Helmholtz International School “HEAVY QUARK PHYSICS”

MASSES OF HEAVY BARYONS IN THE RELATIVISTIC QUARK MODEL

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INTRODUCTION

(lectures by R.Faustov, V.Lyubovitskij, M.Ivanov, A.Likhoded, F.Stancu, C.Roberts, S.Gerasimov)

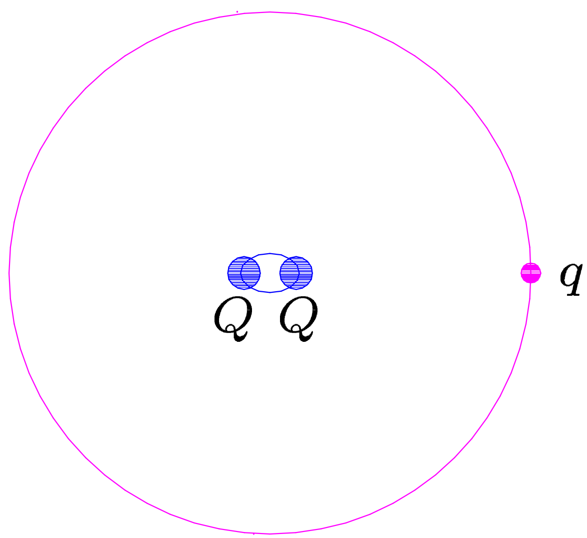
Heavy baryons (qqQ)

$J=1/2$	Λ_c	Λ_b	Ξ_c	Ξ_b	Σ_c	Σ_b	Ξ'_c	Ξ'_b	Ω_c	Ω_b
$J=3/2$					Σ_c^*	Σ_b^*	Ξ_c^*	Ξ_b^*	Ω_c^*	Ω_b^*
	$[ud]c$	$[ud]b$	$[us]c$	$[us]b$	$\{ud\}c$	$\{ud\}b$	$\{us\}c$	$\{us\}b$	$\{ss\}c$	$\{ss\}b$
			$[ds]c$	$[ds]b$	$\{dd\}c$	$\{dd\}b$	$\{ds\}c$	$\{ds\}b$		
					$\{uu\}c$	$\{uu\}b$				
isospin	0	0	1/2	1/2	1	1	1/2	1/2	0	0

Doubly heavy baryons (QQq)

Ξ_{bb}	Ξ_{cc}	Ξ_{bc}	Ω_{bb}	Ω_{cc}	Ω_{bc}
$\{bb\}q$	$\{cc\}q$	$\{bc\}q$	$\{bb\}s$	$\{cc\}s$	$\{bc\}s$
		$[bc]q$			$[bc]s$

First indications of Ξ_{cc} from SELEX (Fermilab)? (lectures by F.Stancu & A.Likhoded)



- average QQ separation $R \sim 1/m_Q$
- average qQ separation $r \sim 1/m_q$ $\qquad r \gg R$

QUARK-DIQUARK PICTURE

[Anselmino et al. (1993); Jaffe (2005), Wilczek (2004)]

⋆ Heavy baryons (qqQ): heavy-quark–light-diquark

⋆ Doubly heavy baryons (QQq): light-quark–heavy-diquark

Different ways to consider diquark:

• completely phenomenological object

• bound qq system

Spectator heavy quark influences dynamics of the light quarks forming the diquark. We assume that such influence is considerably smaller than the diquark correlation and can be neglected. If it could not be neglected than properties of the light diquark will be different in baryons, tetraquarks, pentaquarks etc., due to different number of spectators.

Three body calculation $\longrightarrow$ two-step two body calculation

Difference in dynamics of heavy and light quarks:

• slow relative motion of heavy quarks Q (as in heavy quarkonium)

• fast motion of light quark q (as in heavy-light mesons, $v/c \sim 0.7 - 0.8$) $\rightarrow$ light quark must be treated fully relativistically

qq or QQ diquark is in antitriplet colour state

HQS ($m_Q \to \infty$):

heavy baryons: heavy quark spin and mass decouple $\longrightarrow$ heavy baryon properties are determined by light quarks $\longrightarrow$ masses of ground state baryons with spin $1/2$ and $3/2$ containing the axial vector diquark are degenerate

doubly heavy baryons: diquark spin and mass decouple $\longrightarrow$ doubly heavy baryon properties are determined by light quark $\longrightarrow$ similarity to heavy-light mesons $\bar{Q}q$

Diquark is a composite system with $S = 0, 1 \ldots$:

- different splittings when $1/m_Q$ corrections are included

- diquark is not point-like object: Its interaction with gluons is smeared by the form factor expressed through the overlap integral of diquark wave functions

- diquark excitations can contribute to the excitation spectrum

Pauli principle for ground state diquarks:

- (qq') diquark can have $S = 0, 1$ [scalar $[q, q']$ ("good"), axial vector $\{q, q'\}$ ("bad")]
- (qq) diquarks can have only $S = 1$ (axial vector $\{q, q\}$)

RELATIVISTIC QUARK MODEL

Quasipotential equation of Schrödinger type:

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p},\mathbf{q};M)\Psi_M(\mathbf{q})$$

$\mathbf{p}$ - relative momentum of quarks
M - bound state mass ($M = E_1 + E_2$)
μ_R - relativistic reduced mass:

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}$$

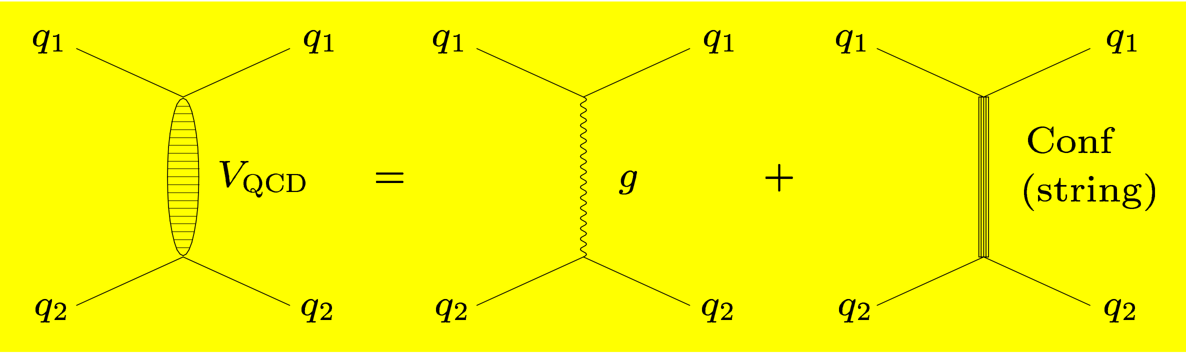
$b(M)$ - cms on-mass-shell relative momentum:

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}$$

$E_{1,2}$ - center of mass energies:

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}$$

- **Quasipotential**



$$V(\mathbf{p},\mathbf{q};M) = \bar{u}_1(p)\bar{u}_2(-p)\left\{\frac{4}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_1^{\mu}\gamma_2^{\nu} + V^V_{\rm conf}(\mathbf{k})\Gamma_1^{\mu}\Gamma_{2;\mu} + V^S_{\rm conf}(\mathbf{k})\right\}u_1(q)u_2(-q)$$

$\mathbf{k} = \mathbf{p} - \mathbf{q}$
$D_{\mu\nu}(\mathbf{k})$ - (perturbative) gluon propagator
$\Gamma_{\mu}(\mathbf{k})$ - effective long-range vertex with Pauli term:

$$\Gamma_{\mu}(\mathbf{k}) = \gamma_{\mu} + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^{\nu},$$

κ - anomalous chromomagnetic moment of quark,

$$u^\lambda(p) = \sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}} \begin{pmatrix} 1 \\ \dfrac{\boldsymbol{\sigma}\mathbf{p}}{\epsilon(p)+m} \end{pmatrix} \chi^\lambda,$$

with $\epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$.

- Lorentz structure of $V_{\rm conf} = V^V_{\rm conf} + V^S_{\rm conf}$

In nonrelativistic limit

$$\left.\begin{array}{rcl} V^V_{\rm conf} & = & (1-\varepsilon)(Ar+B) \\ V^S_{\rm conf} & = & \varepsilon(Ar+B) \end{array}\right\} \quad {\rm Sum:} \quad (Ar+B)$$

ε - mixing parameter

All parameters A, B, κ, ε and quark masses fixed from analysis of meson masses and radiative decays:

$\varepsilon = -1$ from heavy quarkonium radiative decays $(J/\psi \to \eta_c + \gamma)$ and HQET

$\kappa = -1$ from fine splitting of heavy quarkonium ${}^3P_J$ states and HQET

$(1+\kappa) = 0 \Longrightarrow$ vanishing long-range chromomagnetic interaction !

Freezing of α_s for light quarks (Simonov, Badalyan)

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln \frac{\mu^2+M_0^2}{\Lambda^2}}, \quad \beta_0 = 11 - \frac{2}{3}n_f, \quad \mu = \frac{2m_1m_2}{m_1+m_2}$$

Potential parameters:

$A = 0.18$ GeV^2, $B = -0.30$ GeV,

$\Lambda = 0.413$ GeV, $M_0 = 2.24\sqrt{A} = 0.95$ GeV

Quark masses:

$m_b = 4.88$ GeV $m_s = 0.50$ GeV

$m_c = 1.55$ GeV $m_{u,d} = 0.33$ GeV

- **Heavy baryons in quark-diquark picture**

(qq)-interaction:

$$V(\mathbf{p},\mathbf{q};M) = \bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p},\mathbf{q};M)u_1(q)u_2(-q),$$

where

$$\mathcal{V}(\mathbf{p},\mathbf{q};M) = \frac{2}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu + \frac{1}{2}V_{\rm conf}^V(\mathbf{k})\Gamma_1^\mu\Gamma_{2;\mu} + \frac{1}{2}V_{\rm conf}^S(\mathbf{k})$$

(dQ)-interaction: $\qquad d = (qq')$

$$\begin{aligned} V(\mathbf{p},\mathbf{q};M) = {} & \frac{\langle d(P)|J_\mu|d(Q)\rangle}{2\sqrt{E_d(p)E_d(q)}}\bar{u}_Q(p)\frac{4}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma^\nu u_Q(q) \\ & +\psi_d^*(P)\bar{u}_Q(p)J_{d;\mu}\Gamma_Q^\mu V_{\rm conf}^V(\mathbf{k})u_Q(q)\psi_d(Q) \\ & +\psi_d^*(P)\bar{u}_Q(p)V_{\rm conf}^S(\mathbf{k})u_Q(q)\psi_d(Q) \end{aligned}$$

