Loop corrections to heavy-to-light form factors in $B \to Pl\nu$

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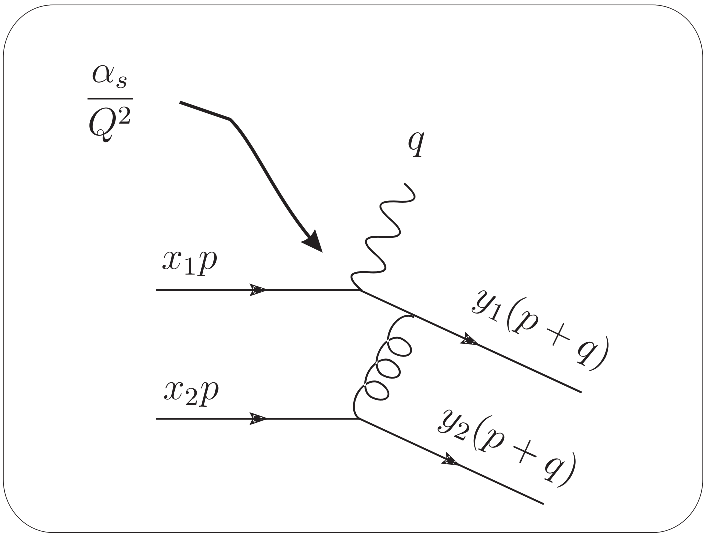
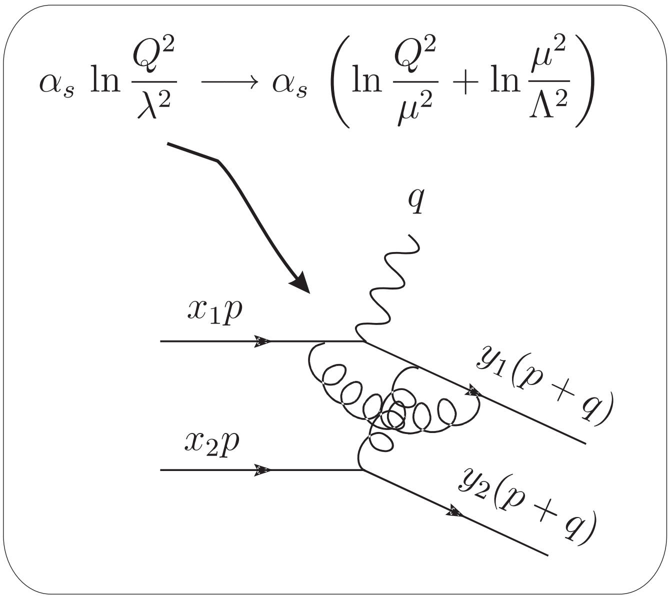
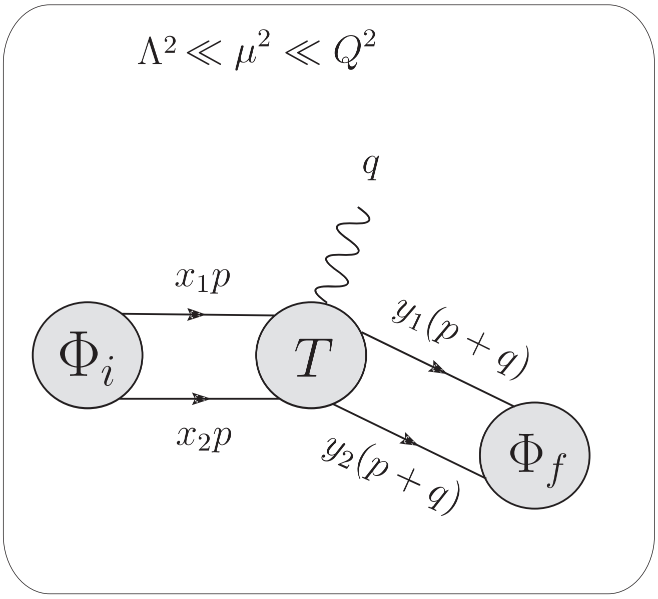
- ✎ Factorization in QCD. General ideas. Exclusive processes.
- ✎ Factorization of heavy-to-light form factors.
- ✎ The $B \to Pl\nu$ decay amplitude and structure of the form factors.
- ✎ The method of expanding by regions and effective theories.
- ✎ Coefficient functions for semileptonic decay.
- ✎ Conclusion.

$$\gamma^\star(Q^2) + \pi \longrightarrow \pi$$







$$f(Q^2) = \int dx dy \; \Phi_f^\dagger(y, \mu^2) \; T(y, x, Q^2, \mu^2) \; \Phi_i(x, \mu^2) = \Phi_f \otimes T \otimes \Phi_i$$

✎ Universality of the light-cone distribution amplitudes.

