Holography and QCD

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outline

- why QCD is a candidate for a holographic description
- dual (holographic) models
- light hadrons - spectroscopy
  - chiral lagrangian
  - shadowing in nuclei
- QCD at finite temperature and density

fifty years of high energy strong interaction

- string theory born in the sixities from hadronic phenomenology   Veneziano
  - high spin particles <-> various oscillation modes of a string
  - -> Regge spectrum, s/t channel duality
  - troubles with large angle scattering behaviour and Bjorken scaling
- discovery of Asymptotic Freedom   Gross, Wilczek, Politzer
  - QCD adopted and strings abandoned to describe strong interaction Physics
  - string theory developed as a theory of gravity
- however:
  - flux tubes act like strings   Wilson, lattice simulations
    - what is the confinement mechanism?
  - Wilson/Polyakov loops in gauge theories similar to boundary of 5D strings
  - perturbative expansion in g_s actually an expansion in $\lambda = g^2 N_c$ and $1/N_c$,
    expansion in $1/N_c$ ~ sum over string-like diagrams   't Hooft

any relation between a 4D gauge theory (QCD) and a (high dimensional) string theory?

Late 90's:

a remarkable connection conjectured between certain string theory in certain curved space-times and certain gauge theories in flat (3+1) dimensional space-time

AdS/CFT (Anti de Sitter/Conformal Field Theory) correspondence conjecture (or Maldacena conjecture)

generalization of this idea useful for strong interaction physics

- gauge theories
- strong coupling regime $g^2_{YM} N_c \gg 1$

AdS/CFT correspondence

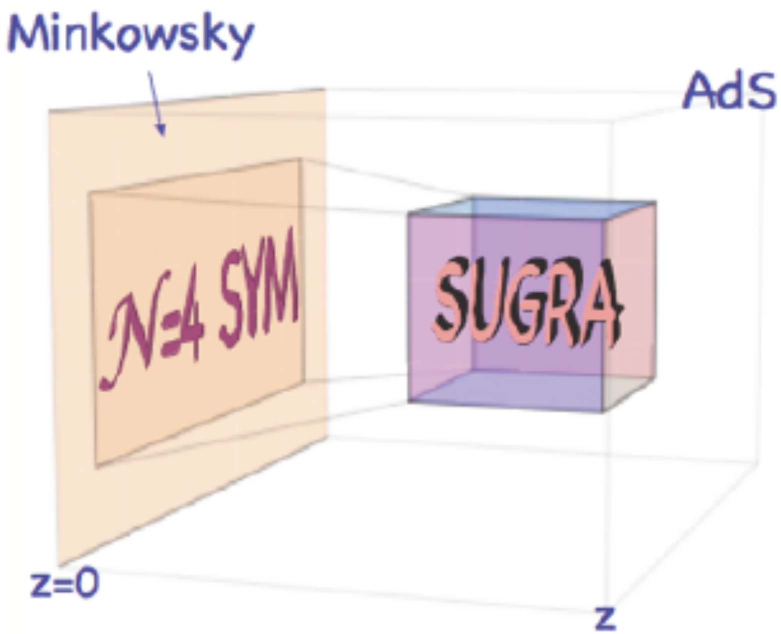
original proposal

correspondence between a low energy supergravity approximation to D=10 Type IIB string theory on $AdS_5 \times S^5$ and a $\mathcal{N}$=4 SYM theory with gauge group $SU(N_c)$ at large N_c

Maldacena '98

more general

equivalence (duality) between a gravity theory defined in $AdS_{d+1} \times C$ (C a compact manifold) and a conformal field theory (CFT) defined on the boundary of AdS_{d+1} (M_d)



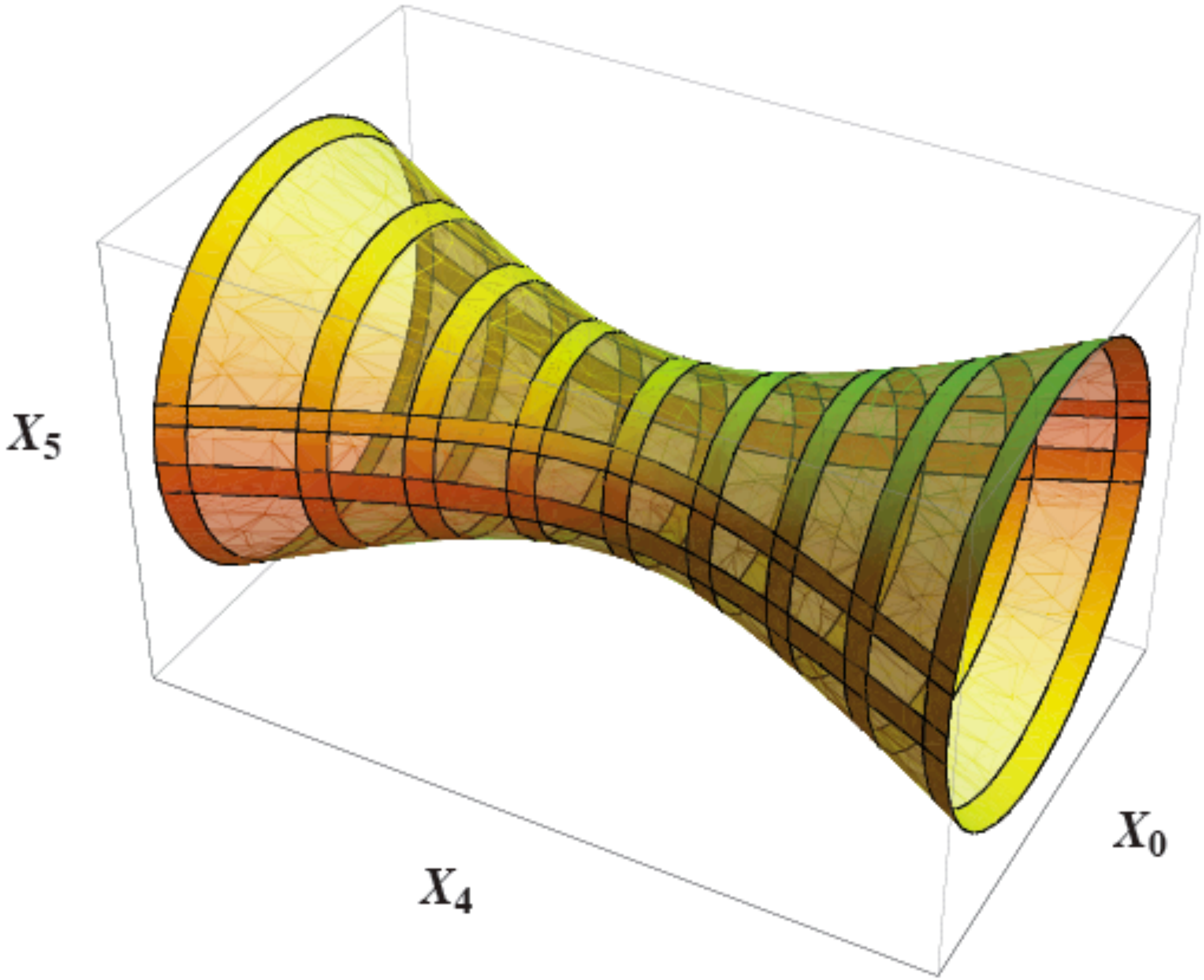
peculiar role of the d+1 dimensional Anti de Sitter (AdS_{d+1}) space:

- solution of Einstein's eqs. in the vacuum with negative cosmological constant
- negative curvature
- embedded in R^{d+2} (with coordinates (X^0,...X^{d+1})) it is defined by

$$\left(X^0\right)^2 - \sum_{i=1}^{d}\left(X^i\right)^2 + \left(X^{d+1}\right)^2 = R^2$$

- group of isometries SO(2,d) ((d+2)(d+1)/2 generators)
- it has a boundary M_d
- on the boundary M_d the coordinate transformation belonging to SO(2,d) are conformal transformations ((d+2)(d+1)/2 generators)

	Transformation	Generator	
Translation	$x'^\mu = x^\mu + a^\mu$	$P_\mu = -i\,\partial_\mu$	d generators
Dilation	$x'^\mu = a\,x^\mu$	$D = -i\,x^\mu\,\partial_\mu$	1 generator
Rotation	$x'^\mu = M^\mu{}_\nu\,x^\nu$	$L_{\mu\nu} = i\,(x_\mu\partial_\nu - x_\nu\partial_\mu)$	d (d-1)/2 generators
Special conf.	$x'^\mu = \dfrac{x^\mu - b^\mu x^2}{1 - 2\,b\cdot x + b^2\,x^2}$	$K_\mu = -i\,(2\,x_\mu x^\nu\partial_\nu - x^2\partial_\mu)$	d generators



F. Giannuzzi

$x \in AdS_{d+1}$ $\quad x=(x^0,x^1,\ldots,x^{d-1},z)$ $\quad ds^2 = \frac{R^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right)$

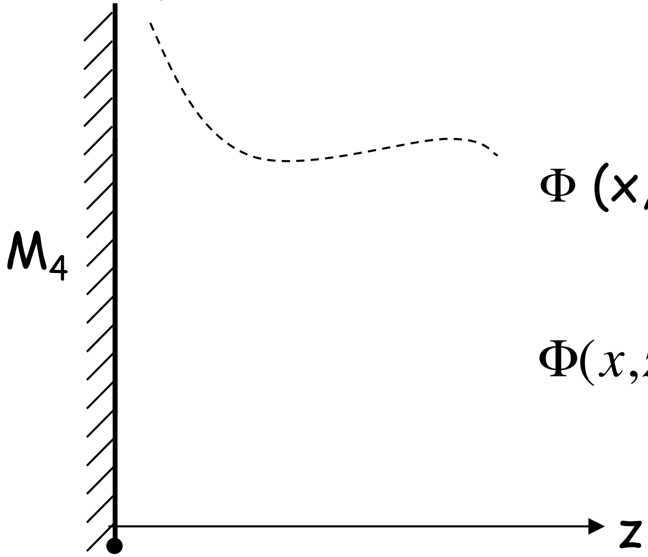
R Anti de Sitter radius

z>0 holographic coordinate

AdS/CFT correspondence

Maldacena,
Gubser, Klebanov and Polyakov
Witten

$\Phi_0(y)$ boundary value



Φ (x,z) scalar massless bulk field in AdS_5

$$\Phi(x,z) = \int_{M_4} d^4y\, K(x,z;y)\, \Phi_0(y)$$

bulk-to-boundary propagator

At Φ_0 (y) an operator O(y) of CFT on M_4 is associated

Φ_0 (y) coupled to O via $\int_{M_4} d^4y\, \Phi_0(y)\, O(y)$

$$\left\langle \exp \int_{M_4} \Phi_0 O \right\rangle_{CFT} \quad vs \quad \exp\left(-S_{AdS}(\Phi)\right)$$

Duality between gravity theory in AdS_{d+1} x C and the large N_c limit of a CFT is given by the generating functionals of the string and CFT correlation functions at the AdS boundary

d-dim generating functional
external source Φ_0

$$Z_{CFT}[\Phi_0] = \int dA \ \exp\left\{-S_{CFT} + \int d^d x \ \Phi_0 O\right\}$$

gravity partition function
in AdS_{d+1} x C with boundary value Φ_0

$$Z_{grav}[\Phi_0] = \int d[\Phi] \ \exp\left\{-S_{grav}[\Phi]\right\}$$

AdS/CFT correspondence conjecture

$$Z_{grav}[\Phi_0] = Z_{CFT}[\Phi_0]$$

Gubser, Klebanov and Polyakov
Witten

relation between the mass m_{d+1}^2 of the field Φ in the bulk and the dimension Δ of a p-form operator O on the boundary:

$$m_{d+1}^2 R^2 = (\Delta - p)(\Delta + p - d)$$

$$\frac{R_{AdS_5}}{\ell_s} = \left(g_{YM}^2 N\right)^{\frac{1}{4}}$$

$R_{AdS_5} >> \ell_s$ { supergravity approximation; large $g_{YM}^2 N$ }

is this relevant for QCD?

