

SPECTROSCOPY AND REGGE TRAJECTORIES OF HEAVY QUARKONIA AND B_c MESONS

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INTRODUCTION

- Significant experimental progress in heavy quarkonium spectroscopy

PDG lists:

- 25 charmonium states
- 14 bottomonium states
- 1 B_c meson state

more experimental candidates are available

- Some of the new states are long-awaited ones, expected by quark models many years ago:

$\eta_c(2S)$, $h_c(1P)$, $\chi_{c2}(2P)$, $\eta_b(1S)$, $\eta_b(2S)$, $h_b(1P)$, $h_b(2P)$, $\Upsilon(1D)$

- Some others, with masses higher than the threshold of the open charm or bottom production, have unexpected decay properties:

$X(3872)$, $X(4260)$, $X(4360)$, $X(4660)$, $X(4140)$ etc.

charged states: $X^\pm(4050)$, $X^\pm(4250)$, $X^\pm(4430)$, $Z_b^\pm(10610)$, $Z_b^\pm(10650)$

- First manifestation of the existence of exotic hadrons (tetraquarks, molecules, hybrids, hadro-quarkonia etc.)?

- Important to study highly excited $Q\bar{Q}$ quarkonium states. Such study allows:

- to single out usual $Q\bar{Q}$ states
- to construct heavy quarkonium Regge trajectories and determine their slopes and intercepts

- Comparison with experiment

Completely relativistic treatment of heavy quarks

RELATIVISTIC QUARK MODEL

Quasipotential equation of Schrödinger type:

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R} \right) \Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M) \Psi_M(\mathbf{q})$$

\mathbf{p} - relative momentum of quarks (diquarks)

M - bound state mass ($M = E_1 + E_2$)

μ_R - relativistic reduced mass:

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}$$

$b(M)$ - on-mass-shell relative momentum in cms:

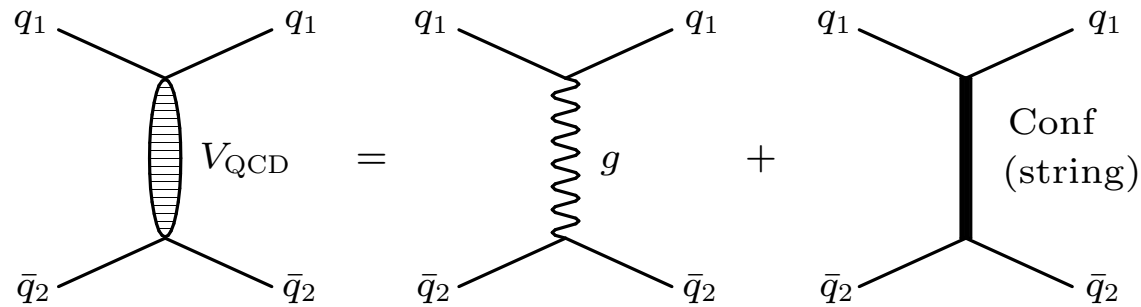
$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}$$

$E_{1,2}$ - center-of-mass energies:

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}$$

- $Q\bar{Q}$ **quasipotential** (meson sector)

(Constructed with the help of off-mass-shell scattering amplitude projected onto positive-energy states)



$$V(\mathbf{p}, \mathbf{q}; M) = \bar{u}_1(p)\bar{u}_2(-p) \left\{ \frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu + V_{\text{conf}}^V(\mathbf{k})\Gamma_1^\mu\Gamma_{2;\mu} + V_{\text{conf}}^S(\mathbf{k}) \right\} u_1(q)u_2(-q)$$

$$\mathbf{k} = \mathbf{p} - \mathbf{q}$$

$D_{\mu\nu}(\mathbf{k})$ - (perturbative) gluon propagator

$\Gamma_\mu(\mathbf{k})$ - effective long-range vertex with **Pauli term**:

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu,$$

κ - anomalous chromomagnetic moment of quark,

$$u^\lambda(p) = \sqrt{\frac{\epsilon(p) + m}{2\epsilon(p)}} \begin{pmatrix} 1 \\ \boldsymbol{\sigma p} \\ \epsilon(p) + m \end{pmatrix} \chi^\lambda,$$

with $\epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$.

- Lorentz structure of $V_{\text{conf}} = V_{\text{conf}}^V + V_{\text{conf}}^S$

In nonrelativistic limit

$$\left. \begin{aligned} V_{\text{conf}}^V(r) &= (1 - \epsilon)(Ar + B) \\ V_{\text{conf}}^S(r) &= \epsilon(Ar + B) \end{aligned} \right\} \text{Sum : } (Ar + B)$$

ϵ - mixing parameter

$$V_{\text{NR}}(r) = V_{\text{Coul}}(r) + V_{\text{conf}}(r)$$

$$V_{\text{Coul}}(r) = -\frac{4\alpha_s}{3r}$$

Parameters A , B , κ , ε and quark masses fixed from analysis of meson masses and radiative decays:

$\varepsilon = -1$ from heavy quarkonium radiative decays ($J/\psi \rightarrow \eta_c + \gamma$) and HQET

$\kappa = -1$ from fine splitting of heavy quarkonium 3P_J states and HQET

$(1 + \kappa) = 0 \implies$ **vanishing long-range chromomagnetic interaction !** (flux tube model)

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln \frac{\mu^2}{\Lambda^2}}, \quad \beta_0 = 11 - \frac{2}{3}n_f, \quad \mu = \frac{2m_1m_2}{m_1 + m_2},$$

