D\bar{D}$ )	$e^+e^- \rightarrow J/\psi X(4160)$		2008
$Y(4260)$	$4264 \pm 12$	$83 \pm 22$	$1^{--}$	$\pi^+\pi^-J/\psi$	$e^+e^-$ (ISR)	BaBar*, CLEO	2005
$Y(4350)$	$4361 \pm 13$	$74 \pm 18$	$1^{--}$	$\pi^+\pi^-\psi'$	$e^+e^-$ (ISR)	BaBar*	2007
$X(4630)$	$4634^{+9}_{-11}$	$92^{+41}_{-32}$	$1^{--}$	$\Lambda_c^+\Lambda_c^-$	$e^+e^-$ (ISR)		2008
$Y(4660)$	$4664 \pm 12$	$48 \pm 15$	$1^{--}$	$\pi^+\pi^-\psi'$	$e^+e^-$ (ISR)		2007
$Z(4050)$	$4051^{+24}_{-23}$	$82^{+51}_{-29}$	?	$\pi^\pm \chi_{c1}$	$B \rightarrow KZ^\pm(4050)$		2008
$Z(4250)$	$4248^{+185}_{-45}$	$177^{+320}_{-72}$	?	$\pi^\pm \chi_{c1}$	$B \rightarrow KZ^\pm(4250)$		2008
$Z(4430)$	$4433 \pm 5$	$45^{+35}_{-18}$	?	$\pi^\pm \psi'$	$B \rightarrow KZ^\pm(4430)$		2007
$Z_b(10610)$	$10,607 \pm 2$	$18.4 \pm 2.4$	$1^+$	$\pi^\pm h_b(1,2P), \pi^\pm Y(1,2,3S)$	$Y_b/Y(5S) \rightarrow Z_b(10610)\pi$		2011
$Z_b(10650)$	$10,652 \pm 2$	$11.5 \pm 2.2$	$1^+$	$\pi^\pm h_b(1,2P), \pi^\pm Y(1,2,3S)$	$Y_b/Y(5S) \rightarrow Z_b(10650)\pi$		2011
$Y_b(10890)$	$10,890 \pm 3$	$55 \pm 9$	$1^{--}$	$\pi^+\pi^-Y(1,2,3S)$	$e^+e^- \rightarrow Y_b$		2008

Light states [PDG] :

$a_0(980)$  in 1965,  $\sigma(600)$  now  $^{500}$  in 1972,  $f_0(980)$  in 1979,  $\kappa(980)$  in 1997  
discussion reopened: [’t Hooft, Isidori, Maiani, Polosa, Riquer, PLB 08]

# Exotica

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$Z_b(10610)$	$10,657 \pm 2$	$18.4 \pm 2.4$	$1^+$	$\pi^\pm h_b(1,2P)$			
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$Y_b(10890)$	$10,890 \pm 3$	$55 \pm 9$	$1^{--}$	$\pi^+\pi^-\Upsilon(1,2)$			

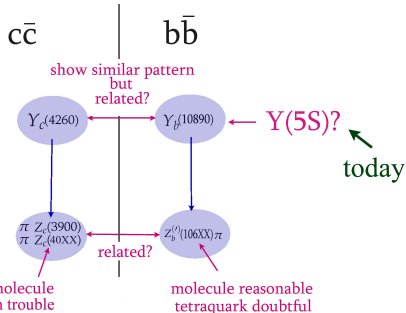
focus here:

$Z_c(3900, 4020, 4025)$

$Y_c(4260)$

$Z_b(106XX)$

$Y_b(10890)$



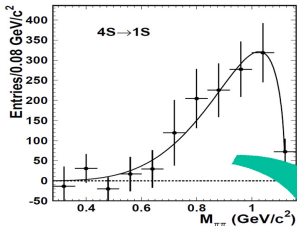
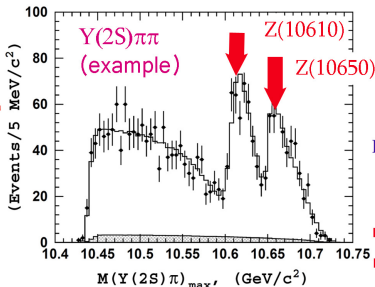
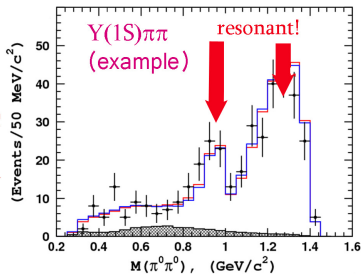
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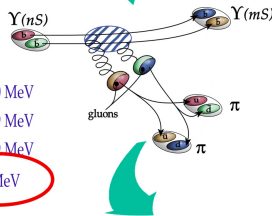
# Why is the data @Y(5S) anomalous?

[Belle Collaboration (2012)]



theory works well (multipole exp.) [Brown, Cahn PRL 75] [Voloshin, JETP 75]

Process:



$$\Gamma(Y(2S) \rightarrow Y(1S)\pi\pi) \approx 0.0060 \text{ MeV}$$

$$\Gamma(Y(3S) \rightarrow Y(1S)\pi\pi) \approx 0.0009 \text{ MeV}$$

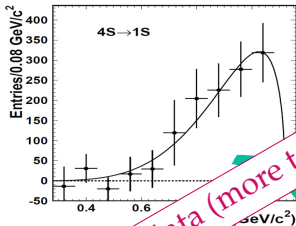
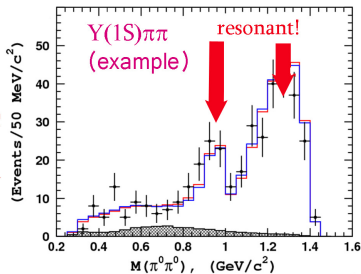
$$\Gamma(Y(4S) \rightarrow Y(1S)\pi\pi) \approx 0.0019 \text{ MeV}$$

$$\Gamma(Y(5S) \rightarrow Y(1S)\pi^+\pi^-) \approx 0.59 \text{ MeV}$$

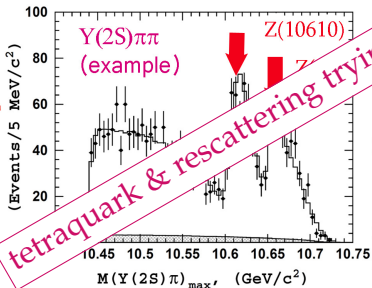
- distinct resonant structure
- differs by two orders of Magnitude!
- NO resonant structure
- Zweig forbidden

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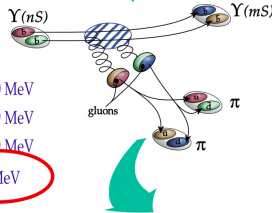
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tetraquark & rescattering trying to explain data (more today)

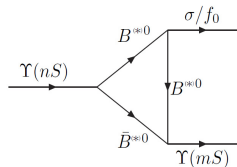
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# Tetraquark vs. Rescattering