- AdS/CFT correspondence involves conformal field theories in M_4

- QCD is not conformal: mass scale Λ_{QCD}

- in the UV, neglecting quark masses and radiative effects, QCD is a nearly conformal theory

  - QCD counting rules, behaviour of form factors at large momentum transfer

  - success of light cone sum rule analyses

QCD a candidate for the AdS/CFT correspondence

- two conditions to be implemented:
  UV -> conformal behaviour -> AdS holographic space
  IR -> modification (at least) of the AdS geometry of the bulk

AdS/QCD: extradimensional models, motivated by the AdS/CFT correspondence conjecture, developed to compute low energy QCD observables

AdS/CFT and QCD

top-down

start from a string theory in high dimensions and try to construct a low dimensional theory with similarities with QCD

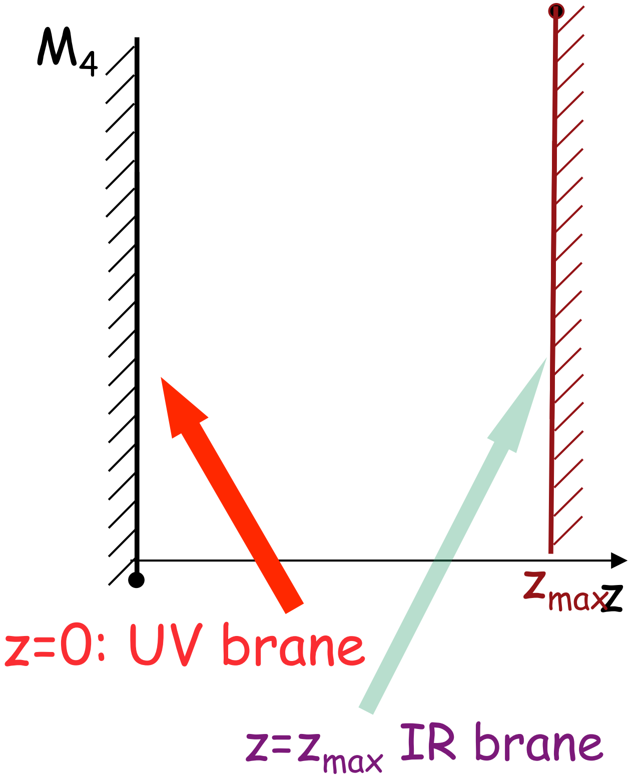
bottom-up

start from QCD and try to understand which feature its gravitational dual (if any) should have

bottom-up: hard wall model

$AdS_5 = M_4$ + radial (holographic) coordinate z

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\mu - dz^2)$$



$x \to \lambda x$

$z \to \lambda z$

maps scale transformations into z

if x is the length scale to which physics (hadrons) is examined, low values of x correspond to low values of z (UV brane)

large distances correspond to large z (IR)

maximum separation -> maximum value of z: z_{max}

non conformal metric
Polchinski, Strassler

↓ confinement

↓ $z_{max} \sim 1/\Lambda_{QCD}$

AdS/QCD dictionary

4D	5D
hadron interpolating operator O	normalizable modes $\psi(x,z)$
hadron mass^2	eigenvalue of a 5D wave eq.
conformal dimension Δ	5D mass m_5
UV	small z
mass scale Λ_{QCD}	z_{max}

AdS/QCD spectrum of light mesons I

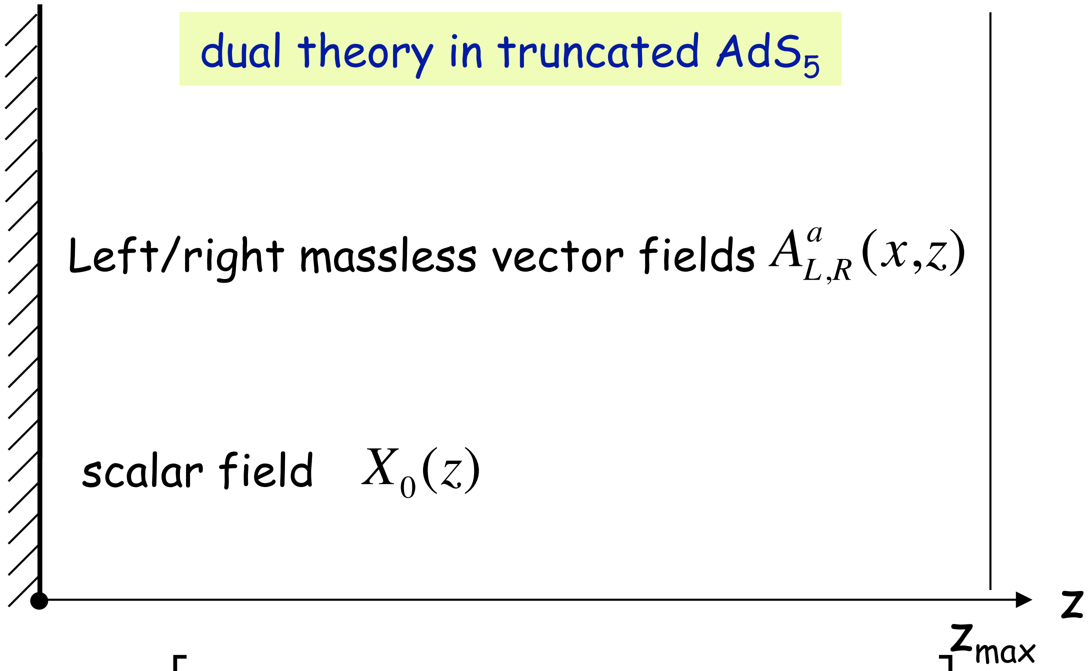
(Erlich, Katz, Son and Stephanov, Pomarol and Da Rold, ...)

- AdS_5 metric $$ds^2 = \frac{R^2}{z^2}(dx^2 - dz^2)$$
- confinement modeled by cutting off the holographic coordinate z to some value $0 < z < z_{\max}$ (hard wall)
- Boundary conditions at $z=z_{max}$
- Chiral symmetry: - two conserved currents in QCD : $J^{\mu}_{L,R} = \bar{q}\gamma^{\mu}\frac{1 \mp \gamma_5}{2}q$
  - two massless bulk fields in 5D : $A_{L,R}$ (x,z)
- Chiral symmetry breaking:
  - in QCD characterized by m_q and the condensate $\langle \bar{q}^{\alpha}_R q^{\beta}_L \rangle \propto \delta^{\alpha\beta}$ $(\alpha,\beta = 1 \ldots N_f)$
  - a massive scalar bulk field in 5D $X^{\alpha\beta}$:
    $X=X_0\, e^{2i\Pi}$
  - at small z: $$X^{\alpha\beta} \underset{z\to 0}{\to} (m_q z + \sigma z^3)\frac{\delta^{\alpha\beta}}{2} \qquad (\sigma \propto \langle \bar{q}q \rangle)$$

QCD in M^4

Left/right currents $j^a_{L,R}(x)$

chiral condensate $\langle \bar{q} q \rangle$



$$S = \frac{1}{k} \int d^5x \sqrt{|g|} \left[|DX|^2 - m_5^2 X^2 - \frac{1}{4 g_5^2} Tr\left(F_L^2 + F_R^2\right) \right]$$

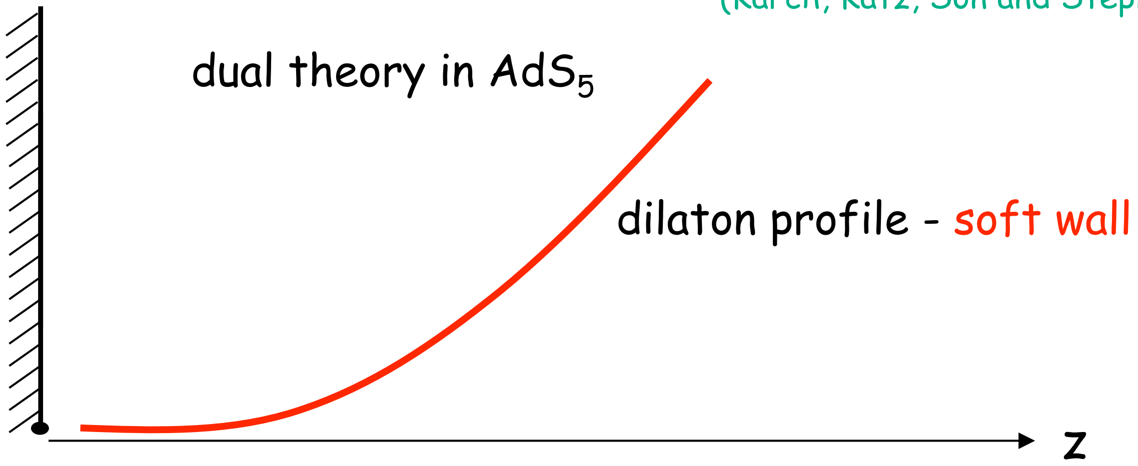
$$D_M X = \partial_M X - i A_{LM} X + i X A_{MR}$$

- a few hadronic parameters reproduced
- Gell-Mann Oakes Renner relation $m_\pi^2 \propto m_q$
- $m(\rho_n)^2 \sim n^2$ not Regge

AdS/QCD spectrum of light mesons II

(Karch, Katz, Son and Stephanov, Andreev,...)

QCD in M^4



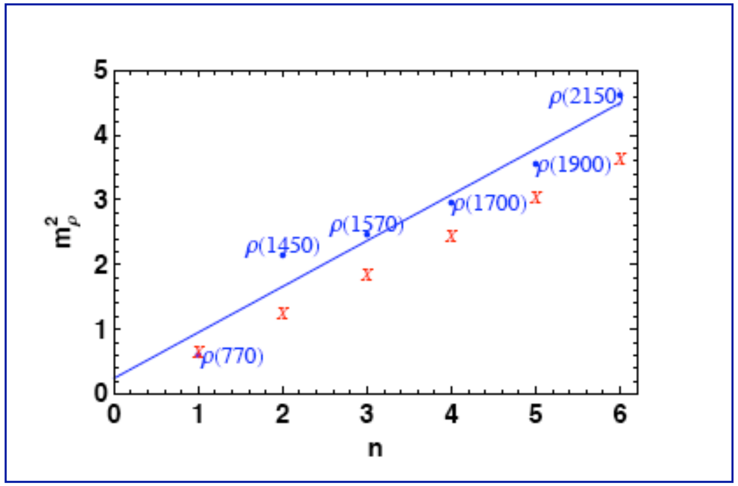
$$S = \frac{1}{k}\int d^5x\sqrt{|g|}\; e^{-\varphi(z)}\left[|DX|^2 - m_5^2X^2 - \frac{1}{4g_5^2}Tr\left(F_L^2 + F_R^2\right)\right]$$

$\varphi = c^2 z^2$ ⟶ $m_n^2 = 4c^2(n+1)$

scale

$c = m_\rho / 2$

Regge behaviour
of vector meson spectrum



light glueballs

De Fazio Giannuzzi Nicotri Jugeau PC, PLB652,73 PRD78,05500

- dilaton background fixed by the vector meson spectrum
- glueball operators - bulk fields correspondence according to the AdS/CFT rules

glueball $J^{PC} = 0^{++}$

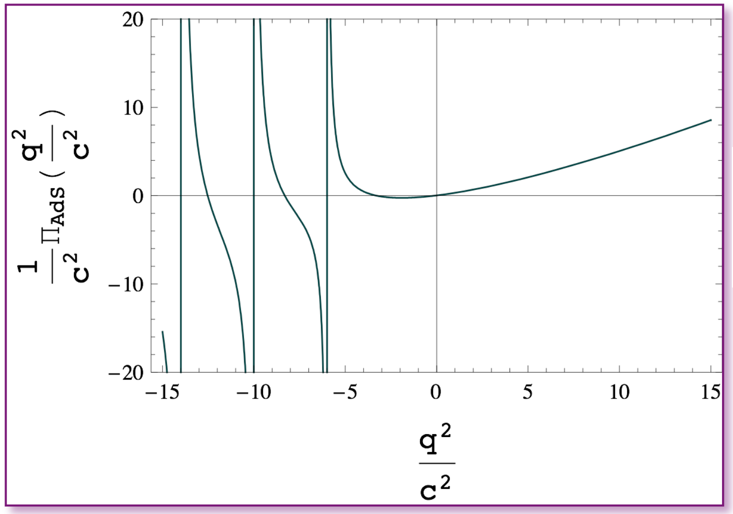
spectrum $m_n^2 = 4c^2(n+2)$

$m_{0^{++}}^2 = 2m_\rho^2$ $m_{0^{-+}}^2 = m_{0^{++}}^2$ $m_{1^{--}}^2 = 3m_\rho^2$

light scalar mesons

$m_n^2 = c^2(4n+6)$ $f_0(980)$ and $a_0(980)$ masses reproduced $\frac{m_{a_0}^2}{m_\rho^2} = \frac{3}{2}$ $(\exp = 1.59)$

$F_n^2 = \frac{3}{\pi^2} c^4 (n+1)$ decay constants compatible with QCD SR estimates



$$\Pi_{QCD}^{AB}(q^2) = i \int d^4x e^{iq\cdot x} \langle 0 | T[\mathcal{O}_S^A(x) \mathcal{O}_S^B(0)] | 0 \rangle$$

chiral lagrangian and low-energy constants

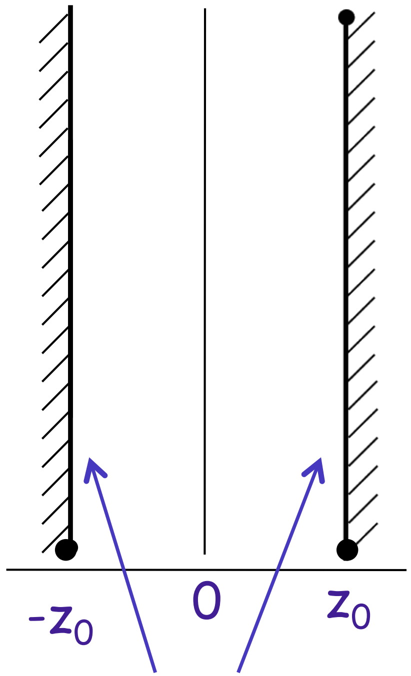
Sanz-Cillero, Zuo, PC JHEP1211 (2012) 012, JHEP1306 (2013) 020, NPB(13)

$$S = S_{\rm YM} + S_{\rm CS}$$

Yang-Mills $$S_{\rm YM} = -\int d^5x \,{\rm tr}\left[-f^2(z)\mathcal{F}_{z\mu}^2 + \frac{1}{2g^2(z)}\mathcal{F}_{\mu\nu}^2\right]$$

Chern-Simons $$S_{\rm CS} = -\kappa \int {\rm tr}\left[\mathcal{A}\mathcal{F}^2 + \frac{i}{2}\mathcal{A}^3\mathcal{F} - \frac{1}{10}\mathcal{A}^5\right].$$