Quasipotential parameters:

$$A = 0.18 \text{ GeV}^2, \quad B = -0.30 \text{ GeV},$$

$$\Lambda = 0.169 \text{ GeV}$$

Quark masses:

$$m_b = 4.88 \text{ GeV}$$

$$m_s = 0.50 \text{ GeV}$$

$$m_c = 1.55 \text{ GeV}$$

$$m_{u,d} = 0.33 \text{ GeV}$$

MASSES OF HEAVY QUARKONIA

$Q\bar{Q}$ potential

$$V_{Q\bar{Q}}(r) = V_{\text{SI}}(r) + V_{\text{SD}}(r)$$

spin-dependent potential

$$V_{\text{SD}}(r) = a_1 \mathbf{L}\mathbf{S}_1 + a_2 \mathbf{L}\mathbf{S}_2 + b \left[-\mathbf{S}_1\mathbf{S}_2 + \frac{3}{r^2}(\mathbf{S}_1\mathbf{r})(\mathbf{S}_2\mathbf{r}) \right] + c \mathbf{S}_1\mathbf{S}_2 + d (\mathbf{L}\mathbf{S}_1)(\mathbf{L}\mathbf{S}_2)$$

where e.g.

$$c = \frac{2}{3E_1E_2} \left[\Delta\bar{V}_{\text{Coul}}(r) + \left(\frac{E_1 - m_1}{2m_1} - (1 + \kappa) \frac{E_1 + m_1}{2m_1} \right) \right. \\ \left. \times \left(\frac{E_2 - m_2}{2m_2} - (1 + \kappa) \frac{E_2 + m_2}{2m_2} \right) \Delta V_{\text{conf}}^V(r) \right]$$

$E_{1,2}$ - center-of-mass energies:

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}$$

★ one-loop radiative corrections are also taken into account

Table 1: Charmonium mass spectrum (in MeV).

State		Theory	Experiment		State		Theory	Experiment	
$n^{2S+1}L_J$	J^{PC}		meson	mass	$n^{2S+1}L_J$	J^{PC}		meson	mass
1^1S_0	0^{-+}	2981	$\eta_c(1S)$	2980.3(1.2)	2^3D_1	1^{--}	4150	$\psi(4160)$	4153(3)
1^3S_1	1^{--}	3096	$J/\psi(1S)$	3096.916(11)	2^3D_2	2^{--}	4190		
2^1S_0	0^{-+}	3635	$\eta_c(2S)$	3637(4)	2^3D_3	3^{--}	4220		
2^3S_1	1^{--}	3685	$\psi(2S)$	3686.09(4)	2^1D_2	2^{-+}	4196	$X(4160)?$	4156($^{29}_{25}$)
3^1S_0	0^{-+}	3989			3^3D_1	1^{--}	4507		
3^3S_1	1^{--}	4039	$\psi(4040)$	4039(1)	3^3D_2	2^{--}	4544		
4^1S_0	0^{-+}	4401			3^3D_3	3^{--}	4574		
4^3S_1	1^{--}	4427	$\psi(4415)$	4421(4)	3^1D_2	2^{-+}	4549		
5^1S_0	0^{-+}	4811			4^3D_1	1^{--}	4857		
5^3S_1	1^{--}	4837			4^3D_2	2^{--}	4896		
6^1S_0	0^{-+}	5155			4^3D_3	3^{--}	4920		
6^3S_1	1^{--}	5167			4^1D_2	2^{-+}	4898		
1^3P_0	0^{++}	3413	$\chi_{c0}(1P)$	3414.75(31)	1^3F_2	2^{++}	4041		
1^3P_1	1^{++}	3511	$\chi_{c1}(1P)$	3510.66(7)	1^3F_3	3^{++}	4068		
1^3P_2	2^{++}	3555	$\chi_{c2}(1P)$	3556.20(9)	1^3F_4	4^{++}	4093		
1^1P_1	1^{+-}	3525	$h_c(1P)$	3525.41(16)	1^1F_3	3^{+-}	4071		
2^3P_0	0^{++}	3870			2^3F_2	2^{++}	4361		
2^3P_1	1^{++}	3906			2^3F_3	3^{++}	4400		
2^3P_2	2^{++}	3949	$\chi_{c2}(2P)$	3927.2(2.6)	2^3F_4	4^{++}	4434		
2^1P_1	1^{+-}	3926			2^1F_3	3^{+-}	4406		
3^3P_0	0^{++}	4301			1^3G_3	3^{--}	4321		
3^3P_1	1^{++}	4319			1^3G_4	4^{--}	4343		
3^3P_2	2^{++}	4354	$X(4350)?$	4351(5)	1^3G_5	5^{--}	4357		

Table 1: (continued)

State		Theory	Experiment		State		Theory	Experiment	
$n^{2S+1}L_J$	J^{PC}		meson	mass	$n^{2S+1}L_J$	J^{PC}		meson	mass
3^1P_1	1^{+-}	4337			1^1G_4	4^{-+}	4345		
4^3P_0	0^{++}	4698			1^3H_4	4^{++}	4572		
4^3P_1	1^{++}	4728			1^3H_5	5^{++}	4592		
4^3P_2	2^{++}	4763			1^3H_6	6^{++}	4608		
4^1P_1	1^{+-}	4744			1^3H_5	5^{+-}	4594		
1^3D_1	1^{--}	3783	$\psi(3770)$	3772.92(35)					
1^3D_2	2^{--}	3795							
1^3D_3	3^{--}	3813	$X(3820)$	3823.5(2.5)					
1^1D_2	2^{-+}	3807							

Table 2: Bottomonium mass spectrum (in MeV).