- **Tetraquark & Rescattering:** Two rivals in explaining Belle enigma

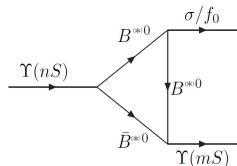
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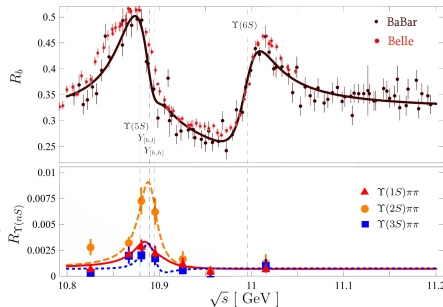
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Rough estimate in Rescattering [Meng, Chao, PRD 08]:

$$\Gamma_{\Upsilon(6S) \rightarrow \Upsilon(1,2,3S)\pi\pi} \gtrsim \Gamma_{\Upsilon(5S) \rightarrow \Upsilon(1,2,3S)\pi\pi}$$

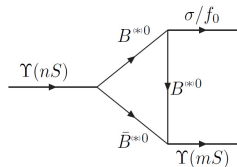
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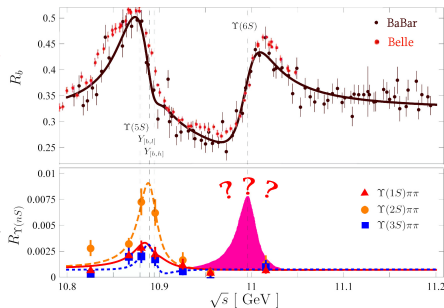
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# Establishing exoticness of $\Upsilon_b$ : Plan A

(isospin)

# Plan A

- [Ali, CH, Aslam, PRL 10] [Ali, CH, Mishima, PRL 11] : model dependent assumption: point-like diquarks  $\rightarrow$  isospin effects in  $e^+e^-$  production
- isodoublet structure  $[bu][\bar{b}\bar{u}]$  and  $[bd][\bar{b}\bar{d}]$  seen in different final states?

## Experiment

channel	$Z_b$ contribution
$Y(1S)\pi\pi$	negligible
$Y(2S)\pi\pi$	large
$Y(3S)\pi\pi$	large
$h_b(1P)\pi\pi$	dominant
$h_b(2P)\pi\pi$	dominant
$BB^*\pi$	dominant?
$B^*B^*\pi$	dominant?

Coupling  $Y_b$  &  $Z_b$  complicated  
 $\rightarrow$  use „unpolluted“  $Y(1S)PP'$   
 final states for  $Y_b$  analysis

theory

$$e^+e^- \rightarrow Y_b \rightarrow Y(1S)PP'$$

$$PP' = \pi^+\pi^-, K^+K^-, \eta\pi^0$$

fit data

$$Y(1S) \pi^+\pi^-$$

predict

$$Y(1S)K^+K^-$$

$$Y(1S)\eta\pi^0$$

# Mixing of flavor states

- $Y_b^{(1)}$  mass eigenstates:

$$Y_{[b,l]} = \cos \theta Y_{[bu]} + \sin \theta Y_{[bd]}$$

$$Y_{[b,h]} = -\sin \theta Y_{[bu]} + \cos \theta Y_{[bd]}$$

- mass breaking:

$$M(Y_{[b,h]}) - M(Y_{[b,l]}) = (7 \pm 3) \cos(2\theta) \text{ MeV}$$

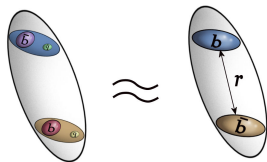
- effective diquark charge:

$$Q_{Y_{[b,l]}} = \frac{1}{3} \cos \theta - \frac{2}{3} \sin \theta$$

$$Q_{Y_{[b,h]}} = -\frac{1}{3} \sin \theta - \frac{2}{3} \cos \theta$$

- $\theta = -45^\circ \rightarrow$  Isospin eigenstates

# Model assumptions



- One gluon exchange model
- ↪ Color factor determines binding:  
Negative sign → Attractive
- Quarks in diquark transform as:
 

$$3 \otimes 3 = \bar{3} \oplus 6$$
✓
✗
- $qq$  bound state color factor:
 

$$t_{ij}^a t_{kl}^a = \underbrace{-\frac{2}{3} (\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj})/2}_{\text{antisymmetric: projects } 3} + \underbrace{\frac{1}{3} (\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj})/2}_{\text{symmetric: projects } 6}$$

- diquarks small  $SU(3)_C$  triplets
  - ↪ model from  $bb$ -onia
- wave function described by Coulomb + linear potential

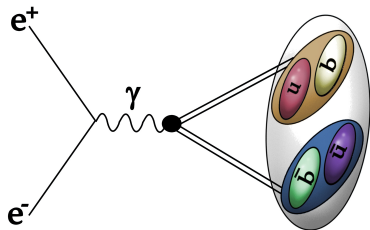
$$V(r) = -a \frac{1}{r} + b r$$

# $Y_b$ Production

- Van Royen-Weisskopf formula  
 $\Rightarrow \Gamma(1^{--} \rightarrow e^+e^-)$

**Assumption: Point-like diquarks**

[Ali, CH, Mishima, PRL (2010)]

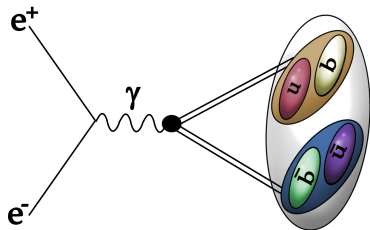


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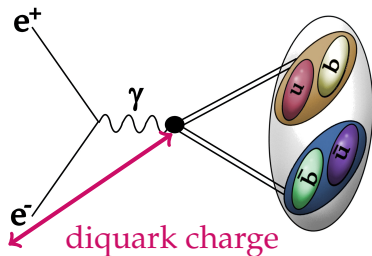
$$\Gamma_{ee}(Y_{[b,l/h]}) = \frac{24\alpha^2 Q_{[b,l/h]}^2}{M_{Y_{[b,l/h]}}^4} \kappa^2 \left| R_{11}^{(1)}(0) \right|^2$$

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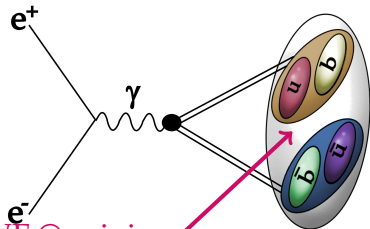


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radial tetraquark WF @ origin

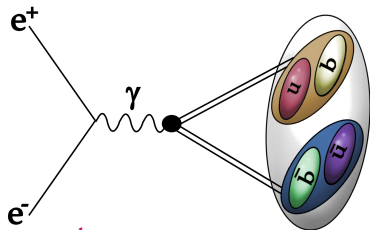
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hadronic size parameter

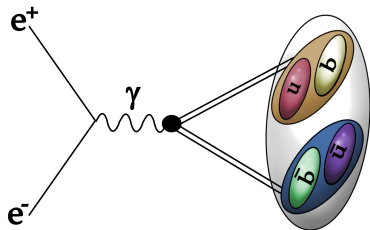
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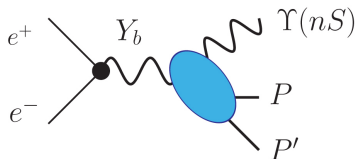
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- Suppressed  $\mathcal{O}(10)$  vs  $Y(5S)$
- Production ratio:  $\Gamma_{Y_{[b,l]}} / \Gamma_{Y_{[b,h]}} = \left( \frac{1-2 \tan \theta}{2+\tan \theta} \right)^2$
- Isospin breaking through production

$$\text{e.g. } \frac{\sigma_{Y(1S)K^+K^-}}{\sigma_{Y(1S)K^0\bar{K}^0}} = \frac{Q_{[bu]}^2}{Q_{[bd]}^2} = \frac{1}{4}$$