✎ Hard scale dependance.

Factorization of the heavy-to-light form factors.

$$M_B \sim m_b \gg \Lambda_{QCD}$$

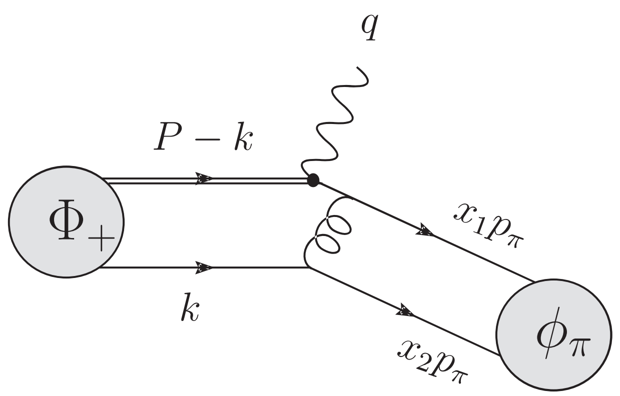
✎ Ch. W. Bauer, S. Fleming, D. Pirjol, I.W. Stewart (2001)

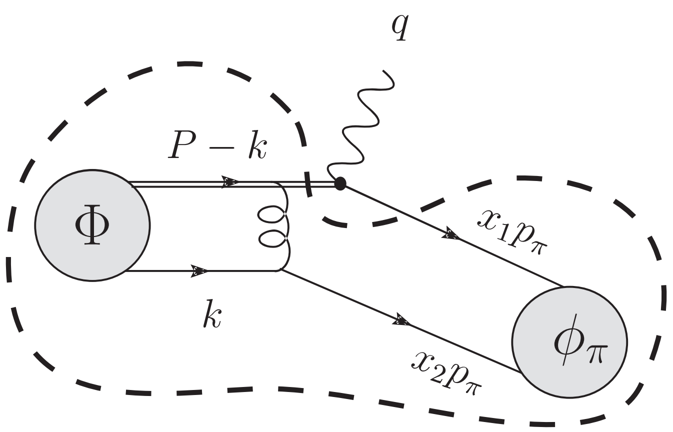
✎ M. Beneke, Th. Feldmann ,Y. Kiyo, D.S. Yang (2002)

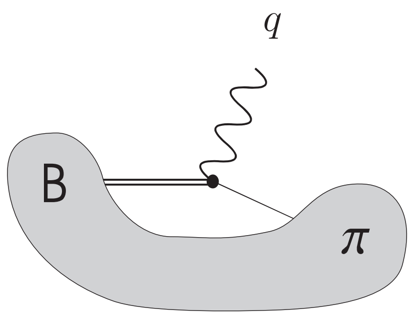
✎ M. Neubert, R.J. Hill, T. Becher, S.J. Lee (2003)

$$B \to light\ particles : B \to \gamma l\nu,\ \underline{B \to Pl\nu},$$
$$B \to Vl\nu,\ B \to PP,\ B \to VV,\ B \to V\gamma$$

$$0 < E_\pi < \frac{M_B}{2} \qquad \Longrightarrow \qquad E_\pi \approx |\mathbf{p}|_\pi \sim M_B$$







The $B \to P l \nu$ decay

$$\langle \pi(p') | \bar{u}\, \gamma^\mu (1+\gamma_5)\, b | B(P) \rangle = f_+(q^2) \left(P^\mu + p'^\mu - \frac{M_B^2 - m_\pi^2}{q^2}\, q^\mu \right) + f_0(q^2)\, \frac{M_B^2 - m_\pi^2}{q^2}\, q^\mu$$

✎ Structure of the form factors

$$f_+(E) = C_+(E)\, \xi(E) + \Phi_+ \otimes T_+(E) \otimes \phi_\pi$$
$$\frac{M_B}{2E}\, f_0(E) = C_0(E)\, \xi(E) + \Phi_+ \otimes T_0(E) \otimes \phi_\pi$$

Universal nonperturbative form factor:

$$\xi(E) = \frac{1}{2E}\, \langle \pi(p') | \bar{\xi}_C\, W_C\, h_v | B(P) \rangle$$