State		Theory	Experiment		State		Theory
$n^{2S+1}L_J$	J^{PC}		meson	mass	$n^{2S+1}L_J$	J^{PC}	
1^1S_0	0^{-+}	9398	$\eta_b(1S)$	9390.9(2.8)	2^3D_1	1^{--}	10435
1^3S_1	1^{--}	9460	$\Upsilon(1S)$	9460.30(26)	2^3D_2	2^{--}	10443
2^1S_0	0^{-+}	9990	$\eta_b(2S)$	9999.0(4.5)	2^3D_3	3^{--}	10449
2^3S_1	1^{--}	10023	$\Upsilon(2S)$	10023.26(31)	2^1D_2	2^{-+}	10445
3^1S_0	0^{-+}	10329			3^3D_1	1^{--}	10704
3^3S_1	1^{--}	10355	$\Upsilon(3S)$	10355.2(5)	3^3D_2	2^{--}	10711
4^1S_0	0^{-+}	10573			3^3D_3	3^{--}	10717
4^3S_1	1^{--}	10586	$\Upsilon(4S)$	10579.4(1.2)	3^1D_2	2^{-+}	10713
5^1S_0	0^{-+}	10851			4^3D_1	1^{--}	10949

State		Theory	Experiment		State		Theory
$n^{2S+1}L_J$	J^{PC}		meson	mass	$n^{2S+1}L_J$	J^{PC}	
5^3S_1	1^{--}	10869	$\Upsilon(10860)$	10876(1)	4^3D_2	2^{--}	10957
6^1S_0	0^{-+}	11061			4^3D_3	3^{--}	10963
6^3S_1	1^{--}	11088	$\Upsilon(11020)$	11019(8)	4^1D_2	2^{-+}	10959
1^3P_0	0^{++}	9859	$\chi_{b0}(1P)$	9859.44(52)	1^3F_2	2^{++}	10343
1^3P_1	1^{++}	9892	$\chi_{b1}(1P)$	9892.78(40)	1^3F_3	3^{++}	10346
1^3P_2	2^{++}	9912	$\chi_{b2}(1P)$	9912.21(40)	1^3F_4	4^{++}	10349
1^1P_1	1^{+-}	9900	$h_b(1P)$	9898.25(1.50)	1^1F_3	3^{+-}	10347
2^3P_0	0^{++}	10233	$\chi_{b0}(2P)$	10232.5(6)	2^3F_2	2^{++}	10610
2^3P_1	1^{++}	10255	$\chi_{b1}(2P)$	10255.46(55)	2^3F_3	3^{++}	10614
2^3P_2	2^{++}	10268	$\chi_{b2}(2P)$	10268.65(55)	2^3F_4	4^{++}	10617
2^1P_1	1^{+-}	10260	$h_b(2P)$	10259.76(1.57)	2^1F_3	3^{+-}	10615
3^3P_0	0^{++}	10521			1^3G_3	3^{--}	10511
3^3P_1	1^{++}	10541	$\chi_b(3P)$	10530(11)	1^3G_4	4^{--}	10512
3^3P_2	2^{++}	10550			1^3G_5	5^{--}	10514
3^1P_1	1^{+-}	10544			1^1G_4	4^{-+}	10513
4^3P_0	0^{++}	10781			1^3H_4	4^{++}	10670
4^3P_1	1^{++}	10802			1^3H_5	5^{++}	10671
4^3P_2	2^{++}	10812			1^3H_6	6^{++}	10672
4^1P_1	1^{+-}	10804			1^3H_5	5^{+-}	10671
1^3D_1	1^{--}	10154					
1^3D_2	2^{--}	10161	$\Upsilon(1D)$	10163.7(1.4)			
1^3D_3	3^{--}	10166					

Bottomonium hyperfine splittings:

- $1S$ and $2S$ states

★ from $\Upsilon(3S) \rightarrow \eta_b(1S)\gamma$, $\Upsilon(2S) \rightarrow \eta_b(1S)\gamma$ and $\Upsilon(2S) \rightarrow \eta_b(2S)\gamma$ (CLEO, BaBar)

$$\Delta M_{\text{hfs}}(\eta_b(nS)) \equiv M_{\Upsilon(nS)} - M_{\eta_b(nS)}$$

$$\Delta M_{\text{hfs}}(\eta_b(1S)) = 69.3 \pm 2.8 \text{ MeV} \quad \Delta M_{\text{hfs}}(\eta_b(2S)) = 48.7 \pm 2.3 \pm 2.1 \text{ MeV}$$

★ from $h_b(1P) \rightarrow \eta_b(1S)\gamma$ and $h_b(2P) \rightarrow \eta_b(2S)\gamma$ (Belle)

$$\Delta M_{\text{hfs}}(\eta_b(1S)) = 57.9 \pm 2.3 \text{ MeV} \quad \Delta M_{\text{hfs}}(\eta_b(2S)) = 24.3_{-4.5}^{+4.0} \text{ MeV}$$

Our prediction

$$\Delta M_{\text{hfs}}^{\text{theor}}(\eta_b(1S)) = 62 \text{ MeV} \quad \Delta M_{\text{hfs}}^{\text{theor}}(\eta_b(2S)) = 33 \text{ MeV}$$

- $1P$ and $2P$ states

$$\Delta M_{\text{hfs}}(nP) \equiv \langle M(n^3P_J) \rangle - M(n^1P_1),$$

where spin-averaged centroid of the triplet states

$$\langle M(^3P_J) \rangle = [M(\chi_{b0}) + 3M(\chi_{b1}) + 5M(\chi_{b2})]/9 \quad M(^1P_1) = M(h_b)$$

★ $1P$ states

$$\Delta M_{\text{hfs}}(1P) = (0.8 \pm 1.1) \text{ MeV}$$

★ $2P$ states

$$\Delta M_{\text{hfs}}(2P) = (0.5 \pm 1.2) \text{ MeV}$$

⇒ vanishing of the long-range chromomagnetic interaction in heavy quarkonia (e.g. flux tube model)

Table 3: B_c meson mass spectrum (in MeV).