$J_{d,\mu}$ – effective long-range vector vertex of diquark:

$$J_{d;\mu} = \begin{cases} \dfrac{(P+Q)_\mu}{2\sqrt{E_d(p)E_d(q)}} & \text{for scalar diquark} \\ \dfrac{(P+Q)_\mu}{2\sqrt{E_d(p)E_d(q)}} + \dfrac{i\mu_d}{2M_d}\Sigma_\mu^\nu k_\nu & \begin{array}{l}\text{for axial vector} \\ \text{diquark } (\mu_\mathrm{d} = 2)\end{array} \end{cases}$$

μ_d - total chromomagnetic moment of axial vector diquark
diquark spin matrix: $(\Sigma_{\rho\sigma})_\mu^\nu = -i(g_{\mu\rho}\delta_\sigma^\nu - g_{\mu\sigma}\delta_\rho^\nu)$
$\mathbf{S}_d$ - axial vector diquark spin: $(S_{d;k})_{il} = -i\varepsilon_{kil}$

$\psi_d(P)$ – diquark wave function:

$$\psi_d(p) = \begin{cases} 1 & \text{for scalar diquark} \\ \varepsilon_d(p) & \text{for axial vector diquark} \end{cases}$$

$\varepsilon_d(p)$ – polarization vector of axial vector diquark

$\langle d(P)|J_\mu|d(Q)\rangle$ – vertex of diquark-gluon interaction:

$$\langle d(P)|J_\mu(0)|d(Q)\rangle = \int \frac{d^3p\, d^3q}{(2\pi)^6} \bar{\Psi}^d_P(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi^d_Q(\mathbf{q}) \Rightarrow F(k^2)$$

Γ_μ – two-particle vertex function of the diquark-gluon interaction:

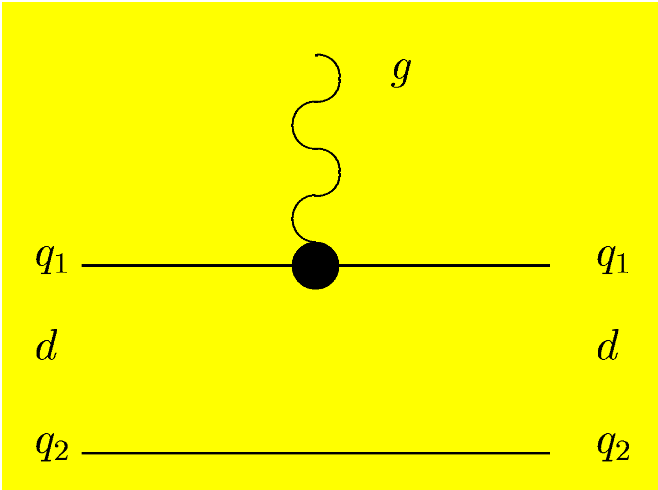


Figure 1: The vertex function Γ of the diquark-gluon interaction in the impulse approximation. The gluon interaction only with one quark is shown.

DIQUARKS

• Masses of light mesons and diquarks (qq)

The quasipotential of qq interaction is extremely nonlocal in configuration space for arbitrary quark masses. To make it local

⋆ heavy quarks: nonrelativistic v/c or heavy quark $1/m_Q$ expansion

⋆ light quarks: highly relativistic

$$\epsilon_q(p) \equiv \sqrt{m_q^2 + \mathbf{p}^2} \to E_q = \frac{M^2 - m_{q'}^2 + m_q^2}{2M}$$

$q\bar{q}$ potential

$$V_{q\bar{q}}(r) = V_{\rm SI}(r) + V_{\rm SD}(r)$$

qq potential

$$V_{qq} = \frac{1}{2} V_{q\bar{q}}(r) = \frac{1}{2}[V_{\rm SI}(r) + V_{\rm SD}(r)]$$

spin-independent potential for S-states ($\mathbf{L}^2 = 0$)

$$V_{\rm SI}(r) = V_{\rm Coul}(r) + V_{\rm conf}(r) + \frac{(E_1^2 - m_1^2 + E_2^2 - m_2^2)^2}{4(E_1+m_1)(E_2+m_2)}\Bigg\{\frac{1}{E_1E_2}V_{\rm Coul}(r)$$
$$+\frac{1}{m_1m_2}\Bigg(1 + (1+\kappa)\Bigg[(1+\kappa)\frac{(E_1+m_1)(E_2+m_2)}{E_1E_2}$$
$$-\left(\frac{E_1+m_1}{E_1} + \frac{E_1+m_2}{E_2}\right)\Bigg]\Bigg)V_{\rm conf}^V(r) + \frac{1}{m_1m_2}V_{\rm conf}^S(r)\Bigg\}$$
$$+\frac{1}{4}\left(\frac{1}{E_1(E_1+m_1)}\Delta\tilde{V}_{\rm Coul}^{(1)}(r) + \frac{1}{E_2(E_2+m_2)}\Delta\tilde{V}_{\rm Coul}^{(2)}(r)\right)$$
$$-\frac{1}{4}\left[\frac{1}{m_1(E_1+m_1)} + \frac{1}{m_2(E_2+m_2)} - (1+\kappa)\left(\frac{1}{E_1m_1} + \frac{1}{E_2m_2}\right)\right]$$
$$\times\Delta V_{\rm conf}^V(r) + \frac{(E_1^2 - m_1^2 + E_2^2 - m_2^2)}{8m_1m_2(E_1+m_1)(E_2+m_2)}\Delta V_{\rm conf}^S(r)$$

spin-dependent potential

$$V_{\rm SD}(r) = \frac{2}{3E_1E_2}\Bigg[\Delta\bar{V}_{\rm Coul}(r) + \left(\frac{E_1-m_1}{2m_1} - (1+\kappa)\frac{E_1+m_1}{2m_1}\right)$$
$$\times\left(\frac{E_2-m_2}{2m_2} - (1+\kappa)\frac{E_2+m_2}{2m_2}\right)\Delta V_{\rm conf}^V(r)\Bigg]\mathbf{S}_1\mathbf{S}_2$$