$f^2(z),\ g^2(z)$ -> geometry



chiral symmetry broken
by the boundary conditions

$$\mathrm{U}(x^\mu) = \mathrm{P}\exp\left\{i\int_{-z_0}^{+z_0} \mathcal{A}_z(x^\mu, z')dz'\right\}$$

$$\mathrm{U}(x) \to g_R(x)\mathrm{U}(x)g_L^\dagger(x)$$

mode expansion

$$\mathcal{A}_\mu(x,z) = \ell_\mu(x)\psi_-(z) + r_\mu(x)\psi_+(z) + \sum_{n=1}^{\infty} B_\mu^{(n)}(x)\psi_n(z)$$

$$\mathcal{A}_\mu(x,-z_0) = \ell_\mu(x)\,, \qquad \mathcal{A}_\mu(x,z_0) = r_\mu(x)\,.$$

$$\mathcal{A}_\mu(x,z) = i\Gamma_\mu(x) + \frac{u_\mu(x)}{2}\psi_0(z) + \sum_{n=1}^{\infty} v_\mu^n(x)\psi_{2n-1}(z) + \sum_{n=1}^{\infty} a_\mu^n(x)\psi_{2n}(z)$$

$$u_\mu(x) \equiv \mathrm{i}\left\{\xi_R^\dagger(x)(\partial_\mu - \mathrm{i}r_\mu)\xi_R(x) - \xi_L^\dagger(x)(\partial_\mu - \mathrm{i}\ell_\mu)\xi_L(x)\right\}$$

$$\Gamma_\mu(x) \equiv \frac{1}{2}\left\{\xi_R^\dagger(x)(\partial_\mu - \mathrm{i}r_\mu)\xi_R(x) + \xi_L^\dagger(x)(\partial_\mu - \mathrm{i}\ell_\mu)\xi_L(x)\right\}$$

$$V(Q,z) = \sum_{n=1}^{\infty} \frac{g_{v^n}\psi_{2n-1}(z)}{Q^2 + m_{v^n}^2}, \qquad A(Q,z) = \sum_{n=1}^{\infty} \frac{g_{a^n}\psi_{2n}(z)}{Q^2 + m_{a^n}^2}$$

$$f_\pm^{\mu\nu} \equiv \xi_L^{-1}\ell^{\mu\nu}\xi_L \pm \xi_R^{-1}r^{\mu\nu}\xi_R$$

chiral Lagrangian at O(p⁴)

Hirn, Sanz, Sakai Sugimoto

$$S_2[\pi] + S_4[\pi] = \int d^4x \Big[\frac{f_\pi^2}{4} < u_\mu u^\mu >$$
$$+L_1 < u_\mu u^\mu >^2 + L_2 < u_\mu u_\nu >< u^\mu u^\nu > + L_3 < u_\mu u^\mu u_\nu u_\nu >$$
$$-iL_9 < f_{+\mu\nu} u^\mu u^\nu > + \frac{L_{10}}{4} < f_{+\mu\nu} f_+^{\mu\nu} - f_{-\mu\nu} f_-^{\mu\nu} >$$
$$+\frac{H_1}{2} < f_{+\mu\nu} f_+^{\mu\nu} + f_{-\mu\nu} f_-^{\mu\nu} > \Big], \qquad (2.23)$$

low energy constants in terms of dual quantities

$$f_\pi^2 = 4\left(\int_{-z_0}^{z_0} \frac{\mathrm{d}z}{f^2(z)}\right)^{-1},$$

$$L_1 = \frac{1}{2}L_2 = -\frac{1}{6}L_3 = \frac{1}{32}\int_{-z_0}^{z_0} \frac{(1-\psi_0^2)^2}{g^2(z)}\,\mathrm{d}z$$

$$L_9 = -L_{10} = \frac{1}{4}\int_{-z_0}^{z_0} \frac{1-\psi_0^2}{g^2(z)}\,\mathrm{d}z\,,$$

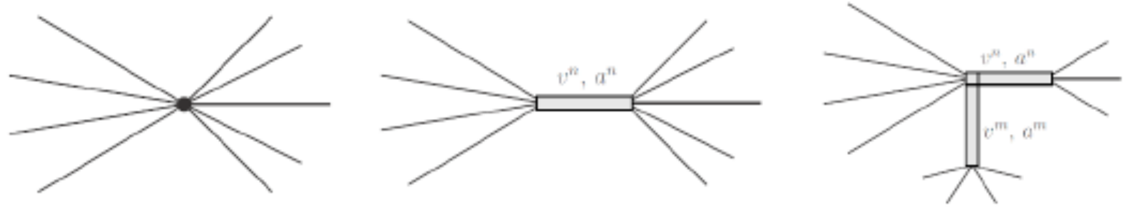
$$H_1 = -\frac{1}{8}\int_{-z_0}^{z_0} \frac{1+\psi_0^2}{g^2(z)}\,\mathrm{d}z\,.$$

	flat [19]	Cosh [19]	hard-wall [24]	Sakai-Sugimoto [23, 25]	χPT [2–4]
$10^3\ L_1$	0.5	0.5	0.5	0.5	0.9 ± 0.3
$10^3\ L_2$	1.0	1.0	1.0	1.0	1.7 ± 0.7
$10^3\ L_3$	−3.1	−3.2	−3.1	−3.1	-4.4 ± 2.5
$10^3\ L_9$	5.2	6.3	6.8	7.7	7.4 ± 0.7
$10^3\ L_{10}$	−5.2	−6.3	−6.8	−7.7	-6.0 ± 0.7
$10^3\ Y$	0.5	0.5	0.5	0.6	—
$10^3\ Z$	0.6	0.8	1.0	1.0	—

chiral Lagrangian at O(p⁶) : about 100 terms $\langle u\cdot u\rangle\langle u_\mu u_\nu u^\mu u^\nu\rangle$ ....

$\langle u_\mu u_\nu u_\rho u^\mu u^\nu u^\rho\rangle$ ....

$\langle u_\mu u_\nu u_\rho\rangle^2$ .... Bijnens, G. Colangelo, Ecker 03



low energy constants in holography

	"Cosh"	DSE [18]	χPT	CQM [8]	Res. Lagr.	VMD
C_{12}^W	-2.1	$-5.13^{+0.15}_{-0.25}$			-4.3 ± 0.3 [63]	
C_{13}^W	-8.8	$-6.37^{+0.18}_{-0.31}$	-70 ± 60 [8]	14 ± 15		-20.0 [8]
			-10 ± 70 [8]	-7 ± 20		
C_{14}^W	-1.3	$-2.00^{+0.06}_{-0.10}$	30 ± 11 [8]	10 ± 8		-6.0 [8]
			1 ± 15 [8]	-1 ± 10		
C_{15}^W	4.4	$4.17^{+0.12}_{-0.20}$	-25 ± 24 [8]	20 ± 7		2.0 [8]
			-3 ± 29 [8]	9 ± 10		
C_{16}^W	-0.2	$3.58^{+0.10}_{-0.17}$				
C_{17}^W	-0.1	$1.98^{+0.06}_{-0.10}$				
C_{19}^W	-7.0	$0.29^{+0.01}_{-0.01}$				
C_{20}^W	-0.4	$1.83^{+0.05}_{-0.09}$				
C_{21}^W	2.6	$2.48^{+0.07}_{-0.12}$				
C_{22}^W	7.9	$5.01^{+0.14}_{-0.24}$	6.5 ± 0.8 [8]	3.9 ± 0.4	8.0 [64]	8.0 [65–67]
			5.1 ± 0.7 [8]		6.5 [68]	
			5.4 ± 0.8 [69]		8.1 ± 0.8 [70]	
			$7.0^{+1.0}_{-1.5}$ [71]			
C_{23}^W	0.9	$2.74^{+0.08}_{-0.13}$				

low energy constants : relations

Sanz-Cillero Zuo PC JHEP 2012

$$
\begin{aligned}
C_{12}^W &= -\frac{N_C}{384\pi^2 f_\pi^2}(40L_1 - Z) \\
C_{13}^W &= \frac{5N_C}{96\pi^2 f_\pi^2}(4L_1 - L_9) \\
C_{14}^W &= \frac{N_C}{64\pi^2 f_\pi^2}(8L_1 - L_9) \\
C_{15}^W &= \frac{N_C}{192\pi^2 f_\pi^2}(52L_1 - L_9) \\
C_{16}^W &= \frac{N_C}{384\pi^2 f_\pi^2}(104L_1 - 8L_9 - 7Z) \\
C_{17}^W &= \frac{N_C}{384\pi^2 f_\pi^2}(72L_1 - 6L_9 - Z) \\
C_{19}^W &= \frac{N_C}{96\pi^2 f_\pi^2}(4L_1 - 3L_9) \\
C_{20}^W &= \frac{N_C}{192\pi^2 f_\pi^2}(80L_1 - 7L_9) \\
C_{21}^W &= \frac{N_C}{192\pi^2 f_\pi^2}(12L_1 + L_9) \\
C_{22}^W &= \frac{N_C}{32\pi^2 f_\pi^2}L_9 \\
C_{23}^W &= \frac{N_C}{96\pi^2 f_\pi^2}(L_9 - 8L_1).
\end{aligned}
$$

relations between, e.g., form factors
in different processes ?

nuclear shadowing

Agozzino, Castorina, PC 2013

Photon-nucleon interaction in the bulk

Polchinski Strassler 2002

contributions to F_2

Brower et al 2010, Strassler et al 2007

$$F_2^N(x,Q^2) = \frac{g_0^2\rho^{3/2}}{32\pi^{5/2}}\int dz dz' \frac{zz'Q^2}{\tau^{1/2}} P_{13}(z,Q^2)P_{24}(z') \times e^{(1-\rho)\tau}\exp\left[\Phi(z,z',\tau)\right] , \qquad (8)$$

$$F_{2ct}^N(x,Q^2,z_0) = \frac{g_0^2\rho^{3/2}}{32\pi^{5/2}}\int dz dz' \frac{zz'Q^2}{\tau^{1/2}} P_{13}(z,Q^2)P_{24}(z') \times e^{(1-\rho)\tau}\, e^{-\frac{\log^2(zz'/z_0^2)}{\rho\tau}}\, G(z,z',\tau). \qquad (15)$$

$$P_{13}(z,Q^2) = \frac{1}{z}(Qz)^2[K_0^2(Qz)+K_1^2(Qz)]$$

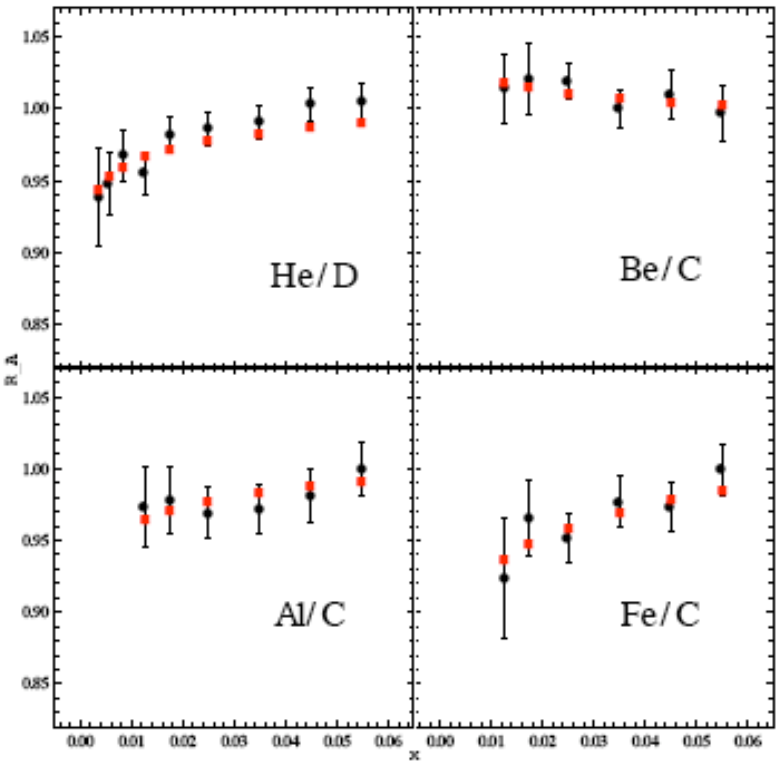
photon in the bulk

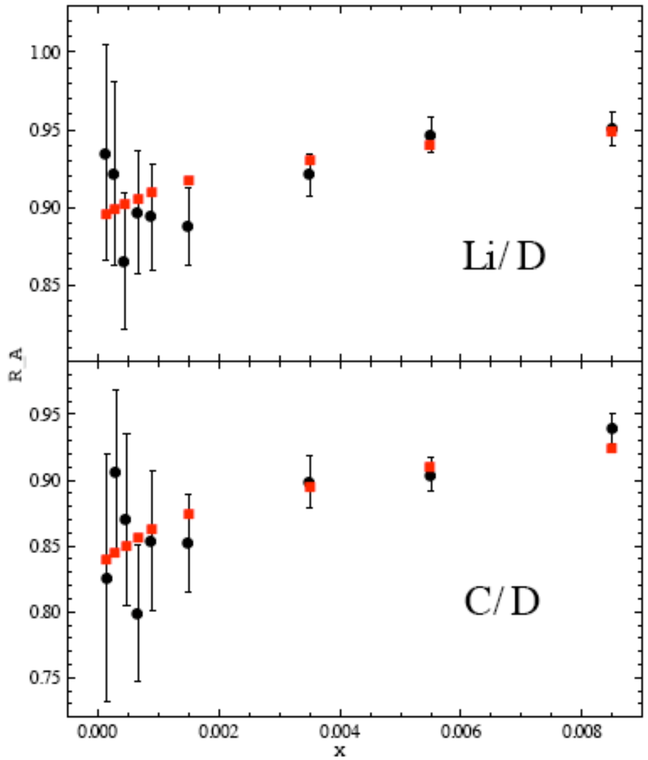
$$P_{24}(z) = \sqrt{-g}\left(\frac{z}{R}\right)^2 \phi^N(z)\phi^N(z) \ .$$

nucleon in the bulk

Q^2 rescaling

$$F_2^A(x,Q^2) = F_{2,cl}^N\left(x,\frac{Q^2}{\lambda_A^2}\right) + F_{2,ct}^N\left(x,\frac{Q^2}{\lambda_A^2},\frac{Q_0^2}{\lambda_A^2}\right)$$





$$R^A = \frac{F_2^A}{F_2^N}$$

A	λ_A(AdS/CFT)	$\lambda_{A,dip}$ [20]	$\lambda_{A,nuc}$ [10]
Li	1.069	1.125	1.045
Be	1.073	1.128	1.077
C	1.111	1.143	1.104
Al	1.184	1.243	1.140
Ca	1.219	1.315	1.137
Fe	1.251	1.387	1.154
Pb	1.327	1.755	1.188

no drawback?