State		Theory	Experiment		State		Theory
$n^{2S+1}L_J$	J^P		meson	mass	$n^{2S+1}L_J$	J^P	
1^1S_0	0^-	6272	B_c	6277(6)	1^3D_1	1^-	7021
1^3S_1	1^-	6333			$1D_2$	2^-	7025
2^1S_0	0^-	6842			$1D_2$	2^-	7026
2^3S_1	1^-	6882			1^3D_3	3^-	7029
3^1S_0	0^-	7226			2^3D_1	1^-	7392
3^3S_1	1^-	7258			$2D_2$	2^-	7399
4^1S_0	0^-	7585			$2D_2$	2^-	7400
4^3S_1	1^-	7609			2^3D_3	3^-	7405
5^1S_0	0^-	7928			3^3D_1	1^-	7732
5^3S_1	1^-	7947			$3D_2$	2^-	7741
1^3P_0	0^+	6699			$3D_2$	2^-	7743
$1P_1$	1^+	6743			3^3D_3	3^-	7750
$1P_1$	1^+	6750			1^3F_2	2^+	7273
1^3P_2	2^+	6761			$1F_3$	3^+	7269
2^3P_0	0^+	7094			$1F_3$	3^+	7268
$2P_1$	1^+	7134			1^3F_4	4^+	7277
$2P_1$	1^+	7147			2^3F_2	2^+	7618
2^3P_2	2^+	7157			$2F_3$	3^+	7616
3^3P_0	0^+	7474			$2F_3$	3^+	7615
$3P_1$	1^+	7500			2^3F_4	4^{+-}	7617
$3P_1$	1^+	7510			1^3G_3	3^-	7497
3^3P_2	2^+	7524			$1G_4$	4^-	7489

Table 3: (continued)

State		Theory	Experiment		State		Theory
$n^{2S+1}L_J$	J^P		meson	mass	$n^{2S+1}L_J$	J^P	
4^3P_0	0^+	7817			$1G_4$	4^-	7487
$4P_1$	1^+	7844			1^3G_5	5^-	7482
$4P_1$	1^+	7853			$2G_4$	4^-	7819
4^3P_2	2^+	7867			2^3G_5	5^-	7817

REGGE TRAJECTORIES

(a) The (J, M^2) Regge trajectory:

$$J = \alpha M^2 + \alpha_0$$

(b) The (n_r, M^2) Regge trajectory:

$$n_r = \beta M^2 + \beta_0,$$

α, β – slopes

α_0, β_0 – intercepts.

$P = (-1)^J$ – natural parity

$P = (-1)^{J-1}$ – unnatural parity

Nonlinear Regge trajectories:

(a) for the parent trajectory in the (J, M^2) plane

$$M^2 = \left(J - \frac{\gamma_1}{(J+2)^2} + \gamma_0 \right) / \gamma,$$

(b) for the $J = 1$ trajectory in the (n_r, M^2) plane

$$M^2 = \left(n_r - \frac{\tau_1}{(n_r+2)^2} + \tau_0 \right) / \tau,$$

γ, τ – slopes, γ_0, τ_0 – intercepts, γ_1, τ_1 – nonlinearity.