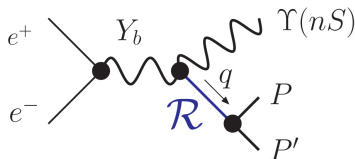
# $Y_b$ decay

Continuum



+

Resonance



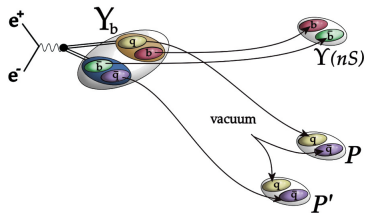
- Breit-Wigner shape for resonance:

$$\frac{1}{(q^2 - M^2) + iM\Gamma}$$

$q^2 \equiv M_{PP'}^2 \rightarrow$  Resonances show in  $M_{PP'}$  spectrum  
Not in  $M_{YP}$  spectrum since  $Z_b$  negligible

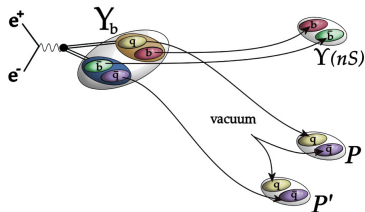
# Continuum Contribution

- Zweig allowed tetraquark continuum [Brown, Cahn, PRL (1975)] :

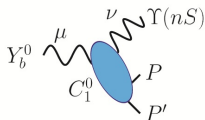


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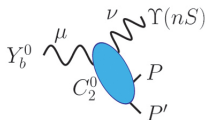


- Possible structures:



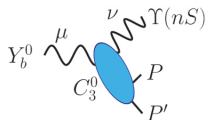
$$\delta$$

$$g_{\mu\nu}$$



$$\delta$$

$$g_{\mu\nu}(\cos^2 \theta - \frac{1}{3})$$

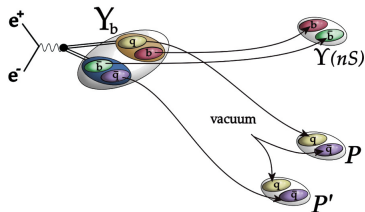


$$\delta$$

$$(k_{1\mu}k_{2\nu} + k_{2\mu}k_{1\nu})$$

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Diagram showing a possible structure for the tetraquark continuum, labeled  $C_1^0$ . It features a photon  $\gamma$  interacting with a quarkonium  $P$  and a quarkonium  $P'$  via a vertex  $C_1^0$ . The interaction is proportional to  $\delta_{\mu\nu}$ .

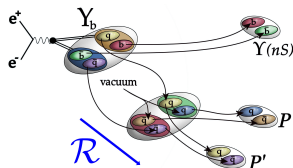
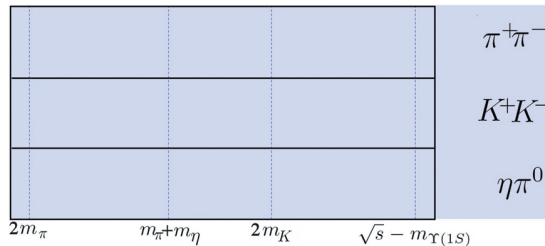
Diagram showing a possible structure for the tetraquark continuum, labeled  $C_2^0$ . It features a photon  $\gamma$  interacting with a quarkonium  $P$  and a quarkonium  $P'$  via a vertex  $C_2^0$ . The interaction is proportional to  $g_{\mu\nu}(\cos^2\theta - \frac{1}{3})$ .

HQ spin interaction  $\propto 1/m_b$

Diagram showing a possible structure for the tetraquark continuum, labeled  $C_3^0$ . It features a photon  $\gamma$  interacting with a quarkonium  $P$  and a quarkonium  $P'$  via a vertex  $C_3^0$ . The interaction is proportional to  $(k_{1\mu}k_{2\nu} + k_{2\mu}k_{1\nu})$ . This diagram is crossed out with a red X.

# Resonances in $M_{PP'}$ Spectrum

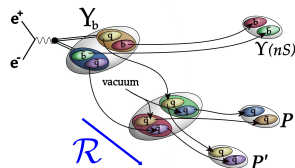
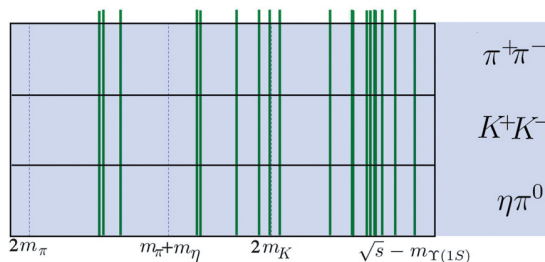
- Resonance  $\mathcal{R}$  contributions for each channel:





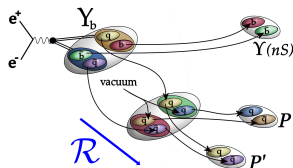
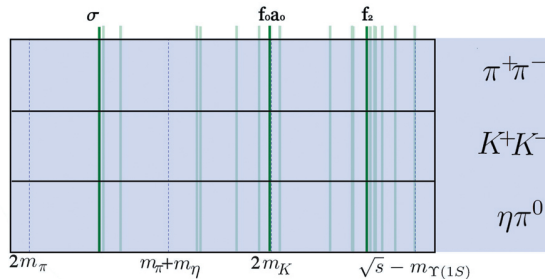
# Resonances in $M_{PP'}$ Spectrum

- Resonance  $\mathcal{R}$  contributions for each channel:



# Resonances in $M_{PP'}$ Spectrum

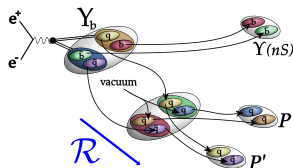
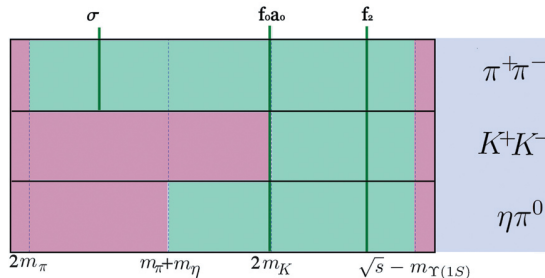
- Resonance  $\mathcal{R}$  contributions for each channel:



- Only  $0^{++}$  and  $2^{++}$  allowed

# Resonances in $M_{PP'}$ Spectrum

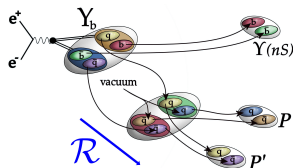
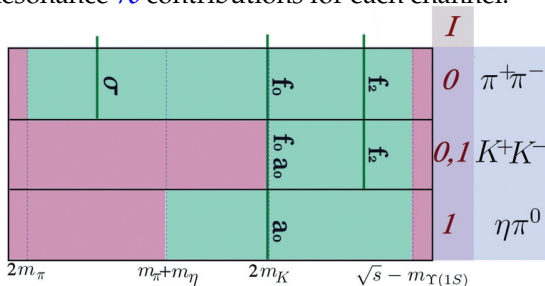
- Resonance  $\mathcal{R}$  contributions for each channel:



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- Kinematical constraints

# Resonances in $M_{PP'}$ Spectrum

- Resonance  $\mathcal{R}$  contributions for each channel:

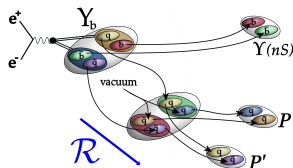


- Only  $0^{++}$  and  $2^{++}$  allowed
- Kinematical constraints
- Final state **isospin**

# Resonances in $M_{PP'}$ Spectrum

- Resonance  $\mathcal{R}$  contributions for each channel:

				$I$	
$2m_\pi$	$\sigma$	$f_0$	$f_2$	$0$	$\pi^+\pi^-$
$m_\pi+m_\eta$	$f_0 a_0$	$f_2$	$f_2$	$0,1$	$K^+K^-$
$2m_K$	$a_0$	$a_0$	$a_0$	$1$	$\eta\pi^0$
					$\sqrt{s} - m_{\Upsilon(1S)}$



- Only  $0^{++}$  and  $2^{++}$  allowed
- Kinematical constraints
- Final state **isospin**
- Threshold effects for  $f_0, a_0, \sigma \rightarrow$  Flatté formalism [Flatte, PLB (1976)]

# Unstable (scalar) particles on thresholds

1 PI insertions:

$$\begin{aligned} \text{---} \bullet \text{---} &= \text{---} + \text{---} \bullet \text{---} + \text{---} \bullet \bullet \text{---} + \dots \\ &= \frac{i}{p^2 - m_0^2} + \frac{i}{p^2 - m_0^2} (-iM^2) \frac{i}{p^2 - m_0^2} + \dots \\ &= \frac{i}{p^2 - m_0^2 - M^2(p^2)} \end{aligned}$$

unstable  $\rightarrow M^2(p^2)$  imaginary, set  $m^2 - m_0^2 - \text{Re}M^2(p^2) = 0$

$$\text{---} \bullet \text{---} = \frac{iZ}{p^2 - m^2(p^2) - iZ \text{Im}[M^2(p^2)]}$$

optical theorem:

$$Z \text{Im}[M^2(p^2)] = -\text{Im}[\mathcal{M}(a \rightarrow a)] = \frac{1}{2} \sum_f \int d\Pi_f |\mathcal{M}(a \rightarrow f)|^2 = -m\Gamma$$

# Unstable (scalar) particles on thresholds

1 PI insertions:

$$\begin{aligned}
 \text{---} \bullet \text{---} &= \text{---} + \text{---} \bullet \text{---} + \text{---} \bullet \bullet \text{---} + \dots \\
 &= \frac{i}{p^2 - m_0^2} + \frac{i}{p^2 - m_0^2} (-iM^2) \frac{i}{p^2 - m_0^2} + \dots \\
 &= \frac{i}{p^2 - m_0^2 - M^2(p^2)}
 \end{aligned}$$

unstable  $\rightarrow M^2(p^2)$  imaginary, set  $m^2 - m_0^2 - \text{Re}M^2(p^2) = 0$

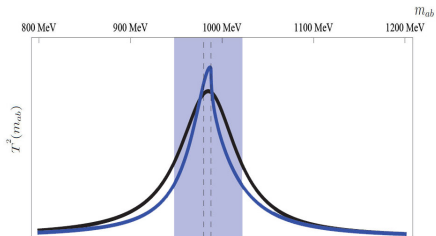
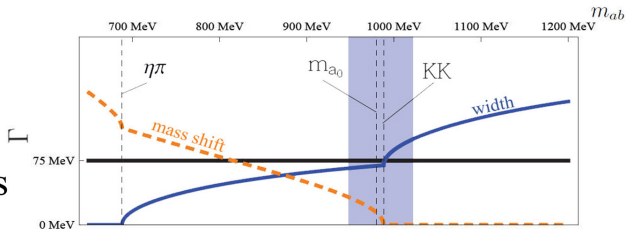
$$\text{---} \bullet \text{---} \quad \boxed{\text{Flatté: } m_0 \tilde{\Gamma}(p^2) \equiv M^2(p^2), \tilde{\Gamma} \text{ complex}}$$

optical theorem:

$$Z \text{Im}[M^2(p^2)] = -\text{Im}[\mathcal{M}(a \rightarrow a)] = \frac{1}{2} \sum_f \int d\Pi_f |\mathcal{M}(a \rightarrow f)|^2 = -m\Gamma$$

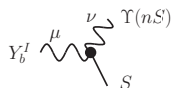
# Unstable (scalar) particles on thresholds

shape changes  
&  
mass shifts,  
depending on  
couplings

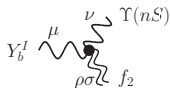




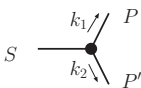
# Effective Lagrangian



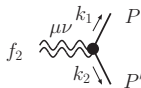
$$ig_{Y_b^I Y S} g_{\mu\nu}$$



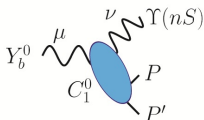
$$ig_{Y_b^I Y f_2} g_{\mu\rho} g_{\nu\sigma}$$



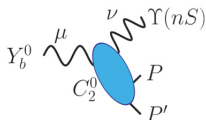
$$-ig_{SPP'} k_1 \cdot k_2$$



$$ig_{f_2 PP'} \frac{1}{2} \times (k_1 - k_2)_\mu (k_1 - k_2)_\nu$$



$$g_{\mu\nu} (\mathcal{M}_1^C)^2$$



$$g_{\mu\nu} (\mathcal{M}_2^C)^2 \left( \cos^2 \theta - \frac{1}{3} \right)$$

unknown transitions encoded in effective couplings  $g, \mathcal{M}$

# $Y_b$ Analysis

Different interfering contributions:

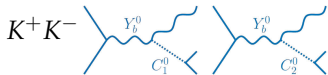
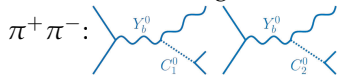
$$\pi^+ \pi^-:$$

$$K^+ K^-:$$

$$\eta \pi^0:$$

# $Y_b$ Analysis

Different interfering contributions:

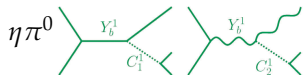
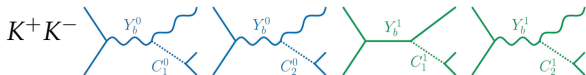
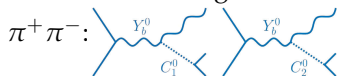


$\eta \pi^0$ :

●  $I = 0$  continuum

# $Y_b$ Analysis

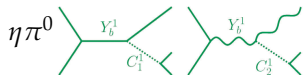
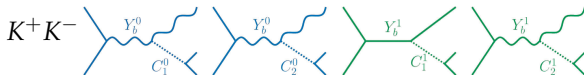
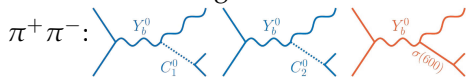
Different interfering contributions:



●  $I = 0$  continuum      ●  $I = 1$  continuum

# $Y_b$ Analysis

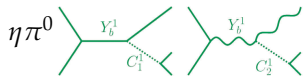
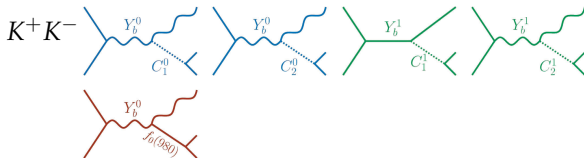
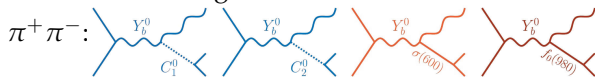
Different interfering contributions:



●  $I = 0$  continuum      ●  $I = 1$  continuum  
●  $\sigma(600)$

# $Y_b$ Analysis

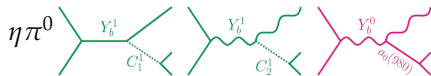
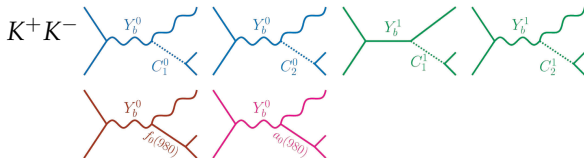
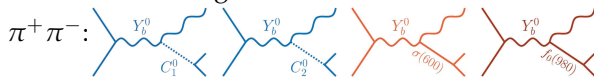
Different interfering contributions:



●  $I = 0$  continuum      ●  $I = 1$  continuum  
●  $\sigma(600)$       ●  $f_0(980)$

# $Y_b$ Analysis

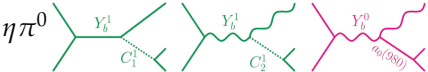
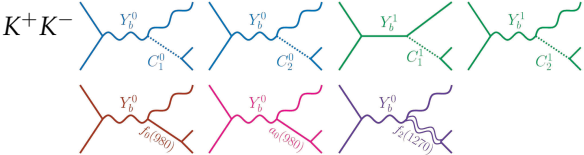
Different interfering contributions:



●  $I = 0$  continuum      ●  $I = 1$  continuum  
●  $\sigma(600)$     ●  $f_0(980)$       ●  $a_0^0(980)$

# $Y_b$ Analysis

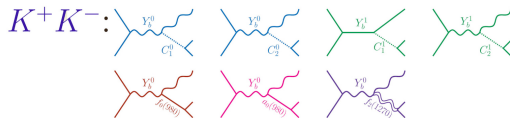
Different interfering contributions:



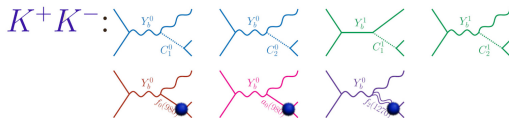
- $I = 0$  continuum
- $\sigma(600)$     ●  $f_0(980)$
- $I = 1$  continuum
- $a_0^0(980)$     ●  $f_2(1270)$



# Couplings: Overview



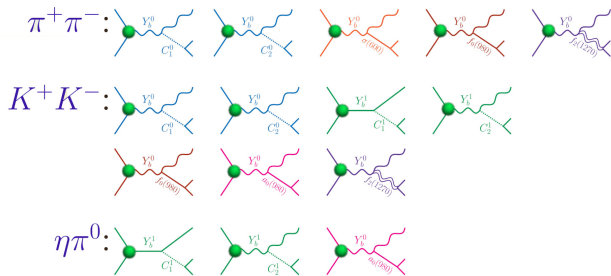
# Couplings: Overview



BES, CB, KLOE

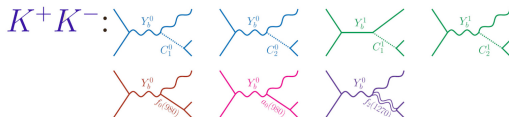
- **known**: Experimental input

# Couplings: Overview



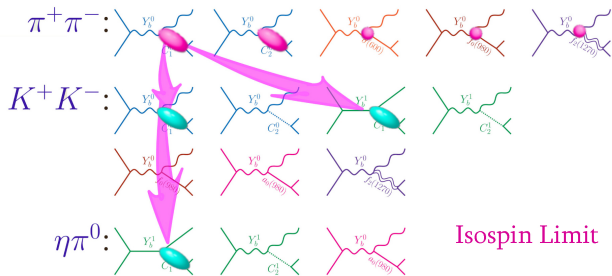
- **known:** Experimental input
- **known:** Our Van Royen-Weisskopf formula

# Couplings: Overview



- **known**: Experimental input
- **known**: Our Van Royen-Weisskopf formula
- **unknown**: Fit parameters

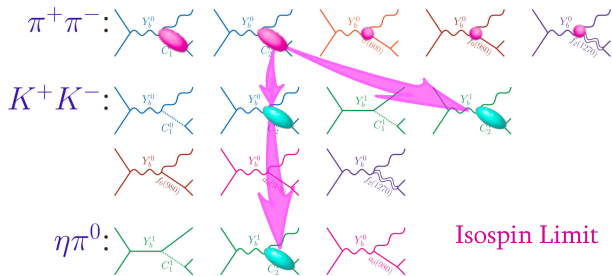
# Couplings: Overview



- **known**: Experimental input
- **known**: Our Van Royen-Weisskopf formula
- **unknown**: Fit parameters  $\rightarrow$  ● **predicted**:

$$A', B', g'_{Y_b^0 Y(1S)\sigma} (= g'_{Y_b^0 Y(1S)f_0}), g'_{Y_b^0 Y(1S)f_2}, \varphi_\sigma, \varphi_{f_0}, \varphi_{f_2}$$

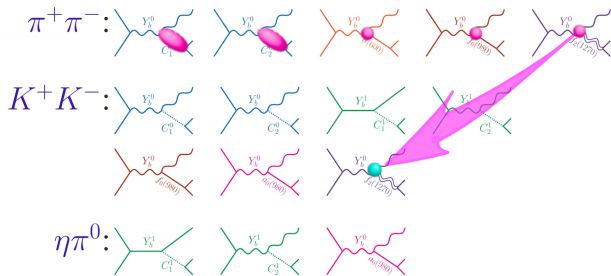
# Couplings: Overview



- **known**: Experimental input
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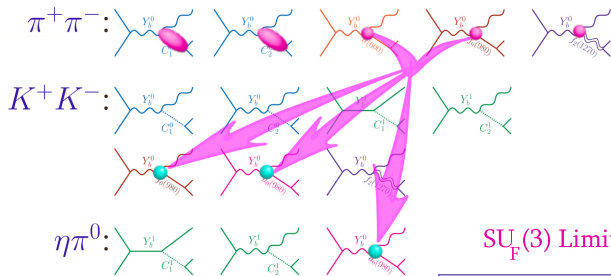
# Couplings: Overview



- **known**: Experimental input
- **known**: Our Van Royen-Weisskopf formula
- **unknown**: Fit parameters  $\rightarrow$  ● **predicted**:

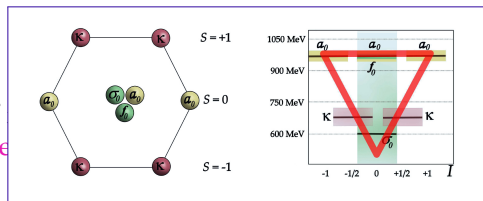
$$A', B', g'_{Y_b^0 Y(1S)\sigma} (= g'_{Y_b^0 Y(1S)f_0}), g'_{Y_b^0 Y(1S)f_2}, \varphi_\sigma, \varphi_{f_0}, \varphi_{f_2}$$

# Couplings: Overview



SU<sub>F</sub>(3) Limit

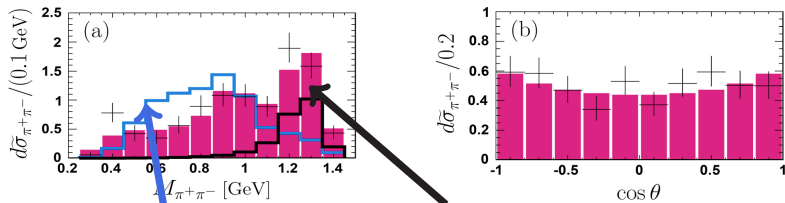
- **known**: Experimental input
- **known**: Our Van Royen-Weisskopf
- **unknown**: Fit parameters → ● **pre**



$$A', B', g'_{Y_b^0 Y(1S)\sigma} (= g'_{Y_b^0 Y(1S)f_0}), g'_{Y_b^0 Y(1S)f_2}, \varphi_\sigma, \varphi_{f_0}, \varphi_{f_2}$$



# Fit to $\sigma(e^+e^- \rightarrow Y_b \rightarrow Y(1S)\pi^+\pi^-)$



$2^{++}$  meson  $f_2(1270)$

$0^{++}$  tetraquarks  $\sigma(500) + f_0(980)$

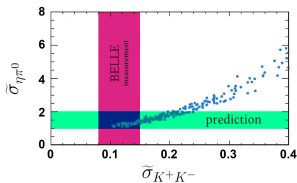
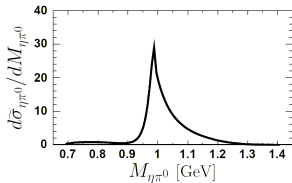
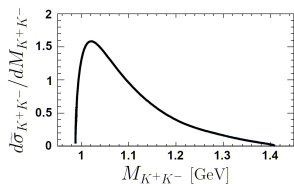
- Fit results, data from [Belle, PRL 08]
- $\chi^2/\text{d.o.f.} = 21.5/15 \rightarrow$  Good agreement with data
- Clear resonance dominance!

( $\tilde{\sigma}$ : normalized to measurement)

# Predictions for $Y(1S)(K^+K^-, \eta\pi^0)$

Fit determines couplings (assume  $SU(3)$  flavor symmetry for couplings  $(\sigma(500), f_0(980), a_0(980)) \rightarrow PP'$ , [t Hooft, Isidori, Maiani, Polosa, Riquer, PLB 08])

↪ predictions for spectra:



■ Agreement with  $\tilde{\sigma}_{K^+K^-} = 0.11^{+0.04}_{-0.03}$  (BELLE)

↪  $1.0 \lesssim \tilde{\sigma}_{\eta\pi^0} \lesssim 2.0$  predicted

■ Resonance dominance

↪ Characteristic shape

↪ **Excellent tests (relying on  $Y_b$  has 2 flavor states)**

Establishing exoticness of  $Y_b$ : Plan B  
(hadroproduction)

# Spectroscopy above Threshold

channel	spectrum	integrated	$Z_b$ contribution
$Y(1S)\pi\pi$	✓	✓	negligible
$Y(2S)\pi\pi$	✓	✓	large
$Y(3S)\pi\pi$	✓	✓	large
$Y(1S)KK$	soon?	✓	
$Y(2S)KK$	not yet	not yet	
$Y(3S)KK$	not yet	not yet	
$h_b(1P)\pi\pi$	✓	✓	dominant
$h_b(2P)\pi\pi$	✓	✓	dominant
$Y(1S)\eta\pi^0$	not yet	not yet	
$\eta_b\rho\pi$	not yet	not yet	
$BB^*\pi$	✓	✓	dominant?
$B^*B^*\pi$	✓	✓	dominant?
@ $Y(6S)$	BELLE II	BELLE II	

our prediction

important to confirm exotic state

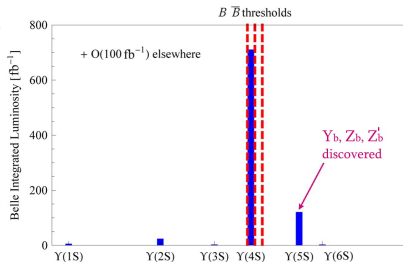
# Spectroscopy above Threshold

channel	spectrum	integrated	$Z_b$ contribution
$Y(1S)\pi\pi$	✓	✓	negligible
$Y(2S)\pi\pi$	✓	✓	large
$Y(3S)\pi\pi$	✓	✓	large
$Y(1S)KK$	soon?	✓	
$Y(2S)KK$	not yet	not yet	
$Y(3S)KK$	not yet	not yet	
$h_b(1P)\pi\pi$	✓	✓	dominant
$h_b(2P)\pi\pi$	✓	✓	dominant
$Y(1S)\eta\pi^0$	not yet	not yet	
$\eta_b\rho\pi$	not yet	not yet	
$BB^*\pi$	✓	✓	dominant?
$B^*B^*\pi$	✓	✓	dominant?
@ $Y(6S)$	BELLE II	BELLE II	

our prediction

important to confirm

exotic state



- multiquark states typically above hadronic thresholds
- few data for  $b\bar{b}$  ( $c\bar{c}$  advantage ISR &  $B$  decays)
- blank area likely full of surprises
- important to understand  $b\bar{b}$  &  $c\bar{c}$  simultaneously

# Spectroscopy above Threshold

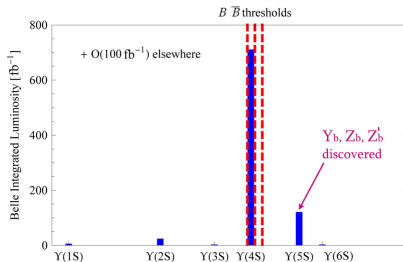
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$Y(2S)\pi\pi$	✓	✓	large
$Y(3S)\pi\pi$	✓	✓	large
$Y(1S)KK$	soon?	✓	
$Y(2S)KK$	not yet	not yet	
$Y(3S)KK$	not yet	not yet	
$h_b(1P)\pi\pi$	✓	✓	dominant
$h_b(2P)\pi\pi$	✓	✓	dominant
$Y(1S)\eta\pi^0$	not yet	not yet	
$\eta_b\rho\pi$	not yet	not yet	
$BB^*\pi$	✓	✓	dominant?
$B^*B^*\pi$	✓	✓	dominant?
@ $Y(6S)$	BELLE II ??	BELLE II ??	

our prediction

important to confirm

exotic state

maybe sooner?