A test of the AdS/QCD soft-wall model: AV*V correlation function

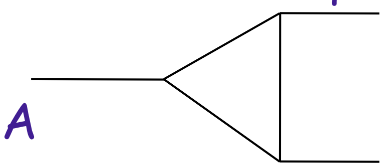
$J_\mu = \bar{q}\gamma_\mu V q$

$J_\nu^5 = \bar{q}\gamma_\nu \gamma_5 A q$

consider the corr. function $T_{\mu\nu}(q,k) = i\int d^4x\, e^{iq\cdot x}\langle 0|T[J_\mu(x)J_\nu^5(0)]|\gamma(k,\varepsilon)\rangle$

$$T_{\mu\nu}(q,k) = e\varepsilon^\sigma T_{\mu\nu\sigma}(q,k)$$

$$T_{\mu\nu\sigma}(q,k) = i^2\int d^4x\, d^4y\, \mathrm{e}^{\mathrm{i}q\cdot x-\mathrm{i}k\cdot y}\langle 0|T[J_\mu(x)J_\nu^5(0)J_\sigma^{em}(y)]|0\rangle$$



soft em field: $k^2=0$ k->0

$$T_{\mu\nu}(q,k) = -\frac{i}{4\pi^2}Tr[QVA]\{\omega_T(q^2)(-q^2\tilde{f}_{\mu\nu} + q_\mu q^\lambda \tilde{f}_{\lambda\nu} - q_\nu q^\lambda \tilde{f}_{\lambda\mu}) + \omega_L(q^2)q_\nu q^\lambda \tilde{f}_{\lambda\mu}\}$$

transverse ↑ longitudinal ↑

$$\tilde{f}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}f^{\alpha\beta}$$

$$f^{\alpha\beta} = k^\alpha\varepsilon^\beta - k^\beta\varepsilon^\alpha$$

QCD chiral limit

$$\omega_L(Q^2) = \frac{2N_c}{Q^2}$$

Adler Bardeen
Bell Jackiw

$\omega_T(Q^2)$ dynamical quantity
no perturbative corrections to all orders

Vainshtein 2003

chiral limit:

$$\omega_L(Q^2) = \frac{2N_c}{Q^2}$$

OPE
$Q^2 \to \infty$

$$\omega_T(Q^2) = \frac{N_c}{Q^2} + \frac{128\pi^3\alpha_s\chi\langle\bar{q}q\rangle^2}{9Q^6} + O\left(\frac{1}{Q^8}\right)$$

Vainshtein 2003

$$\langle 0|\bar{q}\sigma_{\mu\nu}q|\gamma\rangle = ie\chi\langle\bar{q}q\rangle f_{\mu\nu}$$

χ magnetic susceptibility of the chiral condensate

$$\omega_L(Q^2) = 2\ \omega_T(Q^2)$$

$Q^2 \to \infty$

m > 0 :

$Q^2 \to \infty$

$$\omega_L(Q^2) = \frac{2N_c}{Q^2}\left[1 + \frac{2m^2}{Q^2}\ln\frac{m^2}{Q^2} - \frac{8\pi^2 m\langle\bar{q}q\rangle\chi}{N_c Q^2} + O\left(\frac{m^4}{Q^4}\right)\right]$$

$$2\omega_T(Q^2) = \frac{2N_c}{Q^2}\left[1 + \frac{2m^2}{Q^2}\ln\frac{m^2}{Q^2} - \frac{8\pi^2 m\langle\bar{q}q\rangle\chi}{N_c Q^2} + O\left(\frac{m^4}{Q^4}\right)\right]$$

Czarnecki Marciano Vainshtein

to which extent AdS/QCD models reproduces these QCD results?

5d YM action

$$S_{YM} = \frac{1}{k_{YM}} \int d^5x \sqrt{|g|}\; e^{-\varphi(z)}\; Tr\left[|DX|^2 - m_5^2 |X|^2 - \frac{1}{4g_5^2}\left(F_L^2 + F_R^2\right)\right]$$

$$D^M X = \partial^M X - iA_L^M X + iXA_R^M$$

in terms of vector and axial fields

$$V = (A_L + A_R)/2$$
$$A = (A_L - A_R)/2$$

$$S_{YM} = \frac{1}{k_{YM}} \int d^5x \sqrt{|g|}\; e^{-\varphi(z)}\; Tr\left[|DX|^2 - m_5^2 |X|^2 - \frac{1}{2g_5^2}\left(F_V^2 + F_A^2\right)\right]$$

$$D^M X = \partial^M X - i\left[V^M, X\right] - i\left\{A^M, X\right\}$$
$$F_V^{MN} = \partial^M V^N - \partial^N V^M - i\left[V^M, V^N\right] - i\left[A^M, A^N\right]$$
$$F_A^{MN} = \partial^M A^N - \partial^N A^M - i\left[V^M, A^N\right] - i\left[A^M, V^N\right]$$

matching to QCD in SS corr. funct. → $k_{YM} = \frac{16\pi^2}{N_C}$

VV corr. funct. → $g_5^2 = \frac{3}{4}$

vector meson spectrum $m^2(\rho_n) = 4c^2(n+1)$

axial meson spectrum, mass of scalars, decay constants, strong couplings, …

$U(N_f)_L \times U(N_f)_R$ to describe em interactions

Chern-Simons term

$$S_{CS} = S_{CS}(A_L) - S_{CS}(A_R)$$

$$S_{CS}(A) = k_{CS}\int d^5x \quad Tr\left[AF^2 - \frac{i}{2}A^3F - \frac{1}{10}A^5\right]$$

relevant term

$$S_{CS+b} = 3k_{CS}\varepsilon_{ABCDE}\int d^5x \quad Tr\left[A^A\left\{F_V^{BC}, F_V^{DE}\right\}\right]$$

$$S_{5d}^{eff} = S_{YM} + S_{CS+b}$$

$$\omega_L(Q^2) = -\frac{2N_C}{Q^2}\int_0^\infty dy\, A_{par}(Q^2,y)\,\partial_y V(Q^2,y)$$

integral over
the holographic coordinate

$$\omega_T(Q^2) = -\frac{2N_C}{Q^2}\int_0^\infty dy\, A_{perp}(Q^2,y)\,\partial_y V(Q^2,y)$$

equations of motion (eom) for V(q,z), A_{par}(q,z), A_{perp} (q,z) -> ω_L , ω_T

various cases investigated:

$$m = 0,\ m \neq 0$$

$$\langle \bar{q}q \rangle = 0, \quad \langle \bar{q}q \rangle \neq 0$$

De Fazio, Giannuzzi, Nicotri, Sanz-Cillero, PC,
PRD85, 035013

$m = 0 \qquad \langle \bar{q} q \rangle = 0$

$$A_{par}(Q^2, y) = 1$$

$$A_{perp}(Q^2, y) = V(Q^2, y)$$

$$\omega_L(Q^2) = -\frac{2N_C}{Q^2} \int_0^\infty dy\, A_{par}(Q^2, y)\, \partial_y V(Q^2, y) = \frac{2N_C}{Q^2}$$

$$\omega_T(Q^2) = -\frac{2N_C}{Q^2} \int_0^\infty dy\, A_{perp}(Q^2, y)\, \partial_y V(Q^2, y) = \frac{N_C}{Q^2}$$

$\omega_L (Q^2) = 2\, \omega_T (Q^2)$

AdS/QCD OK

$m=0 \quad \langle \bar{q}q \rangle \neq 0$ $\quad A_{par}(Q^2,y)=1 \quad$ $\omega_L(Q^2)=\frac{2N_C}{Q^2}$ → AdS/QCD fits QCD

AdS/QCD

$$\omega_T(Q^2)=\frac{N_c}{Q^2}-\tau g_5^2\sigma^2\frac{2N_c}{Q^8}+O\left(\frac{1}{Q^{10}}\right)$$

$Q^2 \to \infty$ $\quad \tau = 2.472$

$\sigma \to \langle \bar{q}q \rangle$

QCD

$$\omega_T(Q^2)=\frac{N_c}{Q^2}+\frac{128\pi^3\alpha_s\chi\langle \bar{q}q \rangle^2}{9Q^6}+O\left(\frac{1}{Q^8}\right)$$

$$\langle 0|\bar{q}\sigma_{\mu\nu}q|\gamma\rangle = ie\chi\langle \bar{q}q \rangle f_{\mu\nu}$$

in this AdS/QCD model, terms governed by the magnetic susceptibility of the chiral condensate are missed in ω_T

$m \neq 0 \quad \langle \bar{q}q \rangle \neq 0$

AdS/QCD

$$\omega_T(Q^2)=\frac{N_c}{Q^2}\left[1-\frac{g_5^2m^2}{3Q^2}-\frac{2g_5^2m^2c^2}{5Q^4}+\frac{g_5^4m^4}{6Q^4}-\frac{8g_5^2m\sigma}{5Q^4}\right]+O\left(\frac{1}{Q^8}\right)$$

QCD

$$\omega_T(Q^2)=\frac{N_c}{Q^2}\left[1+\frac{2m^2}{Q^2}\ln\frac{m^2}{Q^2}-\frac{8\pi^2m\langle \bar{q}q \rangle\chi}{N_cQ^2}+O\left(\frac{m^4}{Q^4}\right)\right]$$

AdS/QCD

$$\omega_L(Q^2)=\frac{2N_c}{Q^2}-\left[1-\Pi(Q^2,0)\right]N_c\left[\frac{g_5^2m^2}{Q^4}+\frac{8g_5^2m\sigma}{Q^6}-\frac{2g_5^4m^4}{3Q^6}+O\left(\frac{1}{Q^8}\right)\right]$$

QCD

$$\omega_L(Q^2)=\frac{2N_c}{Q^2}\left[1+\frac{2m^2}{Q^2}\ln\frac{m^2}{Q^2}-\frac{8\pi^2m\langle \bar{q}q \rangle\chi}{N_cQ^2}+O\left(\frac{m^4}{Q^4}\right)\right]$$

the structure of the power corrections is not properly reproduced

in spite of the extreme simplicity and economicity
of the holographic models,
more QCD properties that one could have expected
are reproduced