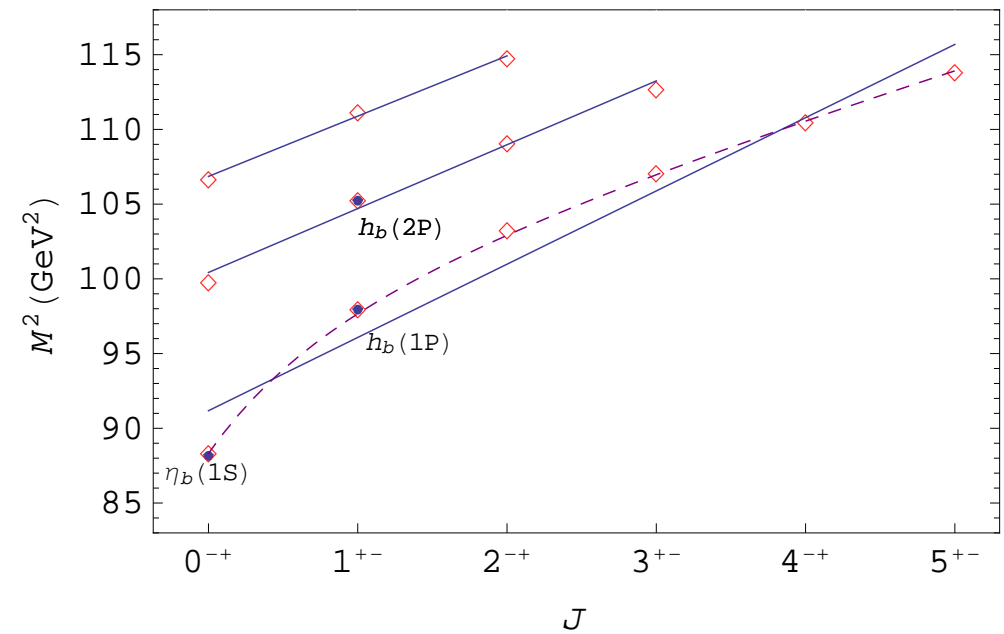
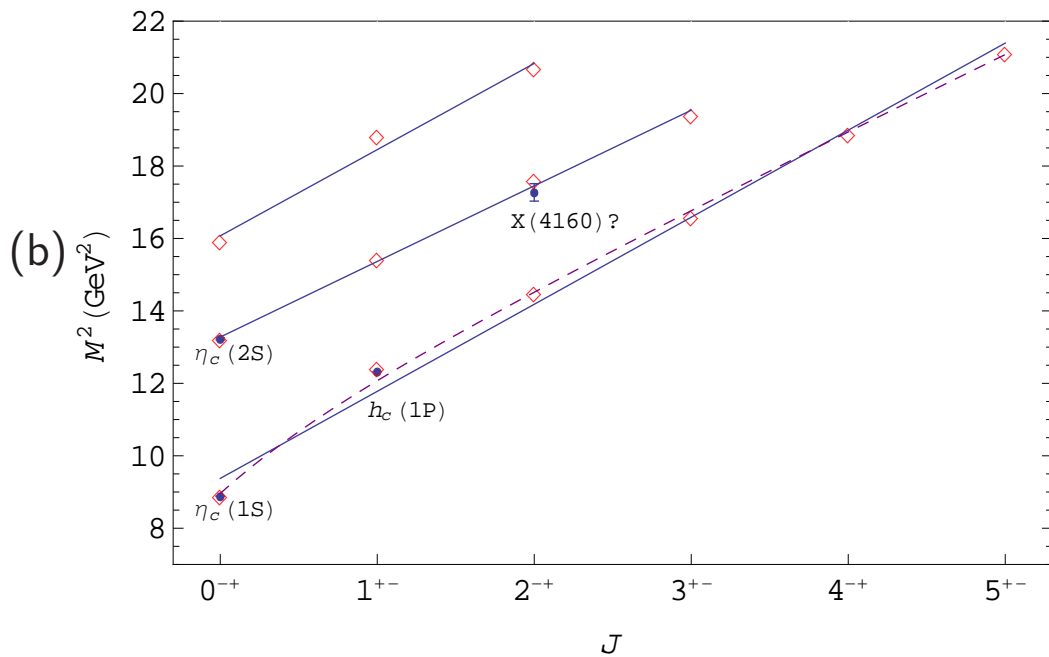
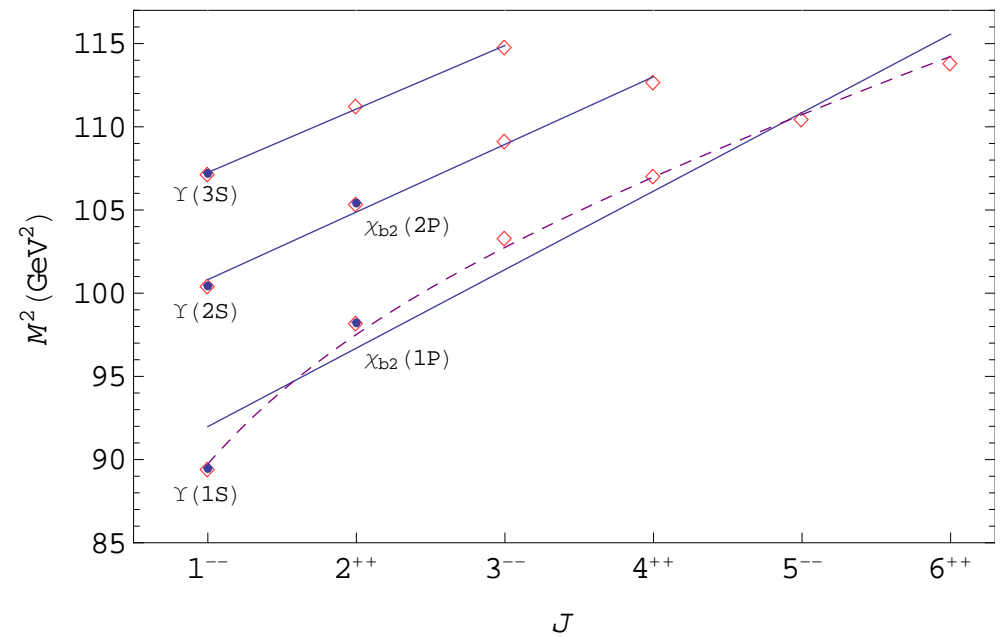