- multiquark states typically above hadronic thresholds
- few data for  $b\bar{b}$  ( $c\bar{c}$  advantage ISR &  $B$  decays)
- blank area likely full of surprises
- important to understand  $b\bar{b}$  &  $c\bar{c}$  simultaneously

# Spectroscopy above Threshold

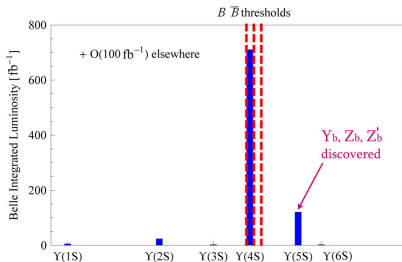
channel	spectrum	integrated	$Z_b$ contribution
$Y(1S)\pi\pi$	✓	✓	negligible
$Y(2S)\pi\pi$	✓	✓	large
$Y(3S)\pi\pi$	✓	✓	large
$Y(1S)KK$	soon?	✓	
$Y(2S)KK$	not yet	not yet	
$Y(3S)KK$	not yet	not yet	
$h_b(1P)\pi\pi$	✓	✓	dominant
$h_b(2P)\pi\pi$	✓	✓	dominant
$Y(1S)\eta\pi^0$	not yet	not yet	
$\eta_b\rho\pi$	not yet	not yet	
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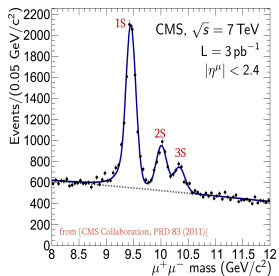


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➔ LHCb promising to map uncharted  $b\bar{b}$  region including more than  $J^{PC} = 1^{--}$

caveat: hadroproduction more complicated than electroproduction

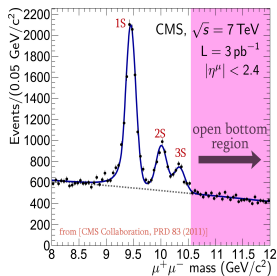
# Hadroproduction of Bottomonia



- $\mu^+\mu^-$  channel: Common particle detection for bottomonia

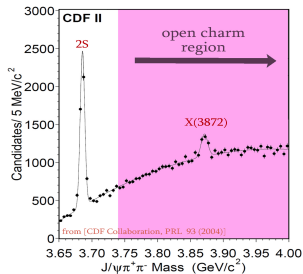
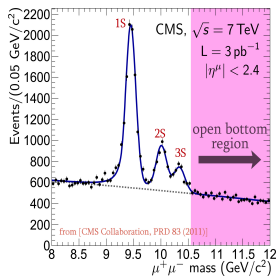


# Hadroproduction of Bottomonia



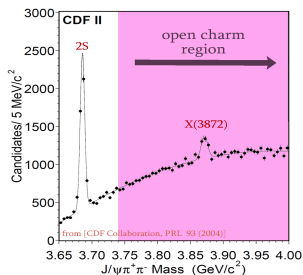
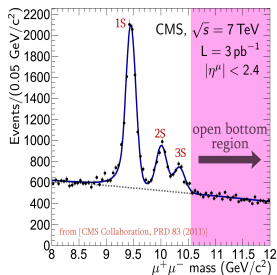
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[Ali, CH, Wang,13] :

Acquire knowledge of bottomonia above hadronic thresholds  
(NRQCD, pNRQCD [Brambilla, Pineda, Soto, Vairo, NP 00] )

➤ clarify nature of observed states

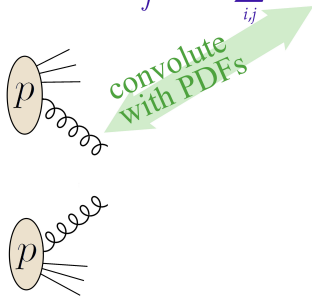
# NRQCD details [see lecture Brambilla [sadly she can't come :-() ]]

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$$\sigma_N(p\bar{p}(p) \rightarrow Y + X)$$

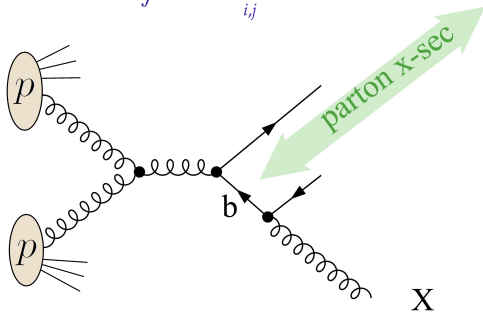
# NRQCD details [see lecture Brambilla [sadly she can't come :- ( ]]

$$\sigma_N(p\bar{p}(p) \rightarrow Y + X) = \int dx_1 dx_2 \sum_{ij} f_i(x_1) f_j(x_2)$$



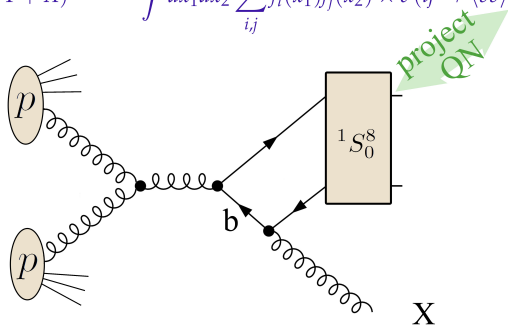
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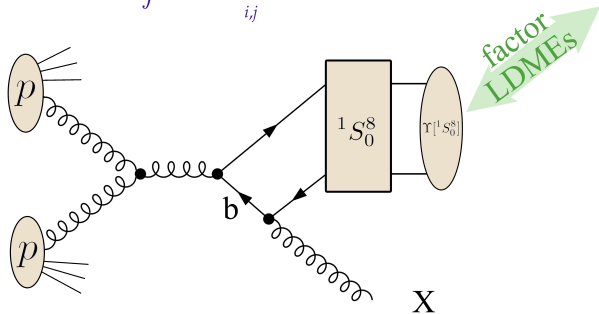
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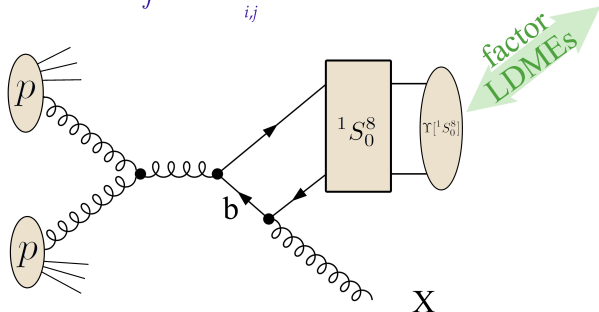
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# NRQCD details [see lecture Brambilla [sadly she can't come :-)]

$$\sigma_N(p\bar{p}(p) \rightarrow Y + X) = \int dx_1 dx_2 \sum_{ij} f_i(x_1) f_j(x_2) \times \hat{\sigma}(ij \rightarrow \langle \bar{b}b \rangle_N + X) \langle O[N] \rangle$$



- partonic channels (include NLO for CS):

$$gg \rightarrow Y[{}^3S_1^1] + g, \quad gg \rightarrow Y[{}^1S_0^8, {}^3S_1^8] + g$$

$$gq \rightarrow Y[{}^1S_0^8, {}^3S_1^8] + q, \quad q\bar{q} \rightarrow Y[{}^1S_0^8, {}^3S_1^8] + g$$