finite temperature and chemical potential

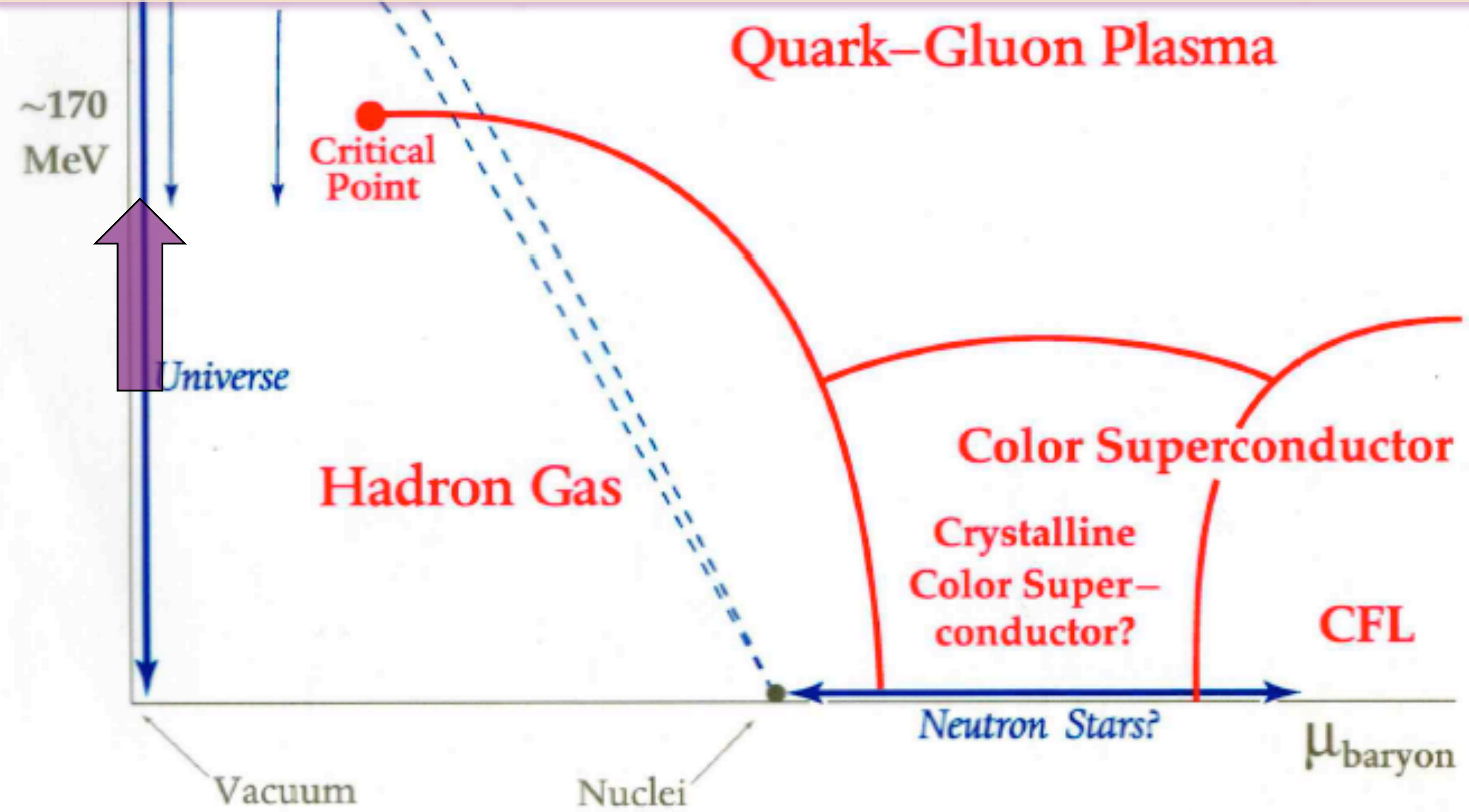
increasing T: chiral symmetry restoration $\langle \bar{q}q \rangle \rightarrow 0$

deconfinement

common expectations: hadron masses change

widths broaden

hadrons melt at some temperature



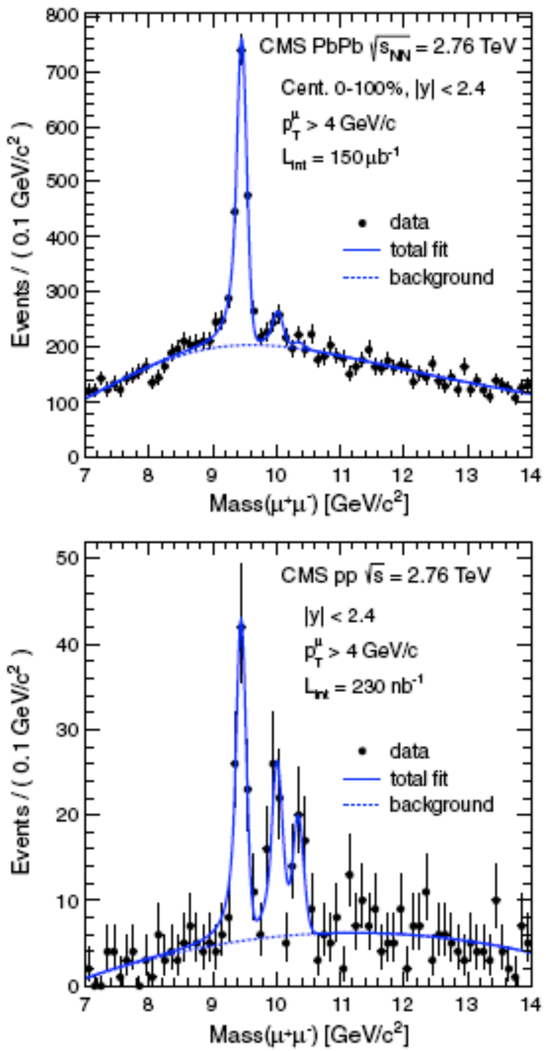
experiments at SpS, RHIC, LHC

common expectations: hadron masses change
widths broaden
hadrons melt at some temperature

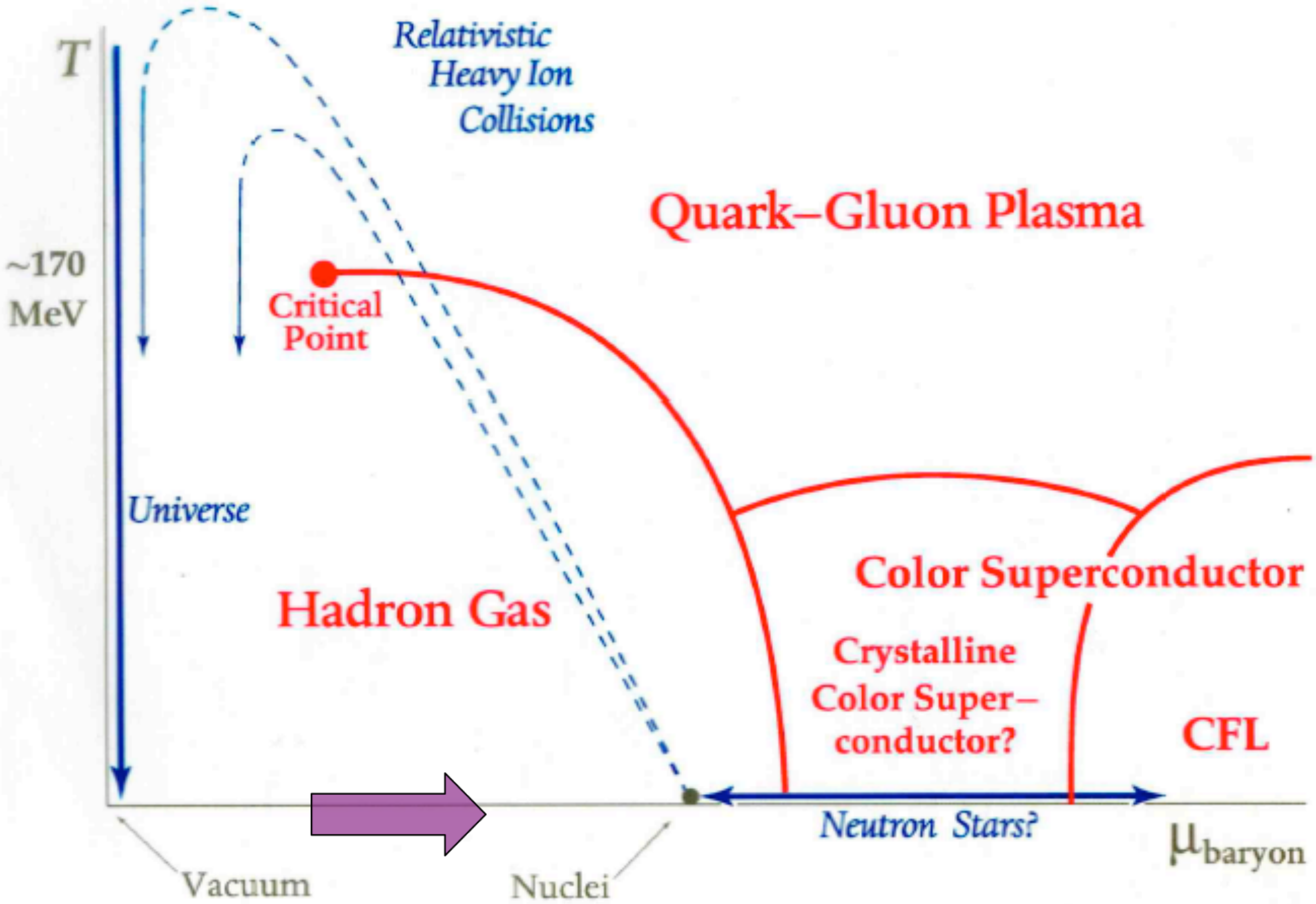
PHYSICAL REVIEW LETTERS week ending 30 NOVEMBER 2012

p_T measurement based
one has a resolution
in this analysis.
extensive forward
uartz-fiber Čerenkov
which cover the range
used for event selec-
PbPb collisions. The
nds to the fraction of
ng at 0% for the most
percentiles of the dis-
n the HF [7,8]. The
sis are 50–100%, 40–
5–10%, and 0–5%,
st HF energy deposit.
described in Ref. [7],
ticipating in the colli-
overlap function (T_{AA})
y class. The T_{AA} factor
nucleon-nucleon (NN)
lementary NN cross
NN-equivalent inte-
ision, at a given event

h their dimuon decay.
th a hardware-based
es in the muon detec-
ty requirements are
the pp online selec-
pidity thresholds are
bPb data, events are
reconstructed primary
s, and the presence of
at least three towers in
These criteria reduce



increasing μ: new order parameters
behaviour of hadrons unknown



experiments at Fair/GSI

The case of finite temperature

($N=4$ at $T\neq 0$ on $S^3 \times S^1$ and $N_c \to \infty$)

two solutions

E. Witten, Adv. Theor. Math. Phys. 2, 505

Thermal-AdS

$$ds^2 = \frac{R^2}{z^2}\left(dt^2 - d\bar{x}^2 - dz^2\right)$$

Black Hole-AdS

$$ds^2 = \frac{R^2}{z^2}\left(f(z)dt^2 - d\bar{x}^2 - \frac{dz^2}{f(z)}\right)$$

$$f(z) = 1 - \frac{z^4}{z_h^4} \qquad 0 < z < z_h$$

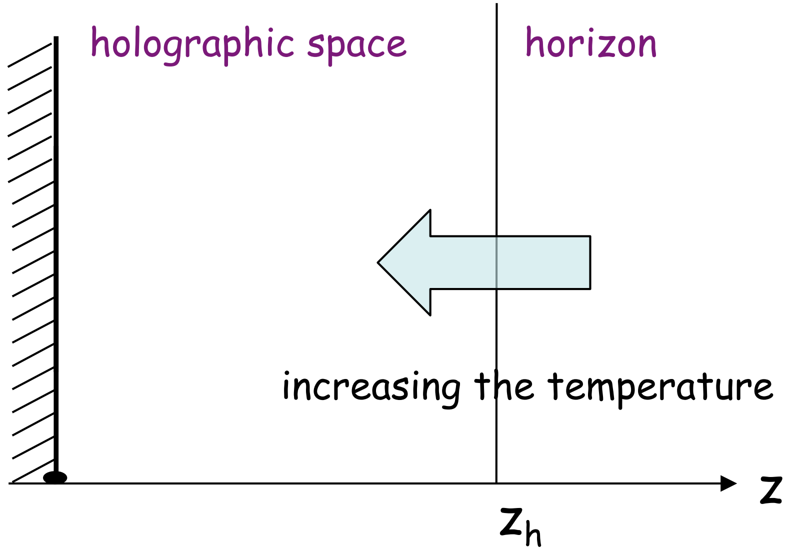
$$0 \leqslant \tau < \beta = \frac{1}{T}$$

black hole horizon

$$z_h = \frac{1}{\pi T}$$

Hawking temperature

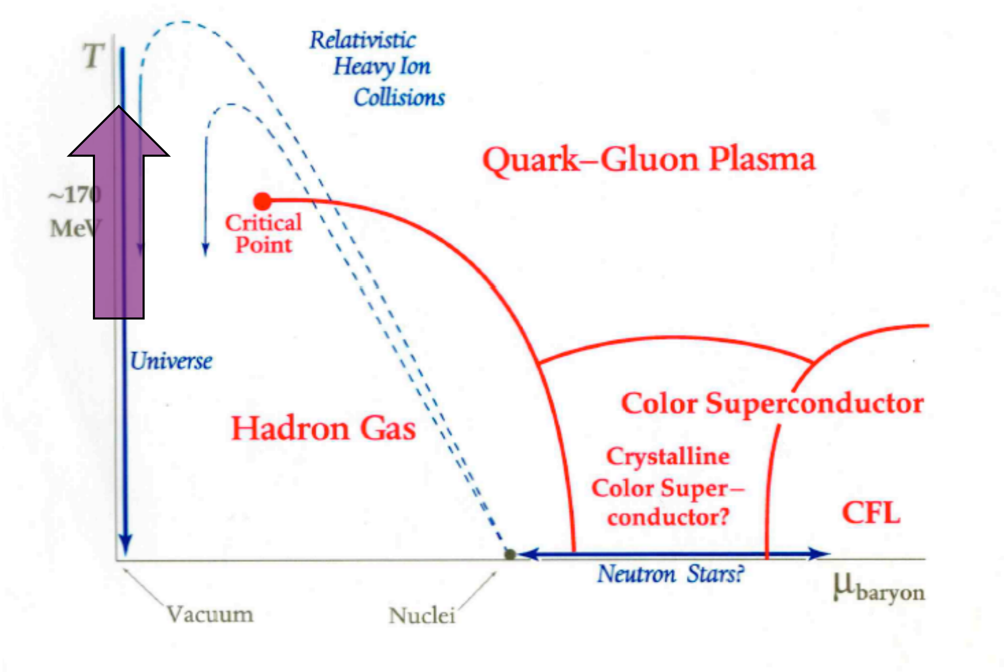
periodic Euclidean time τ extended to β'
$T=1/\beta'$