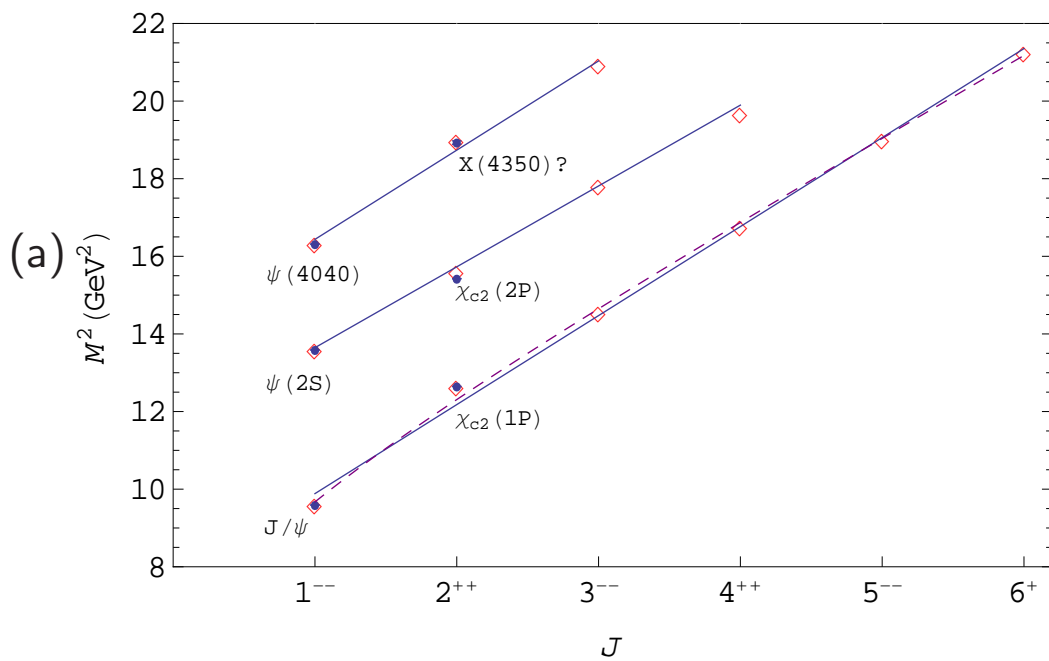
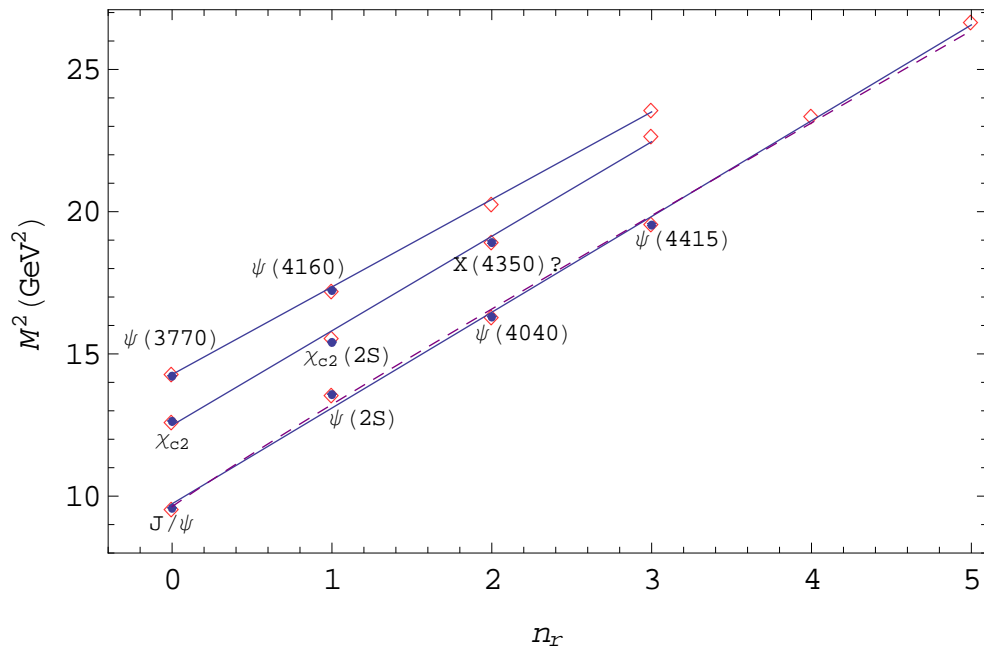
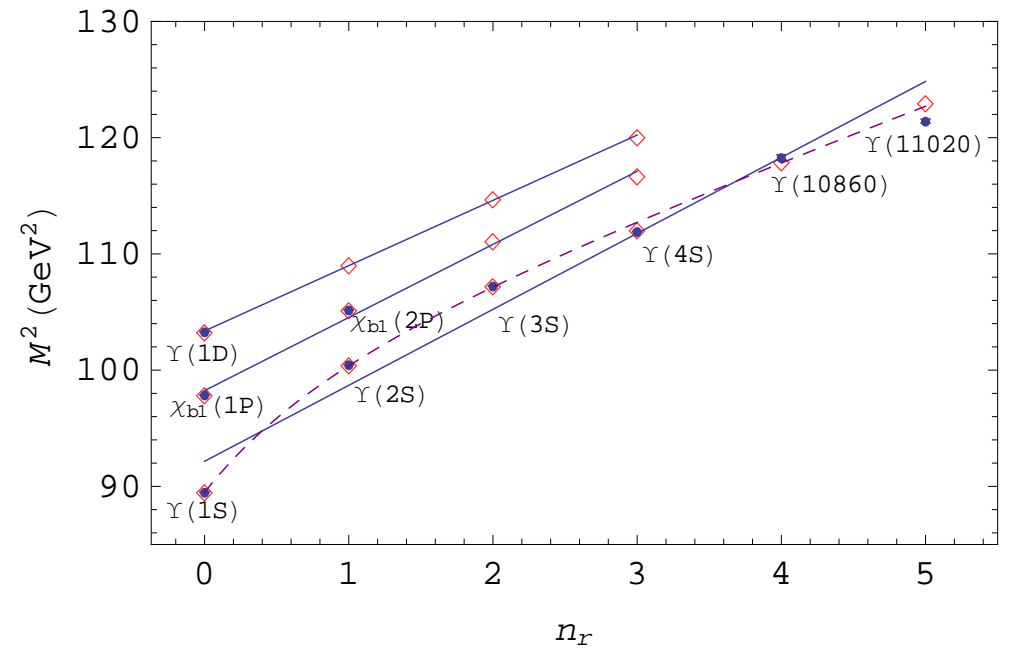