Table 1: Masses of light S-wave mesons (in MeV)

Meson	State $n^{2S+1}L_J$	Theory our	Theory Godfrey Isgur	Theory Maris Roberts	Theory Koll et al.	Experiment PDG
π	1^1S_0	154	150	138	140	139.57
ρ	1^3S_1	776	770	742	785	775.8(5)
π'	2^1S_0	1292	1300		1331	1300(100)
ρ'	2^3S_1	1486	1450		1420	1465(25)
K	1^1S_0	482	470	497	506	493.677(16)
K^*	1^3S_1	897	900	936	890	891.66(26)
K'	2^1S_0	1538	1450		1470	
$K^{*\prime}$	2^3S_1	1675	1580		1550	1717(27)
ϕ	1^3S_1	1038	1020	1072	990	1019.46(2)
ϕ'	2^3S_1	1698	1690		1472	1680(20)

Table 2: Masses of light ground state diquarks (in MeV). S and A denotes scalar and axial vector diquarks antisymmetric $[q, q]$ and symmetric $\{q, q\}$ in flavour, respectively.

Quark content	Diquark type	Mass				
		our RQM	Ebert et al. NJL	Burden et al. BSE (SLK)	Maris BSE (RLK)	Hess et al. Lattice
$[u, d]$	S	710	705	737	820	694(22)
$\{u, d\}$	A	909	875	949	1020	806(50)
$[u, s]$	S	948	895	882	1100	
$\{u, s\}$	A	1069	1050	1050	1300	
$\{s, s\}$	A	1203	1215	1130	1440	

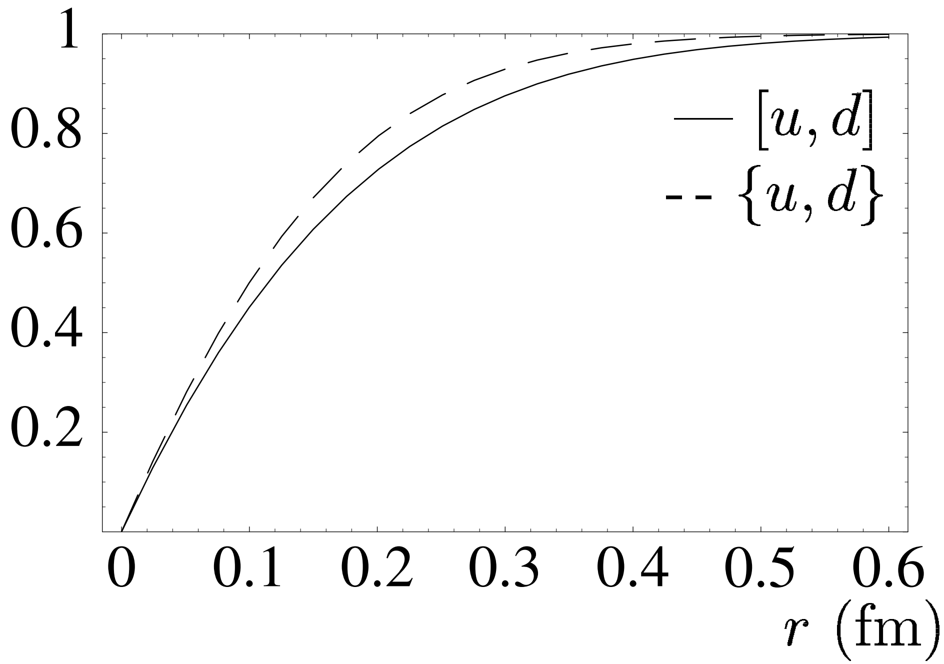


Figure 2: The form factors $F(r)$ for the scalar $[u,d]$ (solid line) and axial vector $\{u,d\}$ (dashed line) diquarks.

DIQUARKS

• Masses of heavy diquarks (QQ)

The quark-quark interaction in the diquark

$$V_{QQ}(r) = V_{QQ}^{\rm SI}(r) + V_{QQ}^{\rm SD}(r).$$

In the nonrelativistic limit (static potential):

$$V_{QQ}^{\rm NR}(r) = \frac{1}{2} V_{Q\bar{Q}}^{\rm NR}(r) = -\frac{2}{3}\frac{\alpha_s(\mu^2)}{r} + \frac{1}{2}(Ar + B).$$

Table 3: Mass spectrum and mean squared radii of the cc diquark.