Hawking-Page phase transition possible between the two geometries

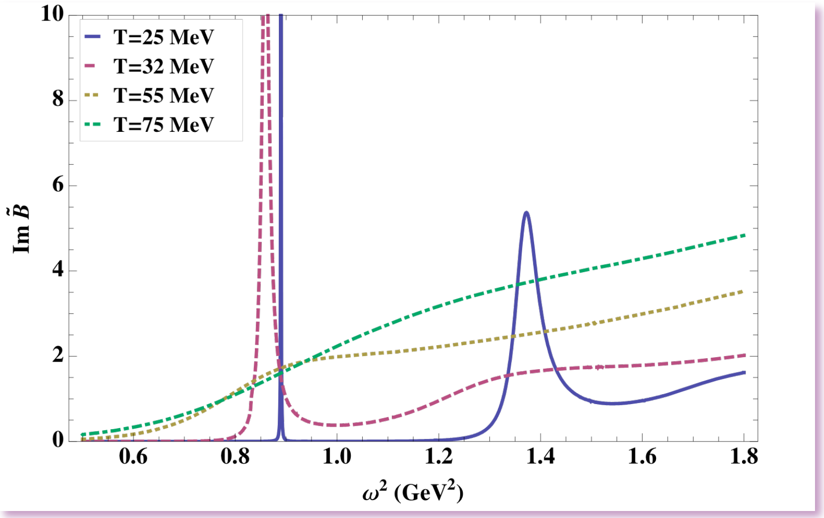
E. Witten, Adv. Theor. Math. Phys. 2, 505

deconfinement in QCD ⟷ phase transition between two geometries in the dual space



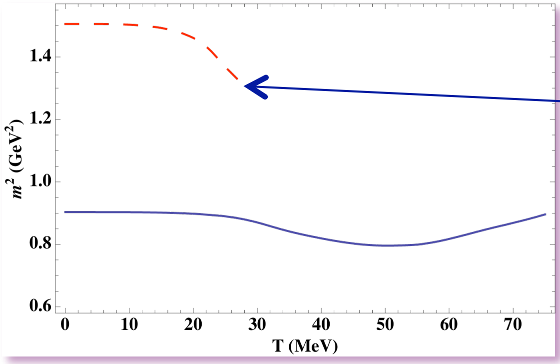
AdS-Black hole

scalar meson
spectral function

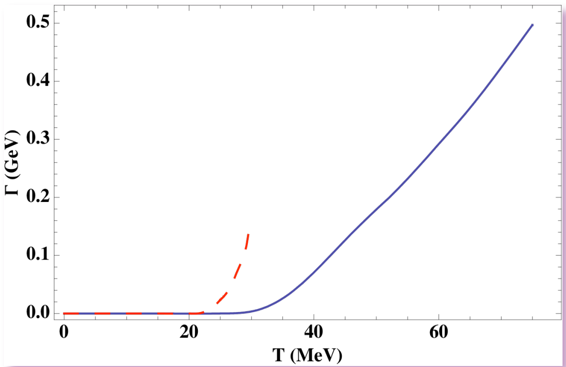


melting temperature

Mass vs temperature



Width vs temperature

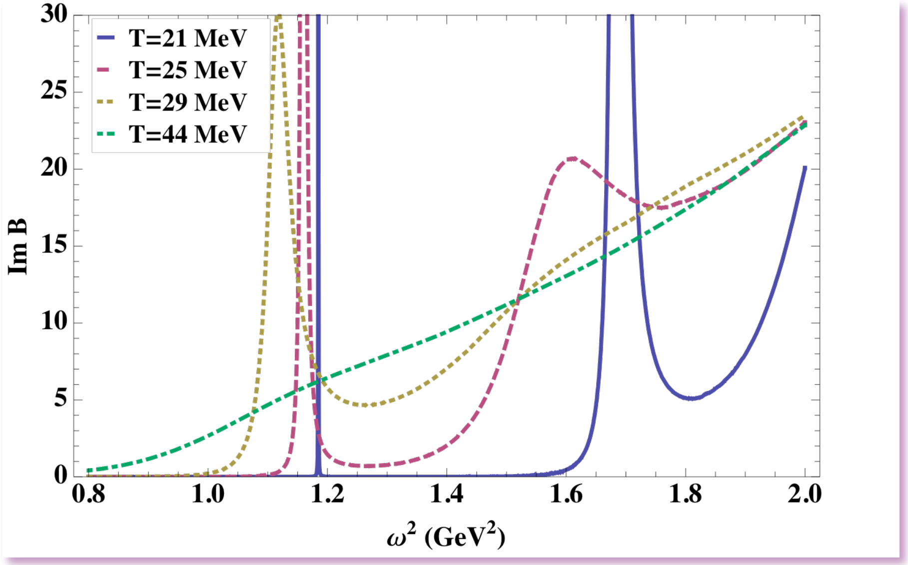


Giannuzzi Nicotri PC, PRD80, 094019

AdS-Black hole

scalar glueball

spectral function
$q=0$
$\omega=q_0$



temperature from the position of the horizon; scale c fixed from the ρ meson mass

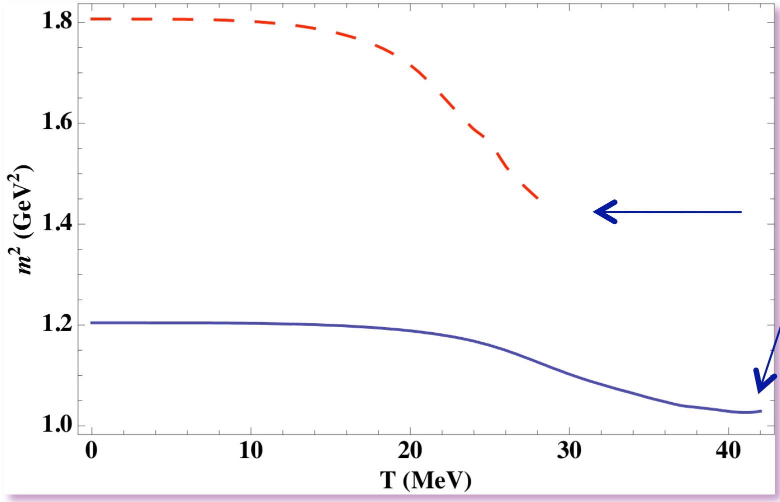
increasing the temperature: masses decrease
widths increase -> melting at particular T

Giannuzzi Nicotri PC, PRD80, 094019

AdS-Black hole

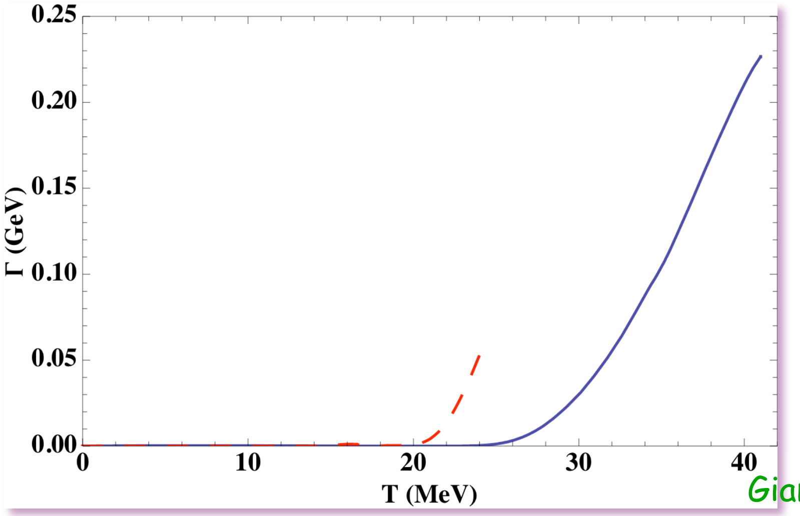
scalar glueball

Mass vs temperature



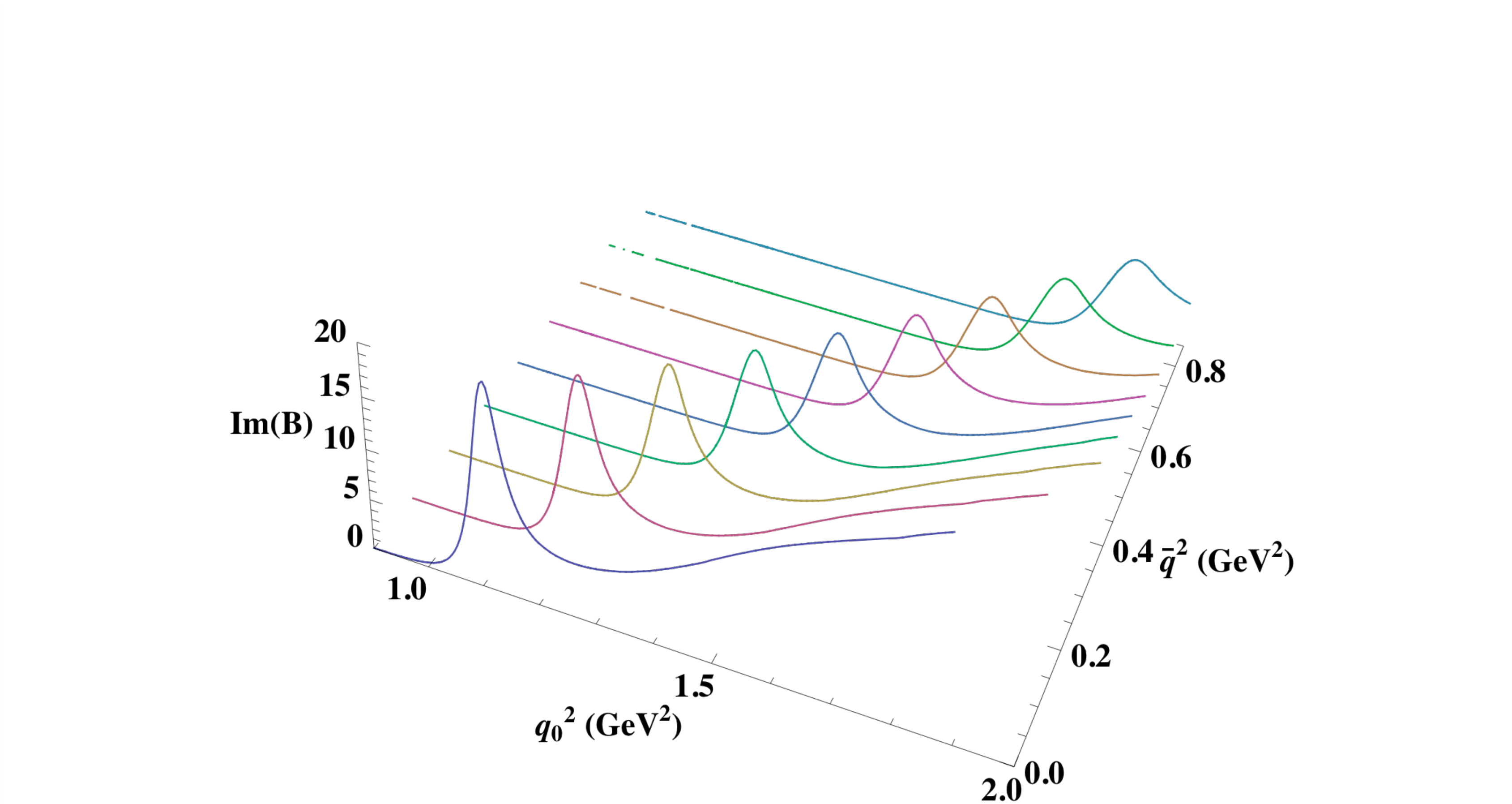
melting temperature

Width vs temperature



Giannuzzi Nicotri PC, PRD80, 094019

spectral function
q not zero



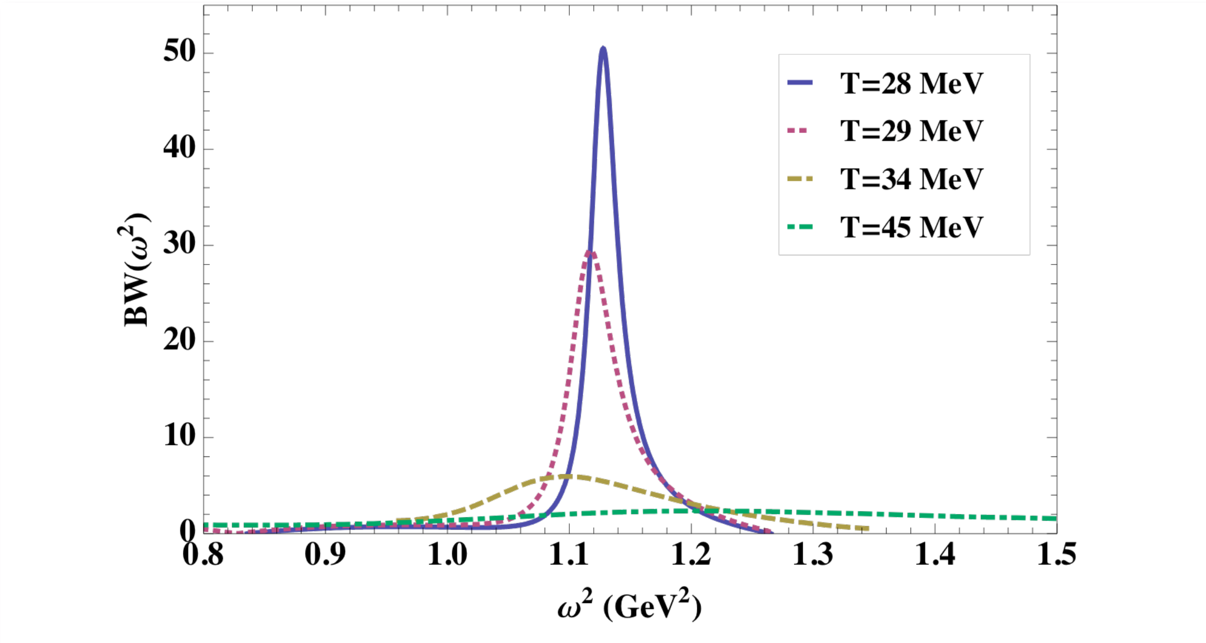
increasing three-momentum: masses increase
widths increase

Giannuzzi Nicotri PC, PRD80, 094019

AdS-Black hole

scalar glueball

spectral function with background subtracted: lightest state



increasing the temperature: masses decrease
widths increase -> melting

similar results in the vector channel

ρ -> J/ψ ?