Figure 1: Parent and daughter (J, M^2) Regge trajectories for charmonium and bottomonium state with natural (a) and unnatural (b) parity. Diamonds are predicted masses. Available experimental data are given by dots with error bars and particle names. The dashed line corresponds to a nonlinear fit.



(a)



(b)

Figure 2: The (n_r, M^2) Regge trajectories for vector (S -wave), tensor and vector (D -wave) charmonium (a) and bottomonium (b) states (from bottom to top).

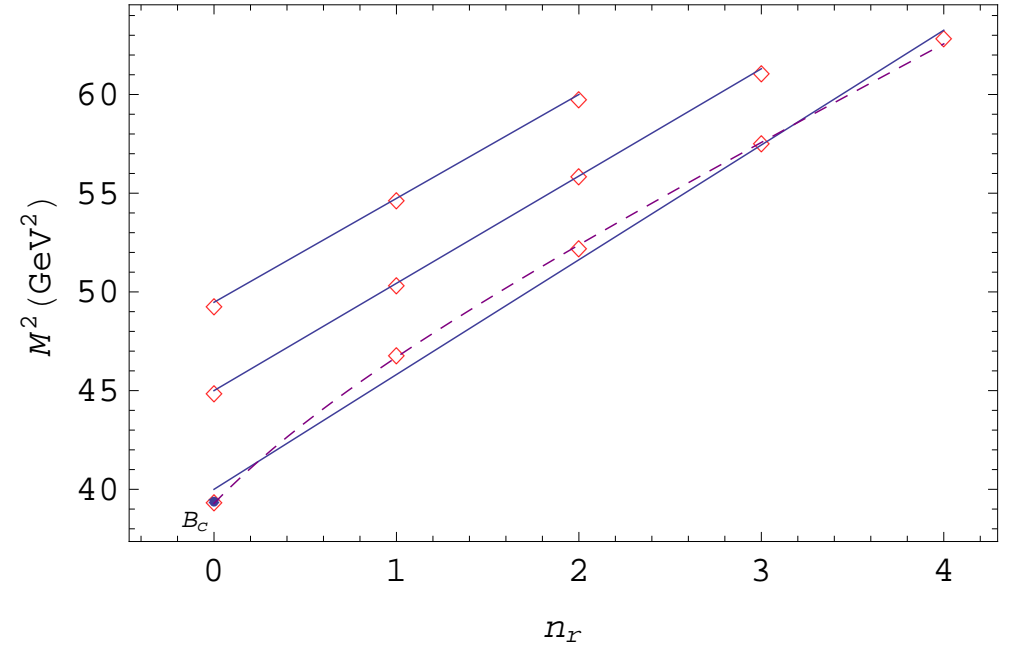
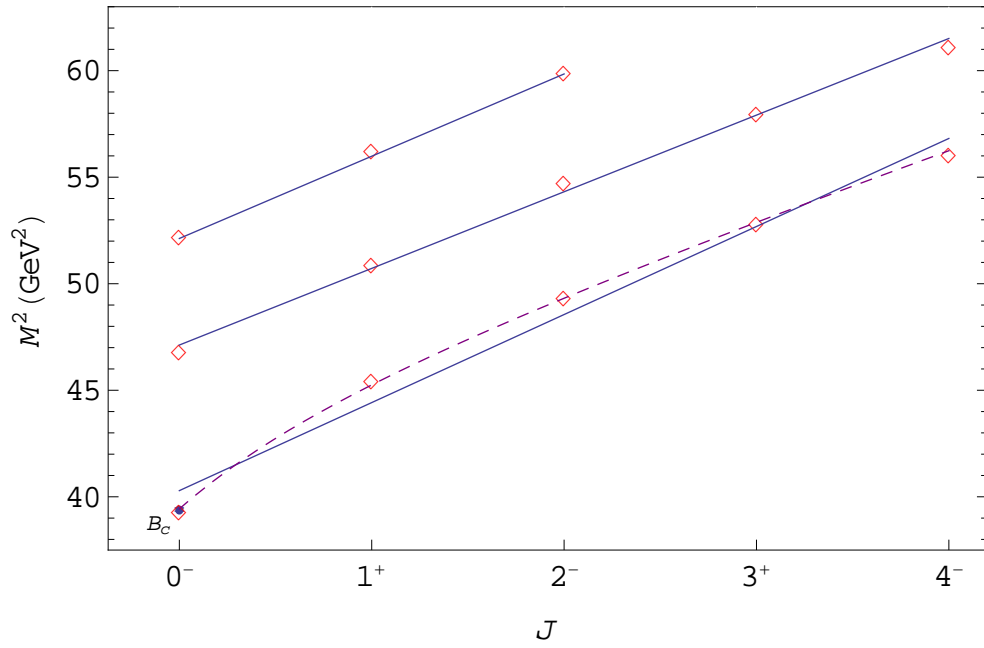


Figure 3: The (J, M^2) and (n_r, M^2) Regge trajectories for the B_c meson states.