- calculate  $p_t$  distribution - take  $p_t > 3 \text{ GeV}$   $\rightarrow$  avoid soft gluon resummation

$$p_Y = \left( \sqrt{M_Y^2 + p_t^2} \cosh(y), p_t, 0, \sqrt{M_Y^2 + p_t^2} \sinh(y) \right)$$

# Collecting parts

- **CS LDMEs** from potential models via VRW (**well-known**)
- currently **CO LDMEs** not known  $\rightarrow$  large uncertainties if contribution sizable
- model dependence enters via  $\mathcal{B}(\Upsilon(6S) \rightarrow \Upsilon(nS)\pi\pi)$  estimate

## CS LDMEs:

at NLO

$$|R(0)|_{\Upsilon(5S)}^2 = 2.37 \text{ GeV}^3$$

$$|R(0)|_{\Upsilon(6S)}^2 = 1.02 \text{ GeV}^3$$

$$\langle O^H \ ^3S_1^1 \rangle = 3|R(0)|^2/(4\pi)$$

## CO LDMEs [ $10^{-2}\text{GeV}^3$ ]:

H	$\langle O^H \ ^1S_0^8 \rangle$	$\langle O^H \ ^3S_1^8 \rangle$
$\Upsilon(1S)$	$11.15 \pm 0.43$	$-0.41 \pm 0.24$
$\Upsilon(2S)$	$3.55 \pm 2.12$	$0.30 \pm 0.78$
$\Upsilon(3S)$	$-1.07 \pm 1.07$	$2.71 \pm 0.13$

[Gong, Wan,Wang, Zhang, 13]

## Branching Ratios:

$\mathcal{B}(\Upsilon(5S) \rightarrow \Upsilon(1S)\pi^+\pi^-)$	$(0.53 \pm 0.06)\%$
$\mathcal{B}(\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-)$	$(0.78 \pm 0.13)\%$
$\mathcal{B}(\Upsilon(5S) \rightarrow \Upsilon(3S)\pi^+\pi^-)$	$(0.48 \pm 0.18)\%$
$\mathcal{B}(\Upsilon(6S) \rightarrow \Upsilon(1S)\pi^+\pi^-)$	$\approx 0.4\%$
$\mathcal{B}(\Upsilon(6S) \rightarrow \Upsilon(2S)\pi^+\pi^-)$	$(0.4 - 1.2)\%$
$\mathcal{B}(\Upsilon(6S) \rightarrow \Upsilon(3S)\pi^+\pi^-)$	$(1.2 - 2.5)\%$
$\mathcal{B}(\Upsilon(1S) \rightarrow \mu^+\mu^-)$	$(2.48 \pm 0.05)\%$
$\mathcal{B}(\Upsilon(2S) \rightarrow \mu^+\mu^-)$	$(1.93 \pm 0.17)\%$
$\mathcal{B}(\Upsilon(3S) \rightarrow \mu^+\mu^-)$	$(2.18 \pm 0.21)\%$

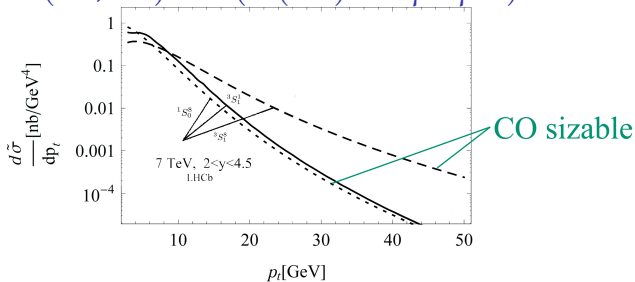
@ $\Upsilon(5S)$

rescattering  
model

data

error estimate

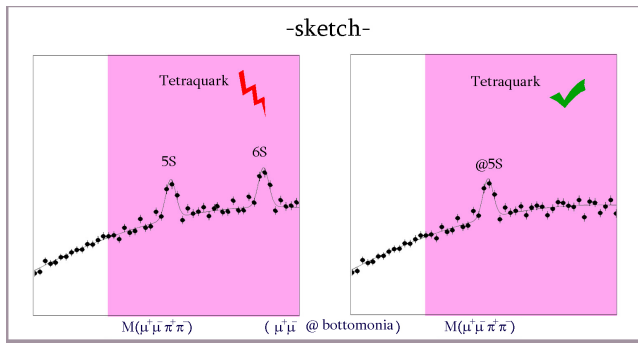
$p\bar{p}(p) \rightarrow Y(5S, 6S) \rightarrow (Y(nS) \rightarrow \mu^+\mu^-)\pi^+\pi^-$  in pb



	Y(5S)			Y(6S)		
	$n = 1$	$n = 2$	$n = 3$	$n = 1$	$n = 2$	$n = 3$
Tevatron	$2 \pm 1$	$2 \pm 1$	$1.2 \pm 0.8$	$1.4 \pm 1.0$	$2 \pm 1$	$4 \pm 3$
LHC 7	$9 \pm 6$	$10 \pm 7$	$7 \pm 5$	$6 \pm 4$	$9 \pm 7$	$23 \pm 17$
LHCb 7	$3 \pm 2$	$3 \pm 2$	$2 \pm 1$	$2 \pm 1$	$3 \pm 2$	$7 \pm 5$
LHC 8	$10 \pm 7$	$12 \pm 8$	$8 \pm 5$	$7 \pm 5$	$11 \pm 8$	$27 \pm 20$
LHCb 8	$3 \pm 2$	$3 \pm 2$	$2.4 \pm 1.6$	$2.7 \pm 2.0$	$3 \pm 2$	$8 \pm 6$
LHC 14	$19 \pm 13$	$22 \pm 15$	$15 \pm 11$	$13 \pm 10$	$20 \pm 15$	$51 \pm 39$
LHCb 14	$6 \pm 4$	$7 \pm 5$	$5 \pm 3$	$4 \pm 3$	$7 \pm 5$	$17 \pm 12$

↪  $\mathcal{O}(\text{pb})$  sufficient to be observed!

# Expectations



Tetraquark refuted:

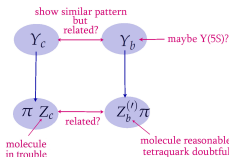
- $\pi^+\pi^-\mu^+\mu^-$  channel opens new possibilities for NRQCD and quarkonia production due to large  $Br$

Tetraquark confirmed:

- important exotic state in  $b\bar{b}$  confirmed
- HQ pattern established

# Conclusions

- Mass estimates strongly model dependent - market is messy
- Important puzzle remains:



- Tetraquarks provide a credible explanation for BELLE anomaly @ $Y(5S)$
- Predicted spectra for  $Y_b(10980)$  provide crucial tests - final proof needed urgently!
- Hadroproduction able to decide disputed nature of  $Y_b(10980)$

# Outlook

- Dedicated lattice studies are important
- Radiative decays may be theoretically treatable in QCDSR - need sufficient statistics
- Hadroproduction of bottomonia & tetraquarks very interesting in the near future!

➡ heaps of new states expected!



Exotica:

\* nobody knows, whats going on

\* good ideas more important, than  
extensive calculations

thank you!