Fukushima et al, 09

finite temperature and density: AdS/RN

Finite density: term added to the QCD lagrangian
in the generating functional $J_D = \mu \psi^+ \psi$

μ : boundary value of the time component of a U(1) gauge field: $A_0(z)$

Reissner/Nordstrom black hole metric

$$ds^2 = \frac{R^2 e^{c^2 z^2}}{z^2} \left(f(z) dt^2 + d\bar{x}^2 + \frac{dz^2}{f(z)} \right) ,$$ Euclidean

$$f(z) = 1 - \left(\frac{1}{z_h^4} + q^2 z_h^2 \right) z^4 + q^2 z^6 ,$$ q charge of the black hole

$$T = \frac{1}{4\pi} \left| \frac{df}{dz} \right|_{z=z_h}$$ Hawking temperature

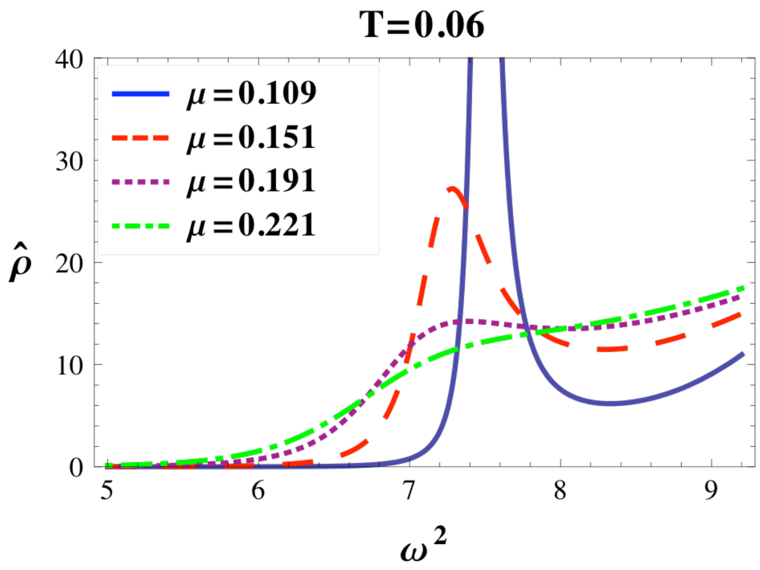
$$A_0(z) = \mu - \eta z^2$$

$$\mu = \kappa \, q \, z_h^2 = \kappa \frac{Q}{z_h}$$ $A_0(z_h)=0$

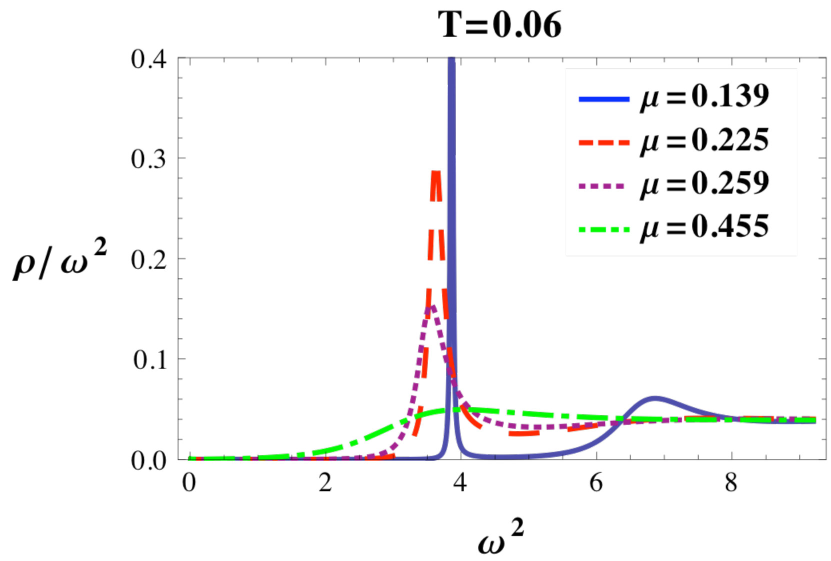
Giannuzzi Nicotri PC, JHEP05 (2012) 076

Finite T and μ
soft-wall model AdS-RN

scalar glueball



vector meson

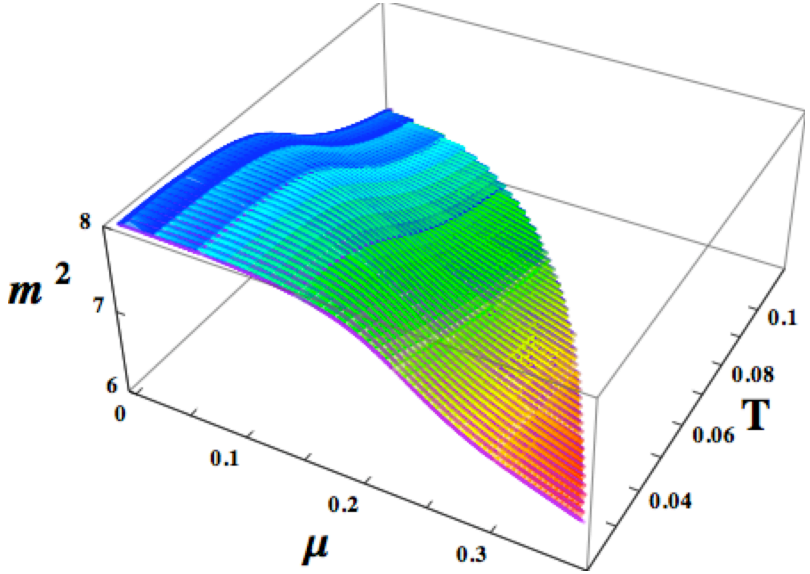


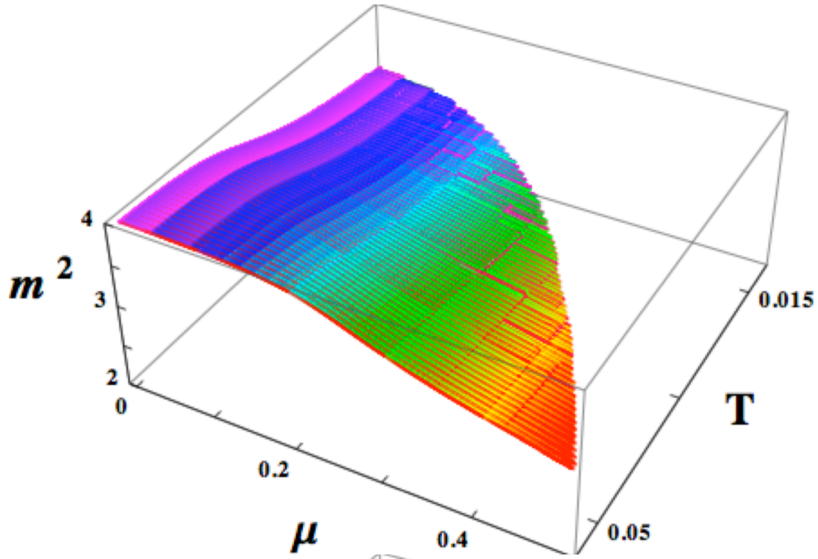
Giannuzzi Nicotri PC, JHEP05 (2012) 076

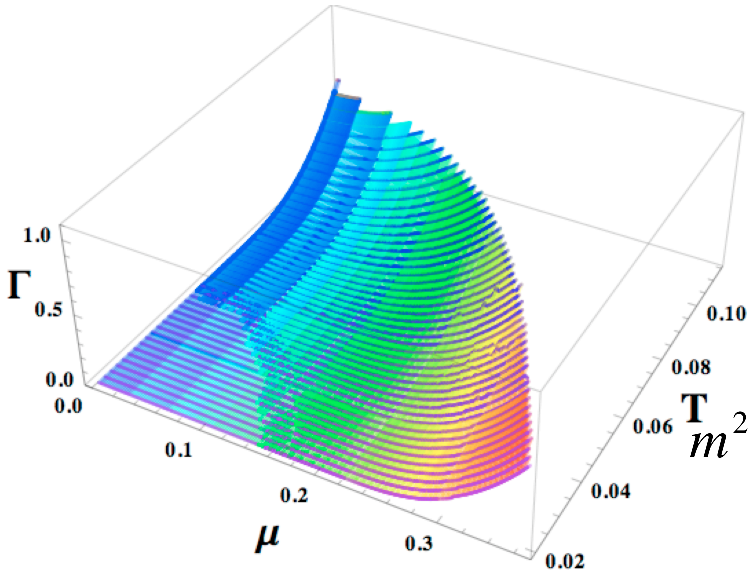
Finite T and μ
soft-wall model AdS-RN

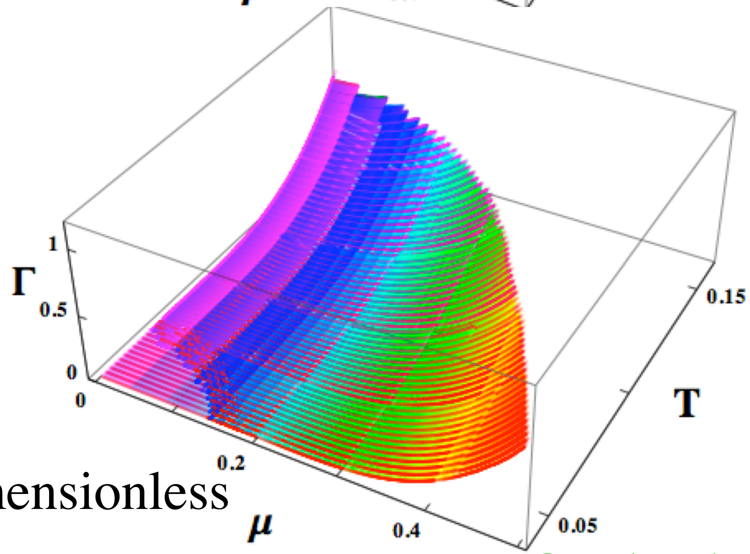
scalar glueball

vector meson







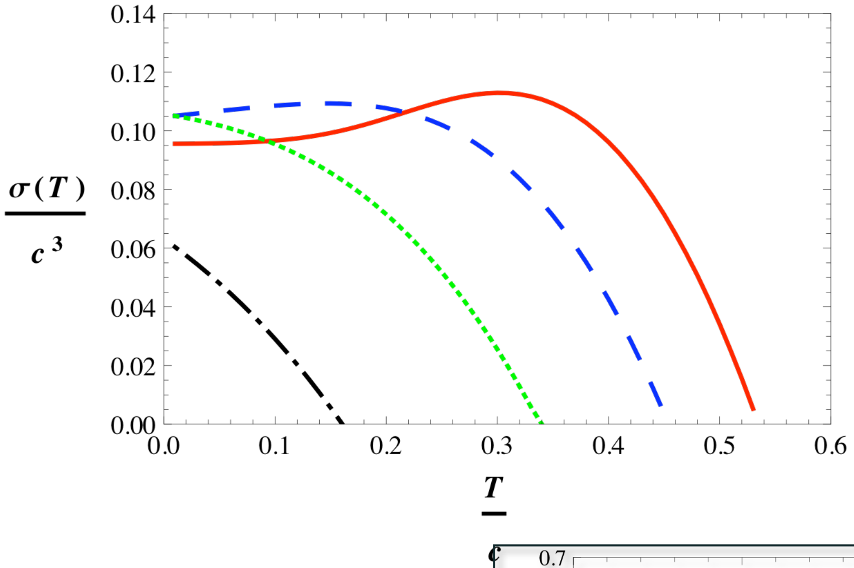


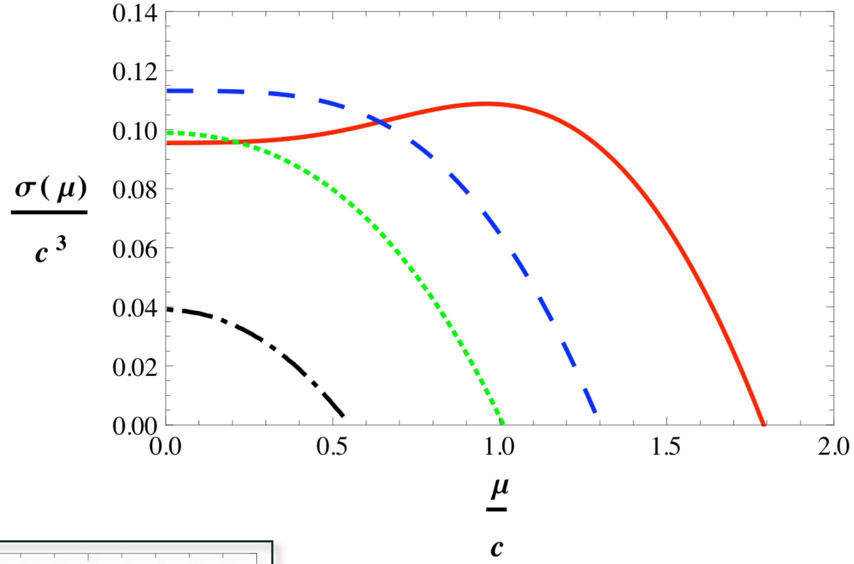
m^2, T, μ dimensionless

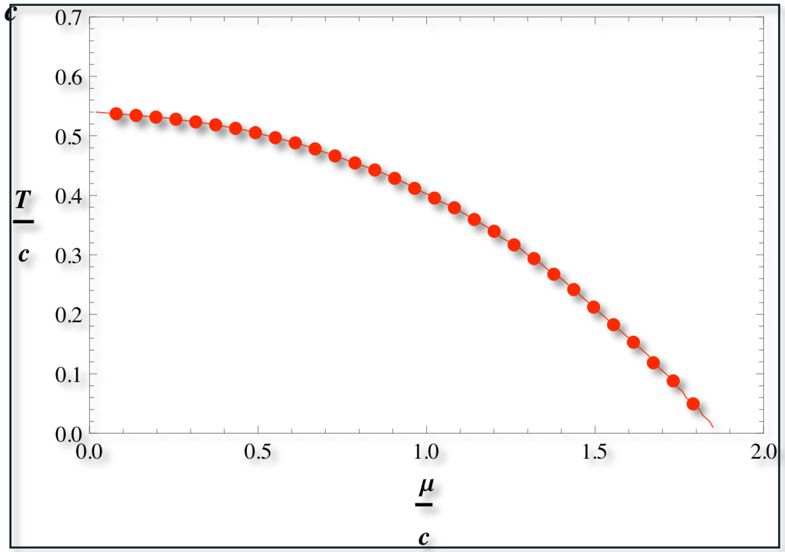
Giannuzzi Nicotri PC, JHEP05 (2012) 076

Finite T and μ
soft-wall model AdS-RN

chiral condensate







Giannuzzi Nicotri Tangorra PC, EPJC72, 2096

finite temperature and density: static potential

Static (infinitely heavy) quark-antiquark pair

$$\langle \mathcal{P}(\vec{x}_1)\mathcal{P}^\dagger(\vec{x}_2)\rangle = e^{-\frac{1}{T}F(r,T)+\gamma(T)}$$

Correlation of Polyakov loops

$$\langle \mathcal{P}\rangle = e^{-\frac{1}{2T}F^\infty(T)}$$

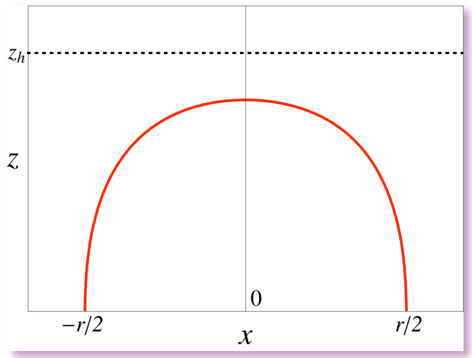