Light-cone distribution amplitudes:

$$\langle 0 | [\bar{q}_\beta Y] (tn)\; [Y^\dagger h_{v\,\alpha}] (0) | B(P) \rangle = -i\, \frac{f_B M_B}{4} \left\{ \frac{1+\hat{v}}{2} \left(n\cdot v\; \hat{n}_+\; \Phi_+(t) + n_+ \cdot v\; \hat{n}\; \Phi_-(t) \right)\; \gamma_5 \right\}_{\alpha\beta}$$

M. Neubert, A.G. Grozin (1997)

$$\left\langle \pi(p') \middle| [\bar{\xi}_{C\,\alpha} W_C]\, (s n_+) \left[W_C^\dagger\, \xi_{C\,\beta} \right] (0) \middle| 0 \right\rangle = \frac{i f_\pi}{4}\, n_+ \!\cdot\! p' \left(\frac{\hat{n}}{2}\, \gamma_5 \right)_{\beta\alpha} \int_0^1 dx\, e^{i x s n_+ \cdot p'} \phi_\pi(x)$$

A.V. Efremov, A.V. Radyushkin (1980), G.P. Lepage, S. J. Brodsky (1980)

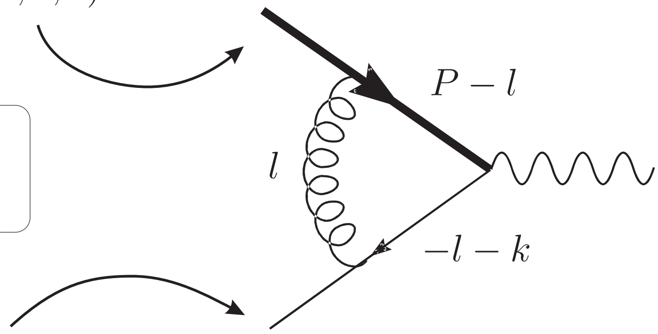
$$C_+(E) \approx C_0(E) \approx 1$$ – Symmetry at large recoil

✎ M. Beneke, Th. Feldmann (2003)

Example: Heavy-light current

$P \approx M\,v = M(1,0,0,0)$

$\lambda^4 = -\frac{k^2}{M^2}$



$-k \sim \Lambda_{QCD} \sim \lambda^2 M$

$$I(\lambda) = i \int [\mathrm{d}l]\, \frac{M\ (l \cdot v)}{(l^2 - 2P\cdot l + i0)\ ((l+k)^2 + i0)\ (l^2 + i0)}$$

<u>Hard region</u> $(l^\mu \sim M,\ l^\mu \to M l^\mu)$

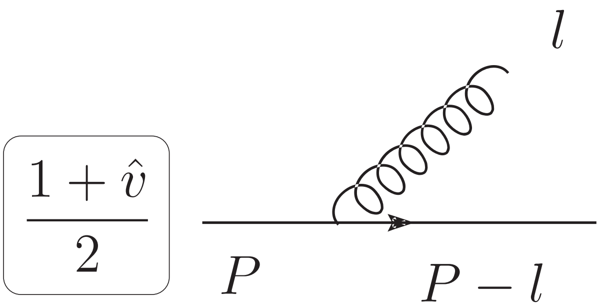
$$I_h = i \left(\frac{\mu^2}{M^2}\right)^\epsilon \int [\mathrm{d}l]\, \frac{(l\cdot v)}{(l^2 - 2\ l\cdot v + i0)\,(l^2+i0)^2} \approx \frac{\alpha_s}{4\pi}\left(-\frac{1}{2\epsilon} + \frac{1}{2}\log\frac{M^2}{\mu^2}\right)$$

<u>Soft region</u> $(l^\mu \sim \lambda^2 M,\ l^\mu \to \sqrt{-k^2}\, l^\mu)$

$$I_s = i \left(-\frac{\mu^2}{k^2}\right)^\epsilon \int [\mathrm{d}l]\, \frac{(l\cdot v)}{(-2v\cdot l + i0)\,((l+k)^2 + i0)\ (l^2+i0)} \approx \frac{\alpha_s}{4\pi}\left(\frac{1}{2\epsilon} + \frac{1}{2}\log\frac{\mu^2}{-k^2} + 1\right)$$

$$\boxed{I \approx I_h + I_s = \frac{\alpha_s}{4\pi}\left(\frac{1}{2}\log\frac{M^2}{-k^2} + 1\right)}$$