State $n^{2S+1}L_J$	Mass (GeV)	$\langle r^2 \rangle^{1/2}$ (fm)	State $n^{2S+1}L_J$	Mass (GeV)	$\langle r^2 \rangle^{1/2}$ (fm)
1^3S_1	3.226	0.56	1^1P_1	3.460	0.82
2^3S_1	3.535	1.02	2^1P_1	3.712	1.22
3^3S_1	3.782	1.37	3^1P_1	3.928	1.54

Table 4: Mass spectrum and mean squared radii of bb diquark.

State $n^{2S+1}L_J$	Mass (GeV)	$\langle r^2 \rangle^{1/2}$ (fm)	State $n^{2S+1}L_J$	Mass (GeV)	$\langle r^2 \rangle^{1/2}$ (fm)
1^3S_1	9.778	0.37	1^1P_1	9.944	0.57
2^3S_1	10.015	0.71	2^1P_1	10.132	0.87
3^3S_1	10.196	0.98	3^1P_1	10.305	1.12
4^3S_1	10.369	1.22	4^1P_1	10.453	1.34

The mass of the ground state of the bc diquark in the axial vector (1^3S_1) state is

$$M_{bc}^A = 6.526 \text{ GeV}$$

and in the scalar (1^1S_0) state is

$$M_{bc}^S = 6.519 \text{ GeV}.$$

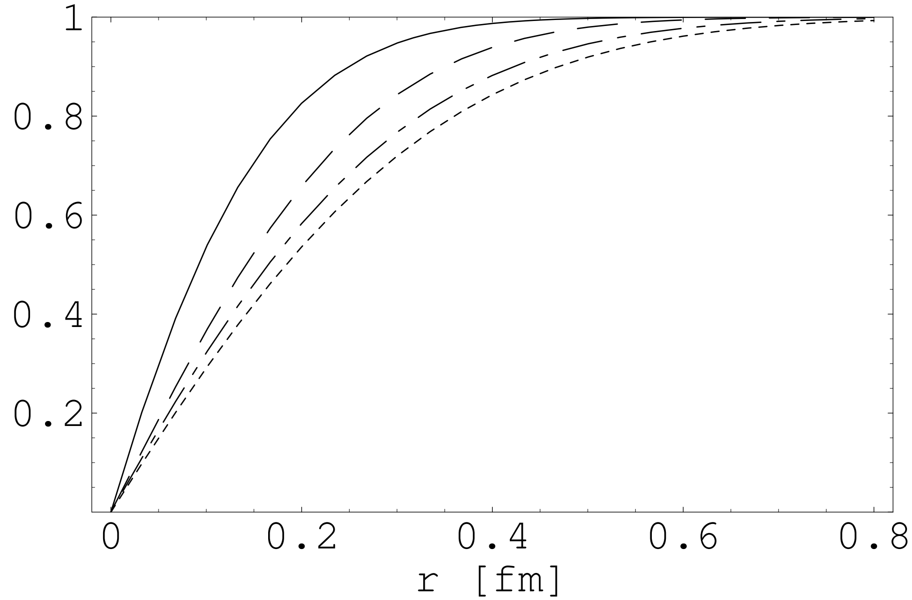


Figure 3: The form factors $F(r)$ for the cc diquark. The solid curve is for the $1S$ state, the dashed curve for the $1P$ state, the dashed-dotted curve for the $2S$ state, and the dotted curve for the $2P$ state.

HEAVY BARYONS

- **Masses of doubly heavy baryons (QQq)**

Limit $M_d \to \infty$ leads to quark-diquark potential $V_{M_d\to\infty}(r)$ very similar to the one for heavy-light mesons.

Bound state equation simplifies

$$\left(\frac{E_q^2 - m_q^2}{2E_q} - \frac{\mathbf{p}^2}{2E_q}\right)\Psi_B(r) = V_{M_d\to\infty}(r)\Psi_B(r), \qquad M_B = M_d + E_q$$

The quasipotential in $M_d \to \infty$ limit

$$V_{M_d\to\infty}(r) = \frac{E_q + m_q}{2E_q}\Bigg[V_{\rm Coul}(r) + V_{\rm conf}(r) + \frac{1}{(E_q+m_q)^2}\Bigg\{\mathbf{p}[V_{\rm Coul}(r) + V_{\rm conf}^V(r) - V_{\rm conf}^S(r)]\mathbf{p} - \frac{E_q+m_q}{2m_q}\Delta V_{\rm conf}^V(r)[1-(1+\kappa)] + \frac{2}{r}\left(V'_{\rm Coul}(r) - V'^S_{\rm conf}(r) - V'^V_{\rm conf}(r)\left[\frac{E_q}{m_q} - 2(1+\kappa)\frac{E_q+m_q}{2m_q}\right]\right)\mathbf{l}\cdot\mathbf{S}_q\Bigg\}\Bigg]$$

- **$1/M_d$ corrections**

(a) scalar diquark

$$\delta V_{1/M_d}(r) = \frac{1}{E_q M_d}\Bigg\{\mathbf{p}\left[V_{\rm Coul}(r) + V^V_{\rm conf}(r)\right]\mathbf{p} + V'_{\rm Coul}(r)\frac{\mathbf{l}^2}{2r} - \frac{1}{4}\Delta V^V_{\rm conf}(r) + \left[\frac{1}{r}V'_{\rm Coul}(r) + \frac{(1+\kappa)}{r}V'^V_{\rm conf}(r)\right]\mathbf{l}\cdot\mathbf{S}_q\Bigg\},$$