Order parameter (in pure gauge theory)

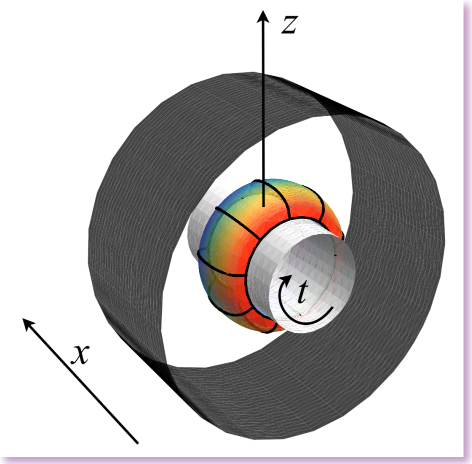
$$S_{\rm NG} = \frac{1}{2\pi\alpha'}\int d^2\xi\sqrt{\det\left[g_{MN}\left(\partial_a X^M\right)\left(\partial_b X^N\right)\right]}$$

Nambu-Goto action

$$F(r,T) = T S_{\rm NG}$$

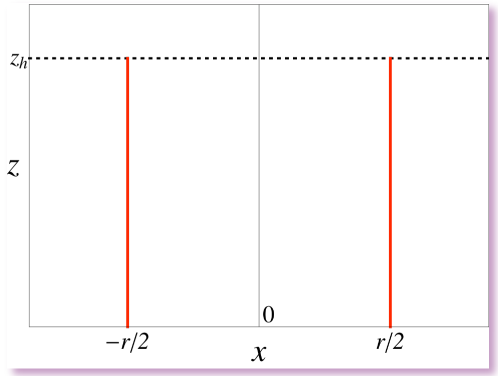
finite temperature and density: AdS/RN

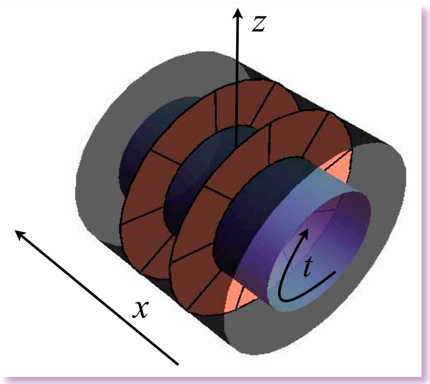




favoured configuration: one brane connecting the two quark lines

-> confinement



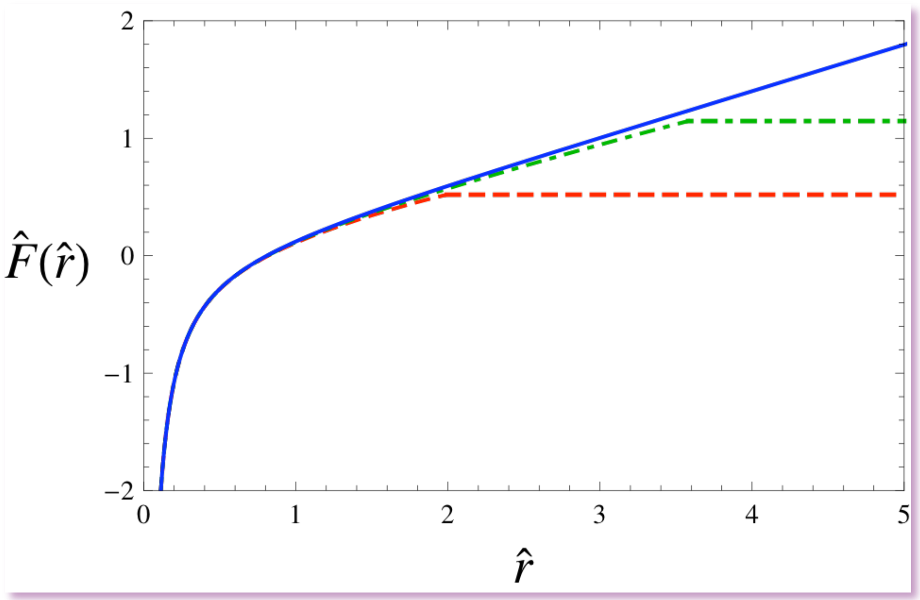


favoured configuration: two branes attached to the horizon

-> deconfinement

Giannuzzi Nicotri PC, PRD83, 035015

Free energy

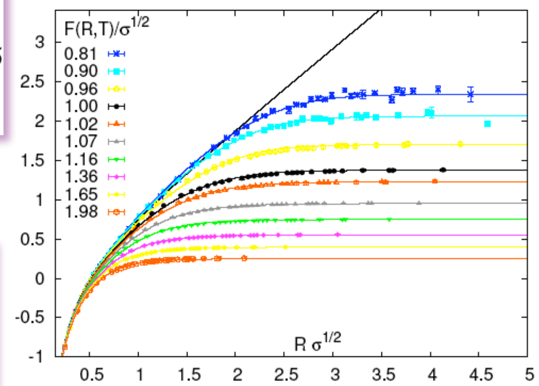


$\hat{\mu} = 0.5 \quad \hat{T} = 0.82T^*$

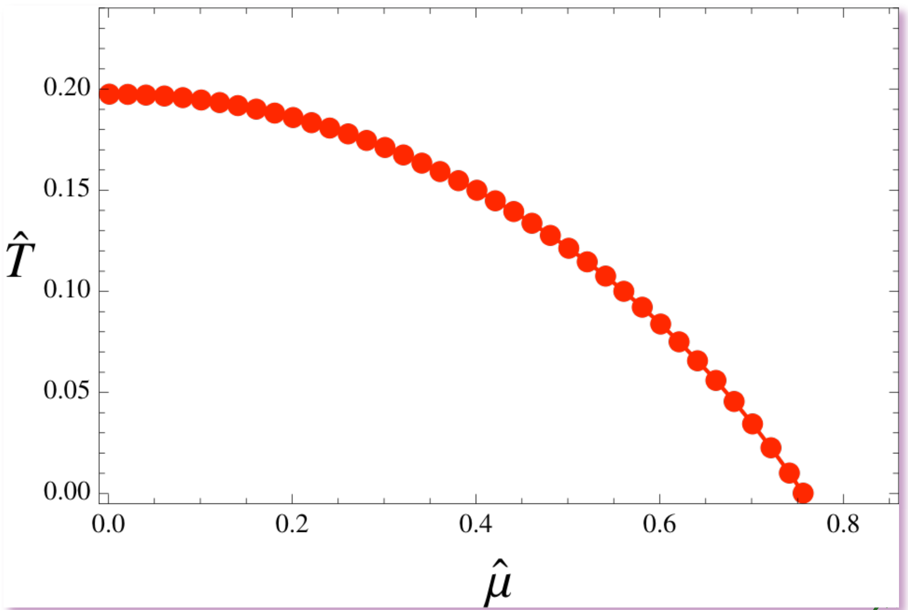
$\hat{\mu} = 0.5 \quad \hat{T} = 1.23T^*$

$\hat{\mu} = 0.5 \quad \hat{T} = 1.65T^*$

$T^* = 0.122$



lattice



Giannuzzi Nicotri PC, PRD83, 035015

dimensionless variables

temperature and density dependence of the gluon condensate

$$\Delta G_2(T,\mu) = G_2(T,\mu) - G_2(0,0)$$

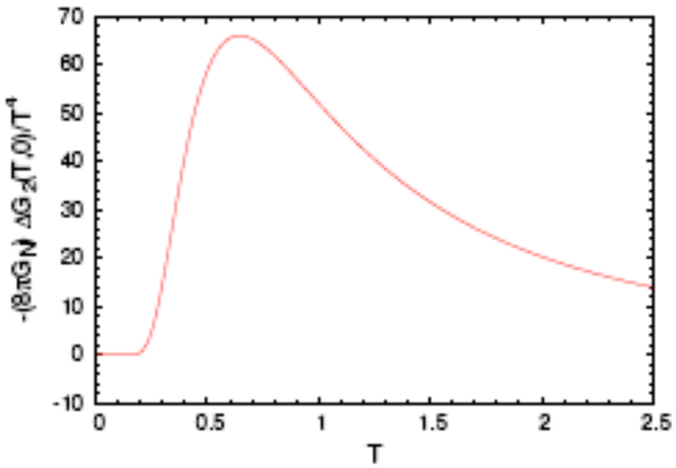
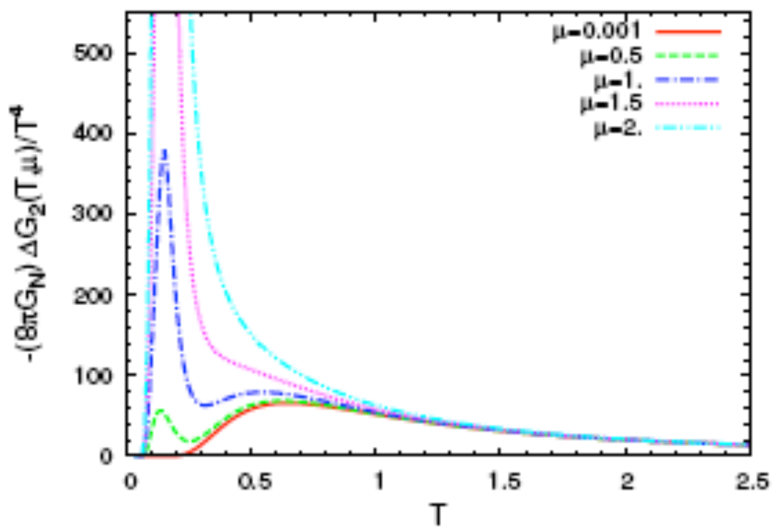


Figure 9: $-\Delta G_2(T,\mu)/T^4$ as a function of T at $\mu = 0$.



Giannuzzi Nicotri PC,
to appear these days in arXiv

conclusions and perspectives

- computational method inspired to the AdS/CFT correspondence
- hadron properties reproduced
- a few drawbacks identified (OPE in AVV)

- T>0 - the AdS/BH phase reproduces qualitatively the commonly expected behaviour for hadron masses and widths
- hadron properties at finite density
- chiral condensate, heavy quark free energy and gluon condensate in the plane T/μ

- role of the Hawking-Page transition under scrutiny

  hadrons melt in the deconfined phase

approach to face many non trivial QCD problems