Heavy quark effective theory (HQET)



$$\frac{\hat{P}-\hat{l}+M}{l^2-2P\cdot l+i0}\,\gamma^\mu\,\frac{1+\hat{v}}{2}\approx\frac{M\ (1+\hat{v})}{-2M\ v\cdot l+i0}\,\gamma^\mu\,\frac{1+\hat{v}}{2}=\frac{1}{-l\cdot v+i0}\,v^\mu\,\frac{1+\hat{v}}{2}$$

Variables: $Q_v(x)=e^{iMv\cdot x}Q(x)$

$$L=\bar{Q}\left(i\hat{D}-M\right)Q\approx\bar{Q}_v\left(i\hat{D}_{us}-M\ (1-\hat{v})\right)Q_v$$

$$Q_v=\left(\frac{1+\hat{v}}{2}+\frac{1-\hat{v}}{2}\right)\ Q_v=h_v+H_v\implies H_v\sim\lambda^2 h_v$$

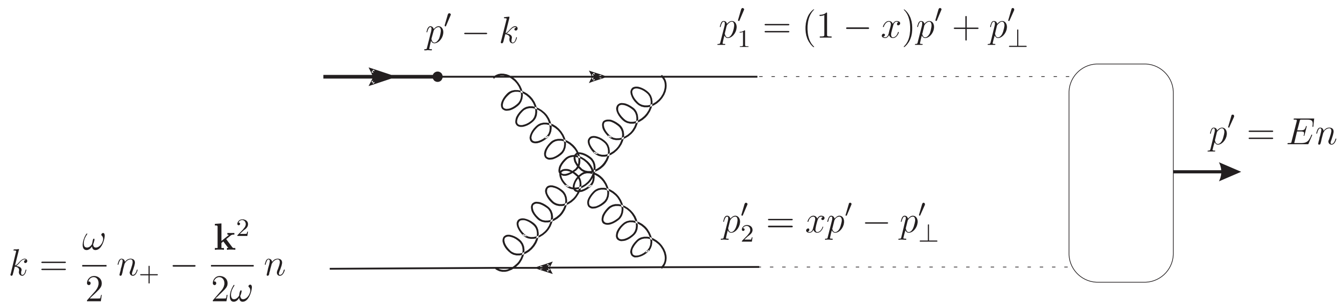
Power counting:

$$h_v\sim\lambda^3,\ H_v\sim\lambda^5$$

Lagrangian:

$$L_{(0)}=\bar{h}_v\left(i\hat{D}_{us}-M\ (1-\hat{v})\right)h_v=\bar{h}_v\,(i\ v\cdot D_{us})\ h_v$$

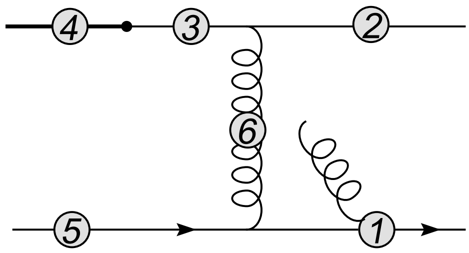
$$\boxed{L_{(0)}=\bar{h}_v\left(i\ v\cdot D_{us}+\frac{(i\hat{D}_{us})^2}{2M}+\mathcal{O}(\lambda^6)\right)\ h_v}$$