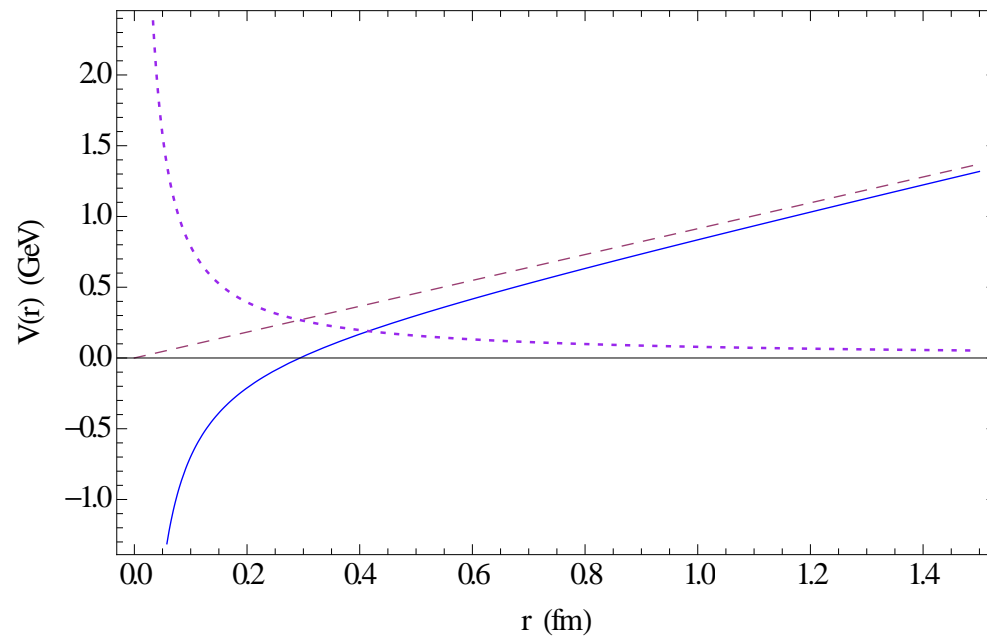


Figure 4: Static potential of the quark-antiquark interaction without the constant term (solid line). Dashed line shows the linear confining potential, dotted line corresponds to the modulus of the Coulomb potential.

Table 4: Mean square radii $\sqrt{\langle r^2 \rangle}$ for the spin-singlet ground and excited states of charmonia, B_c mesons and bottomonia (in fm).

State	$\sqrt{\langle r^2 \rangle}_\psi$	$\sqrt{\langle r^2 \rangle}_{B_c}$	$\sqrt{\langle r^2 \rangle}_\Upsilon$
$1S$	0.37	0.33	0.22
$1P$	0.59	0.53	0.41
$2S$	0.71	0.63	0.50
$1D$	0.74	0.67	0.54
$2P$	0.87	0.79	0.65
$1F$	0.87	0.79	0.65
$3S$	0.94	0.87	0.72
$1G$	0.98	0.89	0.75

CONCLUSIONS

- Mass spectra of heavy quarkonia were calculated in the relativistic quark model up to high radial and orbital excitations.
- Heavy quarks were treated fully relativistically without application of the nonrelativistic v/c and heavy quark $1/m_Q$ expansions.
- The Regge trajectories were constructed both in (J, M^2) and (n_r, M^2) planes. Different behaviour was found for parent and daughter trajectories:
 - ★ daughter trajectories are almost linear, parallel and equidistant.
 - ★ parent trajectories exhibit some nonlinearity. Such nonlinearity occurs only in the vicinity of ground states and few lowest excitations and is mostly pronounced for bottomonia.
- The origin of this nonlinearity is the importance of the Coulomb potential contribution for the lowest states of heavy quarkonia.
- Parameters of linear and nonlinear Regge trajectories were determined.
- The assignment of the experimentally observed heavy quarkonia to the particular Regge trajectories was carried out.
- It was found that our model predicts $Q\bar{Q}$ masses in good agreement with available experimental data for conventional heavy quarkonia. Most of the exotic candidates can be described in our model as diquark-antidiquark tetraquarks.

Table 5: Fitted parameters of the (J, M^2) parent and daughter Regge trajectories for heavy quarkonia with natural and unnatural parity.

Trajectory	natural parity		unnatural parity	
	α (GeV^{-2})	α_0	α (GeV^{-2})	α_0
$c\bar{c}$	J/ψ		η_c	
parent	0.436 ± 0.014	-3.31 ± 0.22	0.416 ± 0.021	-3.90 ± 0.31
first daughter	0.488 ± 0.011	-5.63 ± 0.18	0.479 ± 0.015	-6.36 ± 0.24
second daughter	0.431 ± 0.036	-6.08 ± 0.68	0.414 ± 0.050	-6.66 ± 0.92
$c\bar{c}$	χ_{c0}		χ_{c1}	
parent	0.431 ± 0.016	-5.07 ± 0.25	0.461 ± 0.008	-4.66 ± 0.12
daughter	0.493 ± 0.031	-7.41 ± 0.53	0.456 ± 0.006	-5.83 ± 0.11
$b\bar{b}$	Υ		η_b	
parent	0.212 ± 0.022	-18.5 ± 2.3	0.184 ± 0.024	-16.7 ± 2.5
first daughter	0.246 ± 0.014	-23.8 ± 1.5	0.234 ± 0.016	-23.5 ± 1.7
second daughter	0.262 ± 0.010	-27.1 ± 1.1	0.248 ± 0.014	-26.5 ± 1.6
third daughter	0.246 ± 0.027	-26.6 ± 3.1	0.241 ± 0.026	-27.0 ± 3.0
$b\bar{b}$	χ_{b0}		χ_{b1}	
parent	0.228 ± 0.021	-22.3 ± 2.2	0.239 ± 0.018	-22.5 ± 1.9
daughter	0.254 ± 0.009	-26.7 ± 1.0	0.267 ± 0.006	-27.1 ± 0.7

Table 6: Fitted parameters of the nonlinear Regge trajectories for heavy quarkonia and B_c mesons.

Meson	γ (GeV^{-2})	γ_0	γ_1	τ (GeV^{-2})	τ_0	τ_1
Υ	0.33	32.2	32.3	0.22	22.2	10.1
η_b	0.33	32.9	15.0			
χ_{b0}	0.33	33.7	6.57			
B_c^*	0.32	13.3	12.5	0.21	9.25	4.00
B_c	0.32	14.2	6.21			
B_{c0}	0.32	15.1	2.99			
J/ψ	0.48	4.25	5.47	0.31	3.19	0.82
η_c	0.48	5.19	3.56			