(b) axial vector diquark

$$\delta V_{1/M_d}(r) = \frac{1}{E_q M_d}\Bigg\{\mathbf{p}\left[V_{\rm Coul}(r) + V^V_{\rm conf}(r)\right]\mathbf{p} + V'_{\rm Coul}(r)\frac{\mathbf{l}^2}{2r} - \frac{1}{4}\Delta V^V_{\rm conf}(r) + \left[\frac{1}{r}V'_{\rm Coul}(r) + \frac{(1+\kappa)}{r}V'^V_{\rm conf}(r)\right]\mathbf{l}\cdot\mathbf{S}_q + \frac{1}{2}\left[\frac{1}{r}V'_{\rm Coul}(r) + \frac{(1+\kappa)}{r}V'^V_{\rm conf}(r)\right]\mathbf{l}\cdot\mathbf{S}_d + \frac{1}{3}\left(\frac{1}{r}V'_{\rm Coul}(r) - V''_{\rm Coul}(r) + (1+\kappa)\left[\frac{1}{r}V'^V_{\rm conf}(r) - V''^V_{\rm conf}(r)\right]\right)\left[-\mathbf{S}_q\cdot\mathbf{S}_d + \frac{3}{r^2}(\mathbf{S}_q\cdot\mathbf{r})(\mathbf{S}_d\cdot\mathbf{r})\right] + \frac{2}{3}\left[\Delta V_{\rm Coul}(r) + (1+\kappa)\Delta V^V_{\rm conf}(r)\right]\mathbf{S}_d\cdot\mathbf{S}_q\Bigg\},$$

where $\mathbf{S} = \mathbf{S}_q + \mathbf{S}_d$ is the total spin, $\mathbf{S}_d$ is the diquark spin.

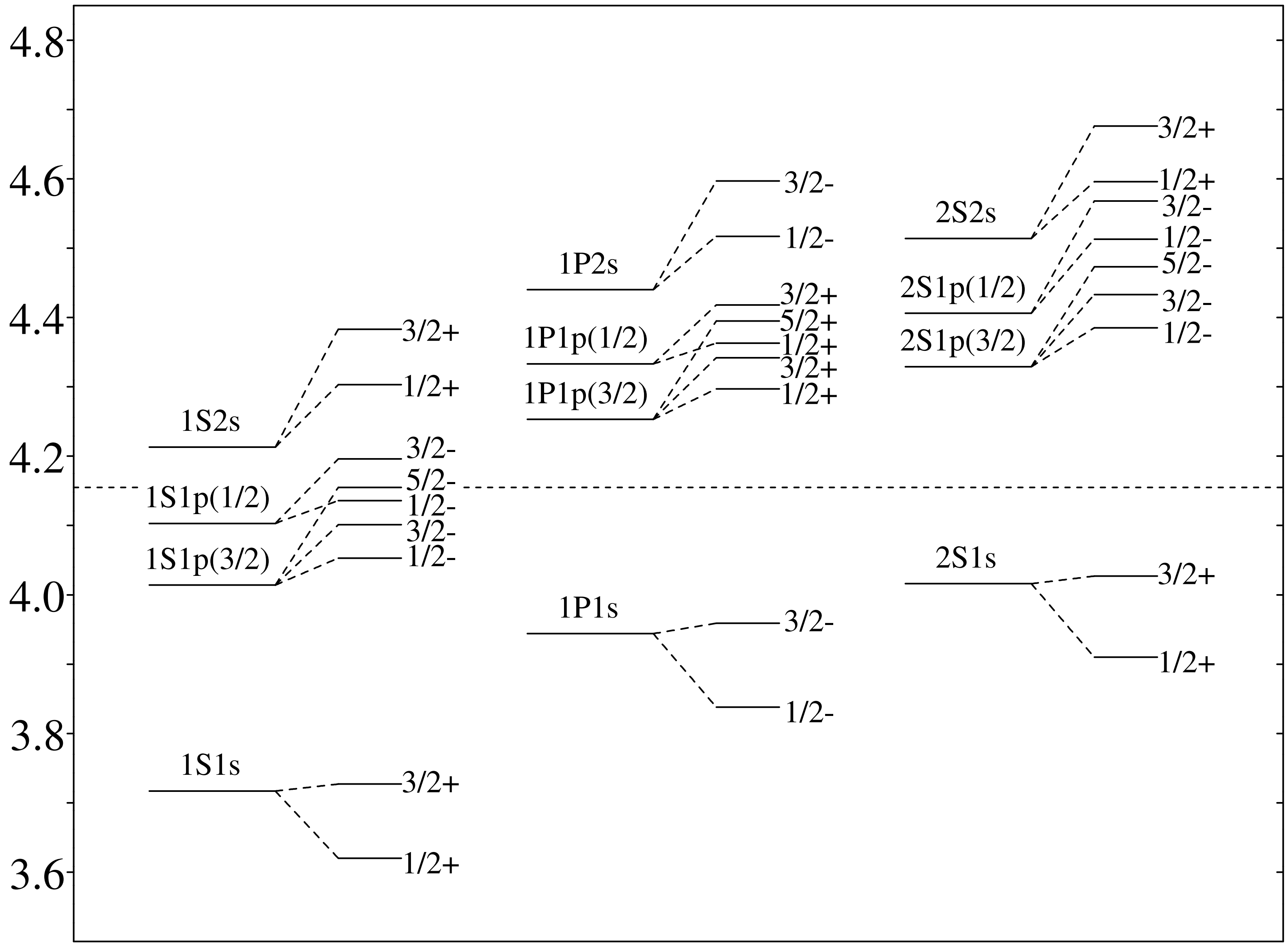


Figure 4: Mass spectrum of Ξ_{cc} baryons. The horizontal dashed line shows the $\Lambda_c D$ threshold.