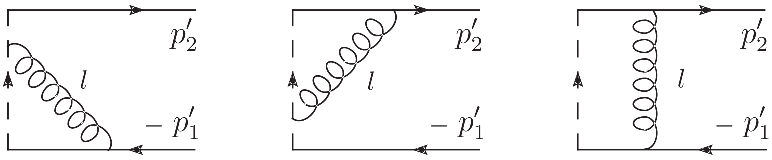


Region	Contribution
Hard collinear $(1, \lambda, \lambda^2)$	$\left(\frac{\mu^2}{2E\omega}\right)^\epsilon \left[\frac{1}{\epsilon^2}\frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} - \frac{1}{\epsilon}\left(\frac{\log x}{\bar{x}} + 2\log\bar{x}\right) + \frac{\log^2 x}{2\ \bar{x}} + \log^2\bar{x}\right]$
Collinear $(1, \lambda^2, \lambda^4)$	$\left(\frac{\mu^2}{\mathbf{p}_\perp^2}\right)^\epsilon \left[-\frac{1}{\epsilon^2}\frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} + \frac{\log x}{\epsilon\ \bar{x}} + \frac{1}{\bar{x}}\left(F(-\bar{x}) - F\left(\frac{\bar{x}}{x}\right)\right)\right]$
The second collinear region	$\left(\frac{\mu^2}{\mathbf{p}_\perp^2}\right)^\epsilon \left[\frac{\log\bar{x}}{\epsilon} + F(-x) - F\left(\frac{x}{\bar{x}}\right)\right]$
Soft $(\lambda^2, \lambda^2, \lambda^2)$	$\left(\frac{\mu^2}{\mathbf{k}^2}\right)^\epsilon \left[-\frac{\Gamma^2(1-\epsilon)}{\epsilon^2\ \Gamma(1-2\epsilon)}\right]$
Ultra-soft-collinear $(1, \lambda^3, \lambda^6)$	$\left(\frac{\mu^2 2E\omega}{\mathbf{k}^2\mathbf{p}_\perp^2}\right)^\epsilon \left[\frac{\Gamma(1-\epsilon)\ \Gamma(1+\epsilon)}{\epsilon^2} + \frac{\log\bar{x}}{\epsilon} + \frac{\log^2\bar{x}}{2}\right]$
The sum	$\log\frac{2E\omega}{\mathbf{k}^2}\log\frac{2E\bar{x}\ \omega}{\mathbf{p}_\perp^2} + \left(2\log\bar{x} + \frac{\log x}{\bar{x}}\right)\ \log\frac{2E\bar{x}\ \omega}{\mathbf{p}_\perp^2}$ $+2\ F(-x) + \frac{2}{\bar{x}}\ F(-\bar{x}) - \log^2\bar{x} + \frac{\pi^2}{3}$

M. Beneke and V.A. Simirnov, NPB 522 (1998) 321, V.A. Simirnov, Applied Asymptotic Expansions In Momenta And Masses, Springer Verlag, Berlin, Germany, 2002.

Factorization of the soft and collinear singularities





$$\Delta f = \Phi^{+}(\omega) \otimes T_0(\omega, x) \otimes \delta\phi_\pi(x)$$

$$\delta\phi_\pi(x,\mu) = \frac{g^2}{4\pi} \log \frac{\mu^2}{\mathbf{p}'^2_\perp} V_\pi^{(1)}(x,x') \otimes \phi_\pi(x',\mu)$$

Let's consider Feynman diagrams with attachment of collinear gluon to collinear antiquark.

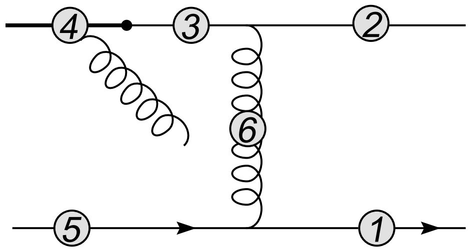
$$A_{1i} = -2i \int [dl] \frac{\tilde{A}_{1i}}{(l^2+i0)\left((q_1-l)^2+i0\right)}$$

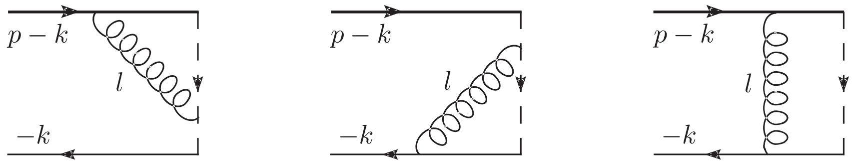
$$\tilde{A}_{15} = \left(C_F - \frac{C_A}{2}\right)\left(-1 + \frac{2Ex}{\beta}\right) \qquad \tilde{A}_{16} = -\frac{C_A}{2}$$

$$\tilde{A}_{14} = \left(C_F - \frac{C_A}{2}\right)\left(-\frac{2E\,x}{\beta} + \frac{2E\,x}{\beta - 2E}\right) \qquad \tilde{A}_{13} = \left(C_F - \frac{C_A}{2}\right)\left(-\frac{2Ex}{\beta - 2E}\right)$$

The sum of these diagram equals the corresponding LCDA correction

$$\tilde{A}_{15} + \tilde{A}_{16} + \tilde{A}_{14} + \tilde{A}_{13} = C_F$$





$$\Delta f = \delta\Phi^+(\omega) \otimes T_0(\omega, x) \otimes \phi_\pi(x)$$

$$\delta\Phi^+(\omega,\mu) = \frac{g^2}{4\pi}\left(\Gamma^{(1)}\,\log^2\frac{\mu^2}{k^2}\,\Phi(\omega,\mu) + \log\frac{\mu^2}{k^2}\,V^{(1)}(\omega,\omega')\otimes\Phi(\omega',\mu)\right)$$

Let's consider gluon radiation by the heavy quark.

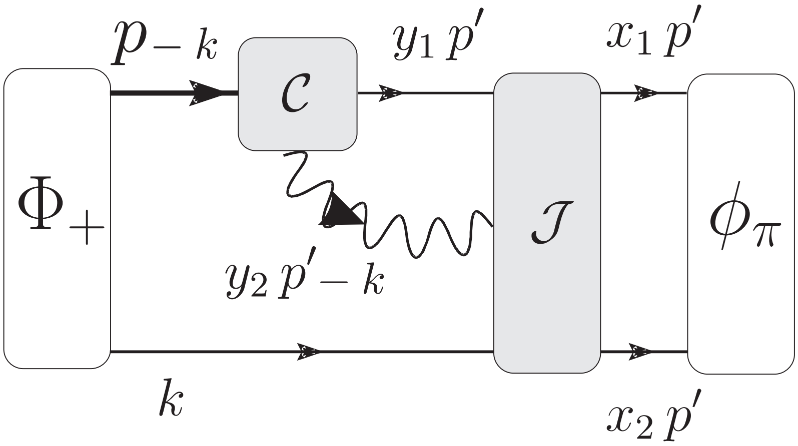
$$A_{4i} = 2i\int [dl]\frac{M\ \tilde{A}_{4i}}{(l^2+i0)\left((p-k-l)^2-m_b^2\right)}\,.$$

$$\tilde{A}_{42} = \left(C_F - \frac{C_A}{2}\right)\frac{1}{\alpha}\,,\quad \tilde{A}_{41} = \left(C_F - \frac{C_A}{2}\right)\frac{-\omega}{(\alpha+\omega)\ \alpha}\,,\quad \tilde{A}_{46} = \frac{C_A}{2}\frac{1}{\alpha+\omega}\,.$$

The sum of these diagram equals the corresponding LCDA correction

$$\tilde{A}_{42}+\tilde{A}_{41}+\tilde{A}_{46} = C_F\,\frac{1}{\alpha+\omega}\,.$$

Structure of the coefficient function

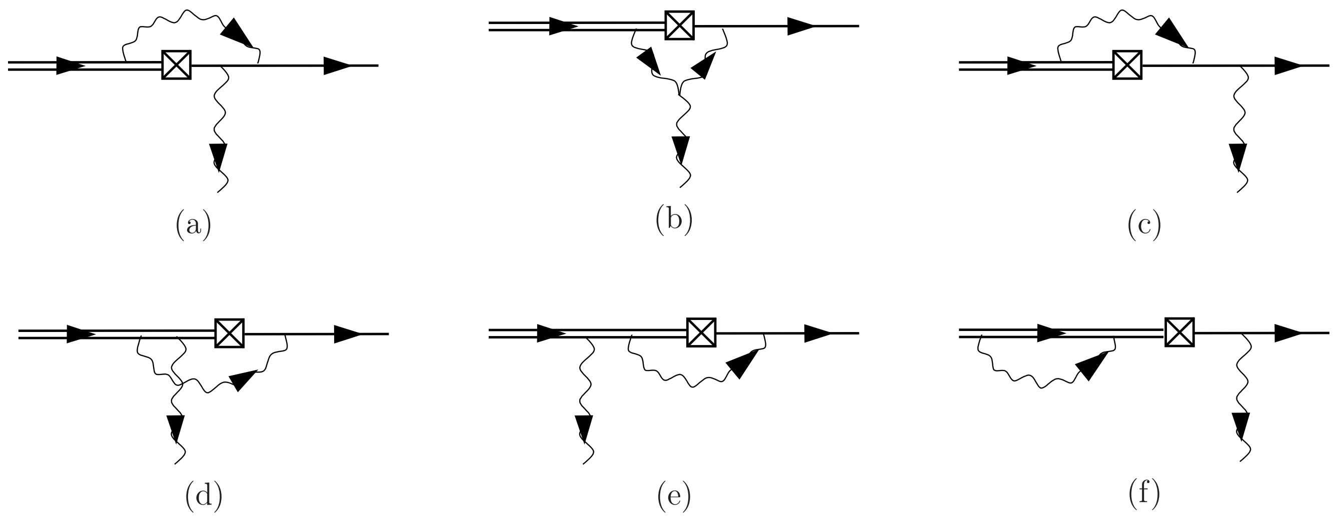


$$M_B^2 \gg 2p' \cdot k \sim \Lambda M_B \gg \Lambda^2$$

$$\ln \frac{M^2}{2p' \cdot k} = \ln \frac{M^2}{\mu_F^2} - \ln \frac{2p' \cdot k}{\mu_F^2}$$

$$T\left(k, x, \mu, E, z = \frac{2E}{M}\right) = \mathcal{C} \otimes \mathcal{J}$$
$$= \int_0^1 dy\, \mathcal{C}\left(y, z, \ln \frac{M^2}{\mu^2}, \alpha_s(\mu)\right)\ \mathcal{J}\left(y, x, \ln \frac{2E\omega}{\mu^2}, \alpha_s(\mu)\right)$$