Table 5: Mass spectrum of ground states of doubly heavy baryons (in GeV). Comparison of different predictions. $\{QQ\}$ denotes the diquark in the axial vector state and $[QQ]$ denotes diquark in the scalar state.

Baryon	Quark content	J^P	our	Gershtein et al.	Ebert et al.	Roncaglia et al.	Körner et al.	Narodetskii Trusov
Ξ_{cc}	$\{cc\}q$	$1/2^+$	3.620	3.478	3.66	3.66	3.61	3.69
Ξ^*_{cc}	$\{cc\}q$	$3/2^+$	3.727	3.61	3.81	3.74	3.68	
Ω_{cc}	$\{cc\}s$	$1/2^+$	3.778	3.59	3.76	3.74	3.71	3.86
Ω^*_{cc}	$\{cc\}s$	$3/2^+$	3.872	3.69	3.89	3.82	3.76	
Ξ_{bb}	$\{bb\}q$	$1/2^+$	10.202	10.093	10.23	10.34		10.16
Ξ^*_{bb}	$\{bb\}q$	$3/2^+$	10.237	10.133	10.28	10.37		
Ω_{bb}	$\{bb\}s$	$1/2^+$	10.359	10.18	10.32	10.37		10.34
Ω^*_{bb}	$\{bb\}s$	$3/2^+$	10.389	10.20	10.36	10.40		
Ξ_{cb}	$\{cb\}q$	$1/2^+$	6.933	6.82	6.95	7.04		6.96
Ξ'_{cb}	$[cb]q$	$1/2^+$	6.963	6.85	7.00	6.99		
Ξ^*_{cb}	$\{cb\}q$	$3/2^+$	6.980	6.90	7.02	7.06		
Ω_{cb}	$\{cb\}s$	$1/2^+$	7.088	6.91	7.05	7.09		7.13
Ω'_{cb}	$[cb]s$	$1/2^+$	7.116	6.93	7.09	7.06		
Ω^*_{cb}	$\{cb\}s$	$3/2^+$	7.130	6.99	7.11	7.12		

HEAVY BARYONS

- **Masses of heavy baryons (qqQ)**

⋆ p/m_Q expansion for heavy quark

⋆ relativistic treatment of light diquark $d = (qq)$:

$$E_d(p) \equiv \sqrt{\mathbf{p}^2 + M_d^2} \to E_d \equiv \frac{M^2 - m_Q^2 + M_d^2}{2M}$$

- leading order in p/m_Q

potential for the S-wave states ($\mathbf{L^2} = \mathbf{0}$, $\mathbf{LS}$=0) is the same for scalar and axial vector diquarks

$$V^{(0)}(r) = \hat{V}_{\rm Coul}(r) + V_{\rm conf}(r),$$

$$\hat{V}_{\rm Coul}(r) = -\frac{4}{3}\alpha_s \frac{F(r)}{r}, \qquad V_{\rm conf}(r) = Ar + B,$$

where $\hat{V}_{\rm Coul}(r)$ is the smeared Coulomb potential (which accounts for the diquark structure).
$M_B(J = 1/2)$ and $M_B(J = 3/2)$ with axial vector diquark are degenerate since $V_{\rm spin-spin} \sim 1/m_Q$.

- p/m_Q corrections up to second order

(a) scalar diquark

$$\delta V(r) = \frac{1}{E_d m_Q}\left\{\mathbf{p}\left[\hat{V}_{\rm Coul}(r) + V^V_{\rm conf}(r)\right]\mathbf{p} - \frac{1}{4}\Delta V^V_{\rm conf}(r)\right\}$$
$$+\frac{1}{m_Q^2}\left\{\frac{1}{8}\Delta\left(\hat{V}_{\rm Coul} + V^S_{\rm conf} - [1-2(1+\kappa)]V^V_{\rm conf}\right) - \frac{1}{2}\mathbf{p}V^S_{\rm conf}\mathbf{p}\right\}$$

(b) axial vector diquark

$$\delta V(r) = \frac{1}{E_d m_Q}\left\{\mathbf{p}\left[\hat{V}_{\rm Coul}(r) + V^V_{\rm conf}(r)\right]\mathbf{p} - \frac{1}{4}\Delta V^V_{\rm conf}(r)\right.$$
$$\left. +\frac{2}{3}\left[\Delta\hat{V}_{\rm Coul}(r) + (1+\kappa)\Delta V^V_{\rm conf}(r)\right]\mathbf{S}_d\mathbf{S}_Q\right\}$$
$$+\frac{1}{m_Q^2}\left\{\frac{1}{8}\Delta\left(\hat{V}_{\rm Coul} + V^S_{\rm conf} - [1-2(1+\kappa)]V^V_{\rm conf}\right) - \frac{1}{2}\mathbf{p}V^S_{\rm conf}\mathbf{p}\right\},$$

where $\mathbf{S}_d$ – light diquark spin, $\mathbf{S}_Q$ – heavy quark spin.