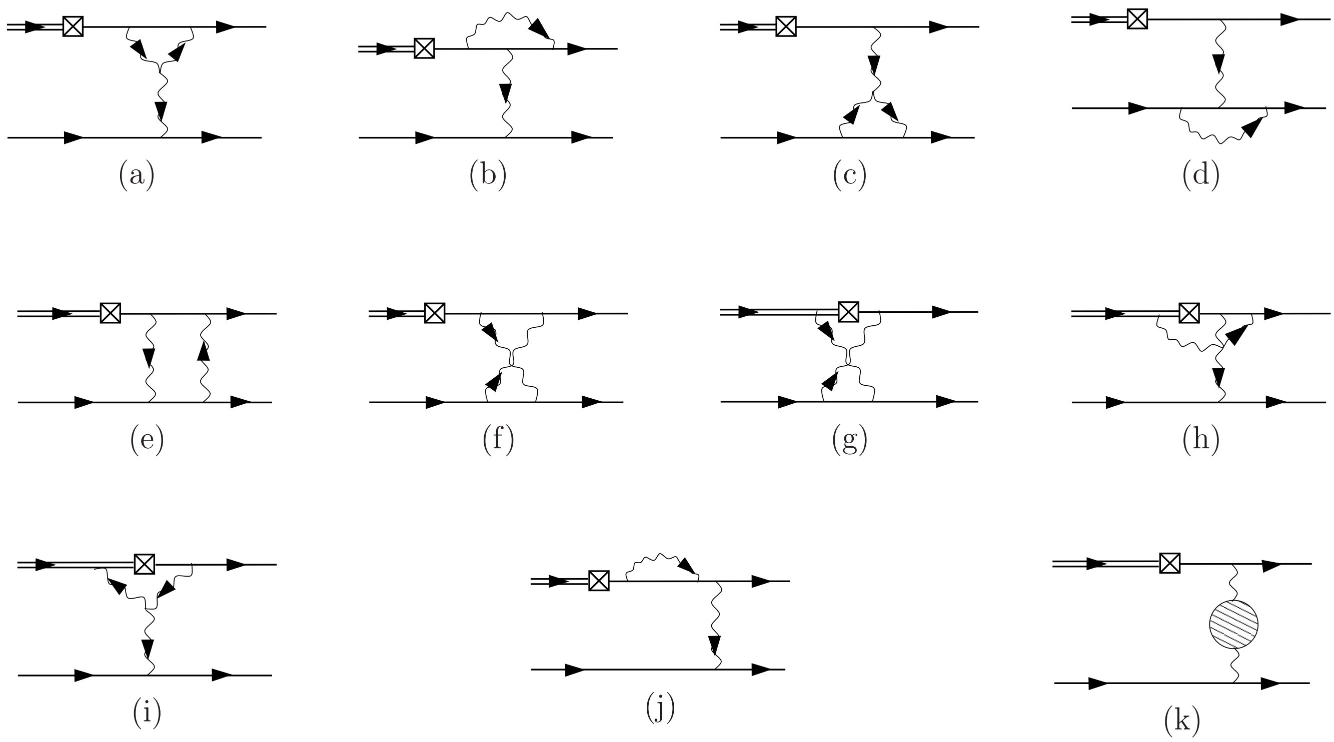
$$\mathcal{J}(y, x) = \frac{\alpha_s}{x\ \omega}\ \left(\delta(x - y) + \frac{\alpha_s}{4\pi}\, \delta\mathcal{J}(y, x)\right)$$



$$
\begin{aligned}
&\delta C(x,z,E,\mu) = \\
&\left(C_F - \frac{C_A}{2}\right) \left[-\frac{2}{x} \left(2\,\log\bar{x} \left(\log\frac{2E}{\mu} - 1 \right) + \log^2\bar{x} + \mathrm{Li}_2\left(1-\bar{x}\,z\right) - \mathrm{Li}_2\left(1-z\right) \right) \right. \\
&\qquad\qquad \left. - \frac{2}{\bar{x}} \left(\frac{\log z}{(1-z)} - \frac{\log x\,z}{(1-x\,z)} - \log x \right) \right] \\
&+ C_F \left(-2\log^2\frac{2E}{\mu} - \frac{\pi^2}{12} + \log\frac{2E}{\mu} - \frac{3\,\log z}{1-z} - 2\,\mathrm{Li}_2\left(1-z\right) - 1 + \frac{z}{(1-z)\,(1-x\,z)} \right. \\
&\qquad\qquad \left. + \frac{1}{\bar{x}} \left(\frac{\log z}{(1-z)^2} - \frac{\log x\,z}{(1-x\,z)^2} + \frac{\log z}{(1-z)} - \frac{\log x\,z}{(1-x\,z)} - 2\,\log x \right) \right) .
\end{aligned}
$$

M. Beneke, Y. Kiyo, D.s. Yang, Nucl. Phys. B **692** (2004) 232 [hep-ph/0402241].

R.J. Hill, T. Becher, S.J. Lee, M. Neubert, JHEP **0407** (2004) 081 [hep-ph/0404217].



$$
\begin{aligned}
\tilde{\mathcal{J}}_f(y,\,x) &= 2\left(C_F - \frac{C_A}{2}\right)\left[\frac{1}{2}\left[\frac{\theta(x-y)}{x-y}\left(\frac{1}{\epsilon} - \log(x-y)\right) + \frac{\theta(y-x)}{y-x}\left(\frac{1}{\epsilon} - \log(y-x)\right)\right]_+\right. \\
&+ \frac{1}{2}\,\mathbf{P}\,\frac{1}{x-y}\left(\frac{1}{\epsilon} - \log(x-y)\right) + \frac{\theta(x-y)}{x-y}\log\frac{x}{y} \\
&- \delta(x-y)\left(\frac{1}{\epsilon^2} - \frac{\log \bar{x}x}{2\epsilon} + \frac{1}{4}\left(\log^2\bar{x} + \log^2 x\right)\right) \\
&- \theta(x-y)\left(\frac{1}{x\,\bar{y}}\left(\frac{1}{\epsilon} - \log\, y\bar{y}\right) + \frac{1}{y}\log\frac{(1-\frac{y}{x})}{\bar{y}}\right) \\
&- \left.\theta(y-x)\left(\frac{1}{y}\left(\frac{1}{\epsilon} - \log\, y\bar{y}\right) + \frac{1}{x\,\bar{y}}\log\frac{(1-\frac{\bar{y}}{\bar{x}})}{y}\right)\right]
\end{aligned}
$$

R.J. Hill, T. Becher, S.J. Lee, M. Neubert, JHEP **0407** (2004) 081 [hep-ph/0404217].

Integrated jet function

$$
\begin{aligned}
\int dy\ \delta\mathcal{J}(y,x) = &\ 2C_F\,\frac{1}{\epsilon^2} + \left(\frac{11}{3}\,C_A - \frac{4}{3}T_F\,n_f\right)\,\frac{1}{\epsilon} + C_A\,\frac{1}{\epsilon}\,\log\frac{1-x}{x} \\
&+ C_F\,\frac{1}{\epsilon}\left(-5 - 2\,\frac{\log(1-x)}{x} - 2\log x\right) \\
&+ C_F\,\left(-13 + \frac{\pi^2}{2} + \frac{\log^2(1-x)}{x} + \left(6 - \frac{1}{1-x}\right)\,\log x + \log^2 x\right. \\
&\left.+ \log(1-x)\left(-\frac{2}{x} + \left(-4 + \frac{2}{x}\right)\log x\right) + 4\left(-2 + \frac{1}{x}\right)\,Li_2(x)\right) \\
&+ T_f\,n_f\,\left(-\frac{20}{9} + \log x\right) + C_A\,\left(-\frac{\pi^2}{3} - \frac{\log^2(1-x)}{2x} + \left(-\frac{11}{3} - \frac{1}{1-x}\right)\log x\right. \\
&\left.+ \log(1-x)\left(\frac{1}{x} + \left(2 - \frac{1}{x}\right)\log x\right) + \left(4 - \frac{2}{x}\right)\,Li_2(x) + \frac{86}{9}\right)
\end{aligned}
$$

$$\boxed{+\frac{C_A}{2}}\ ?$$

R.J. Hill, T. Becher, S.J. Lee, M. Neubert, JHEP **0407** (2004) 081 [hep-ph/0404217].

Conclusion

- ✎ Factorization of the collinear and soft singularities are demonstrated.
- ✎ Loop corrections to the coefficient function are calculated.
- ✎ The hard coefficient function is the same as previous results. The jet function is partly not.