Mass formula

$$\frac{b^2(M)}{2\mu_R} = \frac{\langle \mathbf{p}^2 \rangle}{2\mu_R} + \langle V^{(0)}(r) \rangle + \langle \delta V(r) \rangle.$$

Spin-spin interaction is proportional to

$$\langle \mathbf{S}_d \mathbf{S}_Q \rangle = \frac{1}{2}\left[J(J+1) - S_d(S_d+1) - \frac{3}{4} \right],$$

where $\mathbf{J} = \mathbf{S}_d + \mathbf{S}_Q$ – baryon spin.

Table 6: Masses of the ground state heavy baryons (in MeV).

Baryon	$I(J^P)$	Theory						Experiment
		our	Capstick Isgur	Roncaglia et al.	Savage	Jenkins	Mathur* et al.	PDG
Λ_c	$0(\frac{1}{2}^+)$	2297	2265	2285			2290	2284.9(6)
Σ_c	$1(\frac{1}{2}^+)$	2439	2440	2453			2452	2451.3(7)
Σ_c^*	$1(\frac{3}{2}^+)$	2518	2495	2520	2518		2538	2515.9(2.4)
Ξ_c	$\frac{1}{2}(\frac{1}{2}^+)$	2481		2468			2473	2466.3(1.4)
Ξ_c'	$\frac{1}{2}(\frac{1}{2}^+)$	2578		2580	2579	2580.8(2.1)	2599	2574.1(3.3)
Ξ_c^*	$\frac{1}{2}(\frac{3}{2}^+)$	2654		2650			2680	2647.4(2.0)
Ω_c	$0(\frac{1}{2}^+)$	2698		2710			2678	2697.5(2.6)
Ω_c^*	$0(\frac{3}{2}^+)$	2768		2770	2768	2760.5(4.9)	2752	
Λ_b	$0(\frac{1}{2}^+)$	5622	5585	5620			5672	5624(9)
Σ_b	$1(\frac{1}{2}^+)$	5805	5795	5820		5824.2(9.0)	5847	
Σ_b^*	$1(\frac{3}{2}^+)$	5834	5805	5850		5840.0(8.8)	5871	
Ξ_b	$\frac{1}{2}(\frac{1}{2}^+)$	5812		5810		5805.7(8.1)	5788	
Ξ_b'	$\frac{1}{2}(\frac{1}{2}^+)$	5937		5950		5950.9(8.5)	5936	
Ξ_b^*	$\frac{1}{2}(\frac{3}{2}^+)$	5963		5980		5966.1(8.3)	5959	
Ω_b	$0(\frac{1}{2}^+)$	6065		6060		6068.7(11.1)	6040	
Ω_b^*	$0(\frac{3}{2}^+)$	6088		6090		6083.2(11.0)	6060	

* error estimates of lattice NRQCD calculations are about 50 MeV for charmed baryons and 100 MeV for bottom baryons

Heavy-quark symmetry ($1/m_Q$ expansion) and at lowest-order $SU(3)$ flavour symmetry breaking $\longrightarrow$ equal-spacing rule:

$$J = \frac{1}{2}, \qquad M_{\Sigma_Q} + M_{\Omega_Q} = 2M_{\Xi'_Q},$$
$$J = \frac{3}{2}, \qquad M_{\Sigma^*_Q} + M_{\Omega^*_Q} = 2M_{\Xi^*_Q}, \quad Q = b, c.$$

Table 7: Test of validity of equal-spacing rule (in MeV).

	$J=\frac{1}{2}$		$J=\frac{3}{2}$	
	$Q=c$	$Q=b$	$Q=c$	$Q=b$
$M_{\Sigma_Q} + M_{\Omega_Q}$	5137	11870	5286	11922
$2M_{\Xi_Q}$	5156	11874	5308	11926

Equal-spacing rule for the hyperfine mass splittings:

$$\delta_{\Sigma_Q} + \delta_{\Omega_Q} = 2\delta_{\Xi_Q}, \quad Q = b, c;$$

$$\delta_{\Sigma_Q} = M_{\Sigma^*_Q} - M_{\Sigma_Q}; \quad \delta_{\Xi_Q} = M_{\Xi^*_Q} - M_{\Xi'_Q}; \quad \delta_{\Omega_Q} = M_{\Omega^*_Q} - M_{\Omega_Q}.$$

Table 8: Test of validity of equal-spacing rule for hyperfine mass splittings (in MeV).

	$Q = c$	$Q = b$
$\delta_{\Sigma_Q} + \delta_{\Omega_Q}$	149	52
$2\delta_{\Xi_Q}$	152	52

SUMMARY

- Heavy baryons are studied in quark-diquark approximation. Heavy-quark–light-diquark (dQ) picture for qqQ baryons and light-quark–heavy-diquark (dq) picture for doubly heavy baryons (QQq) are assumed

- Light quarks and light diquarks are treated fully relativistically

- v/c expansion is used only for heavy quarks

- Light and heavy diquarks are not considered as point-like objects. Diquark wave functions are used to calculate diquark-gluon vertex taking diquark structure into account

- The results are obtained with all values of parameters taken from previous considerations of meson properties

- Overall reasonable agreement of our predictions with available experimental data and results of significantly distinct theoretical approaches gives further grounds for the heavy-quark–light-diquark picture of heavy baryons

- Diquark and baryon wave functions can be used for calculation of heavy baryon radiative and weak decays